Towards the automatic calculation of NLO corrections to multi-particle processes

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Introduction

- Collider experiments play a fundamental rôle in particle physics: LEP at CERN, Tevatron at Fermilab and HERA at DESY
- and will in the future: LHC at CERN and ILC.
- The main objective of these experiments is to test the Standard Model of elementary particles,
- which asks for delicate comparisons between theoretical predictions and experimental data. Comparison model and empirical data on statistical level.
  - Need to calculate cross-sections;
  - or perform event generation (simulation),

  for both backgrounds and signals at sufficient accuracy.

Introduction

- Most calculations have to be performed within perturbation theory, for which the accuracy is controled by the number of external particles and the order of the quantum corrections (number of loops).
- Colliders evolve to higher energies serving events from scattering experiments with higher average number of particles involved.
- Also the complexity of the calculation is controled by the number of external particles and the number of loops.
- Complexity, size and amount of the needed calculations asks for them to be performed in an automatic manner.

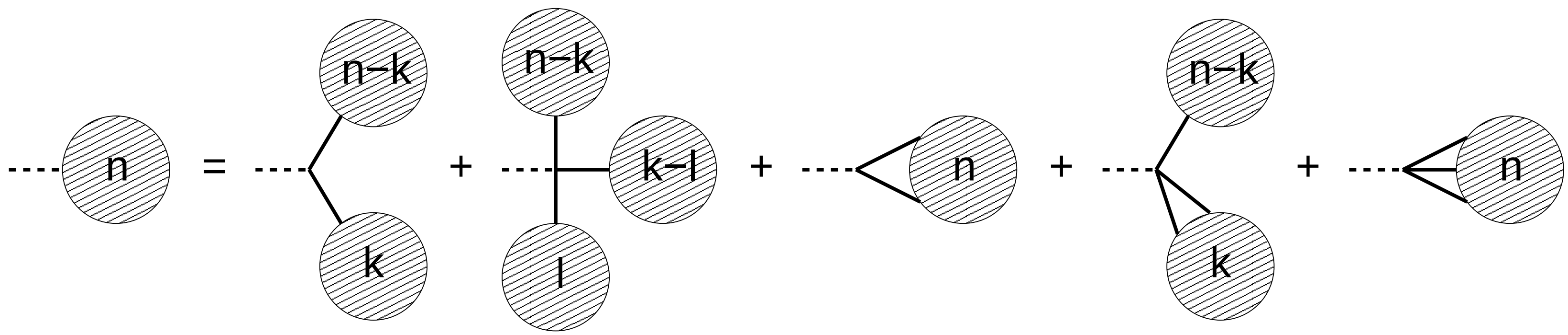
<u>Two of the issues that have to be under controle for NLO calculation</u>

- calculation of tree amplitudes with many external particles;
- calculation of one-loop amplitudes with many external particles.

Recursion relations for QCD

We need to be able to do precise calculations of high energy QCD-processes, *e.g.* to get a handle on the background to interesting signals, including processes with many partons.

Recursive approach: Dyson-Schwinger equation (for gluons)

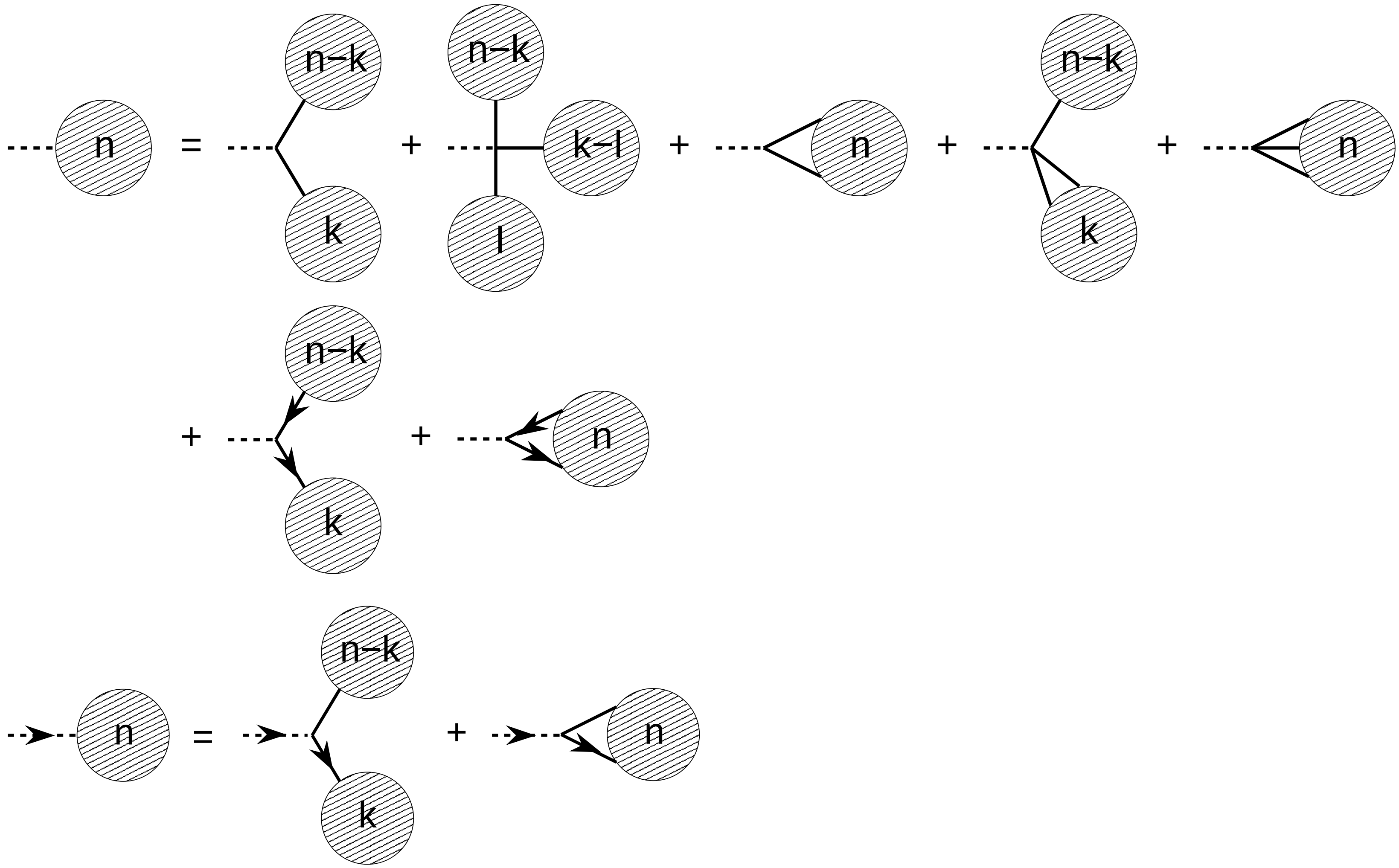


where a summation over all distinct distributions of external particles is understood, and initial (terminal) condition $\cdots\cdot\, i = \varepsilon_i^\mu \times \text{color}_i$.

- Need only first two terms for tree-level calculation.
- The $n+1$-gluon amplitude is the result contracted with

$$\varepsilon_{n+1}^\mu \times \text{color}_{n+1} .$$

Recursion relations for QCD

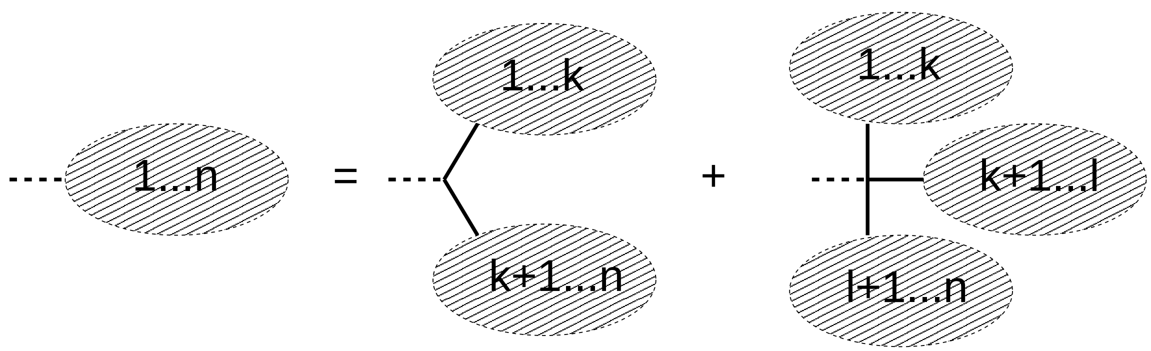


Recursion relations for QCD

Recusion relations are the most efficient way for numerical calculation, but can also easily be put in a computer-algebra system with tree-level result:

$$\mathcal{M} = \sum_{\text{non-cyc}} \delta^{i_1}_{j_2} \delta^{i_2}_{j_3} \cdots \delta^{i_{n-1}}_{j_n} \delta^{i_n}_{j_1} \mathcal{A}(1, 2, \ldots, n) \ .$$

The partial amplitude $\mathcal{A}$ satisfies the color-ordered recursion relation [Berends, Giele 1988]

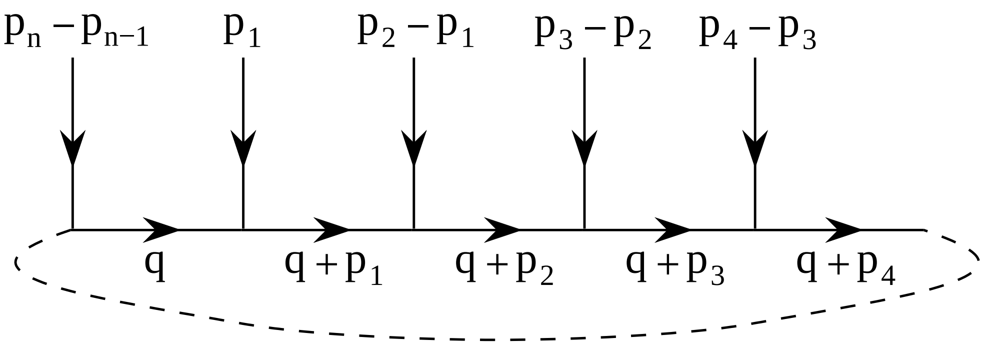


where a summation over all distributions of ordered external particles is understood. Propagators and vertices are without color structure.

Color-decomposition also exists for one-loop amplitudes.

One-loop contributions

In order to reach higher theoretical precision, we want to perform NLO calculations, for which we need to include one-loop contributions.



Inclusion of one-loop contributions demands the calculation of one-loop tensor-integrals

$$I_{n,r}^{\mu_1\mu_2\cdots\mu_r} = \int \frac{d^D q}{i\pi^{D/2}} \frac{q^{\mu_1} q^{\mu_2} \cdots q^{\mu_r}}{q^2 (q+p_1)^2 (q+p_2)^2 \cdots (q+p_{n-1})^2}$$

- consider quarks massless;
- renormalization deals with UV-divergencies;
- IR-divergencies cancel against those from real unresolved particles.

Decomposition in Lorentz-structures

- use Lorentz-invariance to decompose the tensor-integrals (as *e.g.*)

$$
\begin{aligned}
I_{n,3}^{\mu_1\mu_2\mu_3} &= \sum_{i_1\le i_2\le i_3}^{n} (p_{i_1}^{\mu_1} p_{i_2}^{\mu_2} p_{i_3}^{\mu_3})\; I_{n,3}^{i_1,i_2,i_3} \\
&+ \sum_{i_1}^{n} (\eta^{\mu_1\mu_2} p_{i_1}^{\mu_3} + \eta^{\mu_1\mu_3} p_{i_1}^{\mu_2} + \eta^{\mu_2\mu_3} p_{i_1}^{\mu_1})\; I_{n,3}^{0,0,i_1}
\end{aligned}
$$

- calculate coefficient functions, for example using Passarino-Veltman reduction or other recursive techniques.

- zeros of the Gramm-determinant (of the matrix $2\,p_i \cdot p_j$) lead to numerical instabilities which need to be taken care of.

Recursive equations for coefficient functions

[Giele, Glover 2004]

The coefficient functions can be expressed in terms of scalar integrals

[Davydychev 1991]

$$I(D;\{\nu_i\}_{i=1}^n) = \int \frac{d^D q}{i\pi^{D/2}} \frac{1}{s_1^{\nu_1} s_2^{\nu_2} \cdots s_n^{\nu_n}} \;, \quad s_i = (q+p_i)^2, \quad i = 1, \ldots, n$$

Recursion relations for these functions can be derived from the relations

$$\int \frac{d^D q}{i\pi^{D/2}} \frac{\partial}{\partial q^\mu} \frac{\left(\sum_{i=1}^n y_i\right) q^\mu + \left(\sum_{i=1}^n y_i p_i^\mu\right)}{s_1^{\nu_1} s_2^{\nu_2} \cdots s_n^{\nu_n}} = 0$$

via partial integration for any set of parameters $\{y_i\}_{i=1}^n$, and the relation

$$\int \frac{\left(\prod_{j=1}^n x^{\nu_j - 1} dx_j\right) \left(1 - \sum_{j=1}^n x_j\right) \delta\left(1 - \sum_{j=1}^n x_j\right)}{\left(\sum_{j,k}^n S_{jk} x_j x_k\right)^{\sum_{j=1}^n \nu_j - D/2}} = 0 \;,$$

where $S_{ji} = (p_j - p_i)^2$. Choose suitable sets of parameters $\{y_i\}_{i=1}^n$.

Recursive equations for coefficient functions

For example, the scalar functions

$$I(D;\{1\}_{i=1}^{n}) = \int \frac{d^D q}{i\pi^{D/2}} \frac{1}{q^2(q+p_1)^2(q+p_2)^2\cdots(q+p_{n-1})^2}$$

satisfy

$$I(D;\{1\}_{i=1}^{n}) = \sum_{j=1}^{n} b_j^{(n)} I(D;\{1-\delta_{ij}\}_{i=1}^{n})$$

with (remember that $S_{ij} = (p_i - p_j)^2$ and $G_{ij} = 2p_i \cdot p_j$)

$$b_i^{(n=5,6)} = \sum_{j=1}^{n} S_{ij}^{-1} \quad \text{and} \quad b_{i=1,\ldots,n-1}^{(n\geq 7)} = \frac{v_{i5}}{W_5} \;, \quad b_n^{(n\geq 7)} = -\sum_{j=1}^{n-1} b_j^{(n\geq 7)}$$

where

$$W_5 = \frac{1}{2}\sum_{j=1}^{n-1} G_{jj} v_{j5} \quad \text{and} \quad G = u \,\mathrm{diag}\, v^T \;.$$

The functions $I(D;\{1\}_{i=1}^{n=1,2,3,4})$ can be calculated directly in terms of log- and Li_2-functions.

Recursive equations for tensor integrals

[Del Aguila, Pittau 2004]

The momenta p_1, p_2 can be written in terms of massless momenta ℓ_1, ℓ_2 and (complex) parameters α_1, α_2, such that

$$p_1 = \ell_1 + \alpha_1 \ell_2 \;,\quad p_2 = \ell_2 + \alpha_2 \ell_1$$

Define two more massless vectors

$$\ell_3^\mu = \frac{1}{2}\bar{v}(\ell_1)\gamma^\mu(1-\gamma_5)u(\ell_2) \;,\quad \ell_4^\mu = \frac{1}{2}\bar{v}(\ell_2)\gamma^\mu(1-\gamma_5)u(\ell_1) \;,$$

and write

$$q^\mu = \sum_{i=1}^{4} c_i \ell_i^\mu \quad \text{with} \quad c_{1,2} = \frac{q\cdot\ell_{2,1}}{\ell_1\cdot\ell_2} \;,\quad c_{3,4} = \frac{-q\cdot\ell_{4,3}}{4\,\ell_3\cdot\ell_4} \;,$$

then

$$\begin{aligned} q^\mu q^\nu \;&=\; \mathcal{T}^{\mu\nu} + q_\rho T^{\mu\nu\rho} + q^2(\mathcal{T}_0^{\mu\nu} + q_\rho T_0^{\mu\nu\rho}) \\ &+\; (q+p_1)^2 q_\rho T_1^{\mu\nu\rho} + (q+p_2)^2 q_\rho T_2^{\mu\nu\rho} + (q+p_3)^2 q_\rho T_3^{\mu\nu\rho} \end{aligned}$$

where the tensors behave at worst as $1/\sqrt{\Delta_{12}}$ and $1/\sqrt{\Delta_{123}}$.

Recursive equations for tensor integrals

This can be used to reduce tensor integrals in D dimensions by decomposing $q_D = q_4 + q_{D-4}$, which leads to the addition of terms proportional to integrals

$$I^{\mu_1\mu_2\cdots\mu_r}_{n,r;2} = \int \frac{d^Dq}{i\pi^{D/2}} \frac{q^2_{D-4}\, q_4^{\mu_1}\, q_4^{\mu_2} \cdots q_4^{\mu_r}}{q^2(q+p_1)^2(q+p_2)^2\cdots(q+p_{n-1})^2}$$

in the recursive equation.

- Recursive equation becomes a bit simpler for rank $r = 1$.
- Use the “direct” method for the scalar functions.
- Computation on Pentium IV at 2GHz of $I_{n=4,r=4}$ takes 0.005s, $I_{6,6}$ takes 0.09s, $I_{8,8}$ takes 0.5s, $I_{10,10}$ takes 2.4s.

Conclusions

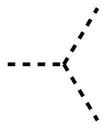
- Need to calculate amplitudes of multi-particle QCD processes;
- at NLO;
- Tree amplitudes under controle with the help of recursion relations;
- One-loop amplitudes seem to be under controle, also with the help of recursion relations.

Recursion relations for QCD

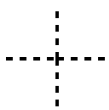
<u>Color-flow representation:</u> [Maltoni, Paul, Stelzer, Willenbrock 2003]

$$(F^{\mu\nu})^i_j = \partial^\mu (A^\nu)^i_j - \partial^\nu (A^\mu)^i_j + \frac{ig}{\sqrt{2}}\left[(A^\mu)^i_k (A^\nu)^k_j - (A^\nu)^i_k (A^\mu)^k_j\right] ,$$

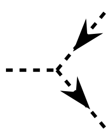
where $(A^\mu)^i_j = \frac{1}{\sqrt{2}} A^\mu_a (\lambda_a)^i_j$, so that



$$= \frac{ig}{\sqrt{2}} \sum_{\text{non-cyc}} \left[\sum_{\text{cyc}} (p_1 - p_2)^{\mu_3} \eta^{\mu_1\mu_2}\right] \delta^{i_1}_{j_2}\delta^{i_2}_{j_3}\delta^{i_3}_{j_1}$$



$$= \frac{-g^2}{2} \sum_{\text{non-cyc}} [2\eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4} - \eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3}]\delta^{i_1}_{j_2}\delta^{i_2}_{j_3}\delta^{i_3}_{j_4}\delta^{i_4}_{j_1}$$



$$= \frac{ig}{\sqrt{2}} \gamma^{\mu_1} \delta^{i_q}_{j_1}\delta^{i_1}_{j_q}$$

$$= \frac{\eta^{\mu_1\mu_2}}{p_1^2}\left(\delta^{i_1}_{j_2}\delta^{i_2}_{j_1} - \frac{1}{N}\delta^{i_1}_{j_1}\delta^{i_2}_{j_2}\right)$$