Extended Interacting Boson Models for Exotic Nuclei

V.K.B. Kota

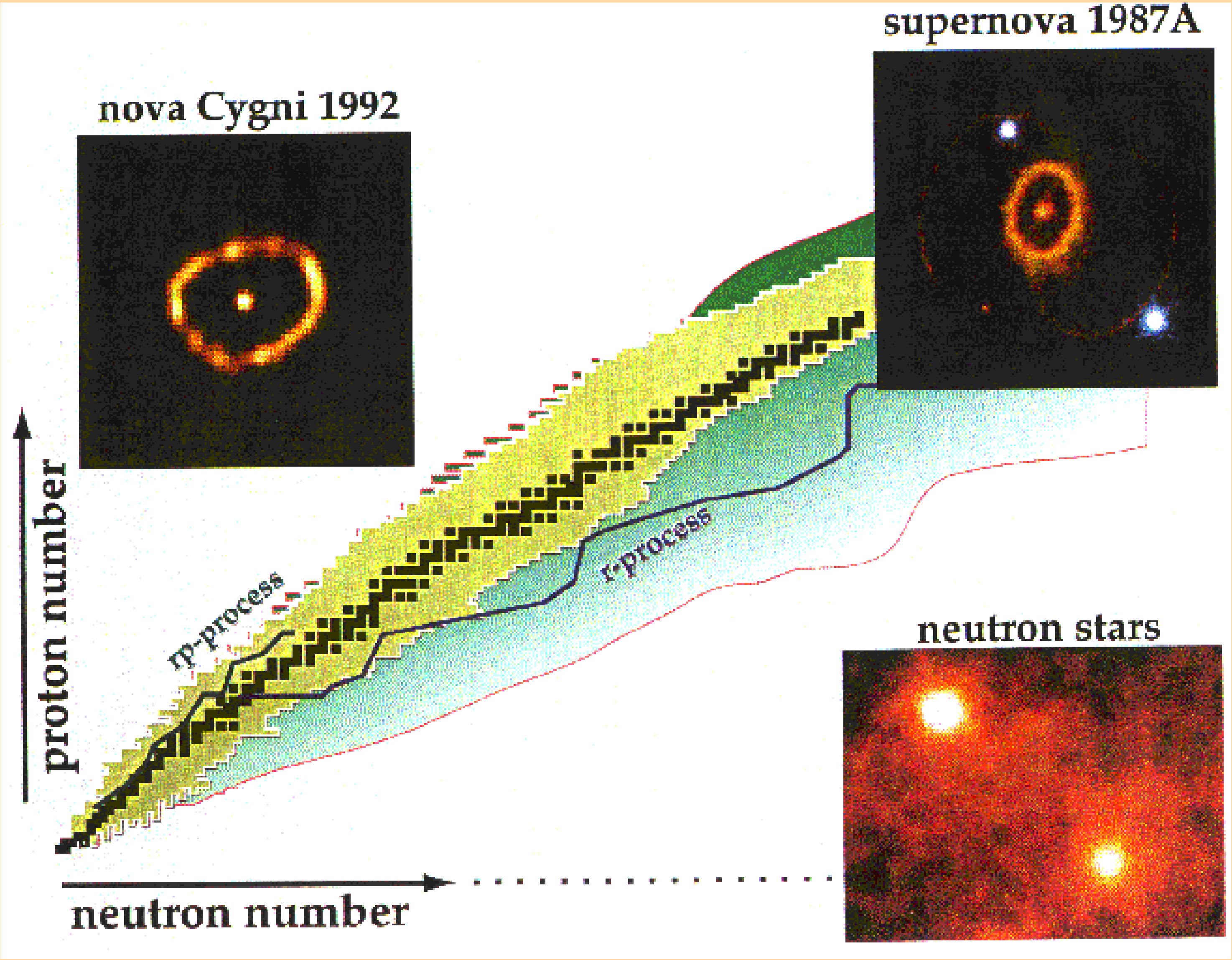
Theoretical Physics Division
Physical Research Laboratory
Ahmedabad 380 009, India

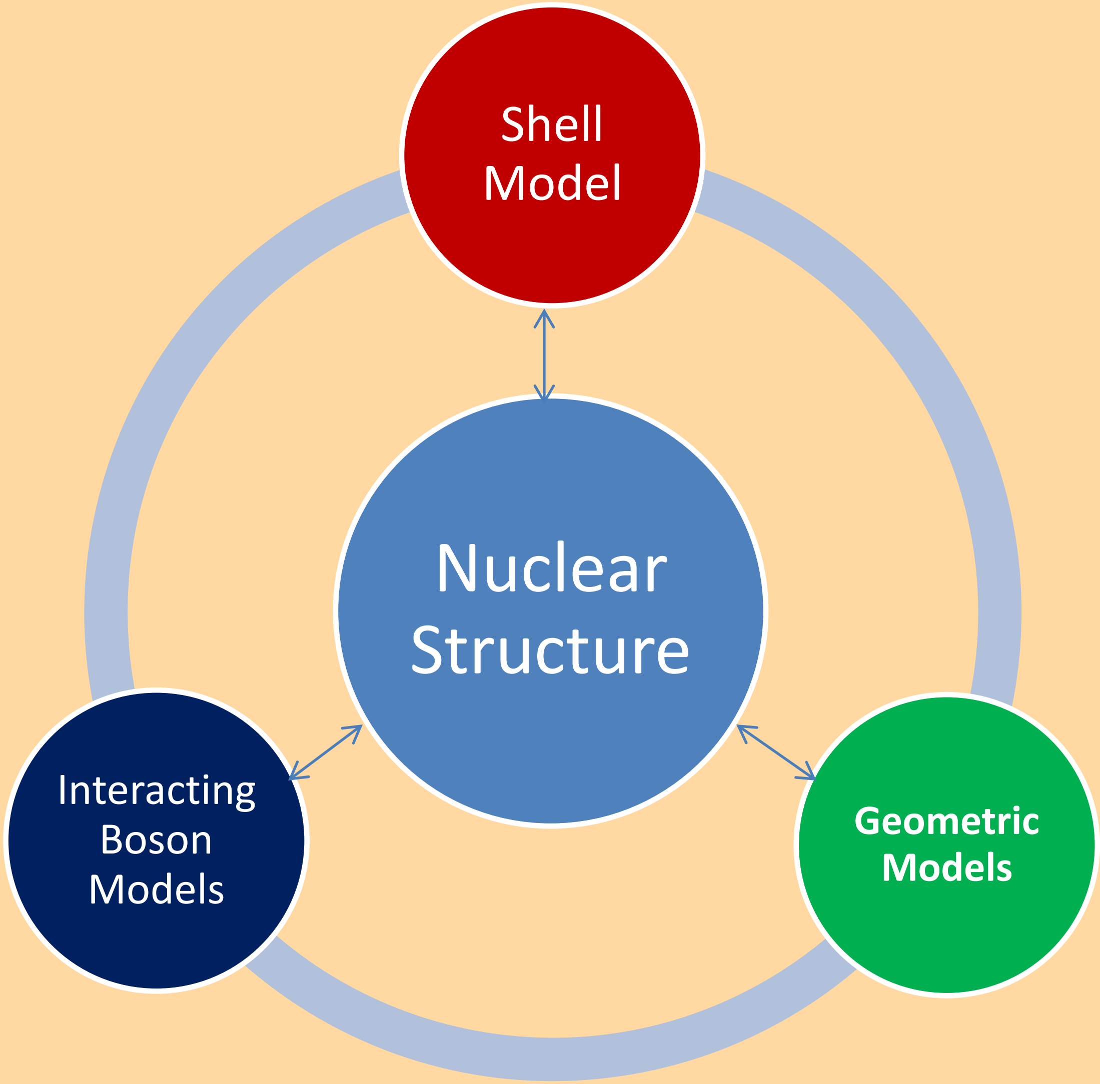
Plan of the Talk

- **Introduction to IBM**
- **IBM $U(6) \oplus U(6) \oplus U(6)$ model for neutron excess nuclei with neutron skin**
- **IBM-*ST* model: Deuteron transfer**
- **IBM with aligned pairs as bosons**
- **Conclusions**

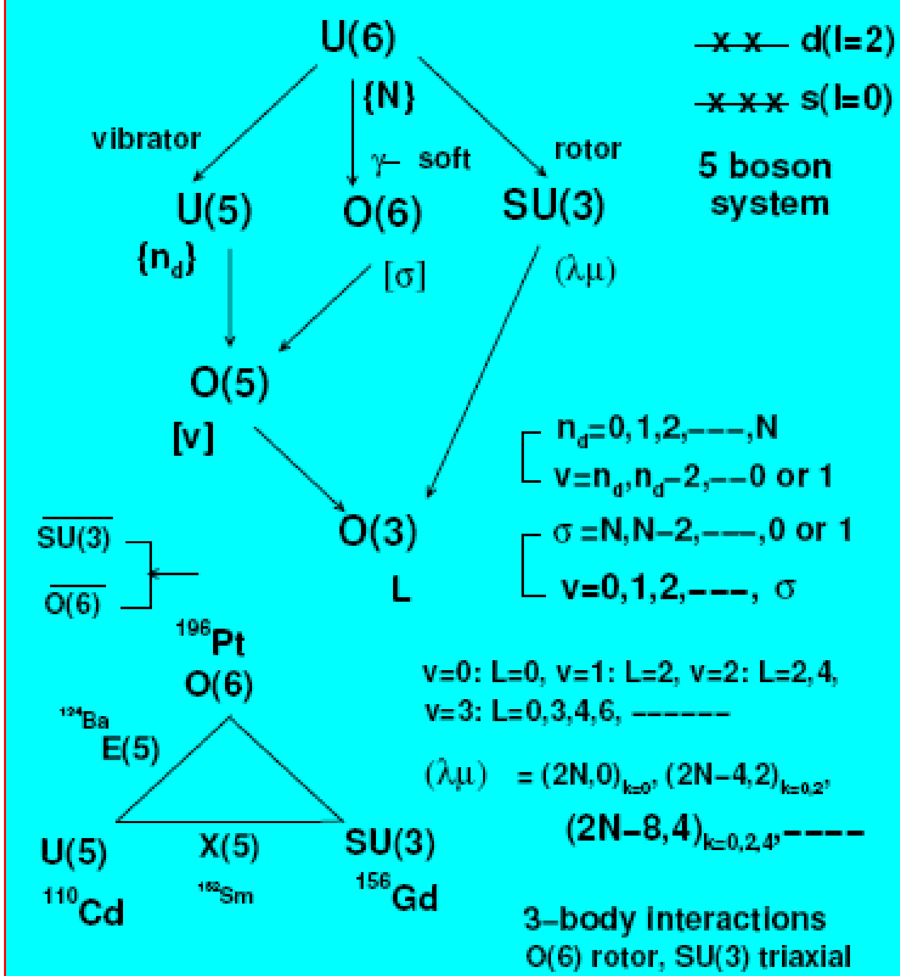
1. INTRODUCTION

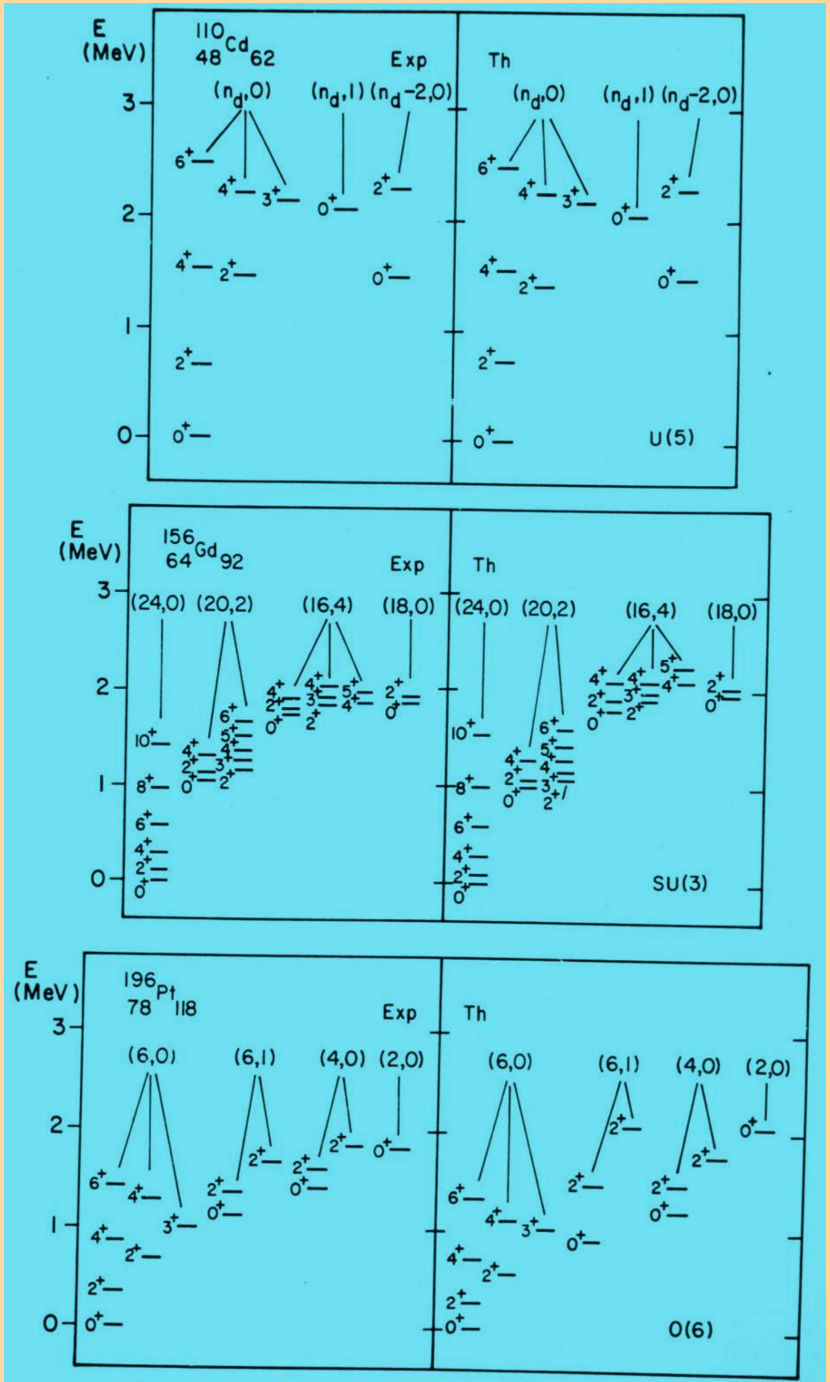
Study of the structure of nuclei near the proton and neutron drip lines has become important as these nuclei exhibit new structures and because they are important for *r* and *rp*-process nucleosynthesis and explosive processes like supernovae and *X*-ray bursts.




Shell
Model
Nuclear
Structure
Interacting
Boson
Models
Geometric
Models

INTERACTING BOSON MODEL
Dynamical Symmetries





Arima and Iachello (first paper:1975)

Proton-Neutron Interacting Boson Model
Complete Classification of Symmetry Schemes

proton bosons (π) and neutron (ν) bosons
with $\ell = 0$ $(s_\pi\,,\ s_\nu)$ and $\ell = 2$ $(d_\pi\,,\ d_\nu)$

SGA: $U(12)$

good quantum numbers: N, L, $M_{F_z} = (N_\pi - N_\nu)/2$

$$U_{sd\pi\nu}(12) \begin{array}{l} \nearrow U_{sd}(6) \otimes SU(2) \\ \nearrow U_\pi(6) \oplus U_\nu(6) \\ \\ \searrow U_s(2) \oplus U_d(10) \\ \searrow O_{sd\pi\nu}(12) \end{array}$$

$$O(12) \begin{array}{l} \nearrow O(6) \otimes O(2) \\ \\ \searrow O(2) \oplus O(10) \end{array} \qquad U(10) \begin{array}{l} \nearrow U(5) \otimes SU(2) \\ \\ \searrow O(10) \supset O(5) \otimes O(2) \end{array}$$

U(6)'s generate U(5), SU(3) and O(6) symmetries
SU(2)'s generate F-spin, O(2)'s generate F_z

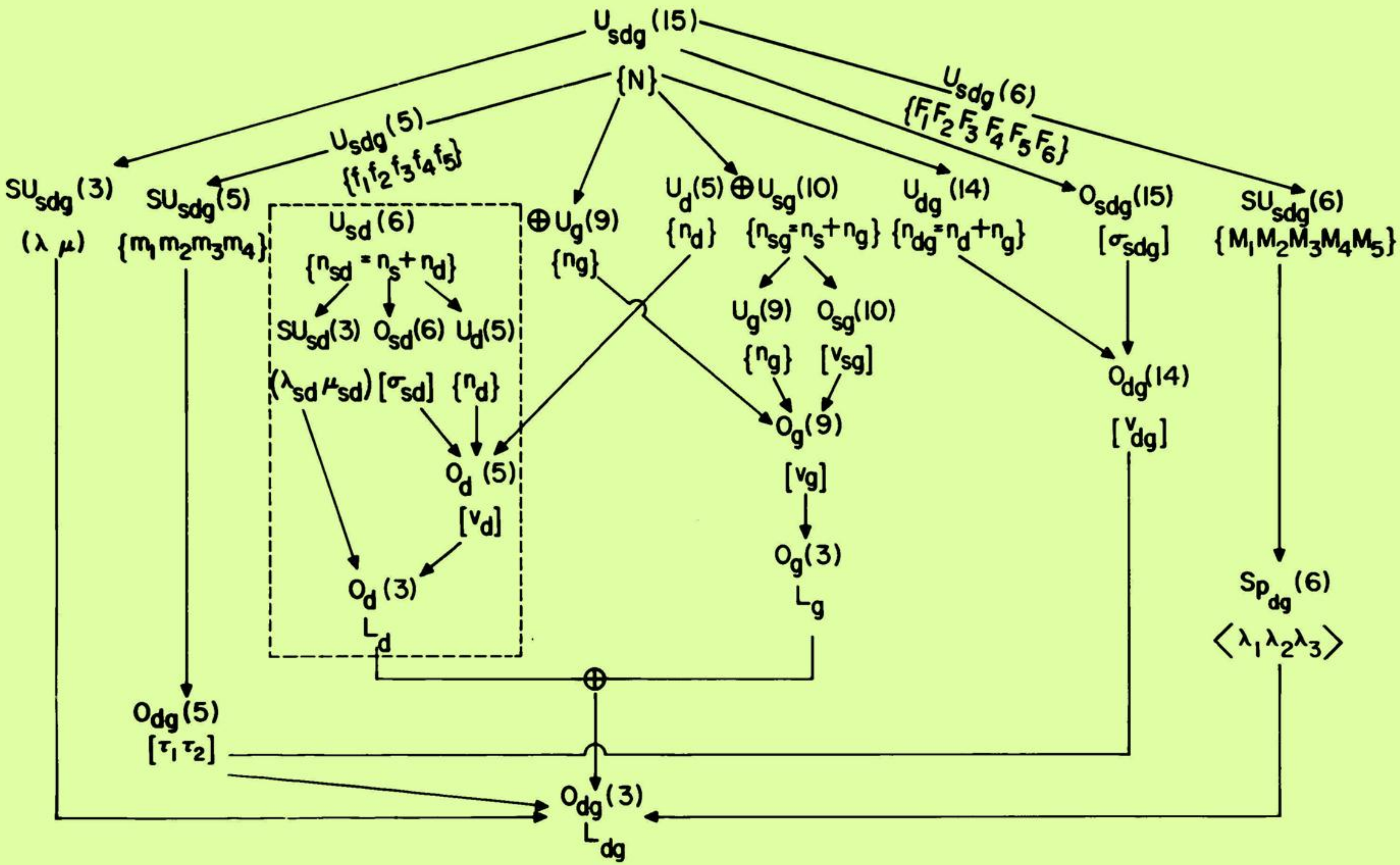
$U(6) \otimes SU(2)$ **generates mixed symmetry states,**
(scissors 1^+ states in rotational nuclei)
seen in ^{156}Gd, ^{196}Pt, ^{134}Ba, ^{94}Mo

$U_\pi(6) \oplus U_\nu(6) \supset SU_\pi(3) \oplus \overline{SU_\nu(3)} \longrightarrow$ **triaxial shapes**

O(12) breaks F-spin symmetry and it is generated
by the pairs: $\left[s_\pi^\dagger s_\nu^\dagger + \sqrt{5}\beta\left(d_\pi^\dagger d_\nu^\dagger\right)^0\right]$, $\beta = \pm 1$

Arima
Iachello
Otsuka
Pietralla
Talmi
Van Isacker
Von Brentano
Scholten
Kota

sdgIBM: Symmetries



Devi, Kota, Yoshinaga, Arima, Kuyucak, Van Isacker

Interacting boson-fermion (IBFM) and boson-fermion-fermion (IBFFM) models for odd-A and odd-odd nuclei:

IBFM symmetry schemes:

$\mathrm{Spin}^{BF}(5)$, $\mathrm{Spin}^{BF}(6)$, $SU^{BF}(5)\otimes SU^{F}(2)$, $SU^{BF}(3)\otimes SU^{F}(2)$, $SO^{BF}(6)\otimes SU^{F}(2)$
$U^{B}(6)\otimes U^{F}(12)$, $U^{B}(6)\otimes U^{F}(21)$, $U^{B}(6)\otimes U^{F}(30)$, --------

IBFFM symmetry schemes:

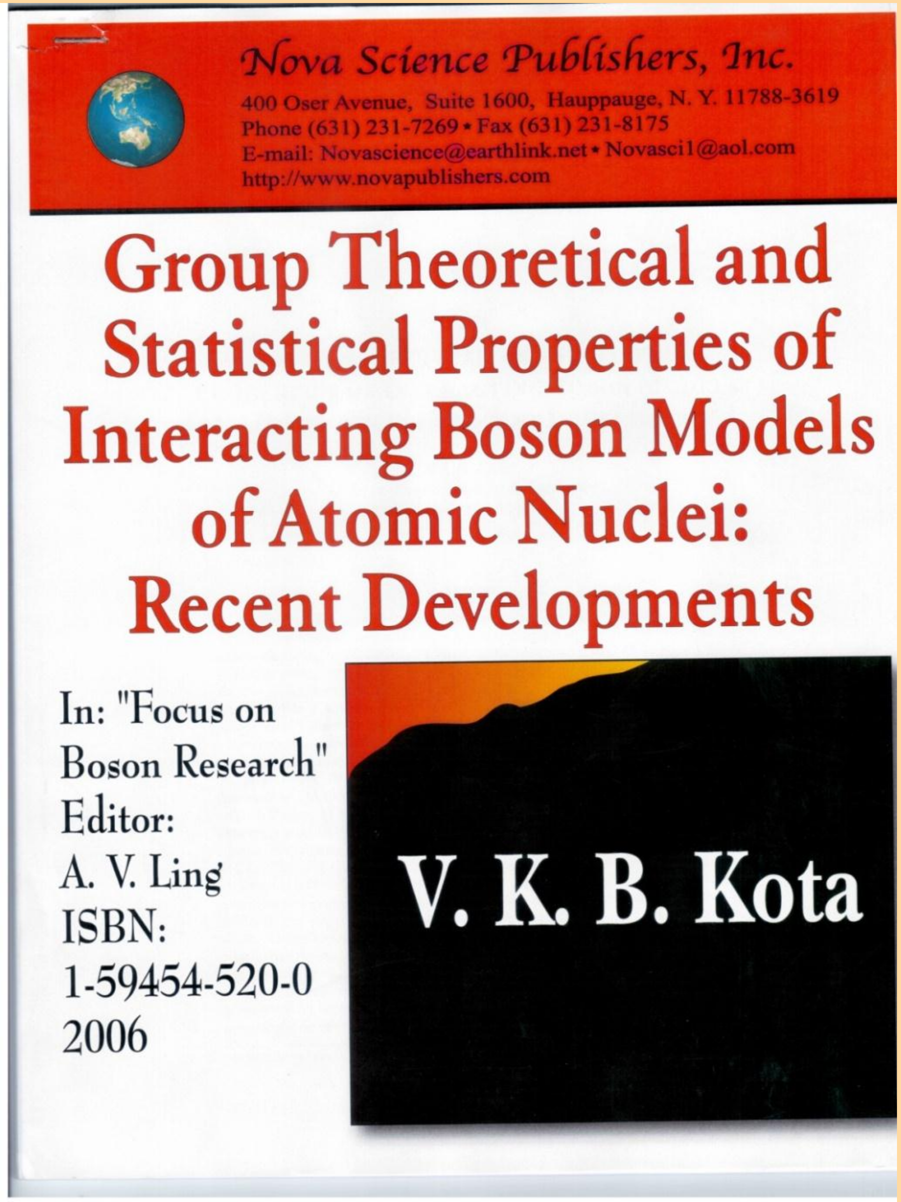
$SU^{BF}(3)\otimes SU^{F}(2j+1)$, $SU^{BFF}(3)$, -------

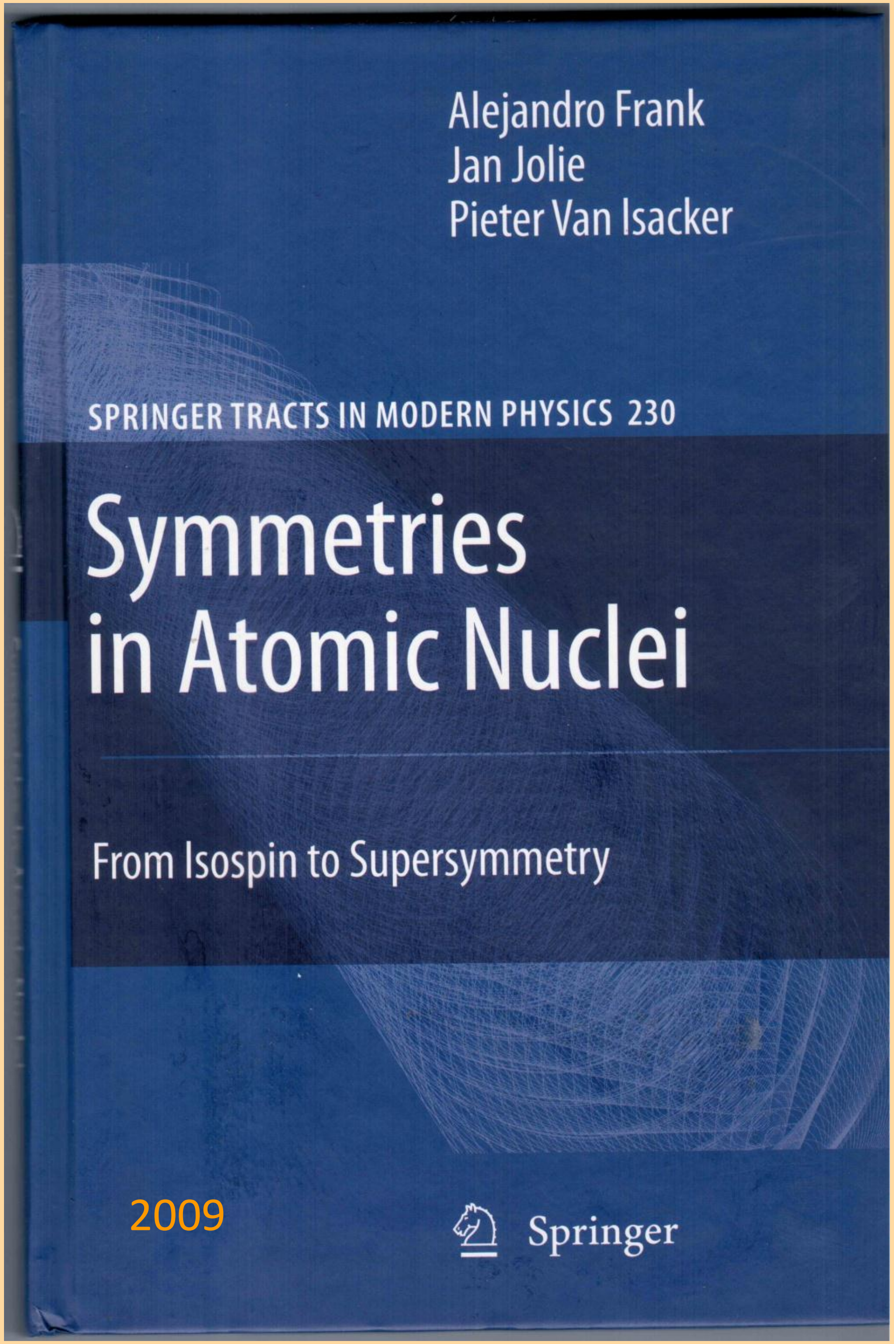
IBM models with isospin and spin-isospin

IBM-3 with U(18) SGA, IBM-4 with U(36) SGA

IBM models with 2 and 4 quasi-particles for high-spins and super deformation,
IBM models with particle-hole excitations and so on

Books by Iachello, Frank, Van Isacker, Jolie


Nova Science Publishers, Inc.
400 Oser Avenue, Suite 1600, Hauppauge, N. Y. 11788-3619
Phone (631) 231-7269 • Fax (631) 231-8175
E-mail: Novascience@earthlink.net • Novasci1@aol.com
http://www.novapublishers.com
Group Theoretical and Statistical Properties of Interacting Boson Models of Atomic Nuclei: Recent Developments
In: "Focus on Boson Research"
Editor:
A. V. Ling
ISBN:
1-59454-520-0
2006
V. K. B. Kota


Alejandro Frank
Jan Jolie
Pieter Van Isacker
SPRINGER TRACTS IN MODERN PHYSICS 230
Symmetries in Atomic Nuclei
From Isospin to Supersymmetry
2009
Springer

New Directions in IBM (since 2000)

SUSY in Nuclei

Jolie, Frank, Van Isacker

Quantum Phase Transitions

Iachello, Casten, Caprio, Jolie, Cejnar, Arias

Neutrinoless Double Beta decay

Iachello, Barea, Kotila

Chaos and regular structures from random Interactions

Frank, Bijker, Zhao, Cejnar, Jolie, Kota

New Symmetries

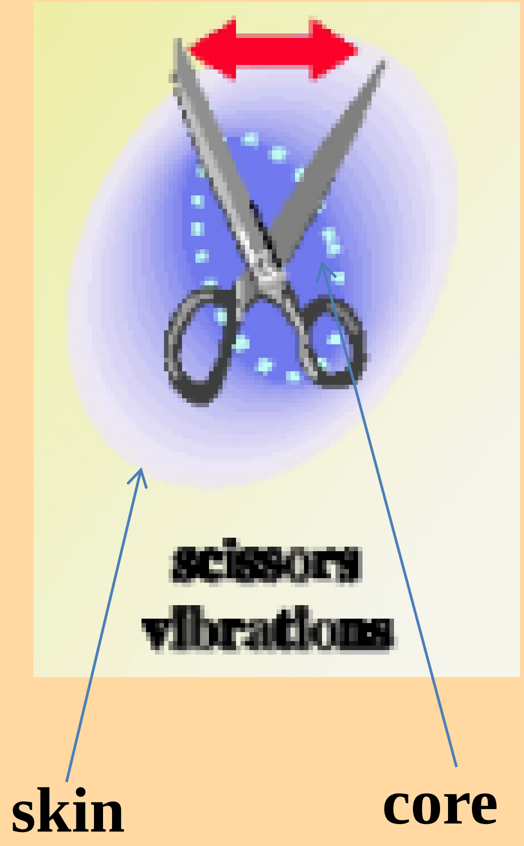
Iachello, Arias, Van Isacker, Kota

2. IBM $U(6)\oplus U(6)\oplus U(6)$ model for neutron excess nuclei with neutron skin

For neutron rich nuclei it is possible to have a core of protons and neutrons and a mantle (or skin) of neutron matter – this has been predicted by some HF calculations.

Then it is possible to have a soft scissors (SS) mode in addition to normal scissors (S).

SS: Out of phase oscillations of skin neutrons against core nucleons (core protons and neutrons move coherently).



IBM model with SS in addition to S was introduced in:
D.D. Warner and P. Van Isacker, Phys. Lett. B 395 (1997) 145-150

$$U_\pi(6) \oplus U_{\nu_c}(6) \oplus U_{\nu_s}(6)$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$N_\pi \qquad N_{\nu_c} \qquad N_{\nu_s}$$

$$N = N_\pi + N_{\nu_s} + N_{\nu_c}$$

$$|S\alpha\rangle_a = |\{N_\pi\}\{N_{\nu_c}\}\{N_\pi + N_{\nu_c} - 1, 1\}; \{N_{\nu_s}\}; \{N-1, 1\}\alpha\rangle$$

$$|SS\alpha\rangle_a = |\{N_\pi\}\{N_{\nu_c}\}\{N_\pi + N_{\nu_c}\}; \{N_{\nu_s}\}; \{N-1, 1\}\alpha\rangle$$

$$|GS\rangle = |\{N_\pi\}\{N_{\nu_c}\}\{N_\pi + N_{\nu_c}\}; \{N_{\nu_s}\}; \{N\}\alpha\rangle$$

$$H = A\hat{M}_{\pi\nu_c} + B\hat{M}_{\pi\nu_s} + C\hat{M}_{\nu_s\nu_c} + D\hat{M}_{\pi\nu}$$

$$|s\alpha\rangle_b = |\{N_\pi\}\{N_{\nu_\rho}\}\{N_\pi + N_{\nu_\rho}\};\{N_{\nu_c}\};\{N-1,1\}\alpha\rangle$$

$$|ss\alpha\rangle_b = |\{N_\pi\}\{N_{\nu_\rho}\}\{N_\pi + N_{\nu_\rho}-1,1\};\{N_{\nu_c}\};\{N-1,1\}\alpha\rangle$$

$$\boxed{|s\alpha\rangle_c = |\{N_\pi\};\{N_{\nu_c}\}\{N_{\nu_\rho}\}\{N_{\nu_c}+N_{\nu_\rho}\};\{N-1,1\}\alpha\rangle}$$

$$|ss\alpha\rangle_c = |\{N_\pi\};\{N_{\nu_c}\}\{N_{\nu_\rho}\}\{N_{\nu_c}+N_{\nu_\rho}-1,1\};\{N-1,1\}\alpha\rangle$$

$a \to b \to c$ w.r.t. $\underline{\underline{U(6)}}$

by $\underline{\underline{\text{U - coefficents}}}$

$$|1^{+}_{sc}\rangle = |s\alpha\rangle_c \,, \quad |1^{+}_{ssc}\rangle = [\;]\,|s\alpha\rangle_c + \cdots |ss\alpha\rangle_c$$

$|1^{+}_{ssc}\rangle \Downarrow |ss\alpha\rangle_a$

$$[\;] = \sqrt{\frac{N_\pi N_{\nu_\rho}}{(N_\pi + N_{\nu_c})(N_{\nu_c}+N_{\nu_\rho})}}$$

$$\hat{C}_2\left(U(6)\right) = \hat{N}\left(\hat{N}+5\right) - 2\hat{M}_{ij}$$

$$\hat{N} = \hat{N}_i + \hat{N}_j$$

$$\langle \hat{M}_{ij} \rangle^{\{N\}} = 0, \quad \langle \hat{M}_{ij} \rangle^{\{N-1,1\}} = N$$

$$|S\alpha\rangle = \sum_{\{t\}} U\left(\{N_\pi\}, \{N_{\nu_c}\}, \{N-1,1\}, \{N_{\nu_s}\}; \{N_\pi + N_{\nu_c} - 1, 1\} \{t\}\right) \\ |\{N_\pi\}; \{N_{\nu_c}\} \{N_{\nu_s}\} \{t\}; \{N-1,1\} \alpha\rangle$$

$$|SS\alpha\rangle = \sum_{\{t\}} U\left(\{N_\pi\} \{N_{\nu_c}\} \{N-1,1\} \{N_{\nu_s}\}; \{N_\pi + N_{\nu_c}\}; \{t\}\right) \\ |\{N_\pi\}; \{N_{\nu_c}\} \{N_{\nu_s}\} \{t\}; \{N-1,1\} \alpha\rangle$$

$U(\cdots) \Longleftrightarrow U(\cdots)$ of $SU(2)$ with $\{N'\} \Rightarrow J = \frac{N'}{2}$, $\{N'-1,1\} \Rightarrow J = \frac{N'}{2} - 1$

construction of H matrix is simple with U-coefficients being easy to Calculate

with B, C $\simeq$ 0 we have

$E(S\alpha) \simeq (A+D)N$

$E(SS\alpha) \simeq DN$

$E(GS) = 0$ $\Rightarrow$ A, D > 0 so that GS is lowest

with A $\simeq$ D

$E(S\alpha) \simeq 2DN = \underline{\underline{3\text{ MeV}}}$ $\Rightarrow$ $E(SS\alpha) \simeq \underline{\underline{1.5\text{ MeV}}}$

Corrections to E(S) and E(SS) Due to B and C terms can be calculated.

$T(M1) = g_\pi L_\pi + g_{\nu c} L_{\nu c} + g_{\nu s} L_{\nu s}$

Expressions for M1 strength:

$$B\left(\mathrm{M1};0_1^+ \rightarrow 1_{\mathrm{S}}^+\right) = \frac{3}{4\pi}\left(g_\nu - g_\pi\right)^2 f(N) N_\nu N_\pi$$

$$B\left(\mathrm{M1};0_1^+ \rightarrow 1_{\mathrm{SS}}^+\right) = \frac{3}{4\pi}\left(g_\nu - g_\pi\right)^2 f(N) \frac{N_{\nu_s} N_\pi^2}{N_\nu + N_\pi}$$

Summary:

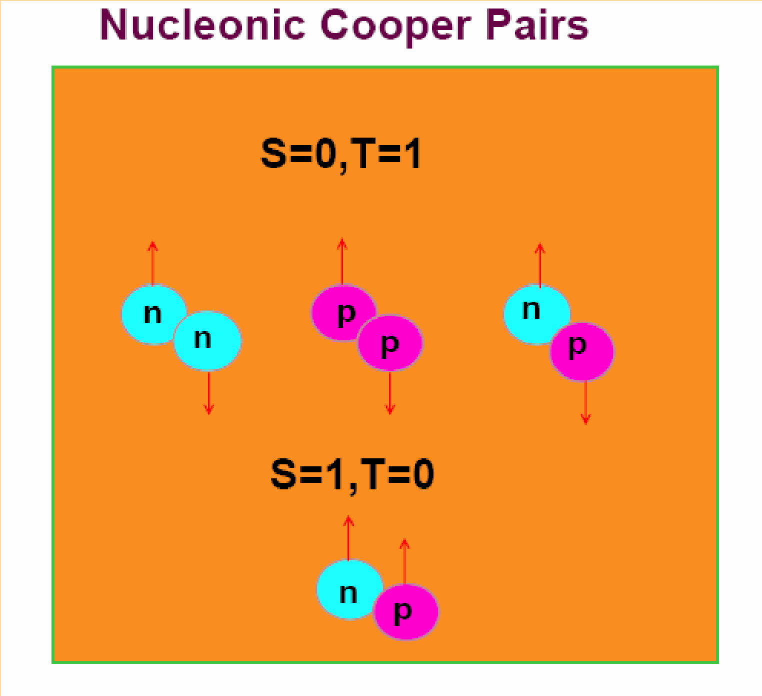
It is shown that IBM approach can be extended to neutron excess nuclei with neutron skin.

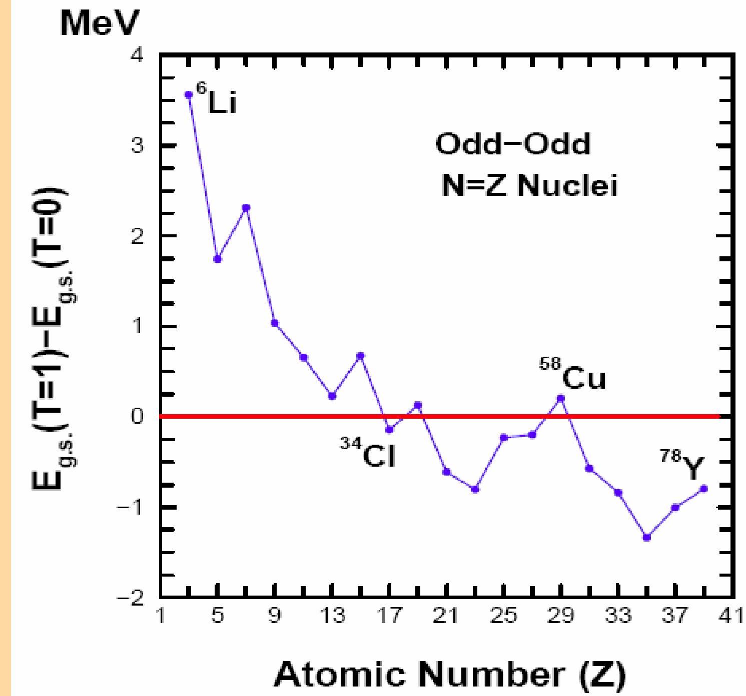
Possibility of soft scissors mode is established and it has analogy with soft dipole mode.

If the extent of the skin is at least comparable to the range of proton-neutron interaction, the notion of a partial decoupling of it from the core becomes valid and soft scissors then possible.

$U(6) \oplus U(6) \oplus U(6)$ with $U(6)$ sub algebras will give more structure to the soft scisors and details of these can be worked out.

3. IBM-*ST* model for N=Z nuclei: deuteron transfer

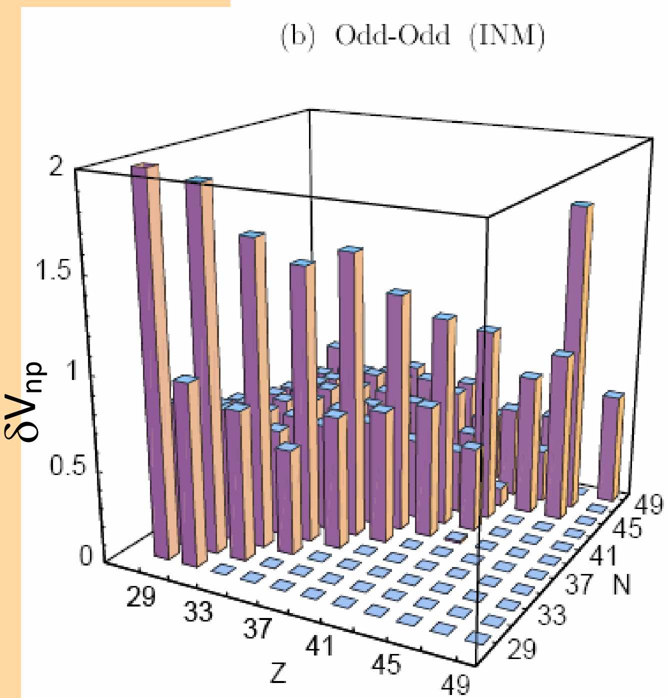




Pair is defined to have L=0 (in LST coupling)

Isovector (T=1),
isoscalar (T=0)
pairs for N=Z nuclei

$\Rightarrow$ SO(8) algebra within shell model



Wigner's SU(4) Symmetry

With a hermitization procedure for the *SO*(8) Hamiltonian, Dyson mapping give a interacting *s*-boson $U_{s:ST}(6)$ model with the bosons carrying $(ST) = (10) \oplus (01)$. This model admits two symmetery limits that are in correspondence with the SU(4) and SO(5) limits of the SO(8) model.

By adding *d*-bosons to the $U_{s:ST}(6)$ model gives the full *sd*IBM-ST or IBM-4 with $U_{sd:ST}(36)$ SGA and this includes deformation effects that are missing in the $U_{s:ST}(6)$ model. With only T=1 bosons we have IBM-3 with $U_{sd:T}(18)$ SGA and this is useful only for even-even nuclei. A group chain of $U_{sd:ST}(36)$ that is proved to useful for heavy N=Z nuclei is [VKBK, Ann. Phys. (N.Y.) 280, 1 (2000)]:

$$U_{sd:ST}(36) \supset SO_{sd:ST}(36) \supset SO_{s:ST}(6) \oplus SO_{d:ST}(30) \supset \dots \supset SO_L(3) \otimes SO_S(3) \otimes SO_T(3)$$

(1) Applies to soft nuclei as this limit is similar to SO(6) limit of IBM-1

(2) Gives a isospin dictated staggering in the B(E2)'s in the yrast *(ST)=(01)* band of N=Z odd-odd nuclei

(3) describes some of the spectroscopic properties of ^{74}Rb : for example alignment spin =1 for the T=0 band

Application of sdIBM-ST to Deuteron Transfer in N=Z Nuclei

It's a Knockout: D.D. Warner, Nature **425**, 570 (2003).
For the reaction (*d,*^{4}*He*) only *T=0* deuteron transfer is possible and both *T=0* (deuteron) and *T=1* (deuteron like) transfers are possible for the reaction (*p,*^{3}*He*).

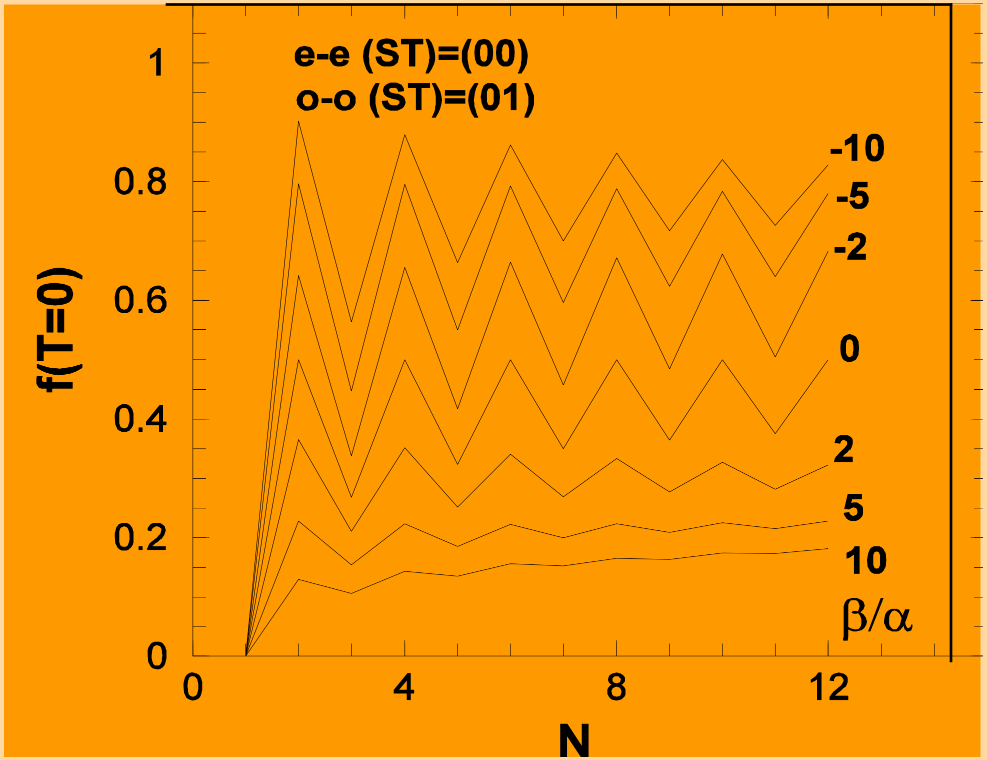
Assuming that the ground states of even-even and odd-odd N=Z nuclei carry orbital angular momentum *L=0*, ground state structure in the *SO(6)* $\oplus$ *SO(30)* scheme is generated by the basis

$$\left| N, \omega = N, \omega_s, \omega_d = 0, (ST) \right\rangle, \;\; \omega_s = N, N-2, \ldots, 0 \text{ or } 1$$

For even-even nuclei (*ST*)=(00) and for odd-odd nuclei (*ST*)=(10) or (01).
Employing the above basis with all allowed ω_s and diagonalizing

$$H = \alpha \left\{ C_2(SO_{s:ST}(6)) + (\beta/\alpha)\hat{n}_{s:S} + (\gamma/\alpha)\hat{n}_d \right\}$$

ground states are determined (they contain on the average ~ 20-25% *d* bosons) and ground to ground deuteron transfer intensities are calculated.



Fractional number of $T=0$ pairs $f(T=0)$ as a function of boson number N for various values of the mixing parameter β/α.
$f(T=0)$ exhibits odd-even staggering in number of $T=0$ pairs in the gs of N=Z nuclei and the staggering is maximum for $|\beta/\alpha|<2$.

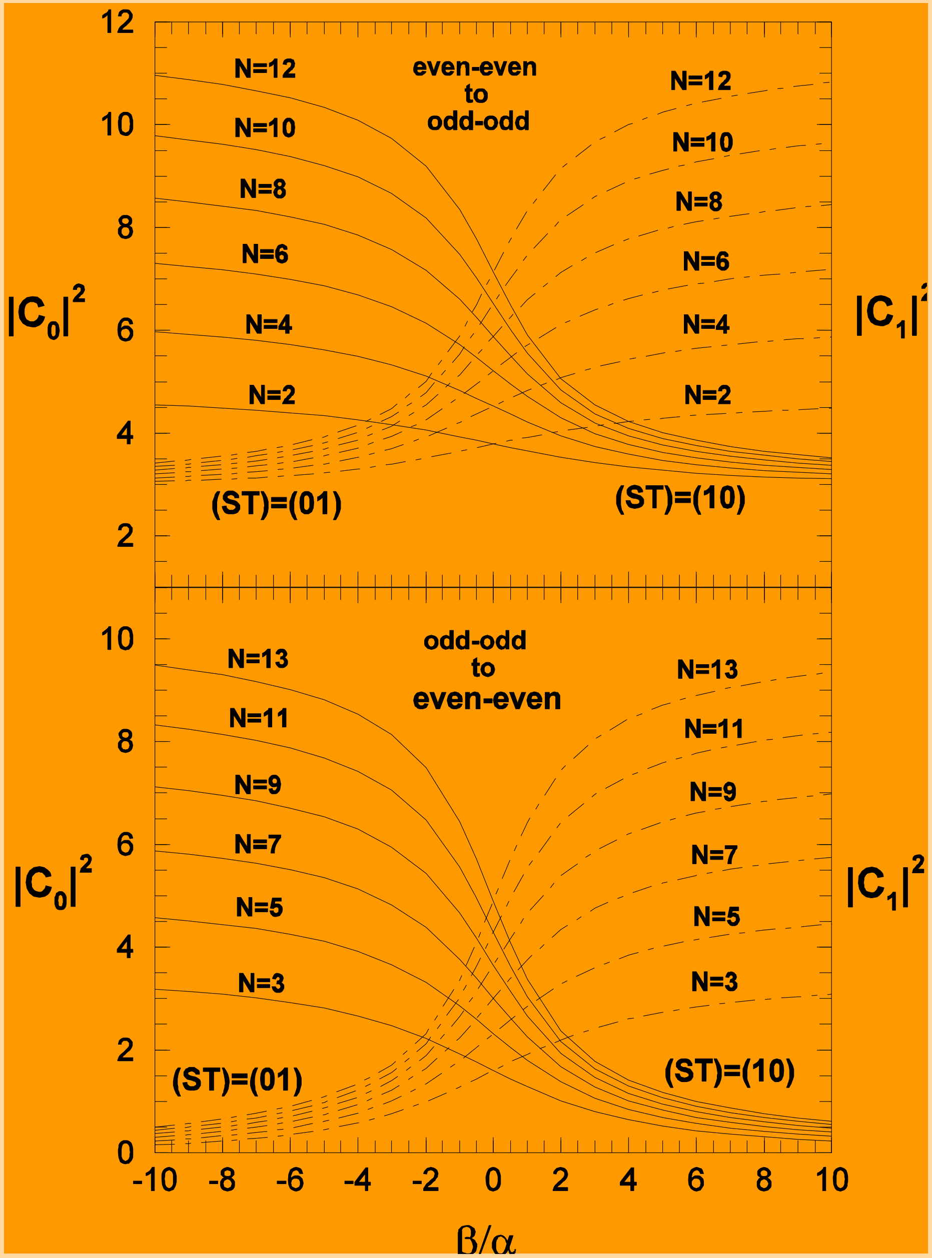
VKBK, Prog. Theo. Phys. 118 (2007) 893.

Significance of the β/α term in the mixing Hamiltonian

Single boson energies:

$$\varepsilon(T_s=0)/\alpha = 5+\beta/\alpha,\ \varepsilon(T_s=1)/\alpha = 5,\ \varepsilon(T_d=0,1)/\alpha = 5+\gamma/\alpha$$

For a *N* boson system with *N* odd $\beta/\alpha<0$ gives *T=0* gs and $\beta/\alpha>0$ gives *T=1* gs and they will be degenerate for $\beta/\alpha=0$. Thus β/α term generates competition between *T=0* and *T=1* pairing correlations.



The *T=0* (continuous curves) and *T=1* (dashed curves) deuteron transfer intensities $|C_T|^2$ between the ground states of N=Z nuclei, in the $SO_{sd:ST}(36) \supset SO_s(6) \oplus SO_d(30)$ scheme of *sd*IBM-*ST*, as a function of the mixing parameter β/α. The *N* values shown are boson numbers for the initial N=Z nucleus and the results are for *N* → *N+1* transfers.

Simple IBM-*ST* [also SO(8)] model predictions for deuteron transfer intensities is now complete. IBM-*ST* results with H interpolating many other symmetry limits of the model are needed for more general predictions.

Within shell model with all oscillator orbits active (ex: sd, pf, sdg, ---), we have

SO(8) pairing

$$U(4\Omega) \supset [U(\Omega) \supset SO(\Omega) \supset ---] \otimes [SU(4) \supset SU_S(2) \otimes SU_T(2)$$

$$U(4\Omega) \supset [U(\Omega) \supset SU(3) \supset ---] \otimes [SU(4) \supset SU_S(2) \otimes SU_T(2)$$

Elliott's rotational SU(3)

T=1 pairing

T=0 pairing

$$H = [SPE] + \alpha Q.Q + \beta P.P + \gamma D.D$$

***J=0* or *1* for the pairs**

J.P. Elliott, Proc. R. Soc. London, 245 (1958) 128, 562; V.K.B. Kota and J.A. Castilho Alcaras, Nucl. Phys. 764 (2006) 181; Y. Lei, S. Pittel, N. Sandulescu, A. Poves, B. Thakur and Y.M. Zhao, Phys. Rev. C 84 (2011) 044318.

4. IBM with aligned pairs as bosons

Low-energy spectroscopy of N=Z nuclei ^{96}Cd and ^{92}Pd gave evidence for spin-aligned neutron-proton pairs (isospin T=0)

Interacting boson model with spin-aligned bosons has been suggested for these nuclei

B. Cederwall et al, Nature 469, January 2011, 68-71
S. Zerguine and P. Van Isacker, Phys. Rev. C 83 (2011) 064314
L. Coraggio, A. Covello, A. Gargano and N. Itaco, Phys. Rev. C 85 (2012) 034335

There is good evidence that N=Z nuclei ^{96}Cd and ^{92}Pd are $g_{9/2}$ nuclei. For example, using Pandya's transform and $g_{9/2}$ - *pp*, *nn* and *pn* matrix elements from realistic interaction indeed gave ^{90}Nb spectrum quite well.

Spectra of N=50 isotones ^{98}Cd , ^{97}Ag , ^{96}Pd are also well reproduced

Yrast levels $_{41}$Nb$_{49}$

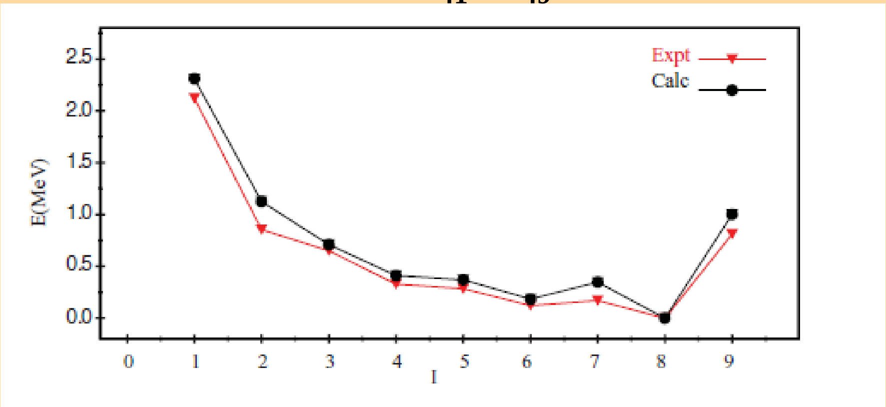


TABLE I. Proton-proton, neutron-neutron, and proton-neutron matrix elements of $V_{\rm eff}$ in the $g_{9/2}$ orbit (in MeV).

J	T	pp	nn	np
0	1	−1.836	−2.224	−2.317
1	0			−1.488
2	1	−0.353	−0.662	−0.667
3	0			−0.440
4	1	0.171	−0.088	−0.100
5	0			−0.271
6	1	0.317	0.083	0.066
7	0			−0.404
8	1	0.459	0.221	0.210
9	0			−1.402

$g_{9/2}$ shell model results with *2p-2n* holes

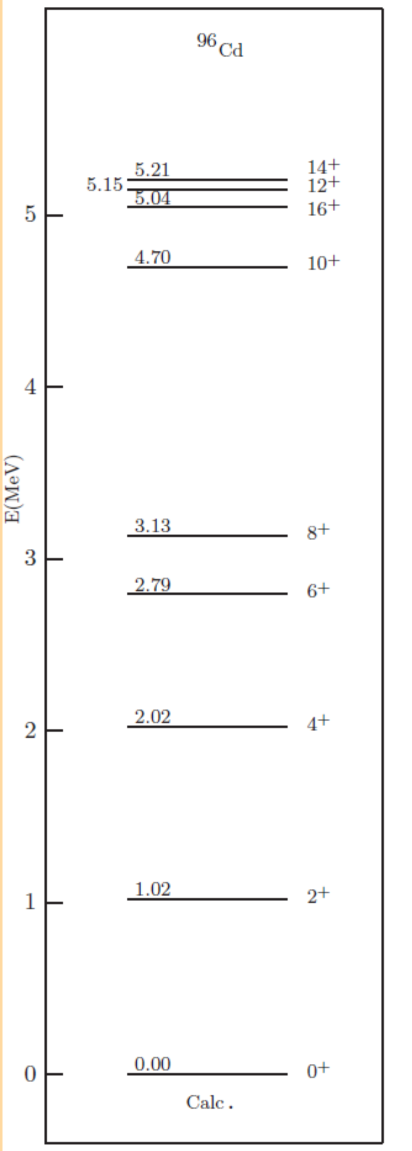


FIG. 4. Calculated spectrum of ^{96}Cd.

TABLE II. Overlap of the calculated $I^{\pi} = 0^+, 2^+, 4^+, 6^+, 8^+$, and 10^+ yrast states in ^{96}Cd with the $[(nn)J_n(pp)J_p]_I$ states, expressed as a percentage. Only components with a percentage >10 are reported.

	(J_n, J_p)									
I^{π}	(0,0)	(0, J)	(J, 0)	(2,2)	(2,4)	(4,2)	(2,8)	(8,2)	(4,6)	(6,4)
0^+	57			30						
2^+		34	32	12						
4^+		29	26	28						
6^+		33	26		16	15				
8^+		39	25				12			
10^+							17	15	18	17

TABLE III. Overlap of the calculated $I^{\pi} = 0^+, 2^+, 4^+, 6^+, 8^+$, and 10^+ yrast states in ^{96}Cd with the $[(np)9(np)9]_I$ state, expressed as a percentage, obtained using (a) $V_9(np)$ in Table I, (b) one-half the original value of $V_9(np)$, and (c) twice the original value of $V_9(np)$.

I^{π}	a	b	c
0^+	90	82	96
2^+	97	94	99
4^+	85	73	94
6^+	48	27	84
8^+	6	3	27
10^+	46	13	94

Therefore aligned *pn*-pairs (then *T=0*) are important than *J=1,T=0* pairs

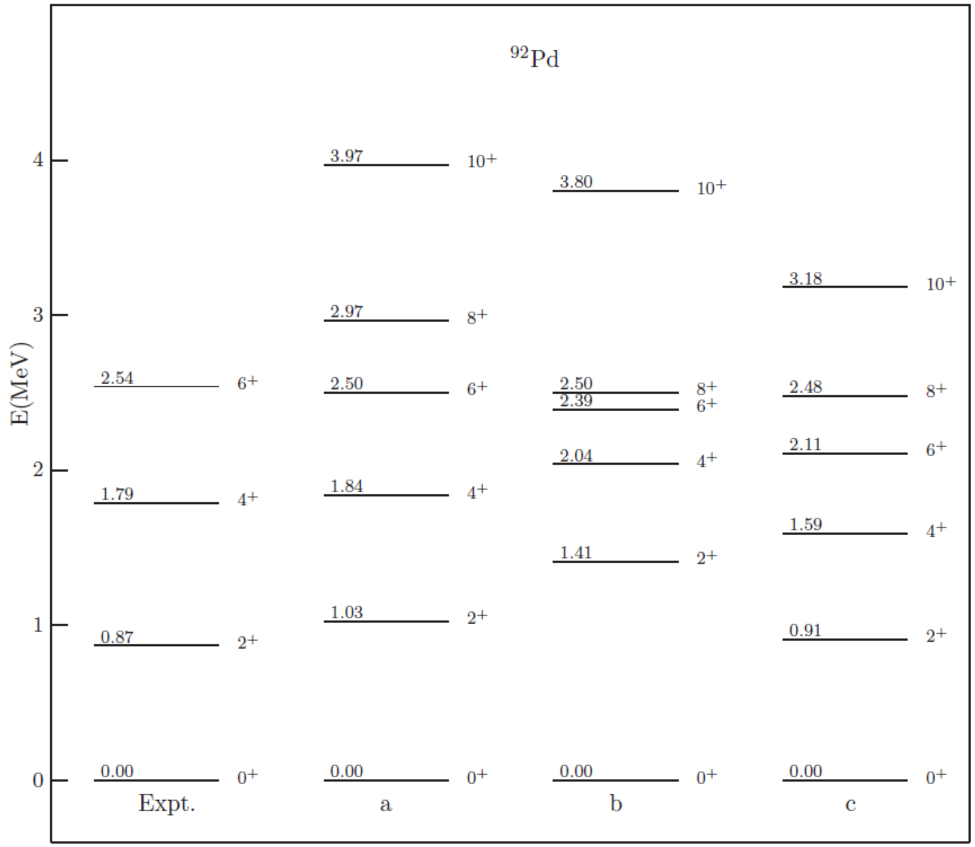


FIG. 5. Experimental spectrum of ^{92}Pd compared with the results of calculations with (a) all matrix elements (see Table I); (b) nn, pp, and np $T = 1$ matrix elements; and (c) nn, pp, and np $T = 0$ matrix elements.

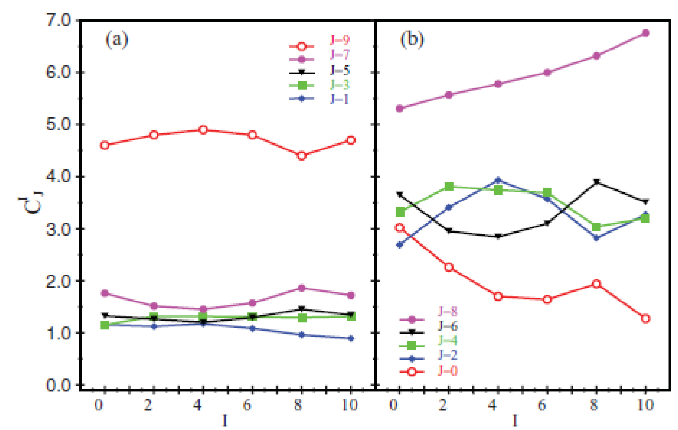


FIG. 6. (Color online) Average number of (a) isoscalar and (b) isovector $(g_{9/2})^2 J$ pairs, C^I_J, as a function of the angular momentum I of the yrast states in ^{92}Pd.

From ^{92}Pd shell model and experimental data, it is quite clear that spin-aligned *pn*-pairs determine the structure of the low-lying levels.

Question: is it possible to construct a IBM with spin-aligned pn-pairs?
Answer: Yes – this we will discuss now (this is useful not only $g_{9/2}$ for nuclei but also for nuclei with valence nucleons in a higher *j*-shell)

IBM Models:

(1) A single fermion pair B with $J = 9, T = 0$, leading to the b-IBM.

(2) Two fermion pairs S and B with $J = 0, T = 1$ and $J = 9, T = 0$, respectively, leading to the sb-IBM.

Used OAI and Democratic mapping for obtaining SPE and TBME of the boson *H*

^{96}Cd is a two boson system

TABLE IV. Overlaps of the $(1g_{9/2})^4$ yrast eigenstates of the SLGT0 interaction with angular momentum J and isospin $T = 0$ with various two-pair states, expressed in percentages.

J	B^2	SP_J	D^2	DG	DI	DK	G^2	I^2	K^2
0	91	80	35				18	7.4	1.9
2	97	85	17	22			1.5	0.0	0.4
4	89	64	42	11	11		0.2	0.2	0.0
6	55	70		43	0.2	4.3	0.0	0.2	0.0
8	5.3	83			7.4	24	1.8	0.2	0.1
10	42					58		6.1	0.5
12	88							57	1.5
14	96								31.4
16	100								100

TABLE V. Energies (in MeV) of $T = 0$ levels for four nucleons in the $1g_{9/2}$ orbit (^{96}Cd) calculated with the shell-model interaction SLGT0 and compared with various versions of IBM obtained by democratic or OAI mapping. E_0 is the binding energy of the ground state.

	SLGT0	b-IBM	sb-IBM	sd-IBM	sdg-IBM
E_0	9.050	8.643	9.041	8.932	9.050
0_1^+	0	0	0	0	0
2_1^+	0.963	0.678	1.077	1.199	1.002
4_1^+	2.100	1.941	2.339	3.754	2.204
6_1^+	3.079	3.302	3.700		4.034
8_1^+	3.449	4.425	4.824		5.688
10_1^+	5.227	5.179	5.578		
12_1^+	5.904	5.572	5.971		
14_1^+	6.056	5.692	6.091		
16_1^+	5.904	5.496	5.895		
18_1^+		∞	∞		
0_2^+	4.594		4.613	4.491	4.594
2_2^+	4.491			4.730	4.554
4_2^+	4.390				4.538

TABLE VII. Energies (in MeV) of $T = 0$ levels for eight nucleons in the $1g_{9/2}$ orbit (^{92}Pd) calculated with the shell-model interaction SLGT0 and compared with various versions of the IBM obtained with two methods of mapping: democratic (Dem) and OAI. E_0 is the binding energy of the ground state.

	SLGT0	*b*-IBM	*sb*-IBM		*sd*-IBM	
			Dem	OAI	Dem	OAI
E_0	18.937	18.135	18.771	18.646	18.624	19.999
0_1^+	0	0	0	0	0	0
2_1^+	0.927	0.637	1.170	0.917	0.728	0.762
4_1^+	1.728	1.104	1.740	1.608	1.561	2.054
6_1^+	2.512	1.965	2.628	2.441	3.155	4.267
8_1^+	3.198	2.836	3.501	3.320	5.486	6.861
10_1^+	4.233	3.683	4.325	4.185		
12_1^+	5.123	4.414	5.050	4.924		

The E2 operator of the *b*-IBM is of the form

$$\hat{T}^{\mathrm{B}}_{\mu}(\mathrm{E2}) = e_{\mathrm{b}}(b^{\dagger} \times \tilde{b})^{(2)}_{\mu}, \tag{49}$$

and is necessarily of scalar character in isospin. Since the mapping implies the equality

$$\langle (1g_{9/2})^2; 9^+ || \hat{T}^{\mathrm{F}}(\mathrm{E2}) || (1g_{9/2})^2; 9^+ \rangle = \langle b || \hat{T}^{\mathrm{B}}(\mathrm{E2}) || b \rangle, \tag{50}$$

$$e_{\mathrm{b}} = -\sqrt{\frac{55}{3\pi}} l_{\mathrm{ho}}^2 \times \sqrt{\frac{266}{187}} (e_\nu + e_\pi). \tag{51}$$

In the following, the factor $\sqrt{55/3\pi}(e_\nu + e_\pi) l_{\mathrm{ho}}^2$ is divided out of all matrix elements, fermionic as well as bosonic.

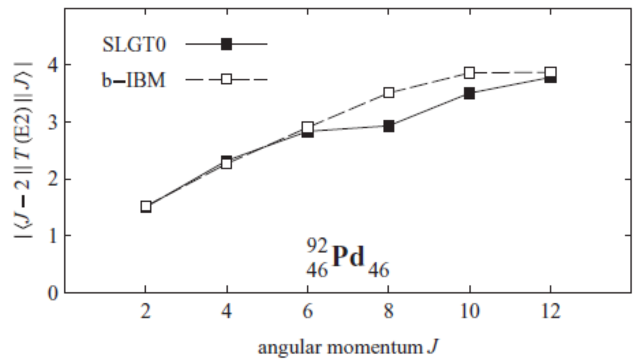


FIG. 5. Same caption as Fig. 2 for eight-nucleon-hole states (^{92}Pd).

***sb*-IBM with *J=0,T=1* and *J=2j, T=0* pairs appears to be a good model for nuclei close to ^{100}Sn. Applications of a similar model in other parts of nuclear chart for N ~ Z nuclei with nucleons in a high-j orbit will be of interest.**

It appears that geometry and dynamics both are important in reproducing expt'l data.

5. Conclusions

(1) $U(6) \oplus U(6) \oplus U(6)$ model for soft scissors mode in neutron rich nuclei with neutron skin
[results for $E(SS)$ and $B(M1)$ are derived].

(2) sd IBM-ST with $(S=0, T=1)$ and $(S=1, T=0)$ pairs generates a $\underset{sdST}{SO(36)} \supset \underset{sST}{SO(6)} \oplus \underset{dST}{SO(30)}$ limit. This limit is used to obtain deuteron transfer intensities in $N=Z$ e-e $\rightleftarrows$ o-o nuclei.

(3) sG-IBM with $(J=0, T=1)$ and $(J=2j, T=0)$ pairs has been developed and studied the structure (energies and $B(E2)$'s) of ^{96}Cd and ^{92}Pd nuclei.

*(4) pn-IBM with F-spin breaking admits $SO(12)$ limit $[U(12) \supset SO(12) \supset SO(6) \otimes SO_{MF}(2)]$
This limit generates low-lying (~ 1.5 MeV excitation) scissors states and also states with p-n boson pairs.