

NLO-GRID

Integration grids for quick NLO QCD cross-section calculations

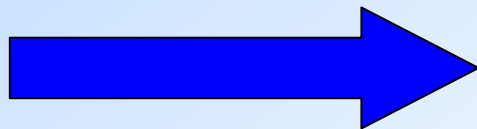
*Dan Clements, Claire Gwenlan, Craig Buttar, G.Salam
Tancredi Carli, Amanda Cooper-Sarkar, Mark Sutton,*

(With the help of Z. Nagy)

Motivation

1. NLO Monte-Carlo QCD calculations are necessary to provide accurate predictions for experimental cross-section measurements.
2. NLO Monte-Carlo calculations however can require long CPU times to achieve sufficient accuracy (typically of order days/weeks).
3. If the PDF is changed the cross-section must be recalculated.

This makes it difficult to use data from hadron-colliders to constrain PDFs using iterative techniques.



Want a method to separate PDFs from the QCD cross-section calculation.....

Scattering Processes At A Hadron-Collider

Hard 2 → 2 scattering

$$\sigma(P_1, P_2) = \sum_{i,j} \int dx_1 dx_2 f_i(x_1, \mu^2) f_j(x_2, \mu^2) \hat{\sigma}_{ij}(x_1 P_1, x_2 P_2, \alpha_s(\mu^2), Q^2 / \mu^2)$$

Diagram annotations:

- PDFs (Parton Distribution Functions) label with arrows pointing to $f_i(x_1, \mu^2)$ and $f_j(x_2, \mu^2)$.
- Parton momenta label with an arrow pointing to $x_1 P_1$ and $x_2 P_2$.
- Parton Level Cross-Section (we call this 'weight' in the following) label with an arrow pointing to $\hat{\sigma}_{ij}$.
- Sum over parton types label with an arrow pointing to the summation index i, j .
- Hadron momenta label with an arrow pointing to P_1 and P_2 .

- The PDFs can be separated from the cross-section by using an integration grid.
- The first step to doing this is to represent the PDF by a set of discrete values on a grid...

Representing a PDF on a grid

- Consider a proton PDF: $q(x, Q^2)$, where 'x' is the parton momentum fraction and 'Q²' is the factorisation scale.
- Instead of defining a grid in x and Q² perform a co-ordinate transform:

$$y(x) = \ln \frac{1}{x} \quad \tau(Q^2) = \ln \ln \frac{Q^2}{\Lambda^2} \quad \Lambda \approx \Lambda_{QCD}$$

- The PDF then becomes:

$$q(x, Q^2) = \sum_{i=0}^n \sum_{j=0}^{n'} q_{k+i, l+j} I_i^n(y(x)) I_j^{n'}(\tau(Q^2)) \quad (1)$$

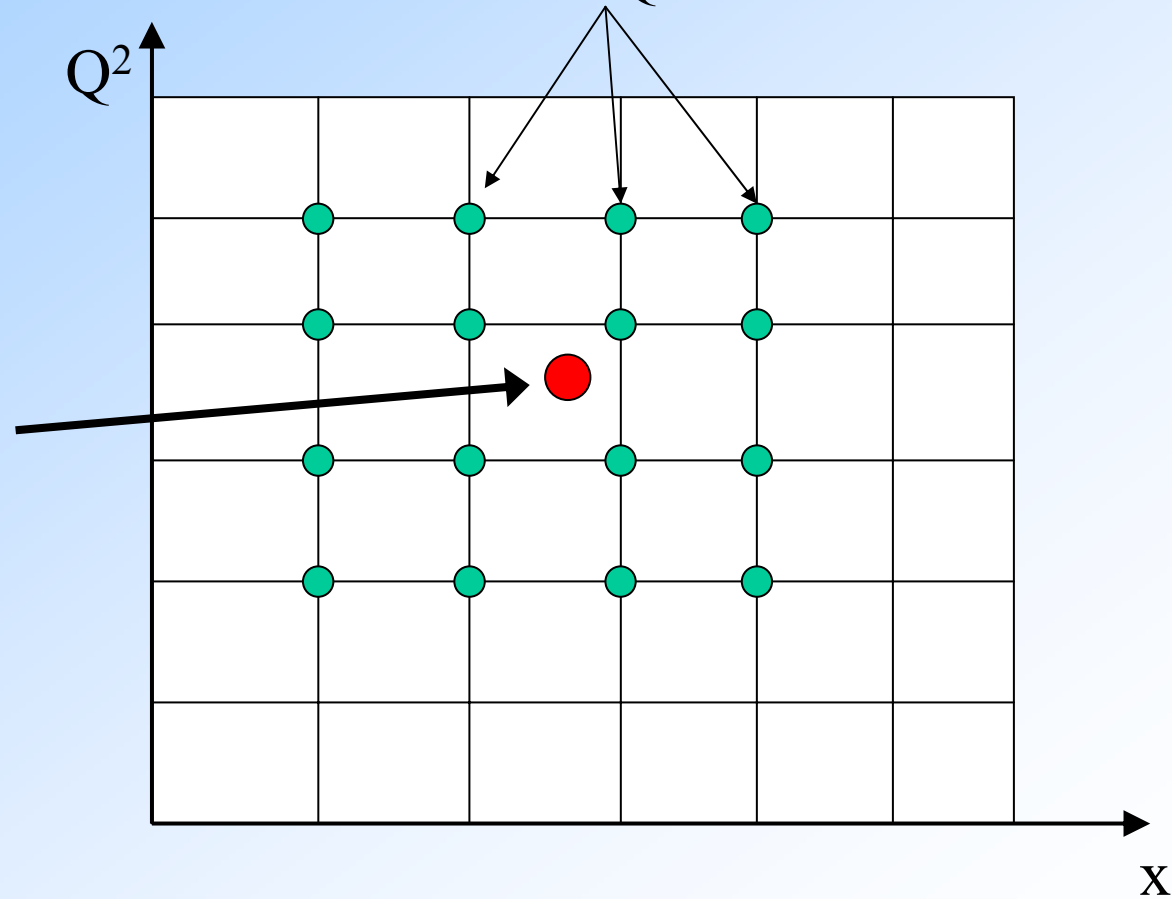
Where the I are interpolation functions of order n and n' .

Representing a PDF on a grid

- What is actually going on....

PDF defined at discrete points in x and Q^2 .


PDF at arbitrary x and Q^2 is described in terms of neighbouring grid points by interpolation.



LO Monte-Carlo Cross-Sections (DIS)

- Consider a Monte-Carlo generator producing N events where each event ' m ' has $x=x_m$ $Q^2=Q_m^2$ and a weight w_m .

- The cross-section is given by:

$$W = \sum_{m=1}^N \left(\frac{\alpha_s(Q_m^2)}{2\pi} \right)^2 w_m \cdot q(x_m, Q_m^2)$$


- But if we believe our interpolation formula we can replace $q(x_m, Q_m^2)$ in the above by (1).

- This is good as the interpolation relies only on the discrete PDF grid points. So we can introduce a weight-grid and for each event update a portion of the grid.....

LO Monte-Carlo Cross-Sections (DIS)

- Define a weight-grid in x and Q^2 similar to the PDF grid and for each event update a portion:

$$W_{k+i,l+j} \rightarrow W_{k+i,l+j} + w_m \cdot I_i^n(y(x_m)) I_j^{n'}(\tau(Q_m^2))$$

n.b. have pushed the PDF interpolation onto the MC weights

- Can now find the result W for an arbitrary PDF after the Monte-Carlo calculation:

$$W = \sum_{i_y} \sum_{i_\tau} \left(\frac{\alpha_s(Q^{2(i_\tau)})}{2\pi} \right) \cdot W_{i_y, i_\tau} \cdot q(x^{(i_y)}, Q^{2(i_\tau)})$$

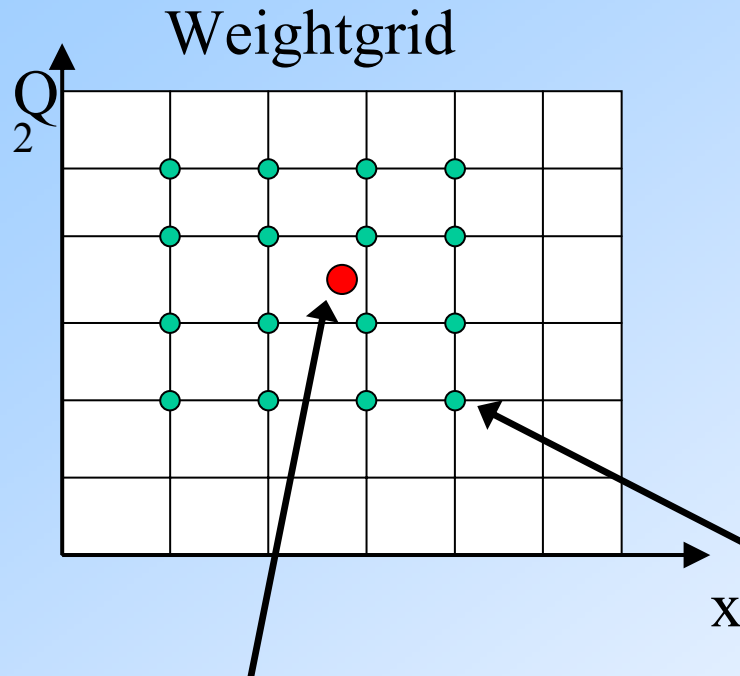
Sum over grid entries

Weightgrid

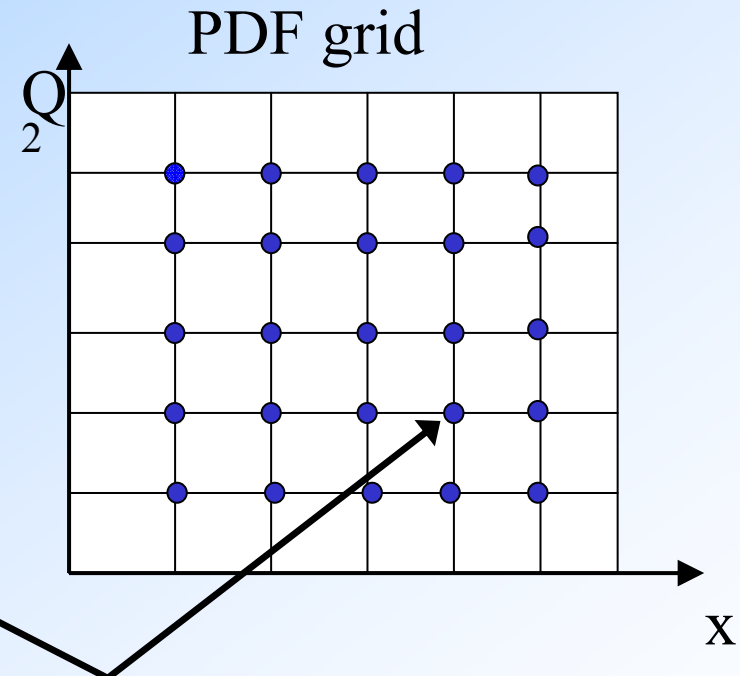
PDF grid

LO Monte-Carlo Cross-Sections (DIS)

What's actually going on:



Monte Carlo weight is split up into several weights on the grid according to the PDF interpolation formula.



The split weight is multiplied by the corresponding value on the PDF grid after Monte-Carlo production.

Moving to NLO Hadron-Hadron Collisions

•For NLO Hadron-Hadron collisions things are a little more complicated:

1. Need to consider two incoming hadrons, hence 2 PDFs and the combinations of partons possible: e.g quark-antiquark, gluon-quark etc, etc, (n.b. use 7 generalised PDF combinations).
2. Need to have grids for both LO and NLO contributions.

•Now the grid generated cross-section looks like.....

$$W = \sum_p \sum_{l=0}^6 \sum_{i_{y1}}^{N_{y1}} \sum_{i_{y2}}^{N_{y2}} \sum_{i_\tau}^{N_\tau} W_{i_{y1}, i_{y2}, i_\tau}^{(p)(l)} \cdot \alpha_s^p(Q_{i_\tau}^2) \cdot F^{(l)}(x_{i_{y1}}, x_{i_{y2}}, Q_{i_\tau}^2)$$

Order

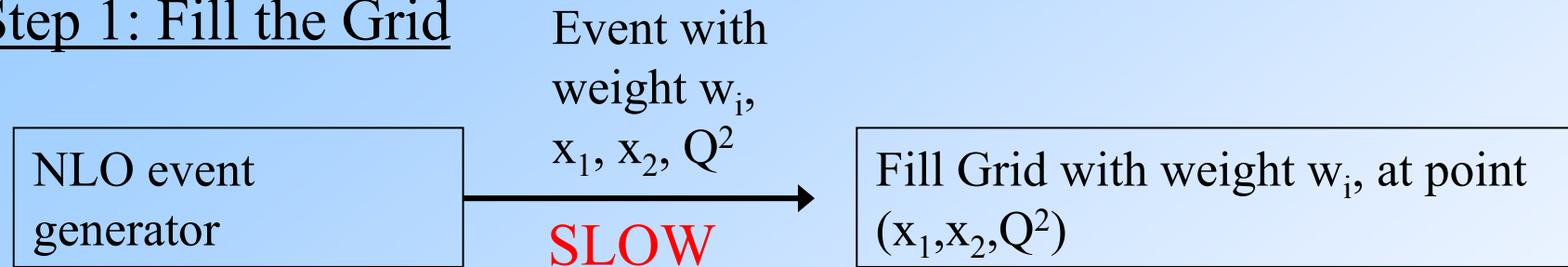
Sum over grid points

3D grids

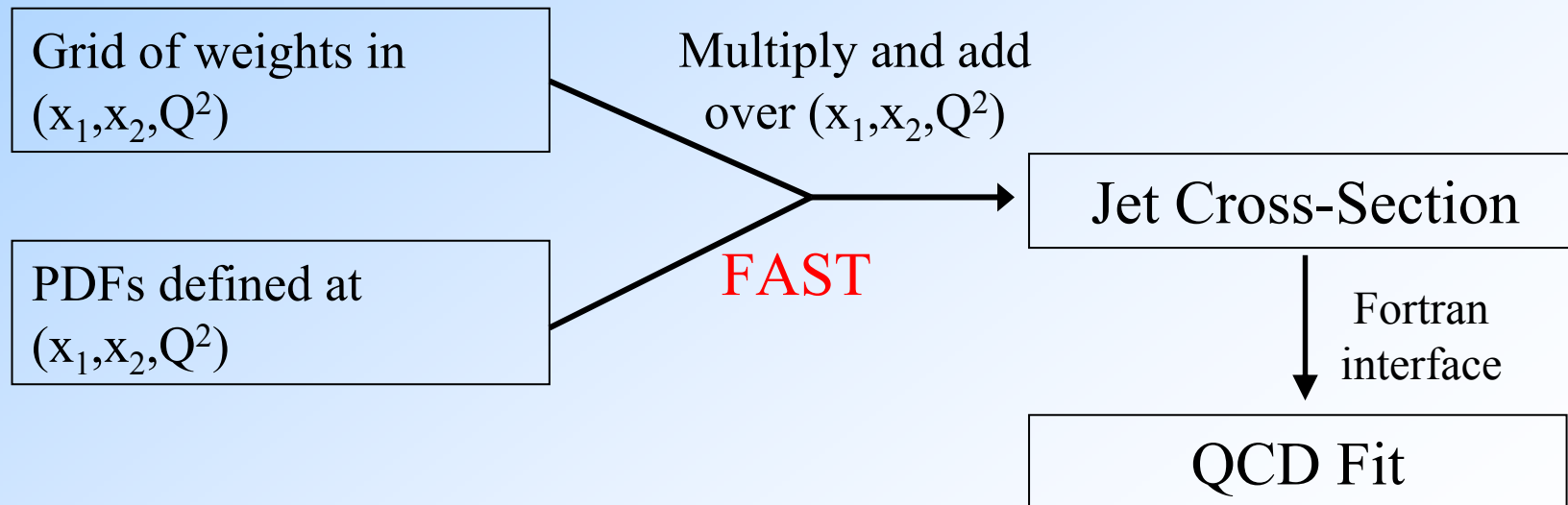
Generalised PDF combinations

Using Integration Grids

Step 1: Fill the Grid



Step 2: Multiply grid by PDFs to generate Cross-Section



Implementing Grids....

- The grid implementation used was developed by Carli, Salam, Siegert and is described in (hep-ph/0510324).
- A set of grids is required for calculation of the jet cross-section in a given observable (e.g in pT) of the cross-section.

Need grids for each order: 2

For each subprocess: 7

Total No. grid per observable: 14

- Each grid is split up into 10 bins in x_1, x_2, Q^2 giving 1000 points.
- Typical file size of grid is 0.1Mb per cross-section and observable bin.
- Code is in C++ and makes use of ROOT libraries, although PDF analyses can be easily carried out in FORTRAN if desired.
- The grid software currently uses the NLOJET++ generator (Z.Nagy)

A Few More Tricks...

- An initial trial run is carried out to optimise the boundaries in x and Q^2 of the grid (for an observable).
- The co-ordinate transform is designed to reflect the kinematics of the process concerned (i.e. LHC, more bins at low x , high Q^2) but this can leave the very high x -region under represented.

1. Implement a variable grid spacing:

$$y = \ln \frac{1}{x} + a(1-x)$$

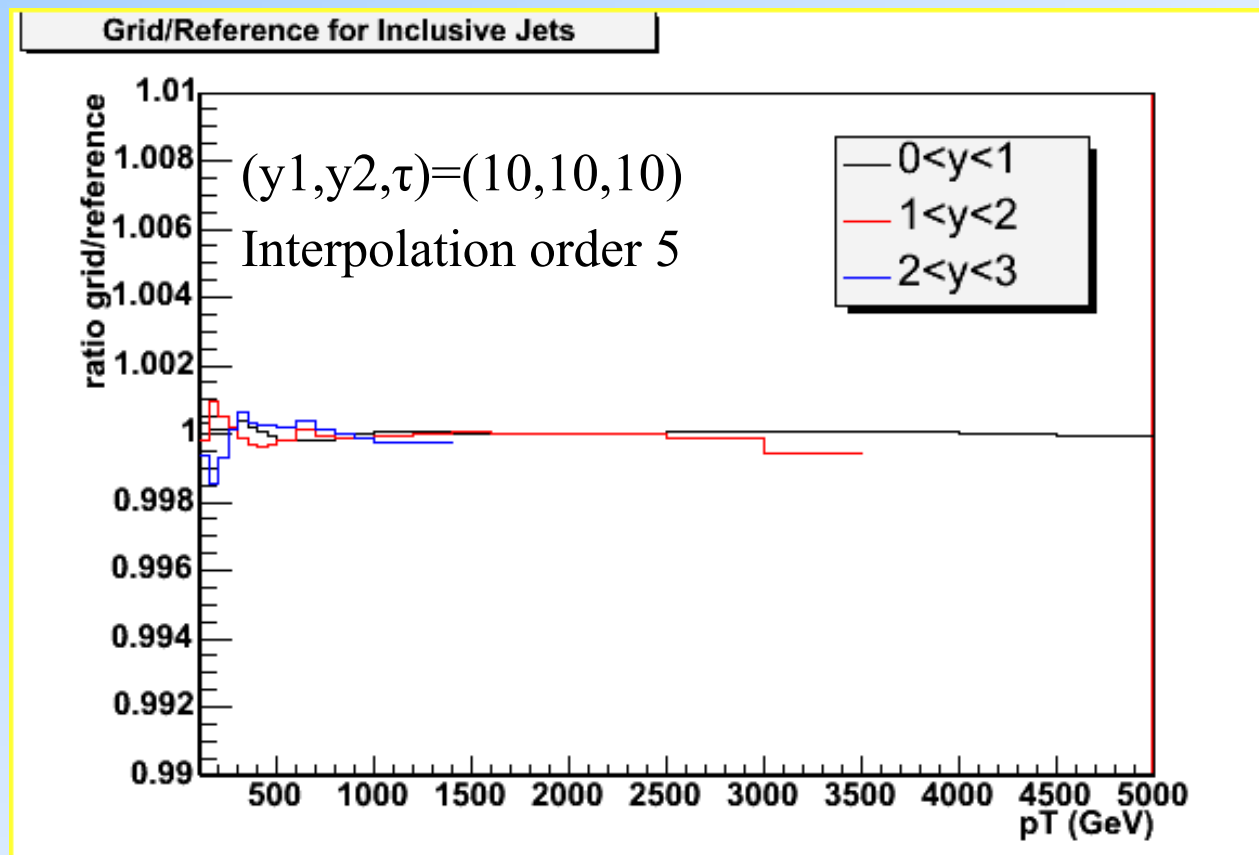
Recall standard co-ordinate transform $y = \ln(1/x)$,
 $\tau = \ln \ln(Q^2/\Lambda)$

2. Employ PDF reweighting to flatten the PDFs in x and hence improve interpolation accuracy.

$$q(x, Q^2) \rightarrow f(x)q(x, Q^2)$$

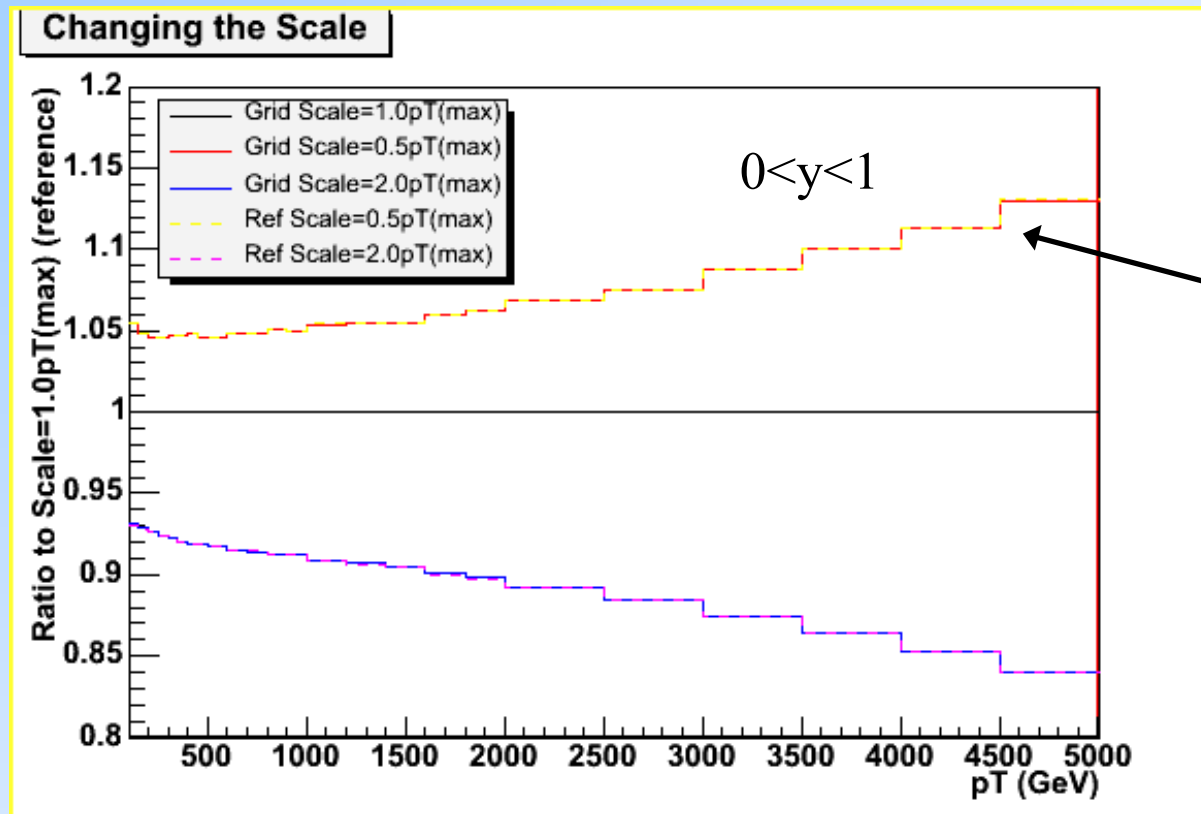
How good are the grids?

- Compared the inclusive jet cross-section at ATLAS as generated using grids and standard NLO calculation (reference) **agreement is better than 0.2%**



Changing the Renormalisation/Factorisation Scales

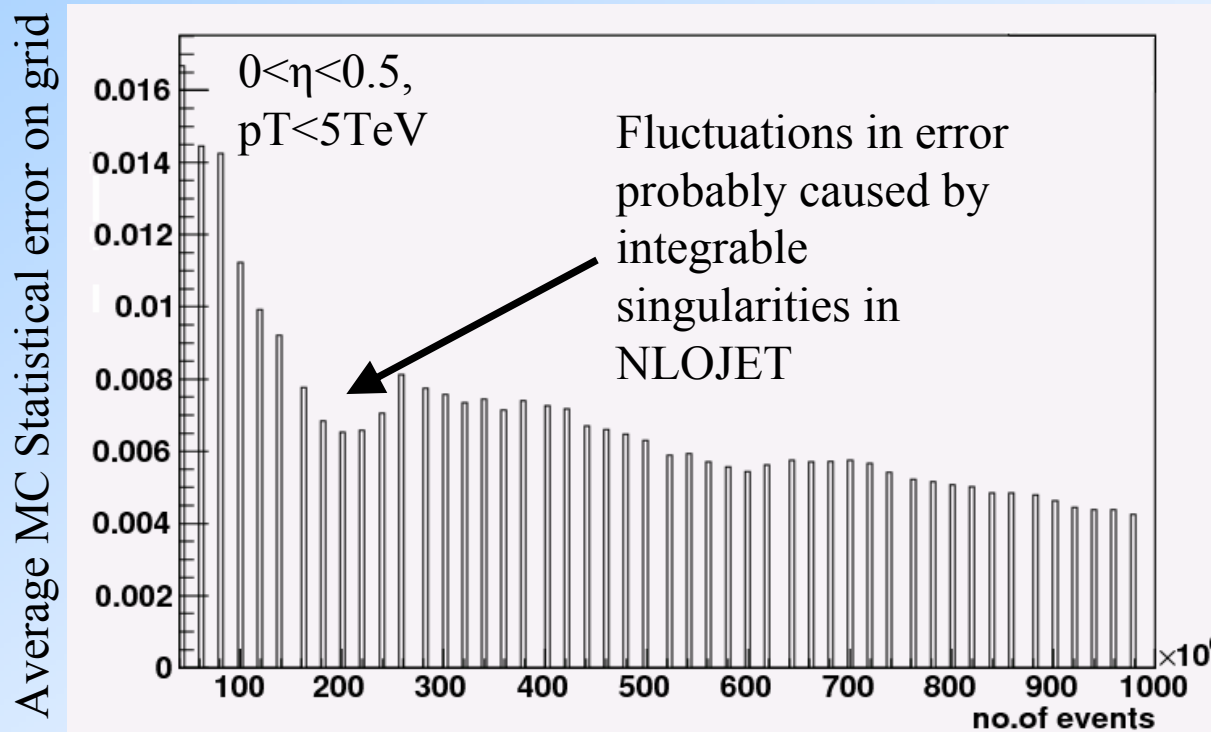
- The renormalisation and factorisation scales can be changed after grid production (hep-ph/0510324).
- **One grid** can be used to generate cross-sections at arbitrary scales.....
- Shown below is the inclusive jet cross-section at ATLAS for 3 scale choices:



Good agreement
between grid
calculation and
reference
calculation
using standard
NLOJET++

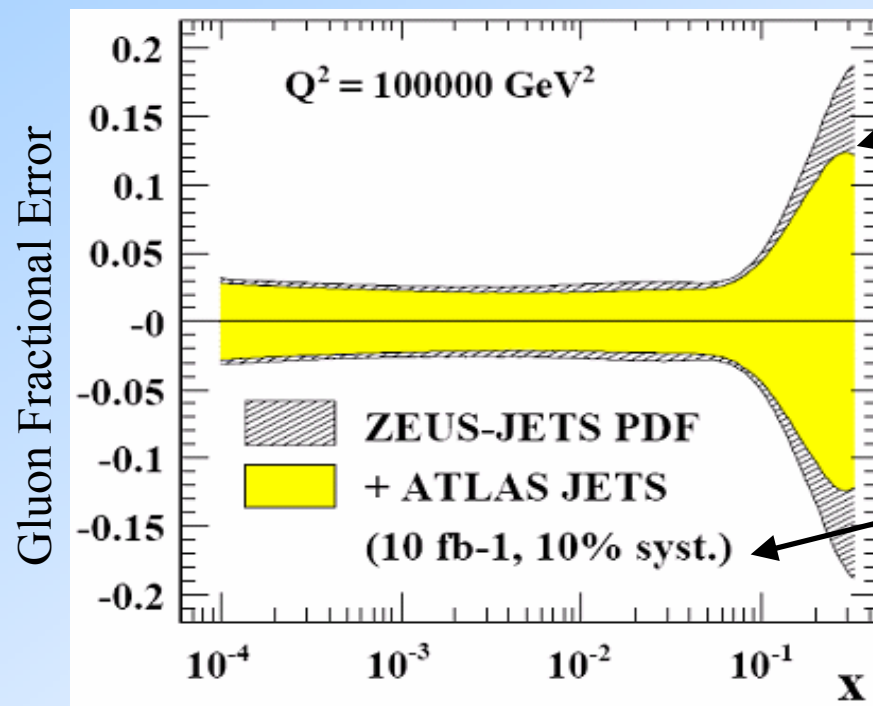
Generating Grids

- Creating grids requires a lot of CPU, values typically of order 0.5CPU years for $\sim 0.2\%$ statistical accuracy.
- Software has been adapted to work on the LCG grid (“The Grid”), to allow for efficient private generation of grids..



PDF Fitting Using Pseudodata

- Grids were generated for the inclusive jet cross-section at ATLAS in the pseudorapidity ranges $0 < \eta < 1$, $1 < \eta < 2$, and $2 < \eta < 3$ up to $p_T = 3 \text{ TeV}$ (NLOJET).
- In addition pseudodata for the same process was generated using JETRAD [4].
- The pseudo-data was then used in a global fit to assess the impact of ATLAS data on constraining PDFs and will include TEVATRON data in the future:

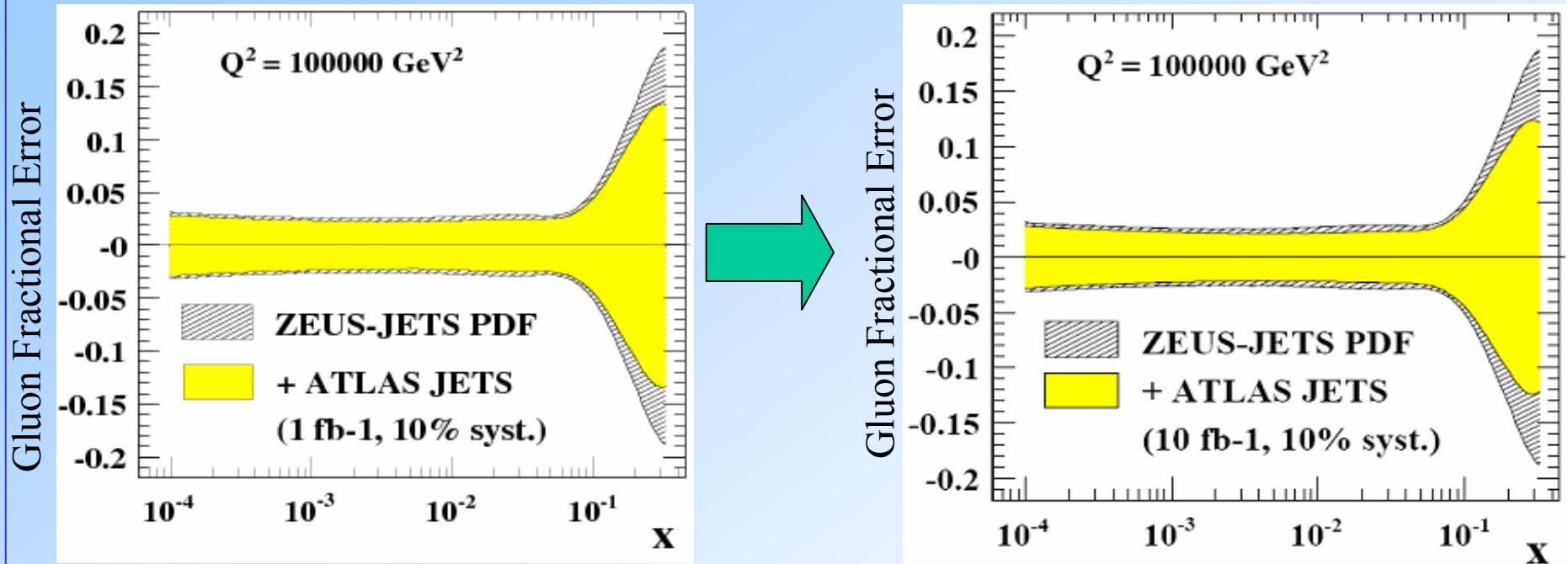


Preliminary indications suggest that ATLAS data can constrain the high x -gluon.

Systematic errors are uncorrelated, $10 \text{ fb}^{-1} = 1$ year of nominal data-taking at $10^{33} \text{ cm}^{-2} \text{ s}^{-1}$

Effect Of Increased Statistics on PDF Fits

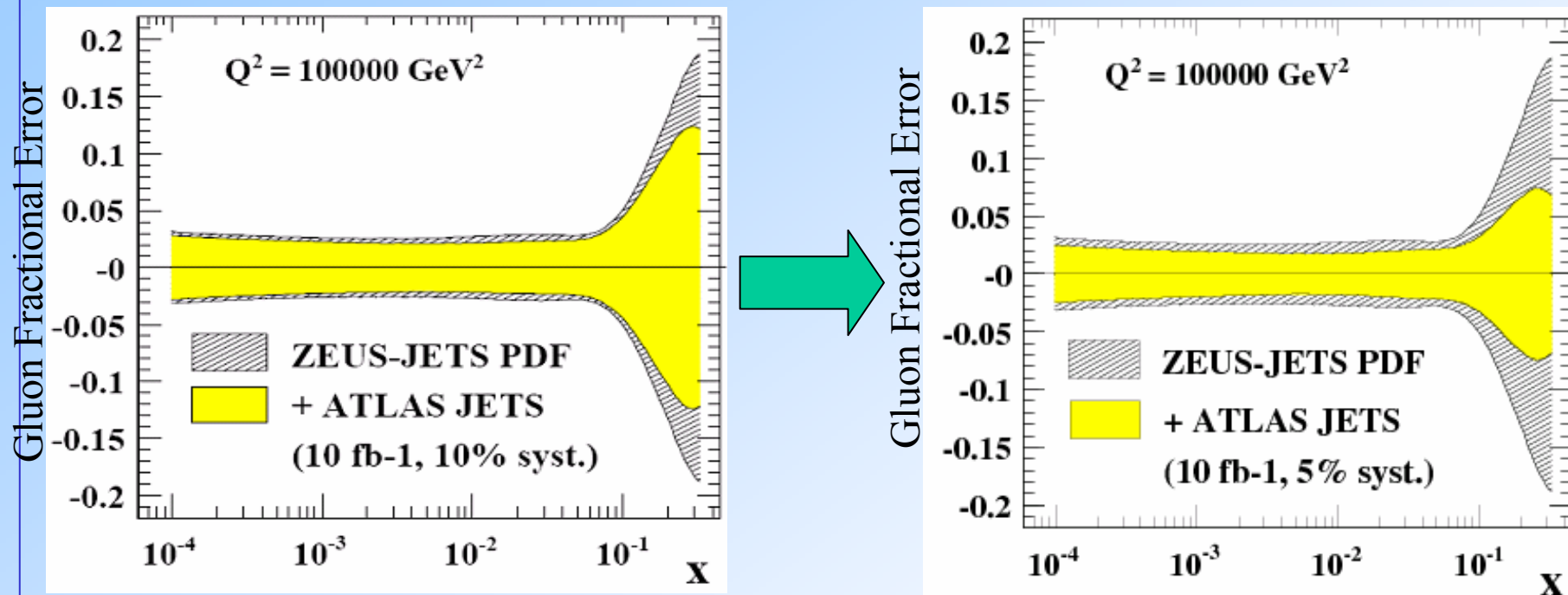
Increase $10\times$ statistics



- Increasing the statistics from 1 fb^{-1} to 10 fb^{-1} has little effect on constraining the PDFs at ATLAS.

Effect Of Decreased Systematic Errors On PDF Fits

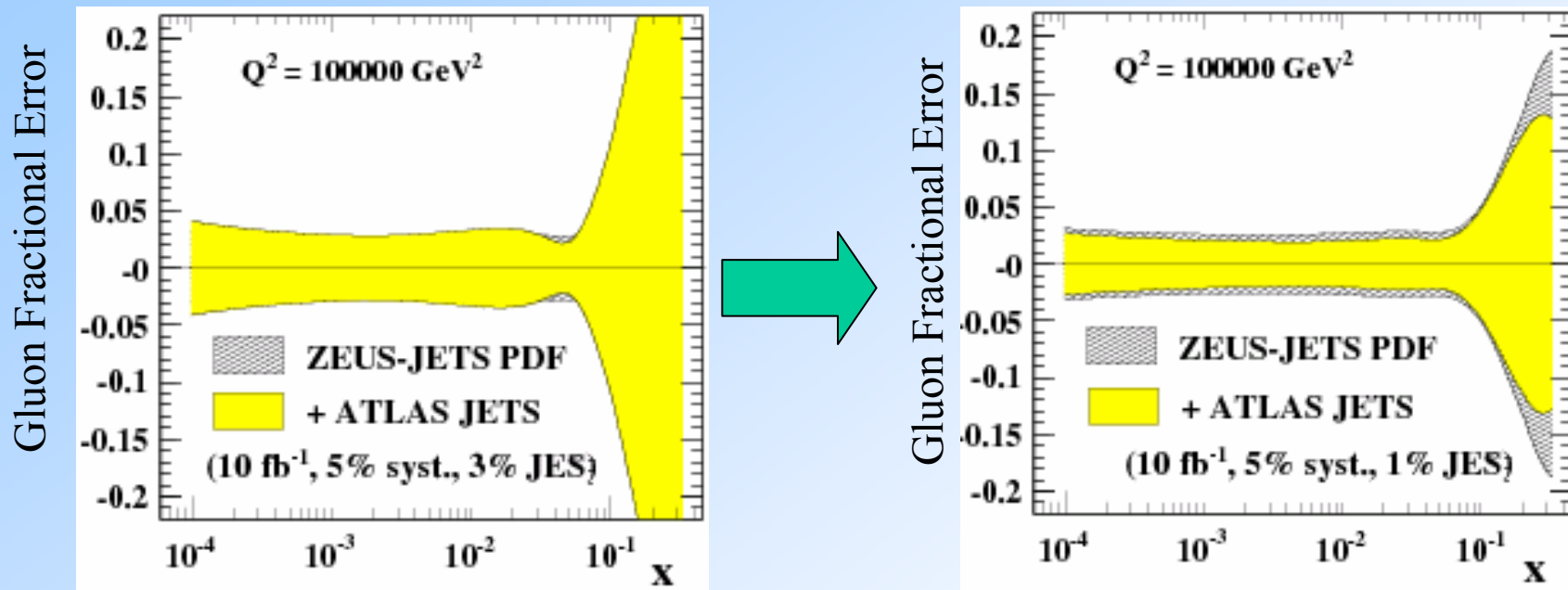
Decrease (uncorrelated) Systematic errors
 $10\% \rightarrow 5\%$



- Decreasing the uncorrelated systematic errors (on the ATLAS experiment) creates a significant improvement in constraining the PDFs.

Effect Of Jet Energy Scale Error On PDF Fits

Decrease Jet Energy Scale error from 3% to 1%



- The ability to constrain the PDFs appears to be strongly connected to the uncertainty on the Jet Energy Scale.
- 1% Jet Energy Scale accuracy at high pT is very challenging!

Summary

- Integration grids using high order interpolation methods can be used to recreate NLO cross-sections to accuracies of better than 0.02%.
- Preliminary results using pseudodata indicate that ATLAS jet data can be useful in constraining the gluon PDF.
- Error on gluon PDF that can be extracted from the jet cross-section is dominated by systematics (particularly JES), the statistical error being negligible even for 1fb^{-1}

Ongoing Developments With Grids And Analysis

- Extension of grid interface to other NLO QCD programs..
- Improved method of storing grids (avoiding unnecessary zero grid points)
- Documentation being prepared for grid software and LCG interface.
- Software freely available from the authors.
- We are willing to provide grids and provide help to interface to existing Fit-package (CTEQ, MRS etc.)

