Super Leading Logarithms in energy-flow observables

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Soft gluons

Resummation needed to account for soft gluon effects in various observables

- $k_\perp$-distributions at small $k_\perp$
- Particle production at kinematical threshold
- Energy flow variables:
  - Event shapes ($\to$ underlying event / hadronisation)
  - Gaps between jets ($\to$ Multijet -/ Higgs - production)

HERA $\to$ LHC: soft emission from 4 instead of 2 (3) partons

Leading logarithms $(\alpha_s L)^n$ in gaps-between-jets

Have to calculate emission of gluons everywhere except into specified interjet region

- Independent gluon emission to all orders

  [Collins, Soper, Sterman et al, 1988-98]

- Non-global contributions [Dasgupta, Salam, 2001]

We have found new class of super-leading logarithms, $\alpha_s^n L^{n+1}$

[J.Forshaw, A.K., M.Seymour, hep-ph/0604094]

Gaps-between-jets

- pp $\rightarrow$ jet jet + soft gluons
- forbidden: real gluons with $k_\perp > Q_0$ in rapidity gap between jets

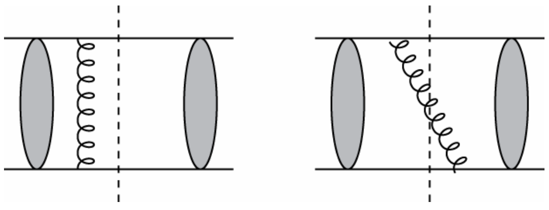
Resum: $$(\alpha_s L)^n, \quad L = Log\frac{Q^2}{Q_0^2}, \quad Q = p_{\perp,jet}$$

Independent emission (Sterman et al) calculated for:

- DIS at HERA ($k_\perp$ jet algorithm) [M.Seymour R.Appleby, 2003]
  + estimate of non-global piece
- Hadron collider [Oderda, Sterman, 1998]

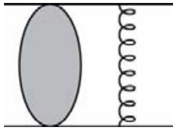
$qq \to qq$: Independent Emission

Leading Logarithms ⇒ gluons strongly ordered in $k_\perp$

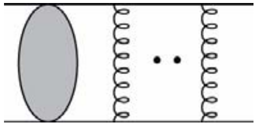


Exact cancellation everywhere except inside gap (and above Q0)

⇒ only have to consider **virtual gluons inside gap**

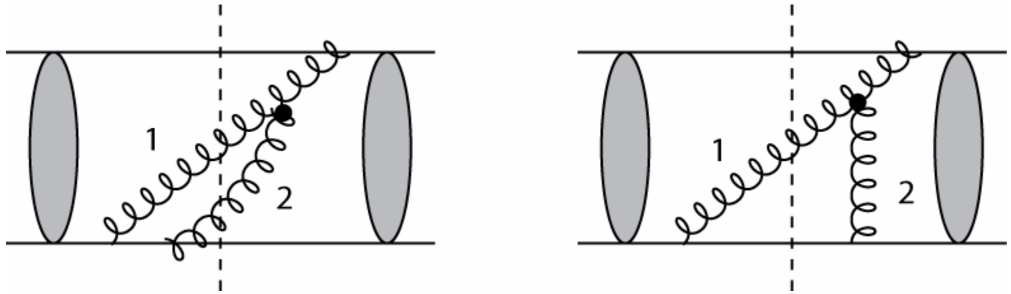


$$\mathbf{M}^{(1)}(Q_0) = -\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_\perp}{k_\perp}\,\mathbf{\Gamma}\,\mathbf{M}_0$$



$$\mathbf{M}(Q_0) = exp\left(-\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_\perp}{k_\perp}\,\mathbf{\Gamma}\right)\,\mathbf{M}_0$$

qq→ qq: Secondary gluons



Non-cancellation if g_1 is outside and g_2 is inside gap

⇒ have to also consider (virt./real) **gluons outside gap with subsequent emission inside gap**

We calculate the all-orders contribution from 1 gluon outside the gap

1-outside the gap

$$\sigma_R = -\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_T}{k_T}\int_{\mathrm{out}}\frac{dy\ d\phi}{2\pi}$$

$$\mathbf{M}_0^\dagger \exp\left(-\frac{2\alpha_s}{\pi}\int_{k_T}^{Q}\frac{dk'_T}{kT'}\boldsymbol{\Gamma}^\dagger\right)\mathbf{D}_\mu^\dagger \exp\left(-\frac{2\alpha_s}{\pi}\int_{Q_0}^{k_T}\frac{dk'_T}{k'_T}\boldsymbol{\Lambda}^\dagger\right)\mathbf{S}_R$$

$$\exp\left(-\frac{2\alpha_s}{\pi}\int_{Q_0}^{k_T}\frac{dk'_T}{k'_T}\boldsymbol{\Lambda}\right)\mathbf{D}^\mu \exp\left(-\frac{2\alpha_s}{\pi}\int_{k_T}^{Q}\frac{dk'_T}{k'_T}\boldsymbol{\Gamma}\right)\mathbf{M}_0$$

$$\sigma_V = -\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_T}{k_T}\int_{\mathrm{out}}\frac{dy\ d\phi}{2\pi}\quad\left[\mathbf{M}_0^\dagger \exp\left(-\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk'_T}{k'_T}\boldsymbol{\Gamma}^\dagger\right)\mathbf{S}_V\right.$$

$$\left.\exp\left(-\frac{2\alpha_s}{\pi}\int_{Q_0}^{k_T}\frac{dk'_T}{k'_T}\boldsymbol{\Gamma}\right)\ \boldsymbol{\gamma}\ \exp\left(-\frac{2\alpha_s}{\pi}\int_{k_T}^{Q}\frac{dk'_T}{k'_T}\,\boldsymbol{\Gamma}\right)\mathbf{M}_0 + \mathrm{c.c.}\right]$$

Λ: evolution of the qq$\rightarrow$ qqg system

$$\mathbf{\Lambda} = \begin{pmatrix} \frac{N}{4}(Y-i\pi)+\frac{1}{2N}i\pi & \left(\frac{1}{4}-\frac{1}{N^2}\right)i\pi & -\frac{N}{4}s_y Y & 0 \\ i\pi & \frac{N}{4}(2Y-i\pi)-\frac{3}{2N}i\pi & 0 & 0 \\ -\frac{N}{4}s_y Y & 0 & \frac{N}{4}(Y-i\pi)-\frac{1}{2N}i\pi & -\frac{1}{4}i\pi \\ 0 & 0 & -i\pi & \frac{N}{4}(2Y-i\pi)-\frac{1}{2N}i\pi \end{pmatrix}$$

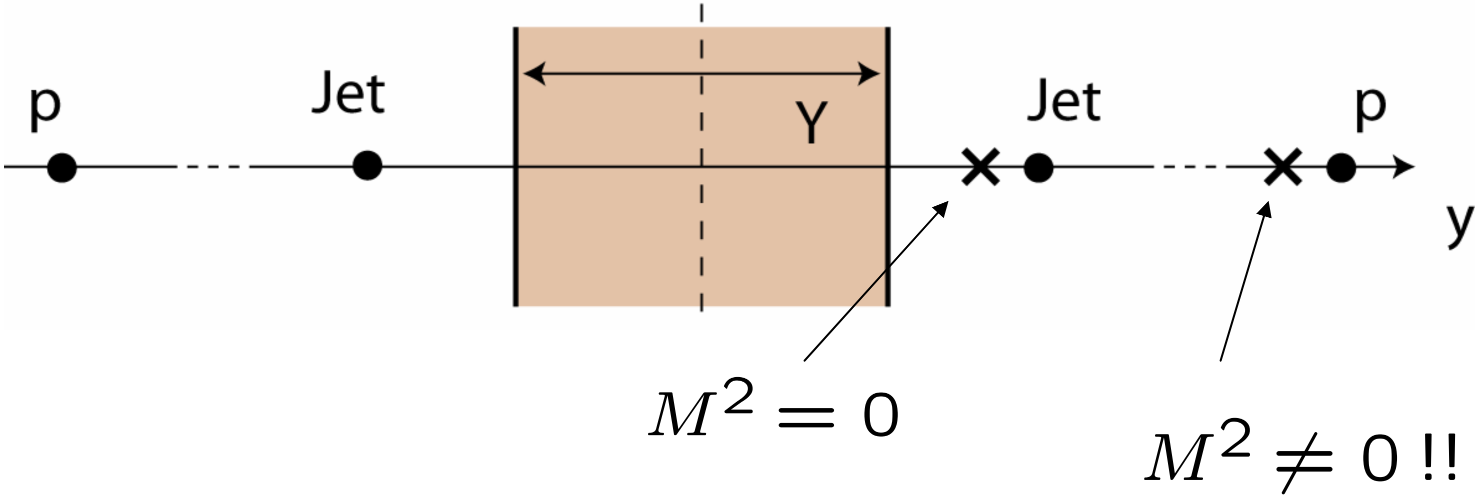
$$+\begin{pmatrix} N & 0 & 0 & 0 \\ 0 & N & 0 & 0 \\ 0 & 0 & N & 0 \\ 0 & 0 & 0 & N \end{pmatrix}\frac{1}{4}\rho(Y,2\,|y|) + \begin{pmatrix} C_F & 0 & 0 & 0 \\ 0 & C_F & 0 & 0 \\ 0 & 0 & C_F & 0 \\ 0 & 0 & 0 & C_F \end{pmatrix}\frac{1}{2}\rho(Y,\Delta y)$$

$$+\begin{pmatrix} \frac{N}{4}\left(-\frac{1}{2}\lambda\right) & 0 & \frac{N}{4}\left(-\frac{1}{2}s_y\lambda\right) & \frac{1}{4}\left(\frac{1}{2}s_y\lambda\right) \\ 0 & \frac{N}{4}\left(-\frac{1}{2}\lambda\right) & 0 & \frac{N}{4}\left(\frac{1}{2}s_y\lambda\right) \\ \frac{N}{4}\left(-\frac{1}{2}s_y\lambda\right) & 0 & \frac{N}{4}\left(-\frac{1}{2}\lambda\right) & \frac{1}{4}\left(-\frac{1}{2}\lambda\right) \\ \frac{1}{2}s_y\lambda & \left(\frac{N}{4}-\frac{1}{N}\right)\left(\frac{1}{2}s_y\lambda\right) & -\frac{1}{2}\lambda & \frac{N}{4}\left(-\frac{1}{2}\lambda\right) \end{pmatrix}$$

[M.Seymour, A.K., JHEP 0601:085,2006]

Out-of-gap gluon in collinear limit: a surprise

$$\sigma_1 = \int_{Q_0}^{Q} \frac{dk_\perp}{k_\perp} \int_{Y/2}^{\infty} dy \, M^2$$



Non-zero contribution from initial state collinear limit

‘Failure of the plus-prescription’

$$\int d^2k_T \int_{out} dy \;\; M^2_{soft} \to \int d^2k_T \left[\int_{Y/2}^{y_{max}} dy \;\; M^2_{soft} + \int_{y_{max}}^{\infty} dy \;\; M^2_{coll} \right]$$

$$\int_{y\text{max}}^{\infty} dy \;\; M^2_{coll} = \int_0^{1-\delta} dz \frac{1}{2} \left(\frac{1+z^2}{1-z} \right) \frac{q(x/z,\mu^2)}{q(x,\mu^2)} A_R + \int_0^{1-\delta} dz \frac{1}{2} \left(\frac{1+z^2}{1-z} \right) A_V$$

$$= \int_0^{1-\delta} dz \frac{1}{2} \left(\frac{1+z^2}{1-z} \right) \left(\frac{q(x/z,\mu^2)}{q(x,\mu^2)} - 1 \right) A_R + \int_0^{1-\delta} dz \frac{1}{2} \frac{1+z^2}{1-z} (A_R + A_V)$$

$$\left(-y_{max} + \Delta y/2 + \ln\left(\frac{Q}{k_T}\right)\right) (A_R + A_V)$$

Super-leading logarithms (SLL)

Out-of-gap gluon :

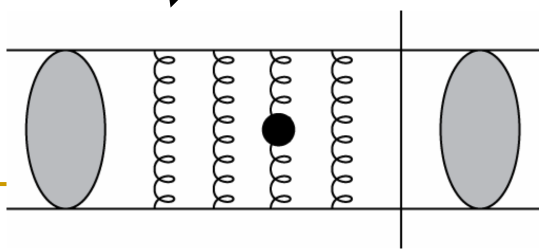
$$\alpha_s \int_{Q_0}^{Q} \frac{dk_\perp}{k_\perp} \, Log\frac{Q}{k_\perp} \sim \alpha_s L^2$$

Each gluon in the gap:

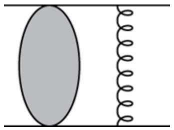
$$\alpha_s \int_{Q_0}^{Q} \frac{dk_\perp}{k_\perp} \int_0^Y dy \sim \alpha_s L$$

$$\sigma_1 = \alpha_s^2 (c_1\, \alpha_s L + c_2\, \alpha_s^2 L^2 + c_3\, \alpha_s^3 L^3 + c_4\, \alpha_s^4 L^5 + \ldots)$$

Failure of plus-prescription above $Q_0 \Rightarrow$ SLL
collinear gluons into pdf only below Q_0



Nature of the SLL

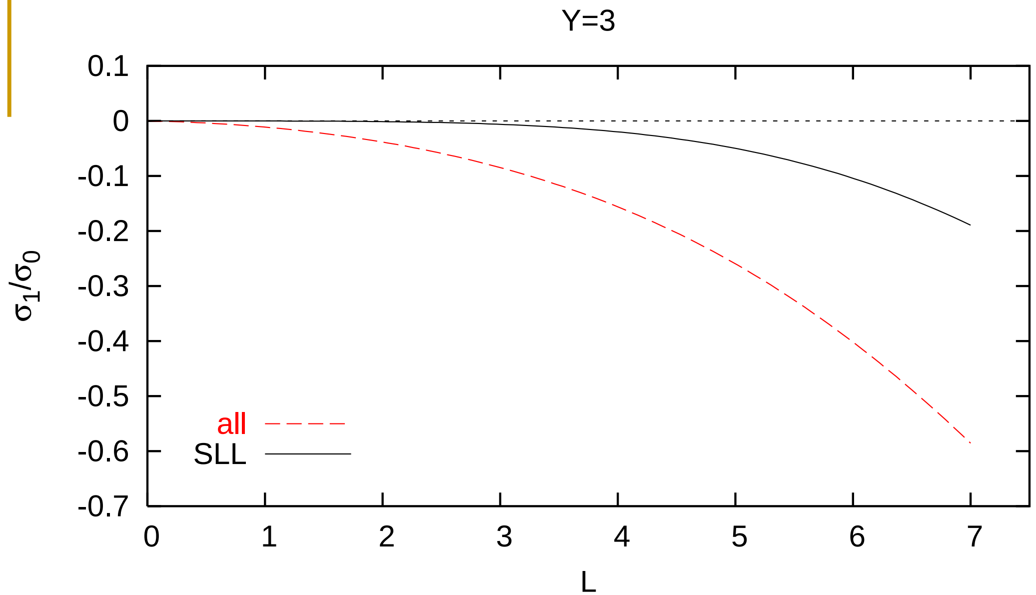
-  $\sim Y + \bar{Y} + c(i\pi)$

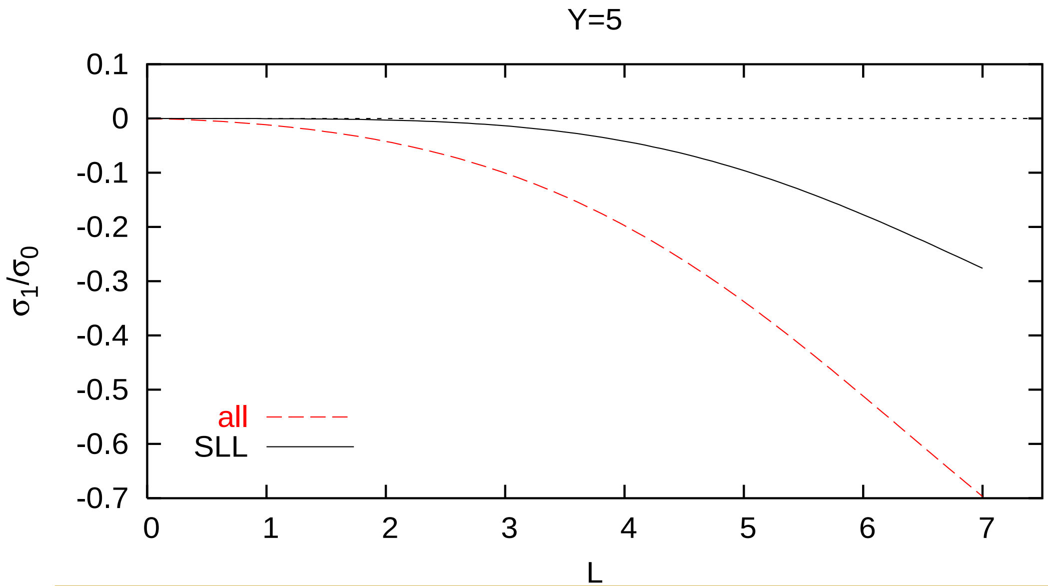
SLL stem from non-cancelling imaginary parts (they vanish if c=0)

- Probable continuation at higher orders:

$$\begin{aligned}\sigma_1 &= \ldots \alpha_s^3 L^3 + \alpha_s^4 L^5 + \alpha_s^5 L^7 + \ldots \\ &= \ldots \alpha_s^3 L^3 + \alpha_s^2 L(\alpha_s^2 L^4 + \alpha_s^3 L^6 + \ldots)\end{aligned}$$

Double-logs $(\alpha_s L^2)^n$ instead of single $(\alpha_s L)^n$



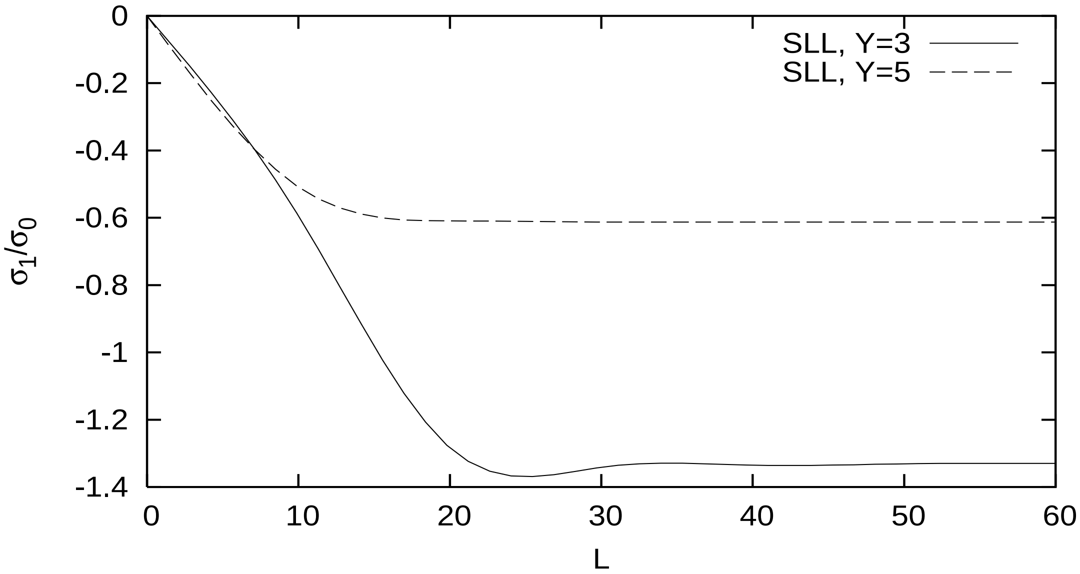


σ_0: independent emission, all orders

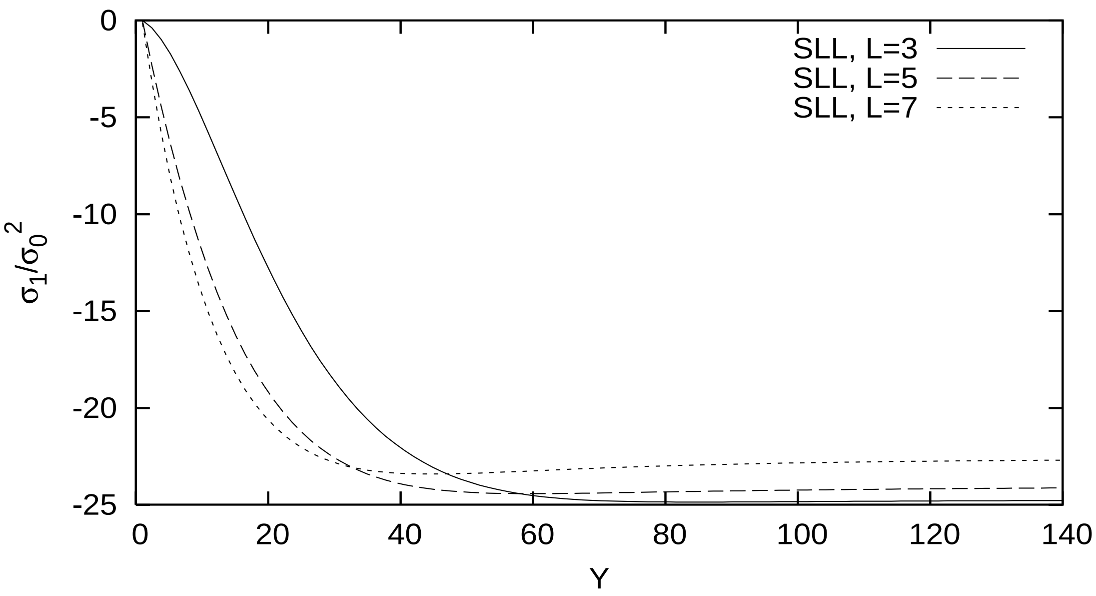
σ_1: 1-outside-the-gap, all orders

SLL relatively small for L<4

But:
2,3,.. gluons outside the gap not yet included



Saturation at large L, Y



large Y:

$\sigma_0 \sim \sigma_{BFKL}$

$\sigma_1 \sim \sigma_0^2$

$\rightarrow$ Exciting hint to higher orders (σ_2, σ_3, ..)

Impact of the SLL's

The 'gap' can originate from kinematical limitations (value of observable < Q_0)

e.g. thrust: $$\sum_i k_{t,i}\, e^{|\eta_i|} < \tau$$

- SLL expected in non-global eventshapes / interjet energy flows
- Global Observables may be effected, too (e.g. transverse thrust).
  SLL suppressed, therefore appear as LL

Summary

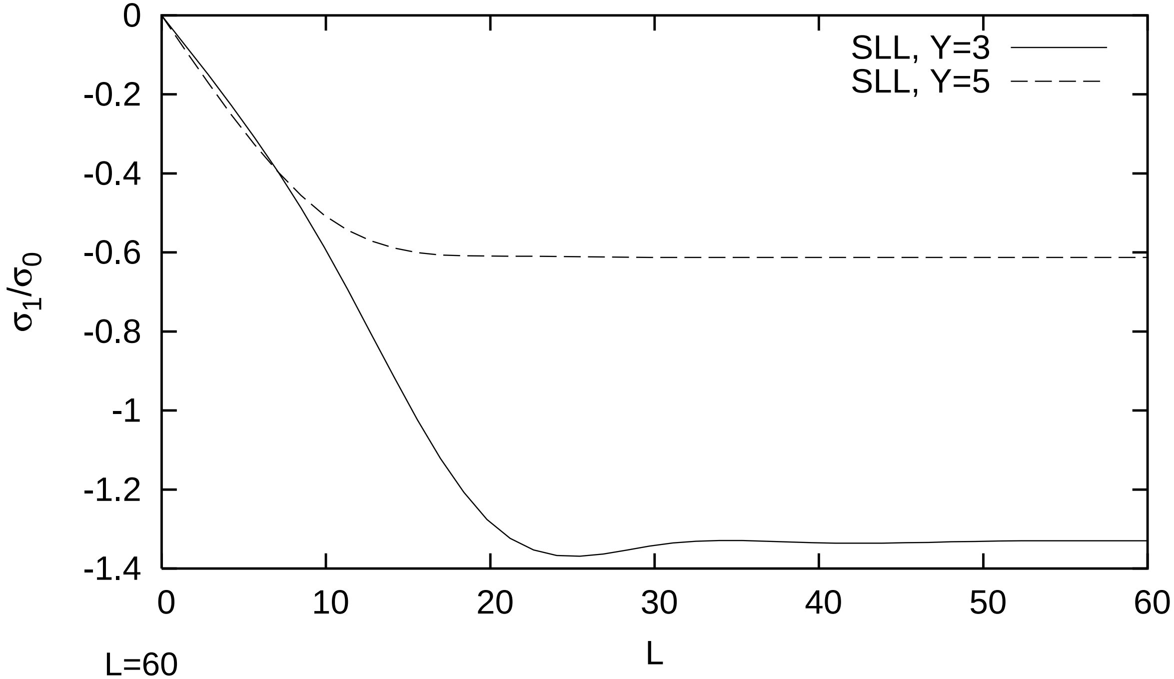
We have found new super-leading logarithms in calculation of
pp $\rightarrow$ jet gap jet

- Stems from region where out-of-gap gluon becomes coll. to initial state particle, originates from Coulomb phase terms
- 'Breakdown of plus-prescription above Q_0' probably gives rise to double logs instead of single ones
- Formally more important than any LL result, numerically modest at LHC, but effect of n gluons outside the gap not yet included
- Saturation at large Y
  $\rightarrow$ deeper link between non-global observables and small-x physics

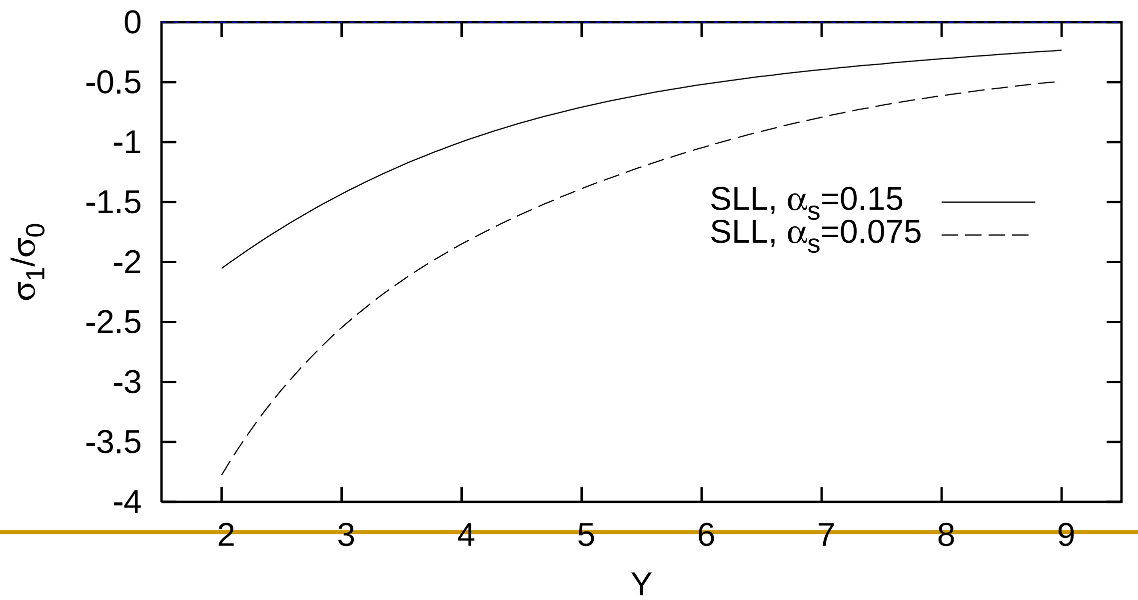
Outlook

- Investigate impact of the SLL contributions on relevant observables at the LHC (in particular eventshapes)
- Have to resum also NLL's to get all LL's
- Resum SLL's (any number of gluons outside the gap) (!)

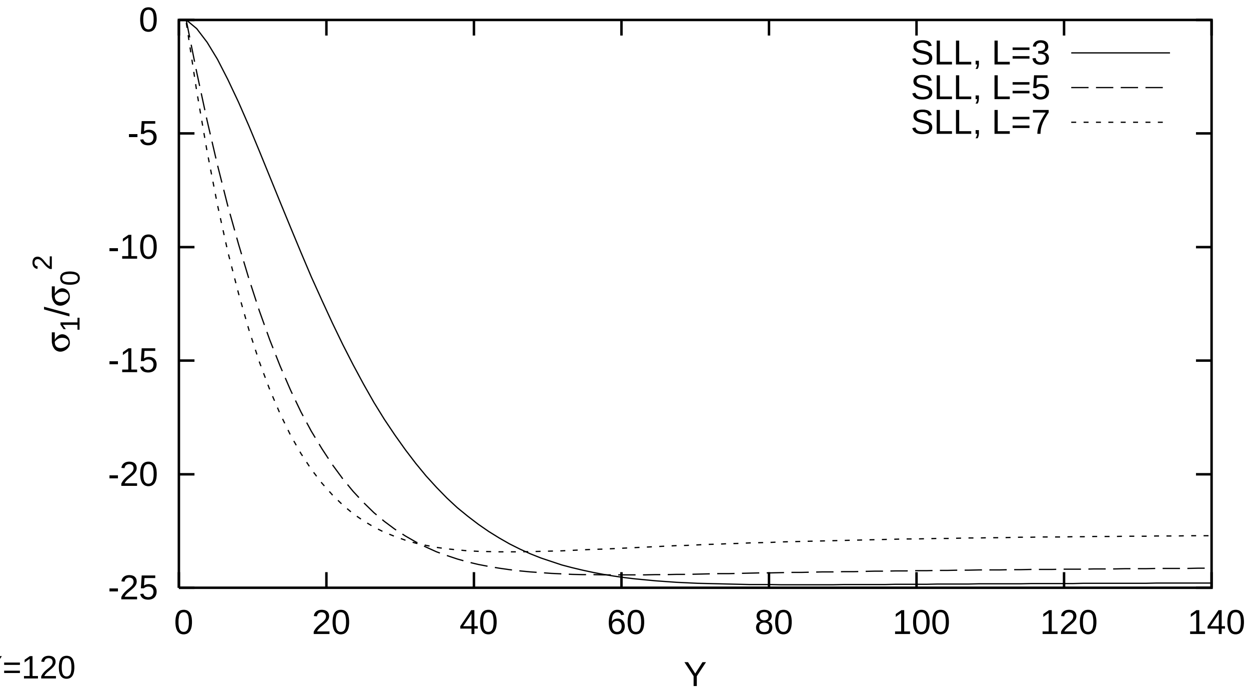
Large L


SLL, Y=3
SLL, Y=5
σ_1/σ_0
L
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
0 10 20 30 40 50 60

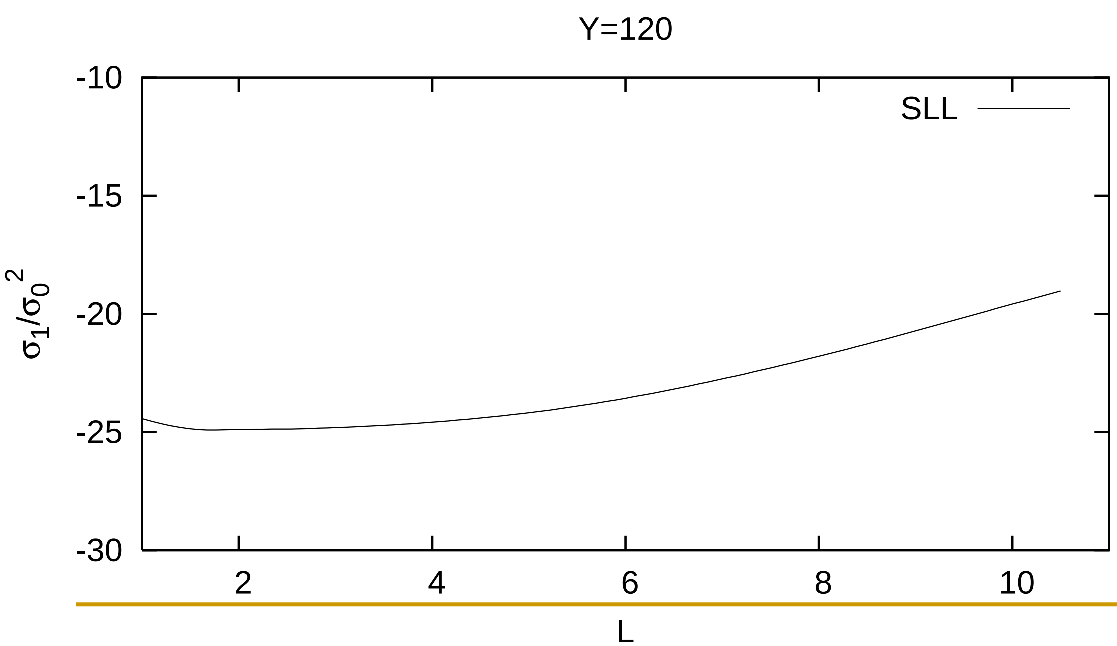
L=60


SLL, α_s=0.15
SLL, α_s=0.075
σ_1/σ_0
Y
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
2 3 4 5 6 7 8 9

Large Y



SLL, L=3
SLL, L=5
SLL, L=7
σ_1/σ_0^2
Y
0
-5
-10
-15
-20
-25
0
20
40
60
80
100
120
140



Y=120
SLL
σ_1/σ_0^2
L
-10
-15
-20
-25
-30
2
4
6
8
10