

Infrared safe definition of jet flavour

Jet-flavour algorithms at parton level

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In collaboration with
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Quark and gluon jets

- In the literature **376 papers** with ‘quark/gluon jet’ in title
- Physically a quark/gluon jet = a jet initiated by a quark/gluon
- Experimentalists try determination of jet flavour
 - Discriminate quark/gluon jets using **kinematical properties**
[jet profile, subjet multiplicity]
 - Jet charge = **weighted charge** of particles in a jet

$$Q_{\text{jet}} = \sum_{i \in \text{jet}} q_{ti} Q_i / \sum_{i \in \text{jet}} q_{ti}$$

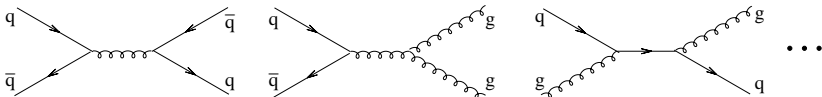
All experimental definitions are practical but **IR unsafe**

- Hints of **theoretical problems** in IR safety and flavour
 - Feynman is alleged to have said “impossible”
 - Flavour insensitive definition of observable suggested

[Nagy, Soper]

Subprocess decomposition

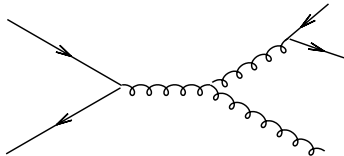
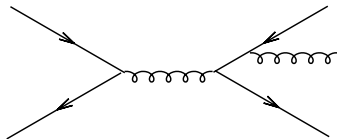
It is useful to decompose a QCD process into subprocesses



- to attribute more physical meaning to higher-order calculations
[which subprocess gets largest contribution]
- to know relative numbers of quark and gluon jets
[multiplicity studies, MC tuning]
- to combine matrix elements and parton shower
[CKKW]
- to match analytical resummations to fixed order
[CAESAR+NLOJET++]

Jet flavour at parton level beyond LO

Problem: map **final state momenta** \rightarrow **Born momenta**



- Flavour decomposition is **ambiguous beyond LO** \Rightarrow Interference
- Cluster of event into **jets** provides a mapping into a Born event
- Jet flavour = **net flavour** of particles in the jet

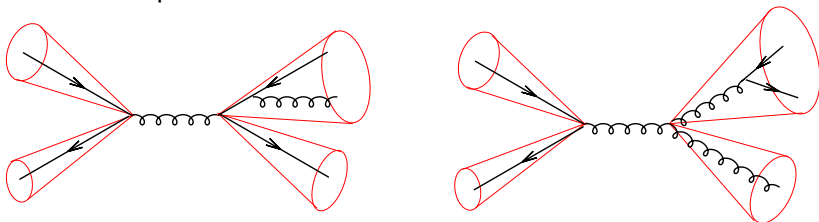
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$$f_{\text{jet}} = u + \bar{u} = (0, 0, 0, 0, 0, 0) = g$$

Ambiguity can be removed only in **soft/collinear limit**

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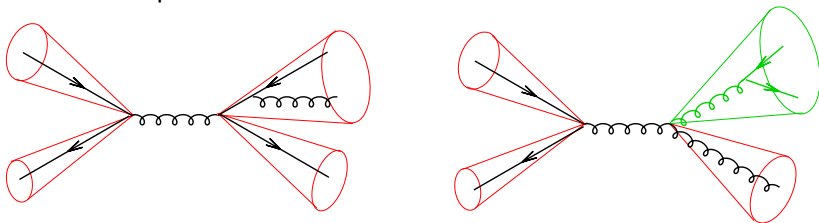
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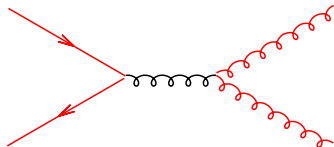
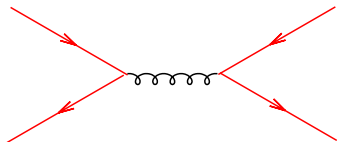
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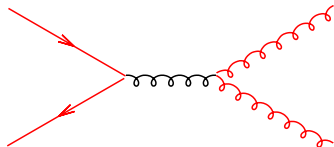
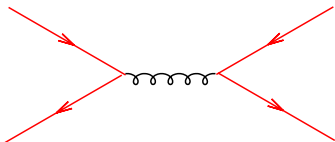
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Jet flavour and infrared safety

At NLO any IR safe jet algorithm is also an **IR safe flavour algorithm**

- **Soft/collinear gluons** do not change the flavour
- **Collinear $q\bar{q}$ pairs** are always recombined together

Beyond NLO **soft large angle $q\bar{q}$ pairs** can be clustered into **different jets** thus spoiling the reconstruction of jet flavour

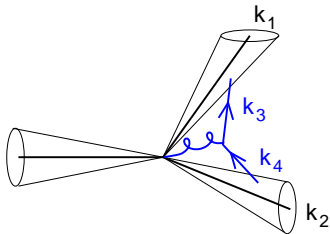
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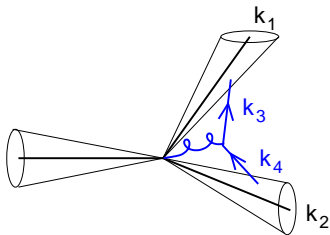
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Jet flavour algorithms in e^+e^-

Durham algorithm

- Soft gluon emission $g \rightarrow g_i g_j$

$$[dk_j] |M^2(k_j)| \sim \frac{dE_j}{\min(E_i, E_j)} \frac{d\theta_{ij}^2}{\theta_{ij}^2}$$

- $d_{ij}^{(D)} \rightarrow 0$ for $\theta_{ij} \rightarrow 0$ and $E_j \rightarrow 0$

$$d_{ij}^{(D)} = 2(1 - \cos \theta_{ij}) \times \min(E_i^2, E_j^2)$$

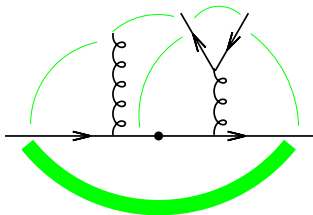
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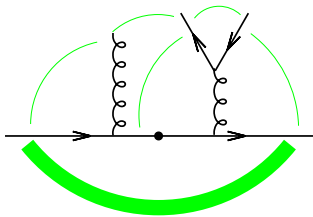
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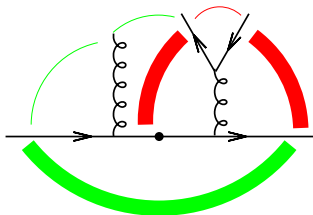
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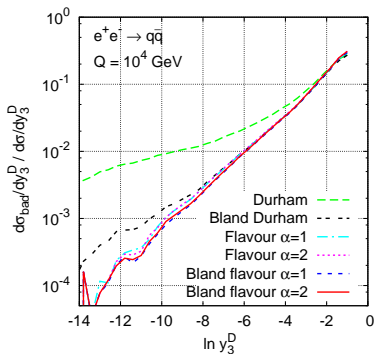
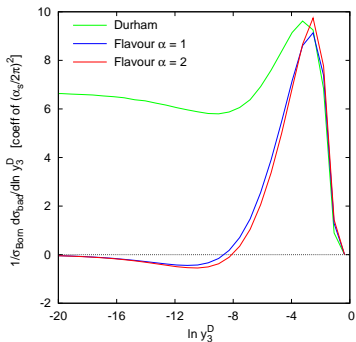
Class of IR safe algorithms identified by $\alpha \in (0, 2]$

$$d_{ij}^{(F)} = 2(1 - \cos \theta_{ij}) \times \begin{cases} \max(E_i, E_j)^\alpha \min(E_i, E_j)^{2-\alpha} & \text{softer of } i, j \text{ flavoured} \\ \min(E_i^2, E_j^2) & \text{softer of } i, j \text{ flavourless} \end{cases}$$

Optional: **flavour blandness** \Leftrightarrow Recombine only $q\bar{q}$ with **no net flavour**

Tests of IR safety in e^+e^- annihilation

- Generate multi-parton configurations in e^+e^- and **cluster to 2 jets**
- Compute **fraction of misidentified events** σ_{bad} as a function of y_3^D
- IR safety at fixed order (EVENT2) \Leftrightarrow σ_{bad} **vanishes** for $y_3 \rightarrow 0$
- IR safety at all orders (HERWIG) \Leftrightarrow **different scalings** for $y_3 \rightarrow 0$



Jet flavour in hadron-hadron collisions

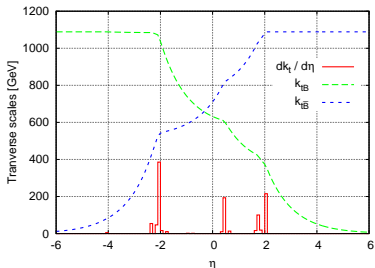
- Distance d_{ij} is modified to have boost invariance

$$d_{ij}^{(F)} = (\Delta\eta_{ij}^2 + \Delta\phi_{ij}^2) \times \begin{cases} \max(k_{ti}, k_{tj})^\alpha \min(k_{ti}, k_{tj})^{2-\alpha} & \text{softer of } i, j \text{ flavoured} \\ \min(k_{ti}^2, k_{tj}^2) & \text{softer of } i, j \text{ flavourless} \end{cases}$$

- Need a distance wrt B ($\eta \rightarrow \infty$) and \bar{B} ($\eta \rightarrow -\infty$)

$$d_{iB}^{(F)} = \begin{cases} \max(k_{ti}, k_{tB}(\eta_i))^\alpha \min(k_{ti}, k_{tB}(\eta_i))^{2-\alpha} & i \text{ flavoured} \\ \min(k_{ti}^2, k_{tB}^2(\eta_i)) & i \text{ flavourless} \end{cases}$$

- $k_{tB}(\eta)$ and $k_{t\bar{B}}(\eta)$ monotonic functions of η that saturate at the **typical hardness** of the event

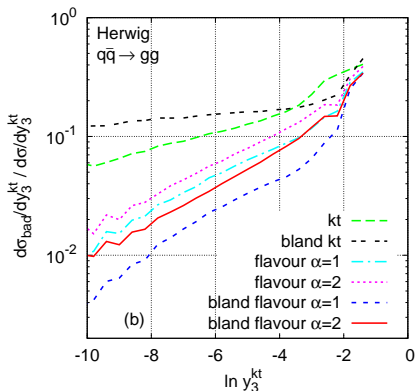
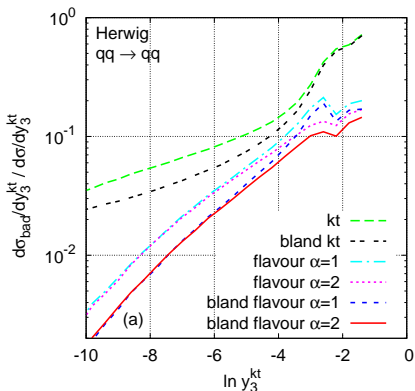


$$k_{tB} = \sum_i k_{ti} (\Theta(\eta_i - \eta) + \Theta(\eta - \eta_i)) e^{\eta_i - \eta}$$

$$k_{t\bar{B}} = \sum_i k_{ti} (\Theta(\eta - \eta_i) + \Theta(\eta_i - \eta)) e^{\eta - \eta_i}$$

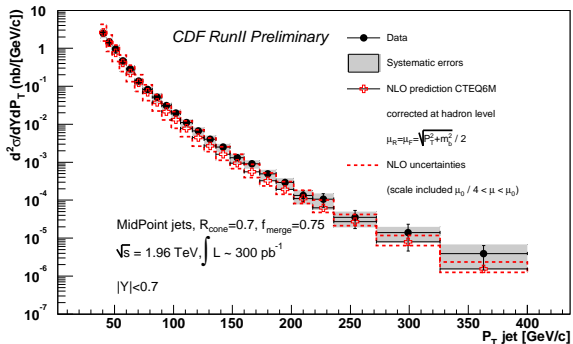
Tests of IR safety in hadron-hadron collisions

- IR safety tests **impossible at fixed order** at the moment
 - Missing **favour information** in fixed order programs
 - Missing **two-loop virtual correction** to each subprocess
- Tests with HERWIG \Rightarrow Importance of **flavour blandness**



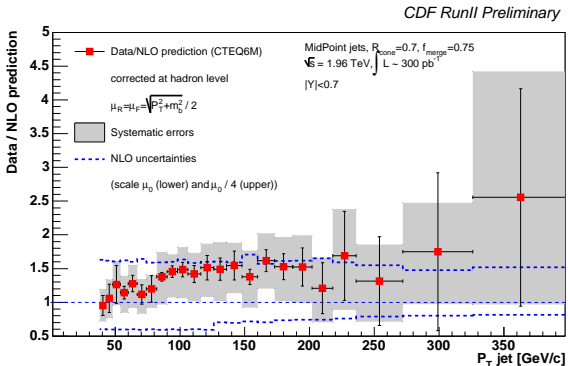
Heavy-flavour jets

- The algorithm has been designed to work at **parton level**
- At hadron level the algorithm can be used for **heavy flavour jets**
- Experimental definition of ***b*-jet** = jet containing ***b*-flavour**
- Comparisons to NLO of inclusive p_T spectra have **large renormalisation scale uncertainties** ($\sim 40 - 50\%$)



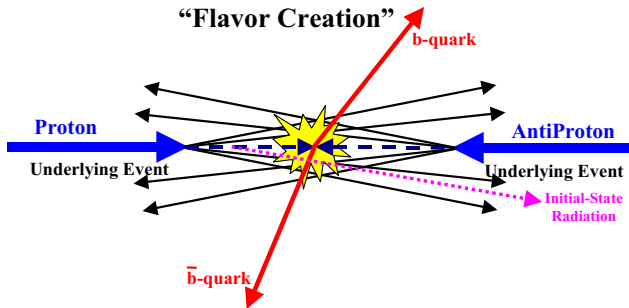
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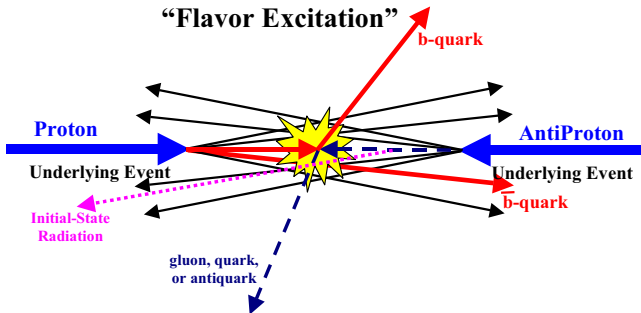
Flavour production mechanisms at hadron colliders

- Flavour creation $\alpha_s^2 + \alpha_s^3 \Rightarrow$ well described by **NLO QCD**
- Flavour excitation $\alpha_s^2 \times \alpha_s \ln E_T/m_b \Rightarrow \alpha_s^n \ln^n E_T/m_b$ in b pdf
- Gluon splitting $\alpha_s^2 \times \alpha_s \ln E_T/m_b \Rightarrow$ all order $\alpha_s^n \ln^{2n-1} E_T/m_b$



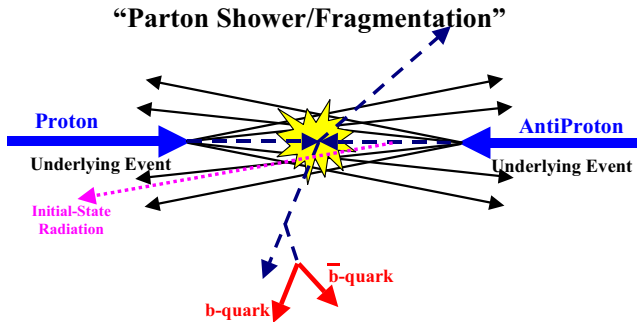
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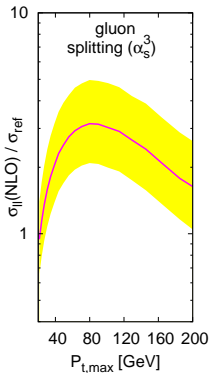
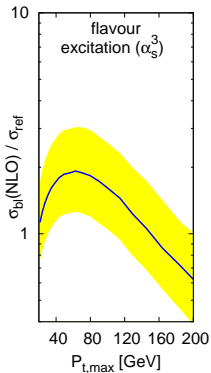
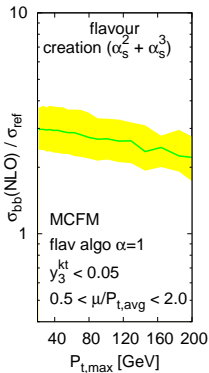
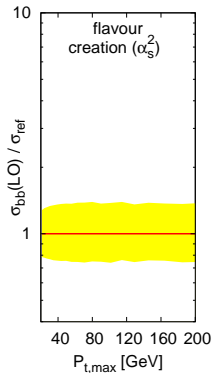
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Theoretical uncertainties in NLO calculation

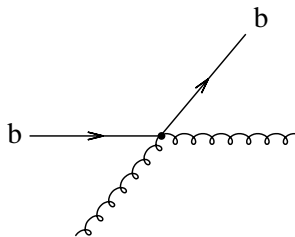
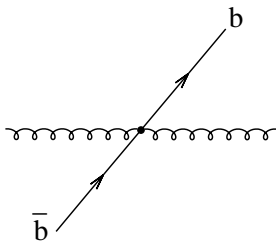
- Tevatron RunII $\sqrt{s} = 1.96\text{TeV}$
- Dijet events with $P_{t,\text{max}} > 30\text{GeV}$ and $|\eta_{\text{jet}}| < 1$
- NLO predictions obtained with **MCFM**



IR safe calculations of b -jet E_T spectra

- IR safe flavour algorithms \Rightarrow **no gluon splitting** contributions
- **Dijet cross sections** calculable at NLO, eventually with MC@NLO
- Reduced theoretical uncertainties in **inclusive jet cross sections**
- IR safe cross sections calculable with NLO programs with massless partons (**i.e. setting $m_b = 0$**)
- New phenomenological studies feasible

[b parton densities, F_2^b , F_2^c , ...]



Summary

- We have a variant of k_t algorithm with **IR safe determination of jet flavour**
- Algorithm acts **differently for quarks and gluons**
 - designed for parton level
 - not practical at hadron level for light flavours (maybe JADE?)
- Matching of analytical resummations as soon as **flavour info** in fixed order programs will become available
[Nagy, work in progress]
- Applications at hadron level for **heavy flavour studies** are under investigation . . .