Shell Structure and Shell Evolution in Neutron-rich Nuclei

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Vanishing of shell closures:

Theory and experiment are now indicating that shell closures may change far from stability.

Issues

- **Neutron-rich Sn isotopes : Depressed first E(2+) & low B(E2); New shell closure predicted at N=90 and the role of three body forces.**
- **Anomalous depression of 5/2$^+$ state in ^{135}Sb**
- **Shell evolution of neutron rich nuclei and the role of pairing – experimental signatures and implications**

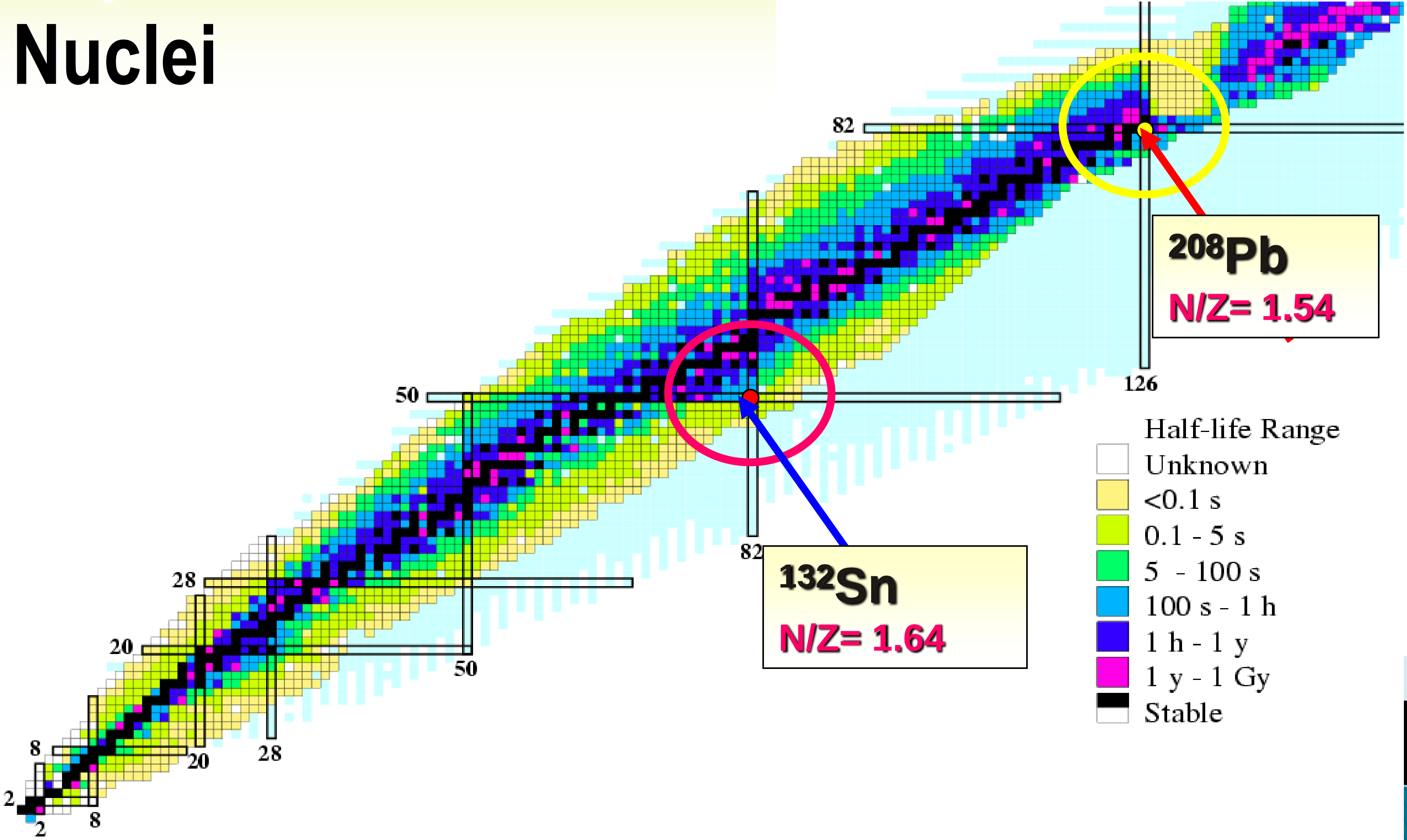
Signatures of changing shell structure away from stability

- The energy of the first 2^+ state ($E(2^+_1)$) of even-even nucleus
  - **Spherical nuclei:**
    - *this state is formed by breaking a pair spending the pair binding energy,*
    - *Or exciting a pair across the energy gap to the next orbit- therefore shows pronounced maxima at the shell closures.*
  - **Deformed nuclei :** *"anomalously" low first excited collective 2+ states are observed.*

  - Three new doubly magic Oxygen isotopes have been observed. (^{14}O, ^{22}O and ^{24}O)
  - For neutron rich nuclei, semi-magic ones like ^{32}Mg, ^{30}Ne have shown erosion of N=20 shell closure with sudden decrease in their $E(2^+_1)$ values and an increase in their corresponding $B(E2, 2^+_1 \rightarrow 0^+_1)$.
  - However some recent measurements of reduced ($E(2^+_1)$) and B(E2) values in highly neutron nuclei having Z=4-18 have raised a serious discussion on hindered E2 strength unexpected for these nuclei phenomenologically as well as theoretically.

Neutron-rich Sn isotopes : New shell closure predicted at N=90 and the role of three body forces.

Experimental Chart of Nuclei



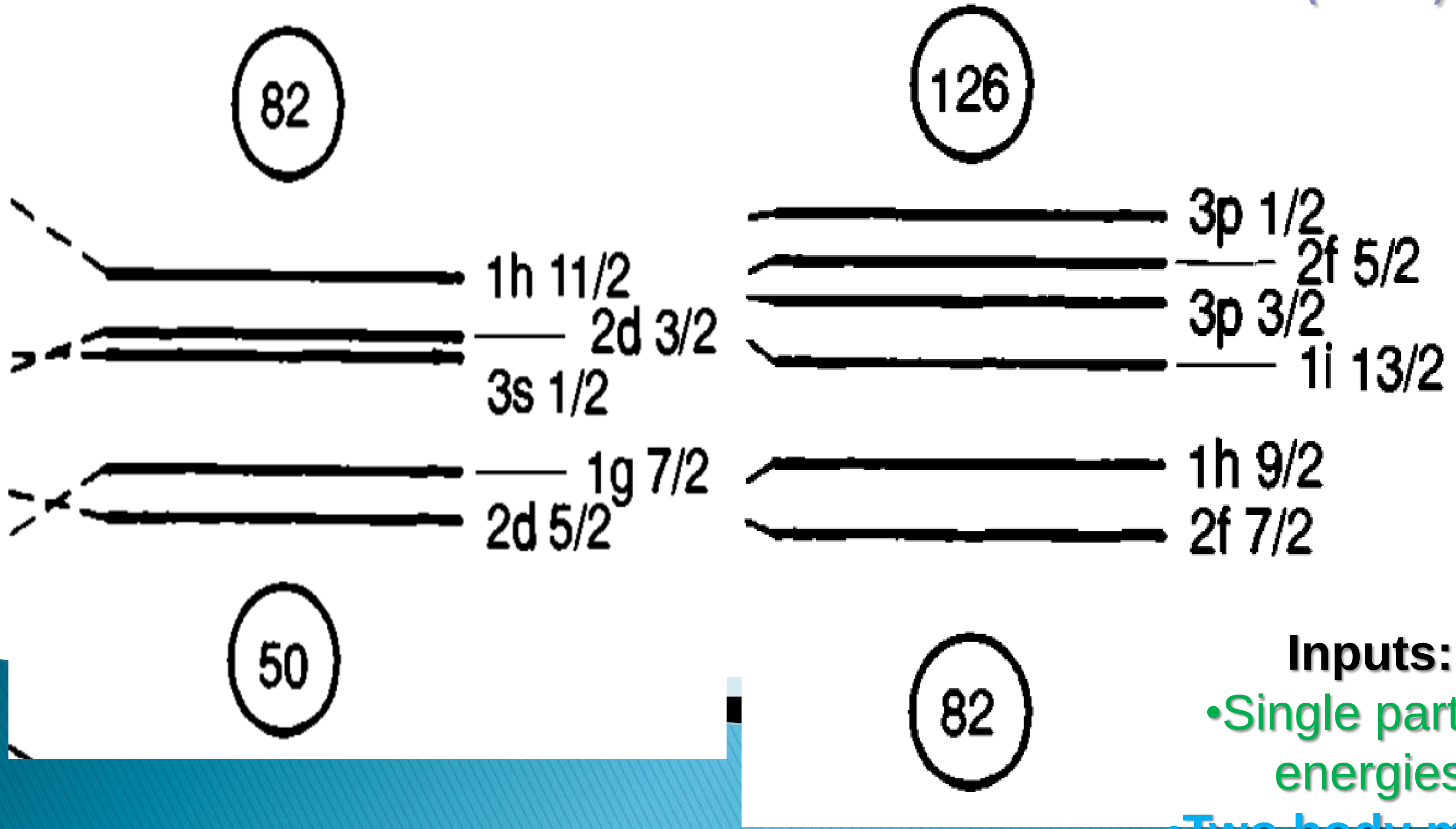
^{208}Pb
N/Z= 1.54

^{132}Sn
N/Z= 1.64

Half-life Range
☐ Unknown
🟨 <0.1 s
🟩 0.1 - 5 s
🟩 5 - 100 s
🟦 100 s - 1 h
🟦 1 h - 1 y
🟪 1 y - 1 Gy
⬛ Stable

The Core and the Valence Space

Collaborator : M Saha Sarkar (SINP)

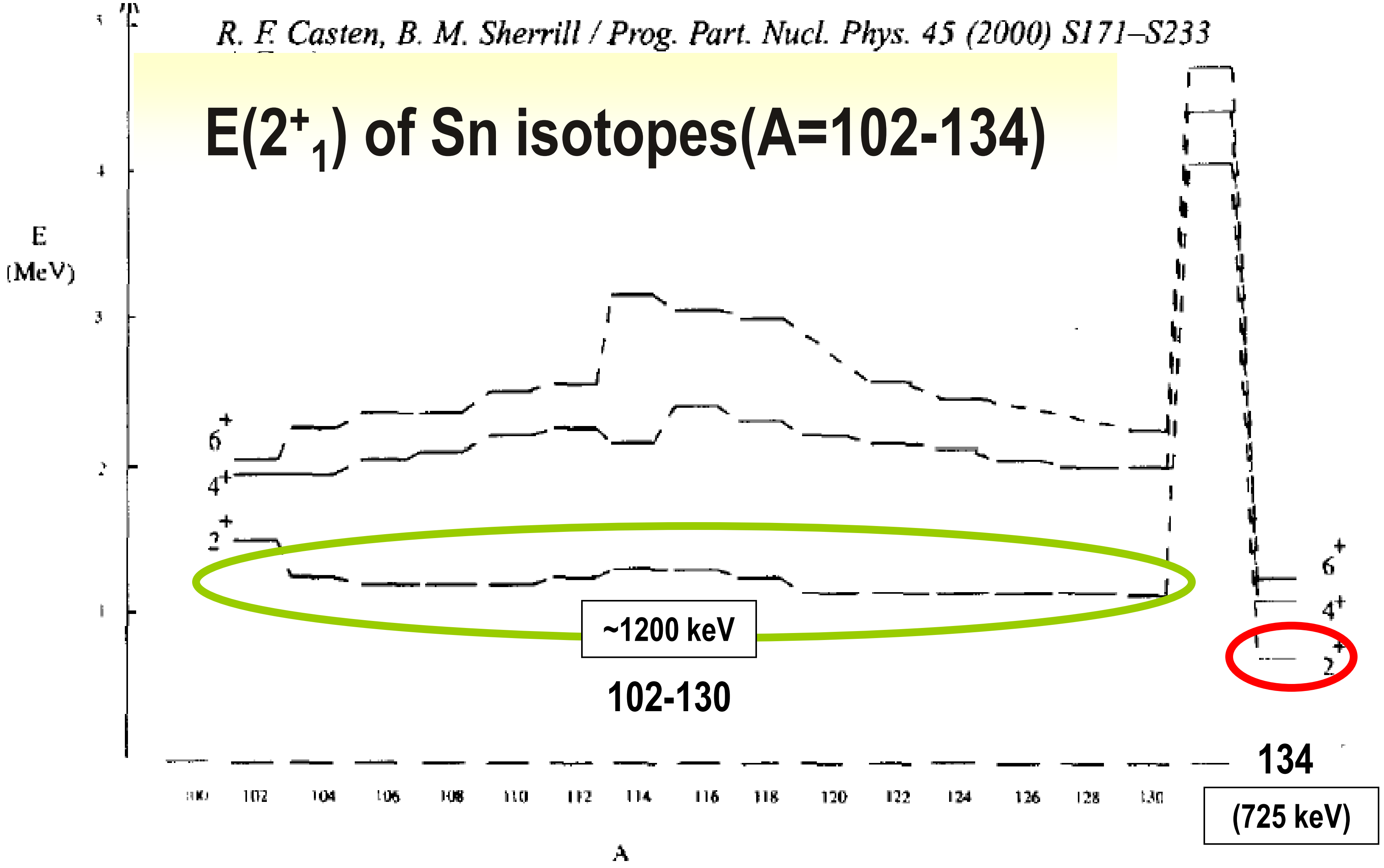


Inputs:

- Single particle energies
- Two body matrix elements

CORE: ^{132}Sn

$E(2^+_1)$ of Sn isotopes(A=102-134)



R. F. Casten, B. M. Sherrill / Prog. Part. Nucl. Phys. 45 (2000) S171–S233

~1200 keV

102-130

134

(725 keV)

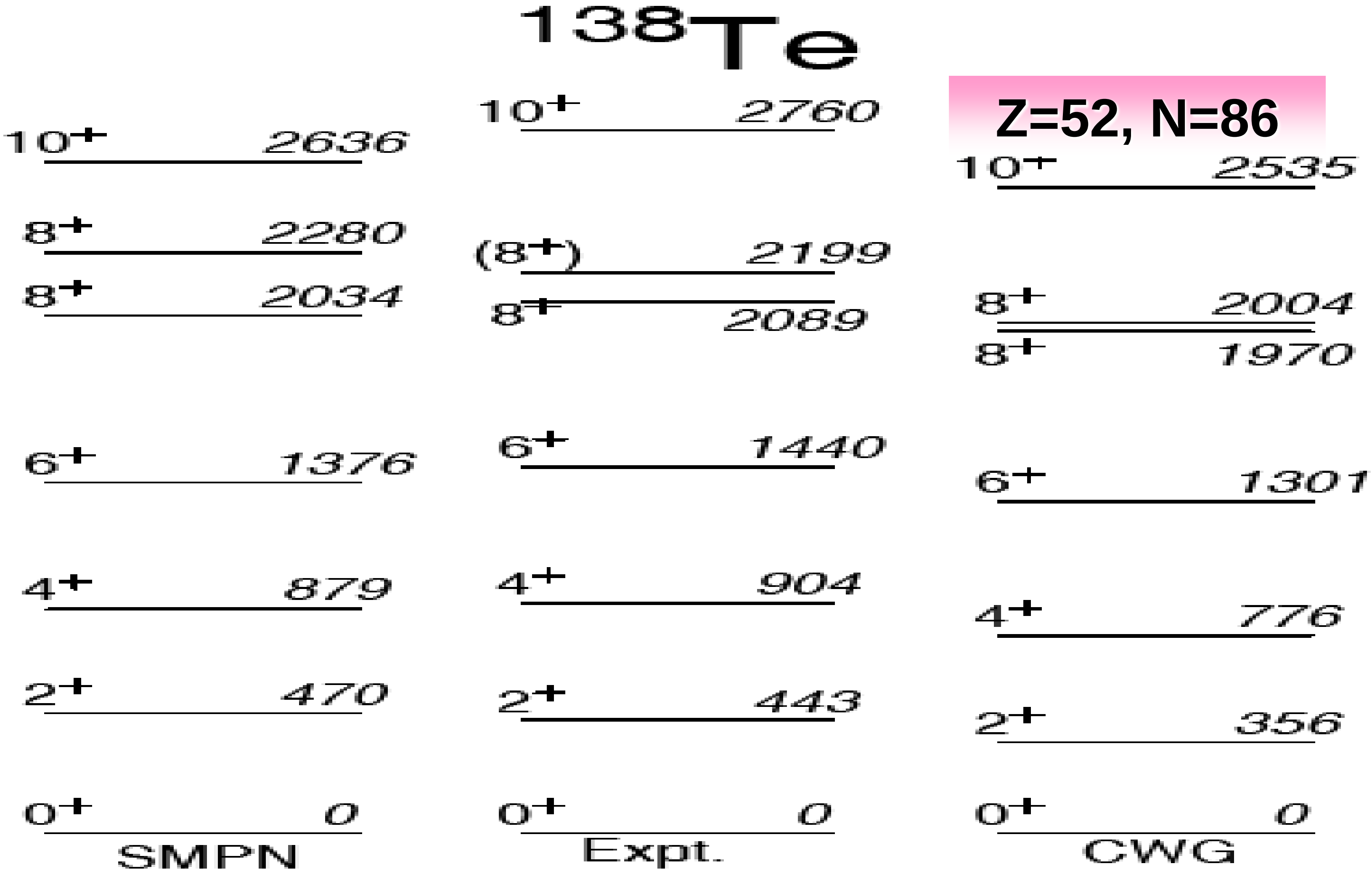
Interactions used

- Primarily two types of interactions used: realistic and empirical
  - Empirical interactions : the interaction derived from ^{208}Pb region (Chou & Warburton) which fails for N > 84: specific matrix elements are tuned to reproduce known experimental levels .

    S. Sarkar, M. Saha Sarkar, Eur. Phys. Jour. A21, 61 (2004).
  - Realistic interactions obtained starting with a *G* matrix derived from the CD-Bonn nucleon-nucleon interaction using the *Q*-box method

    B. A. Brown, N. J. Stone, J. R. Stone, I. S. Towner, and M. Hjorth-Jensen, Phys. Rev. C 71, 044317 (2005).

Both give similar agreement: comparatively better with SMPN; $R_4=E_4/E_2$ indicate vibrational spectrum



Z=52, N=86

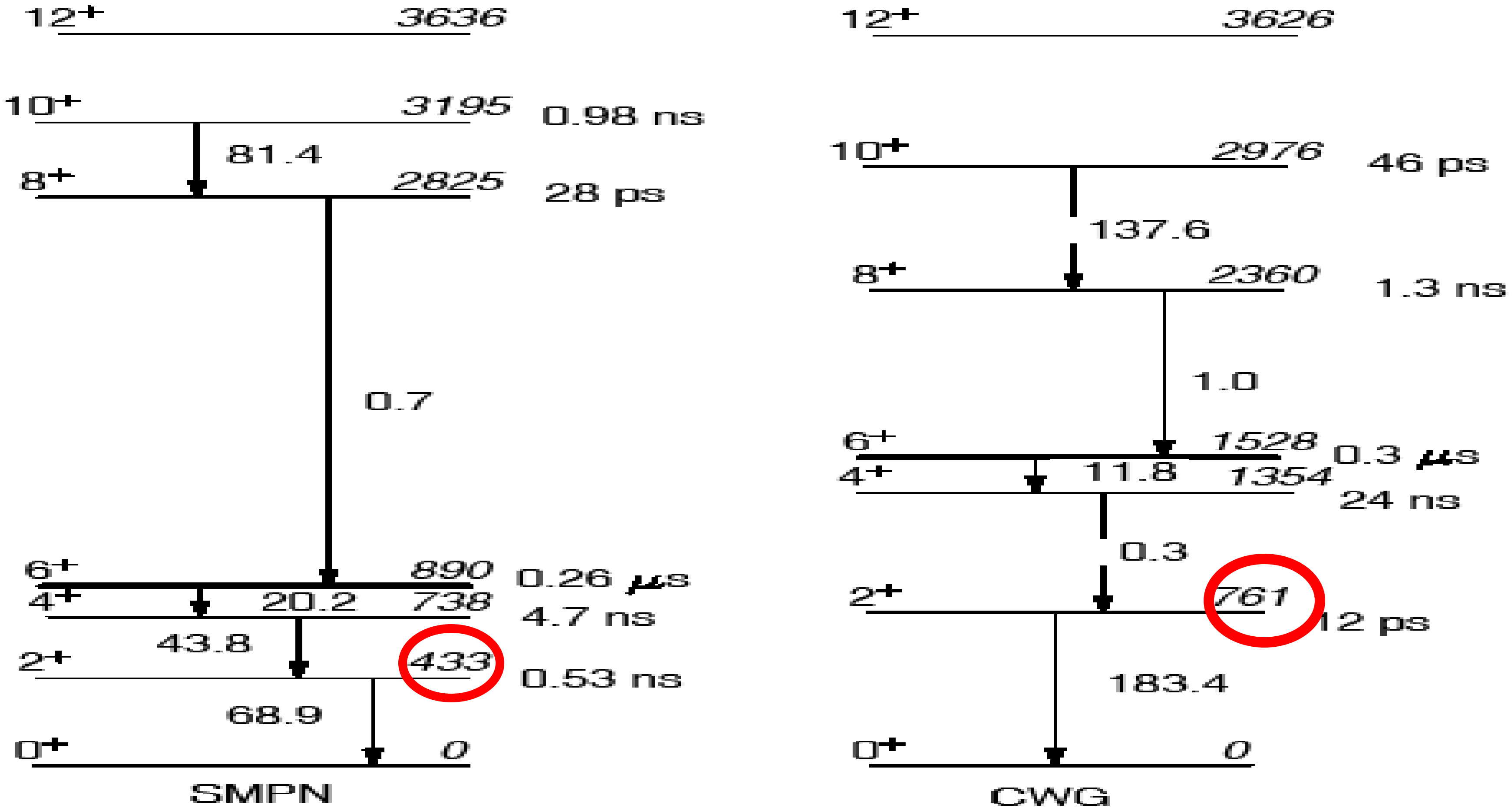
S. Sarkar and M. Saha Sarkar, Phys. Rev. C 78, 024308 (2008).

NEW FEATURE IN THE SEMI-MAGIC neutron –rich isotopes

Depressed 2^+

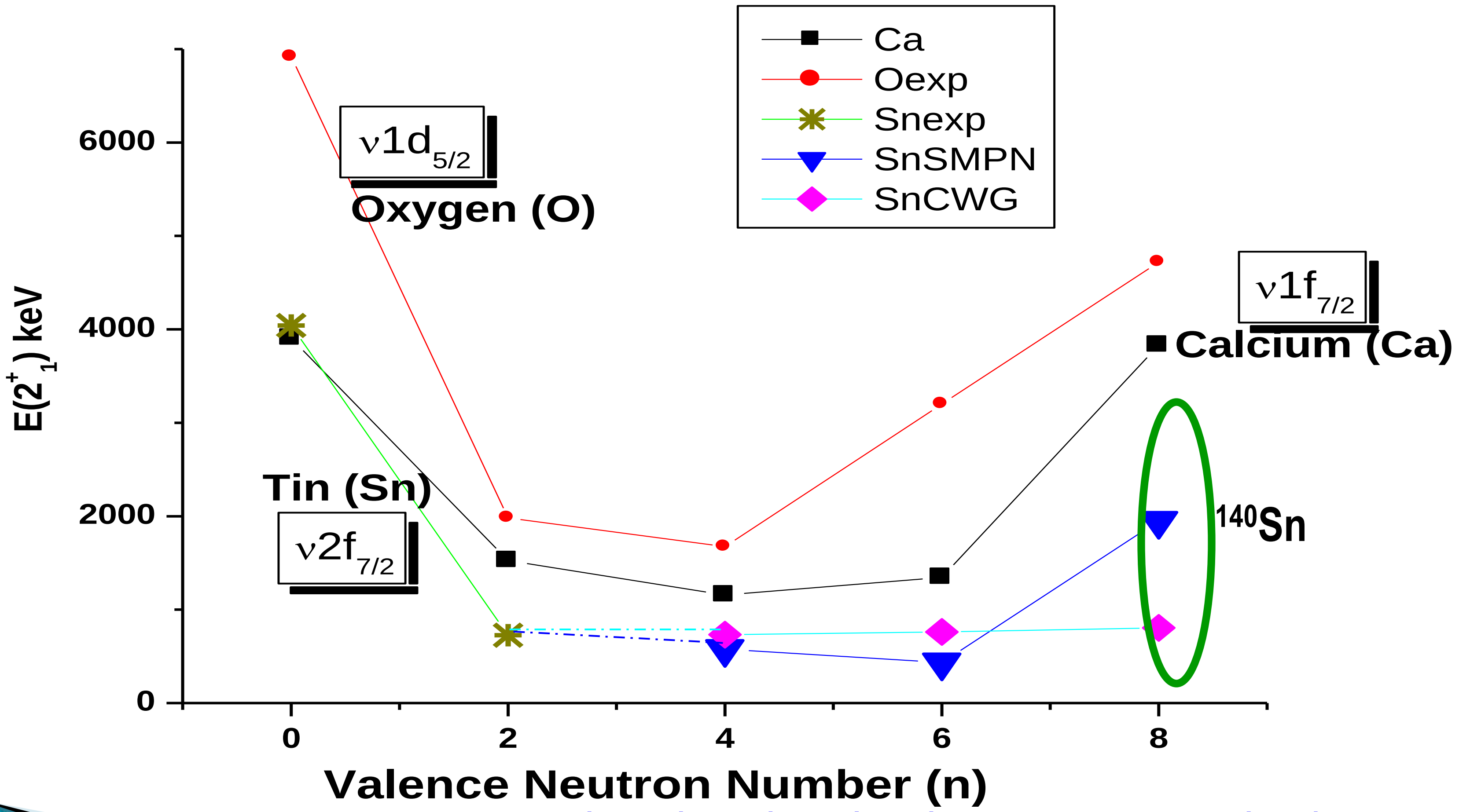
^{138}Sn

Z=50, N=88



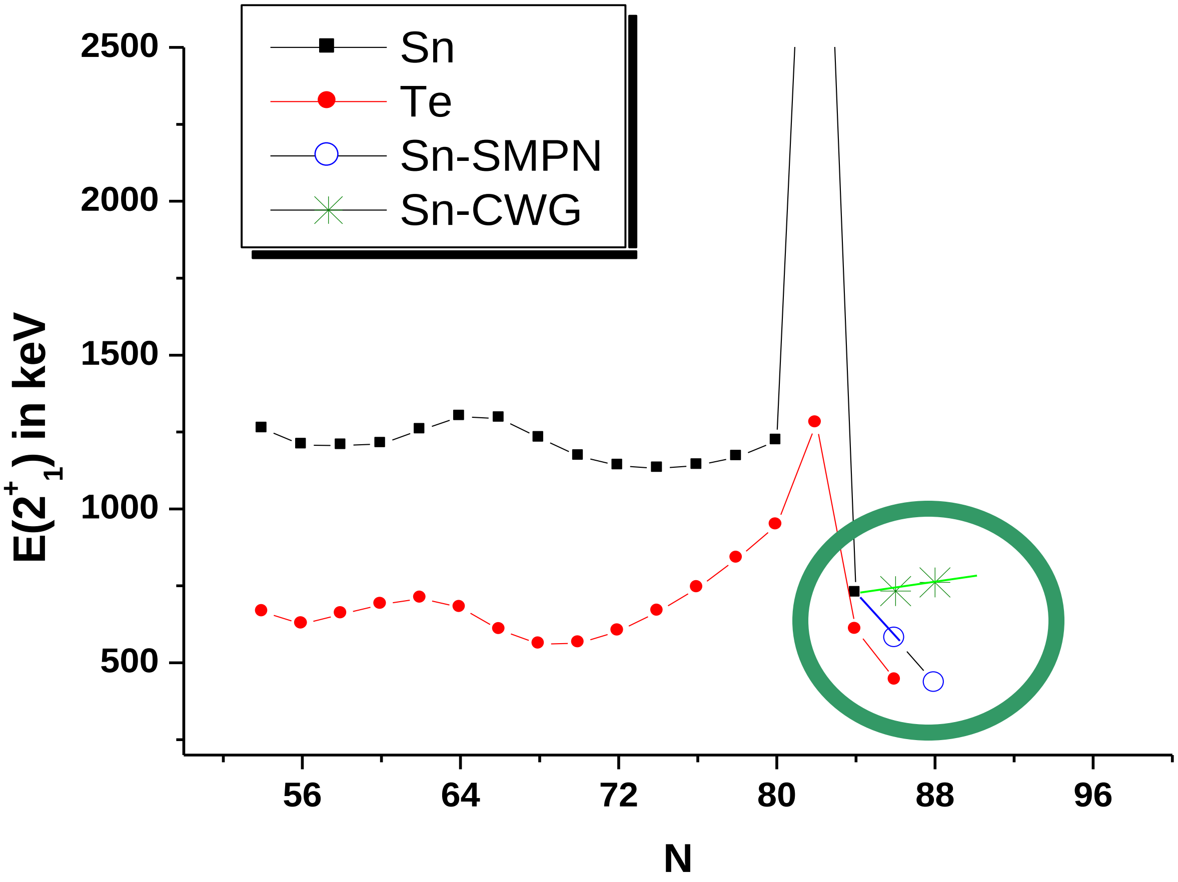
S. Sarkar and M. Saha Sarkar, Phys. Rev. C 78, 024308 (2008).

Results for $^{136\text{-}140}$Sn:
Comparison with other neutron-rich regions



S. Sarkar and M. Saha Sarkar, Phys. Rev. C 81, 064328 (2010).

Casten-Sherrill observation



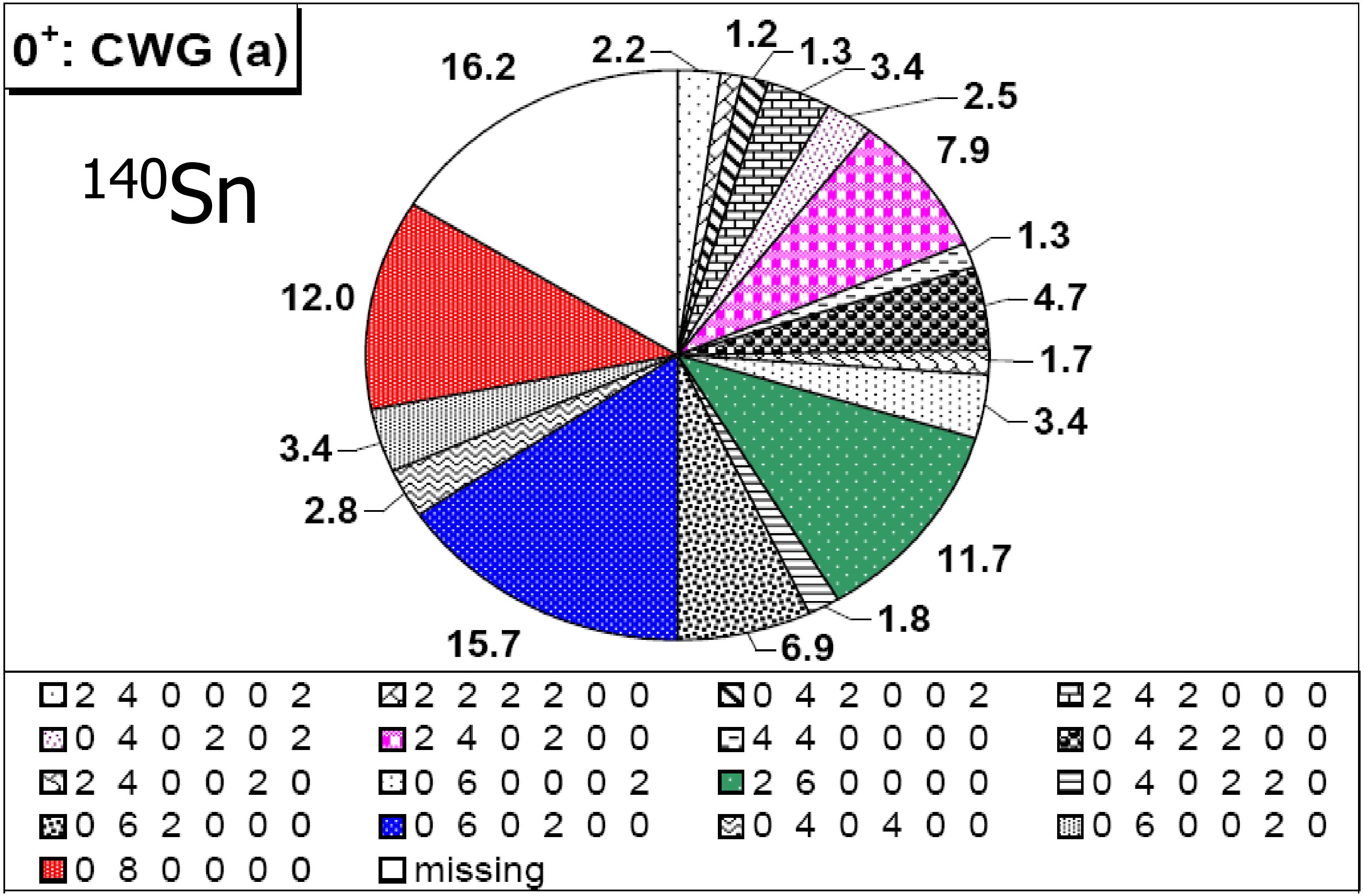
- Casten and Sherill have pointed out that, although $[E(2^+_1)\text{Sn} - E(2^+_1)\text{Te}]$ 400 keV for a given neutron number over most of the *N* = 50–82 shell, the difference is only 119 keV for *N* = 84

- The difference for *N* = 86 is 108 keV with SMPN. It is consistent with the trend discussed by Casten and Sherill. (**Casten-Sherrill Systematics**)

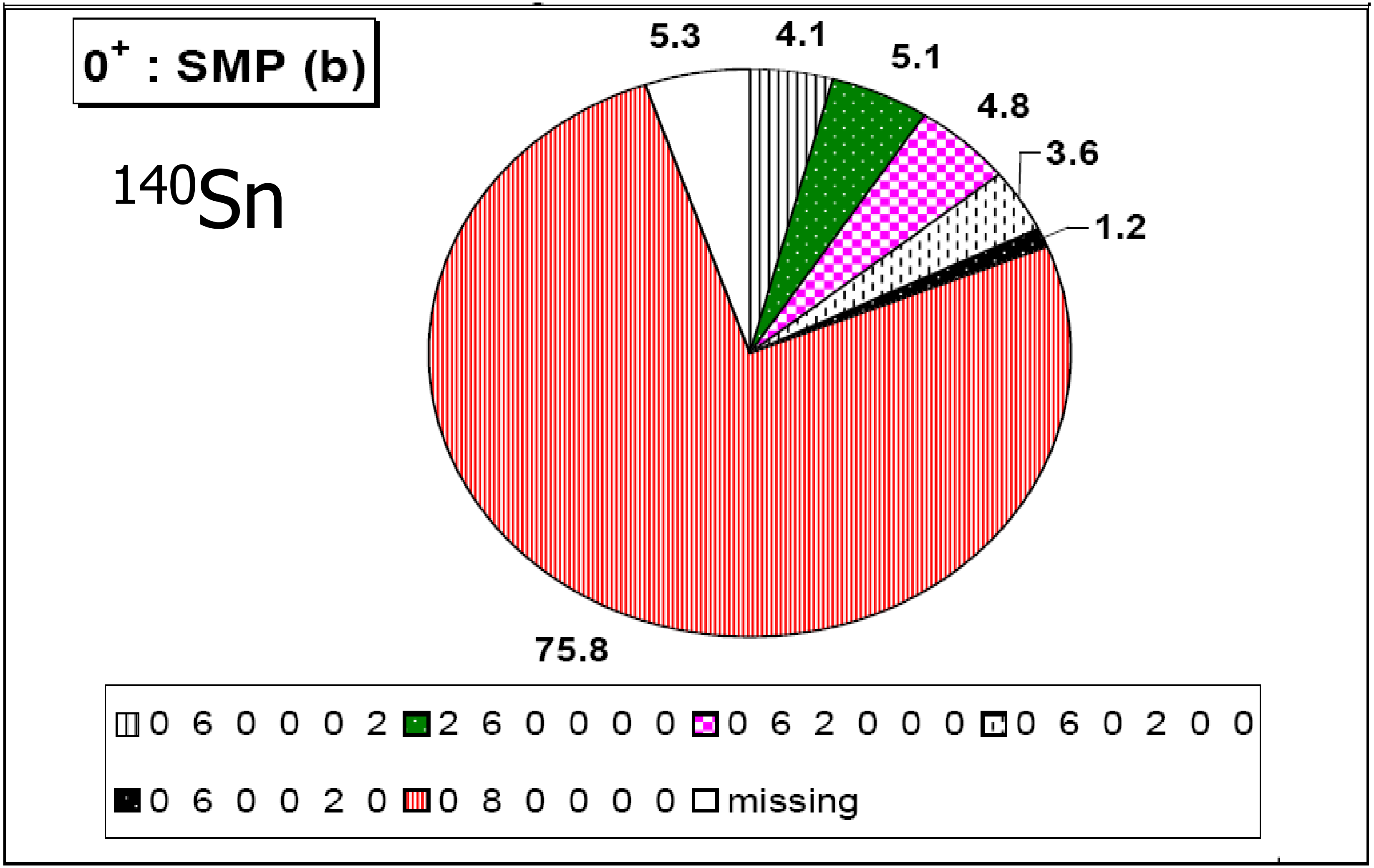
- For CWG, this difference is 733 − 356 = 377 keV for *N* = 86, which deviates from the trend.

R. F. Casten and B. M. Sherrill, Prog. Part. Nucl. Phys. 45, S171 (2000).

Wave function structure for CWG

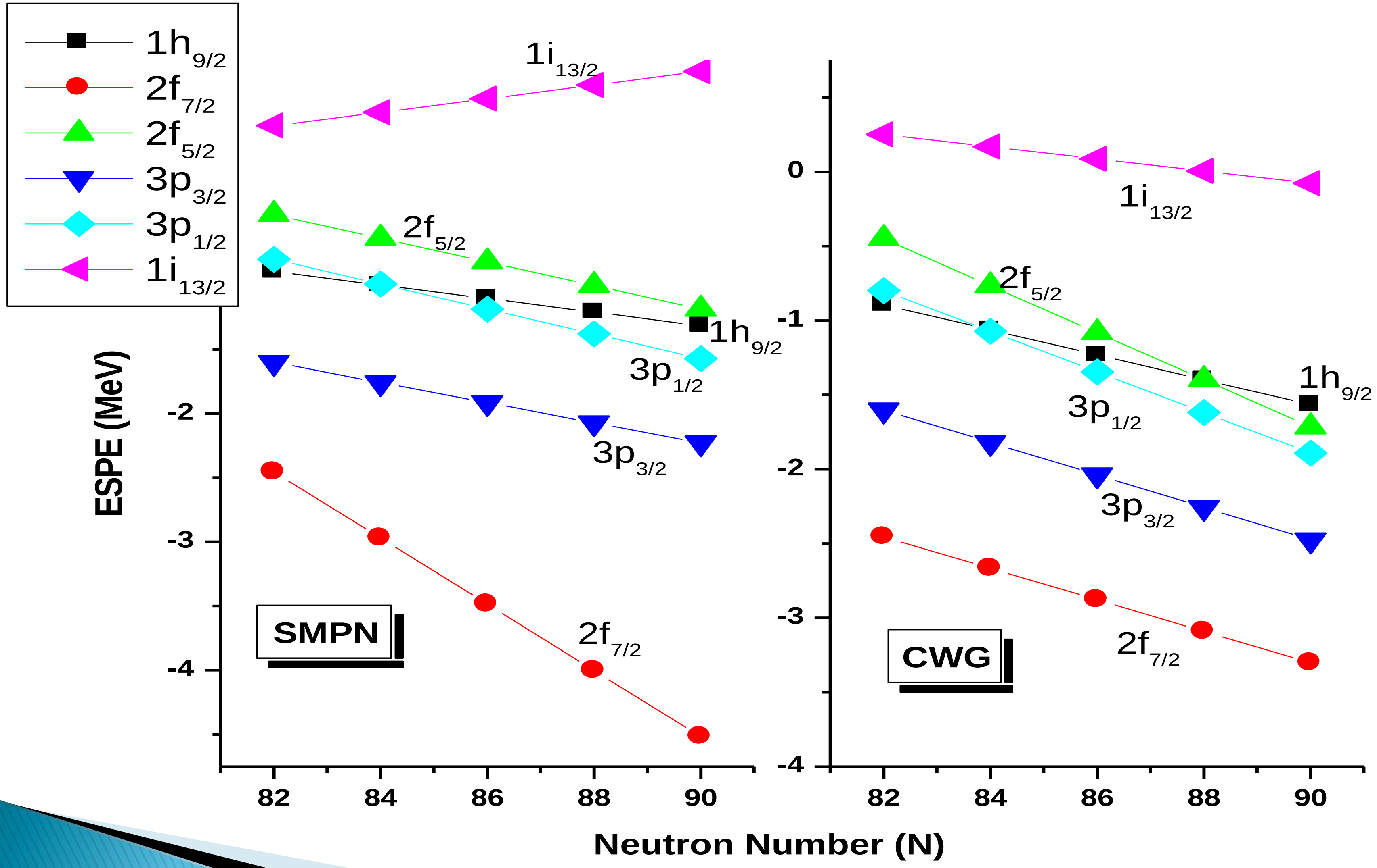


Wave function structure for SMPN

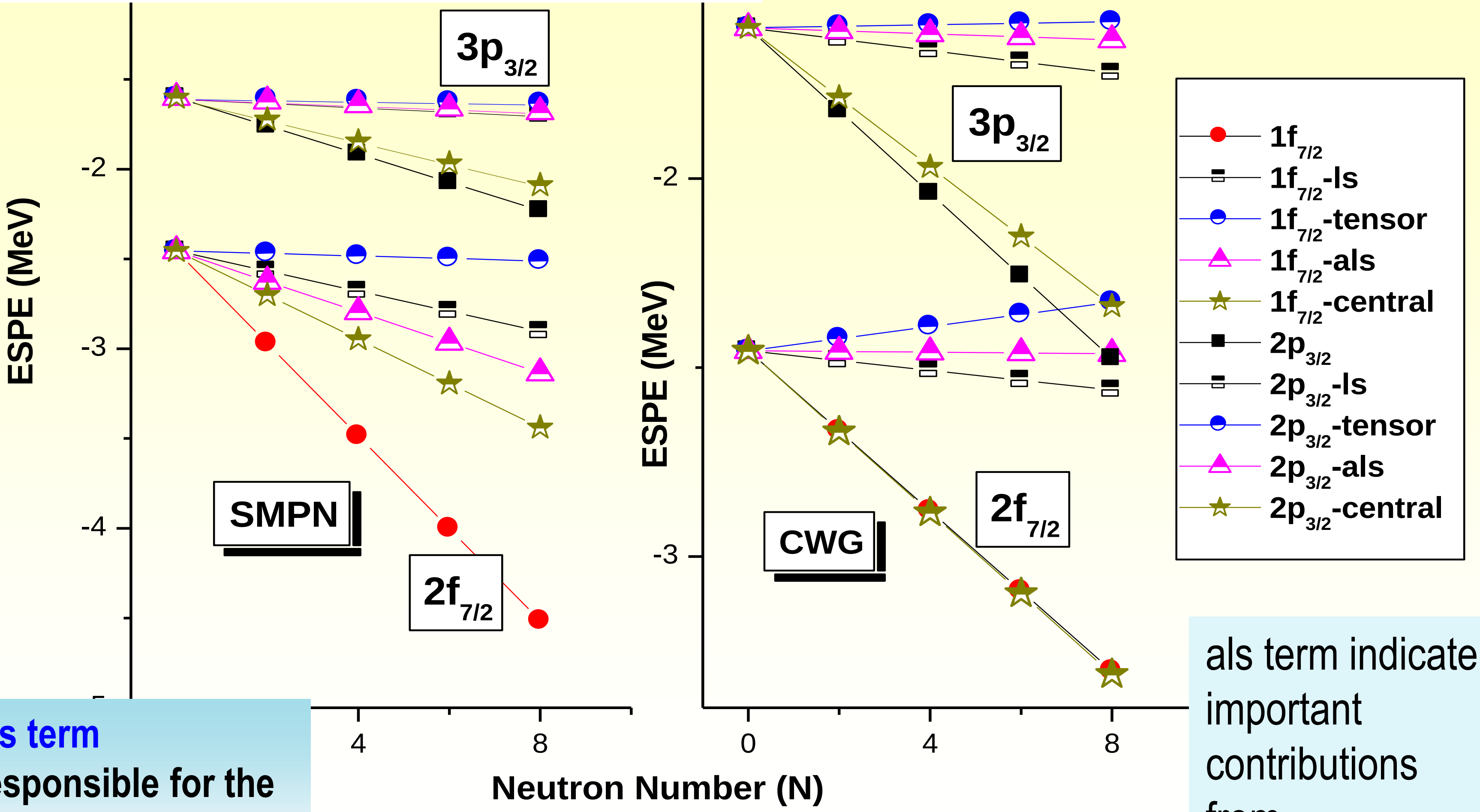


$^{140}\mathrm{Sn}$

Neutron Effective Single Particle Energies (ESPE) for CWG and SMPN interactions with increasing neutron numbers



Spin-Tensor Decomposition

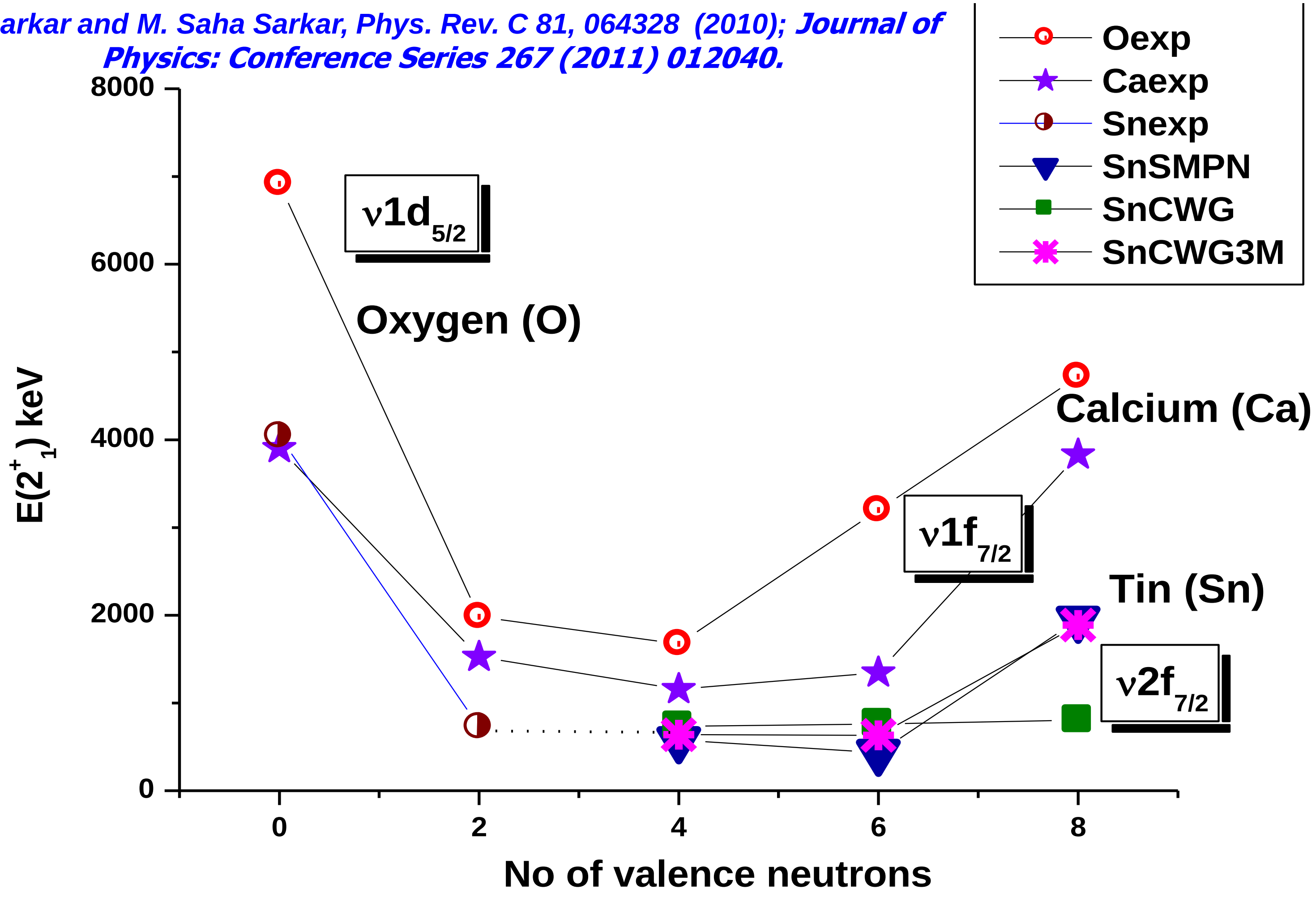


als term **responsible for the observed gap in SMPN**

als term indicate important contributions from **many body forces**

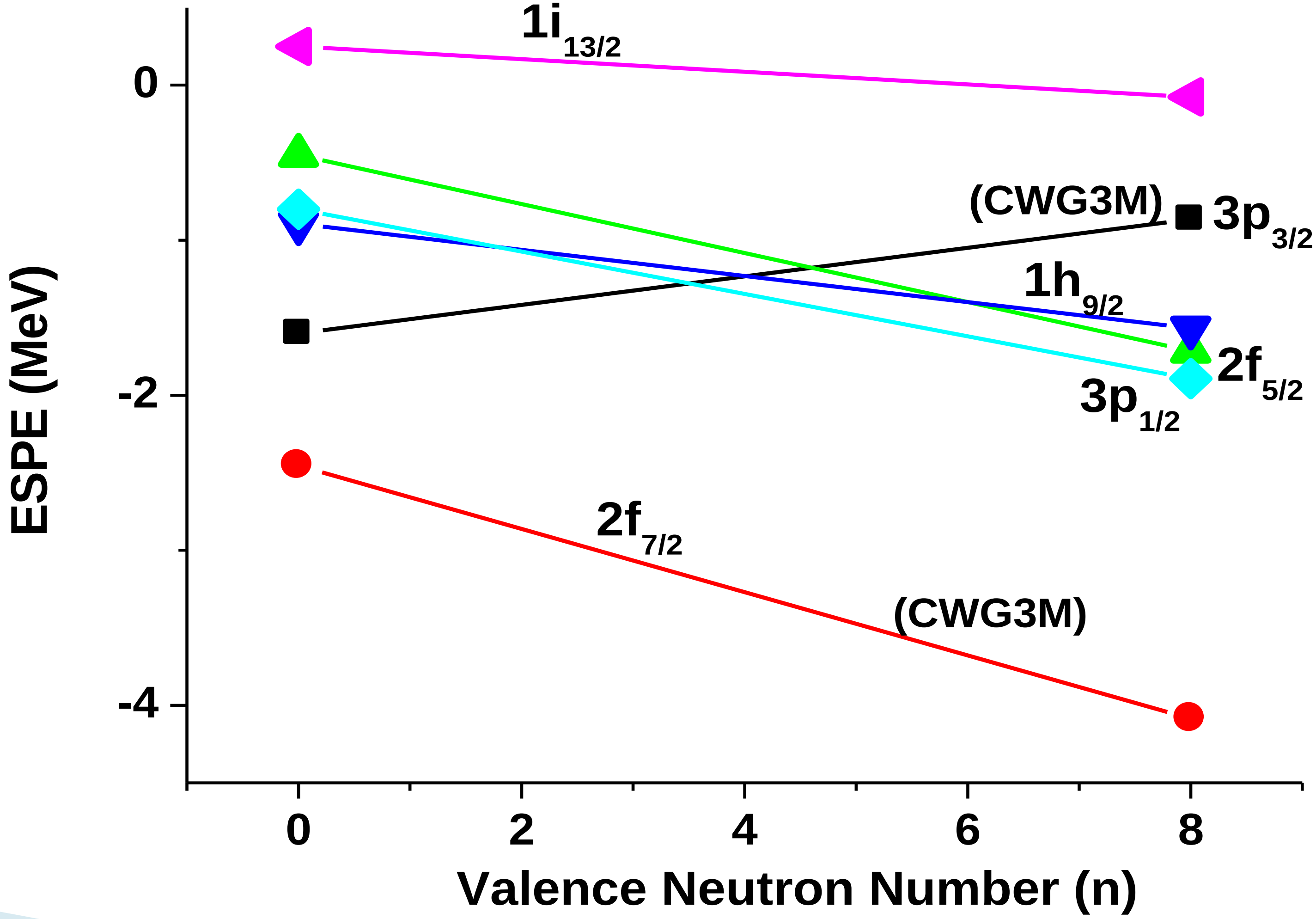
Comparison with neutron-rich isotopes

S. *Sarkar and M. Saha Sarkar, Phys. Rev. C 81, 064328 (2010);* ***Journal of Physics: Conference Series 267 (2011) 012040.***

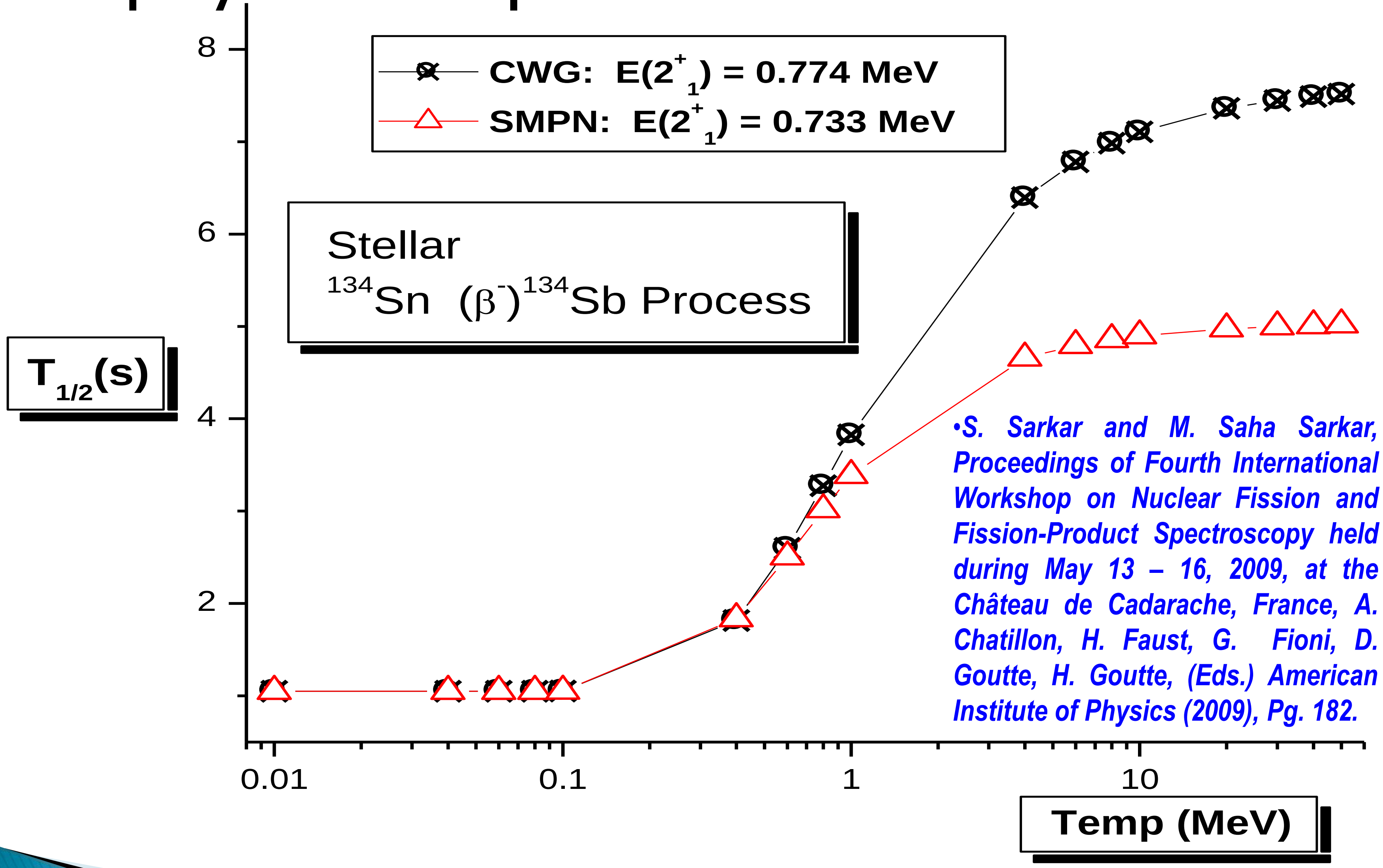


CWG3M: Realistic + 3 body

Neutron ESPEs with CWG3M interactions for increasing neutron numbers



Astrophysical Implication



- *S. Sarkar and M. Saha Sarkar, Proceedings of Fourth International Workshop on Nuclear Fission and Fission-Product Spectroscopy held during May 13 – 16, 2009, at the Château de Cadarache, France, A. Chatillon, H. Faust, G. Fioni, D. Goutte, H. Goutte, (Eds.) American Institute of Physics (2009), Pg. 182.*

Shell evolution for neutron rich nuclei and the role of pairing – experimental signatures and implications

Exotic domain and Grodzin's formula

$$B(E2;0^{+}\rightarrow 2^{+})=14.9\frac{1}{[E_{2^{+}}/\mathrm{keV}]}\frac{Z^{2}}{A}[e^{2}\,\mathrm{b}^{2}], \qquad (1)$$

Does the Grodzin's formula which gives the systematic dependence of $B(E2, 0^+ \rightarrow 2^+)$ on $E(2^+)$, need to be re-formulated in the exotic region of neutron excess?

^{24}Mg : $E(2+)=1.369$ MeV; $B(E2, 0^+ \rightarrow 2^+)=0.0432\ (11) e^2b^2$ ~21 W.U.

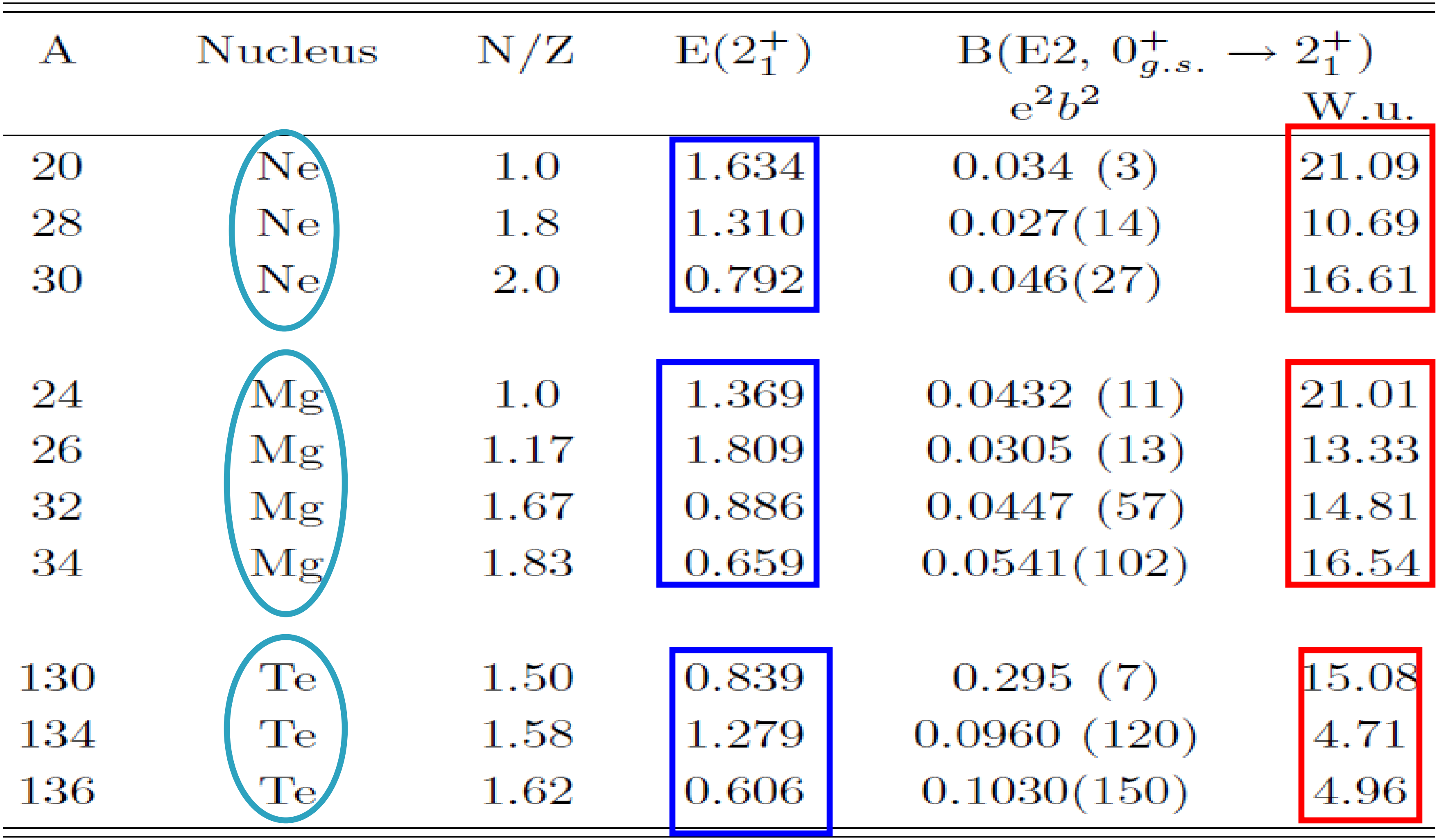
^{32}Mg: $E(2+)=0.886$ MeV; $B(E2, 0^+ \rightarrow 2^+)=0.039\ (7)\ e^2b^2$~13 W.U.

^{134}Te : $E(2+)=1.279$ MeV; $B(E2, 0^+ \rightarrow 2^+)=0.0960\ (120)\ e^2b^2$~4.7 W.u.

^{136}Te: $E(2+)=0.606$ MeV; $B(E2, 0^+ \rightarrow 2^+) =0.1030(150)\ e^2b^2$ ~ 5 W.u

E(2^+_1) and B(E2)

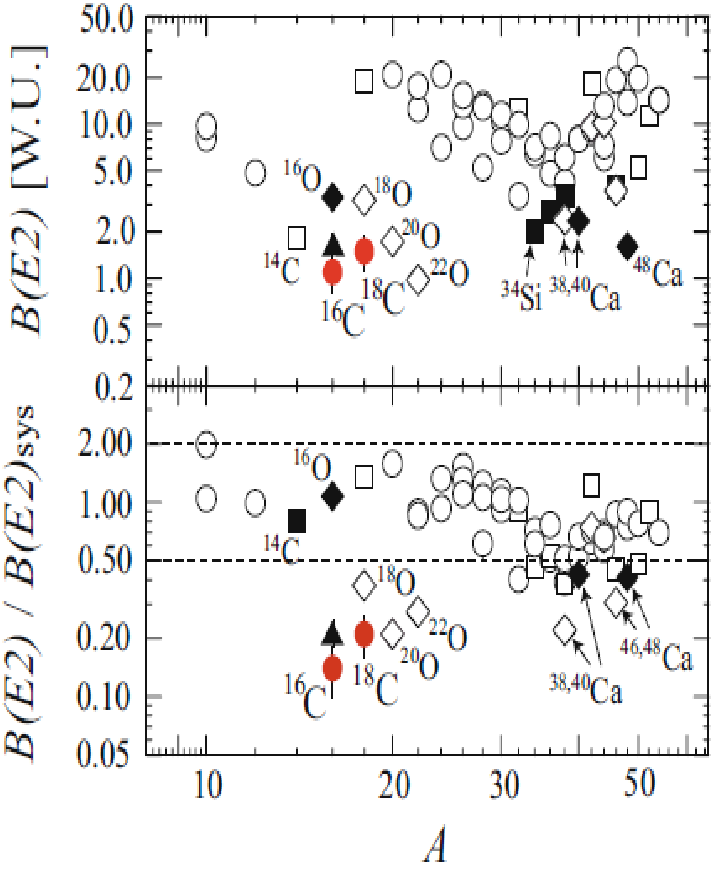
S. Raman, C.W. Nestor, Jr., P. Tikkanen, At. Data Nucl. Data Tables 78, 1 (2001); http://www.nndc.bnl.gov.



A	Nucleus	N/Z	E(2^+_1)	B(E2, $0^+_{g.s.} \rightarrow 2^+_1$)	
				e^2b^2	W.u.
20	Ne	1.0	1.634	0.034 (3)	21.09
28	Ne	1.8	1.310	0.027(14)	10.69
30	Ne	2.0	0.792	0.046(27)	16.61
24	Mg	1.0	1.369	0.0432 (11)	21.01
26	Mg	1.17	1.809	0.0305 (13)	13.33
32	Mg	1.67	0.886	0.0447 (57)	14.81
34	Mg	1.83	0.659	0.0541(102)	16.54
130	Te	1.50	0.839	0.295 (7)	15.08
134	Te	1.58	1.279	0.0960 (120)	4.71
136	Te	1.62	0.606	0.1030(150)	4.96

Anomalous B(E2)s in carbon isotopes

Eur. Phys. J. A **42**, 393–396 (2009)



Umeya et al. / Nuclear Physics A 829 (2009) 13–30

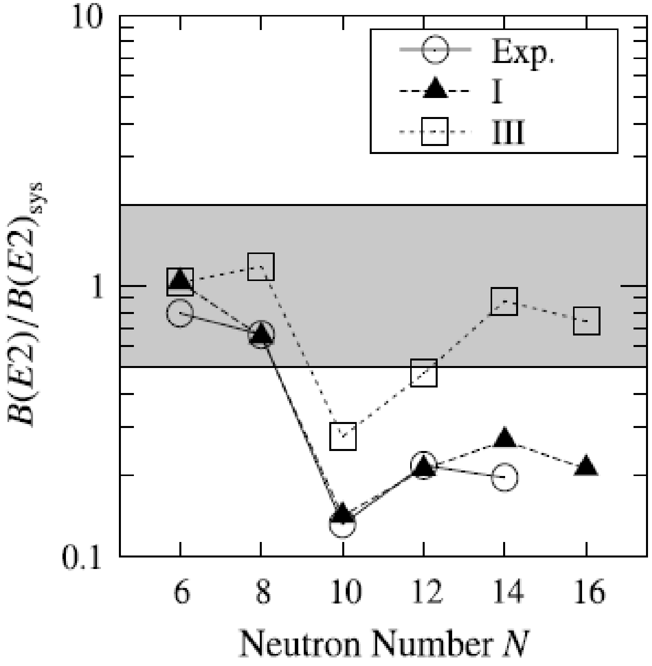


Fig. 4. Ratios of experimental and theoretical $B(\mathrm{E2})$ values to those of the systematics. The theoretical $B(\mathrm{E2})$ values are calculated (I) in the first-order perturbation with the Paris potential and (III) in the $0\hbar\omega$ model space with the traditional effective charges $e_p^{\mathrm{eff}} = 1.3e$ and $e_n^{\mathrm{eff}} = 0.5e$. The experimental value 13.0 e^2 fm^4 [4] is used for ^{16}C.

	$E(2^+_1)$ MeV	$B(E2, 0^+_{g.s.} \rightarrow 2^+_1)$
$^{134}Sn_{84}$	0.733	1.42 W.u.
$^{136}Sn_{86}$	0.578	2.16 W.u.
$^{138}Sn_{88}$	0.433	1.63 W.u.

SMPN

	$E(2^+_1)$ MeV	$B(E2, 0^+_{g.s.} \rightarrow 2^+_1)$
$^{134}Sn_{84}$	0.774	1.51 W.u.
$^{136}Sn_{86}$	0.733	3.00 W.u.
$^{138}Sn_{88}$	0.761	4.33 W.u.

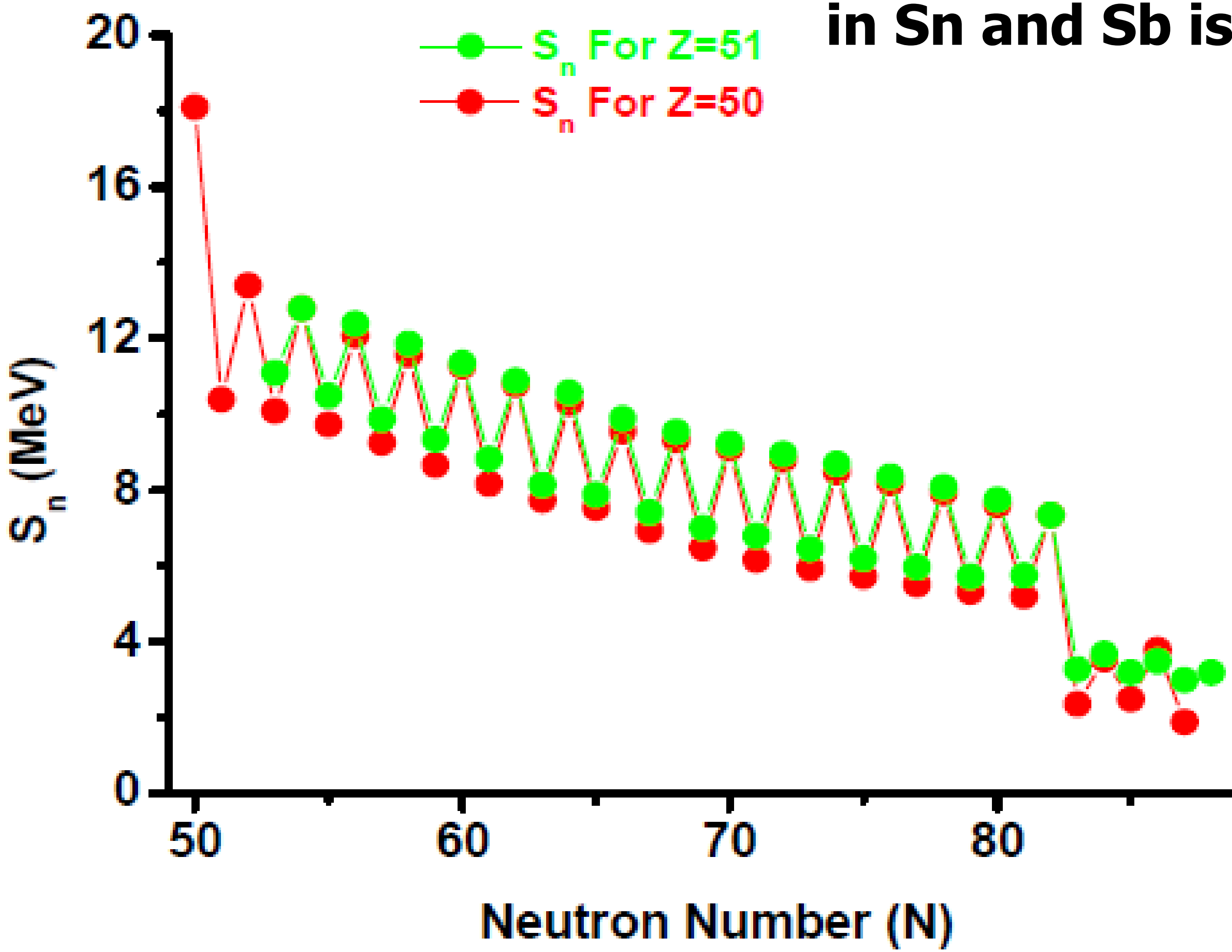
CWG

S. SARKAR AND M. SAHA SARKAR

PHYSICAL REVIEW C 78, 024308 (2008)

Structure of even-even *A* = 138 isobars and the yrast spectra of semi-magic Sn isotopes above the ^{132}Sn core

Odd Even Staggering in Sn and Sb isotopes



reduction of pairing above ^{132}Sn ?

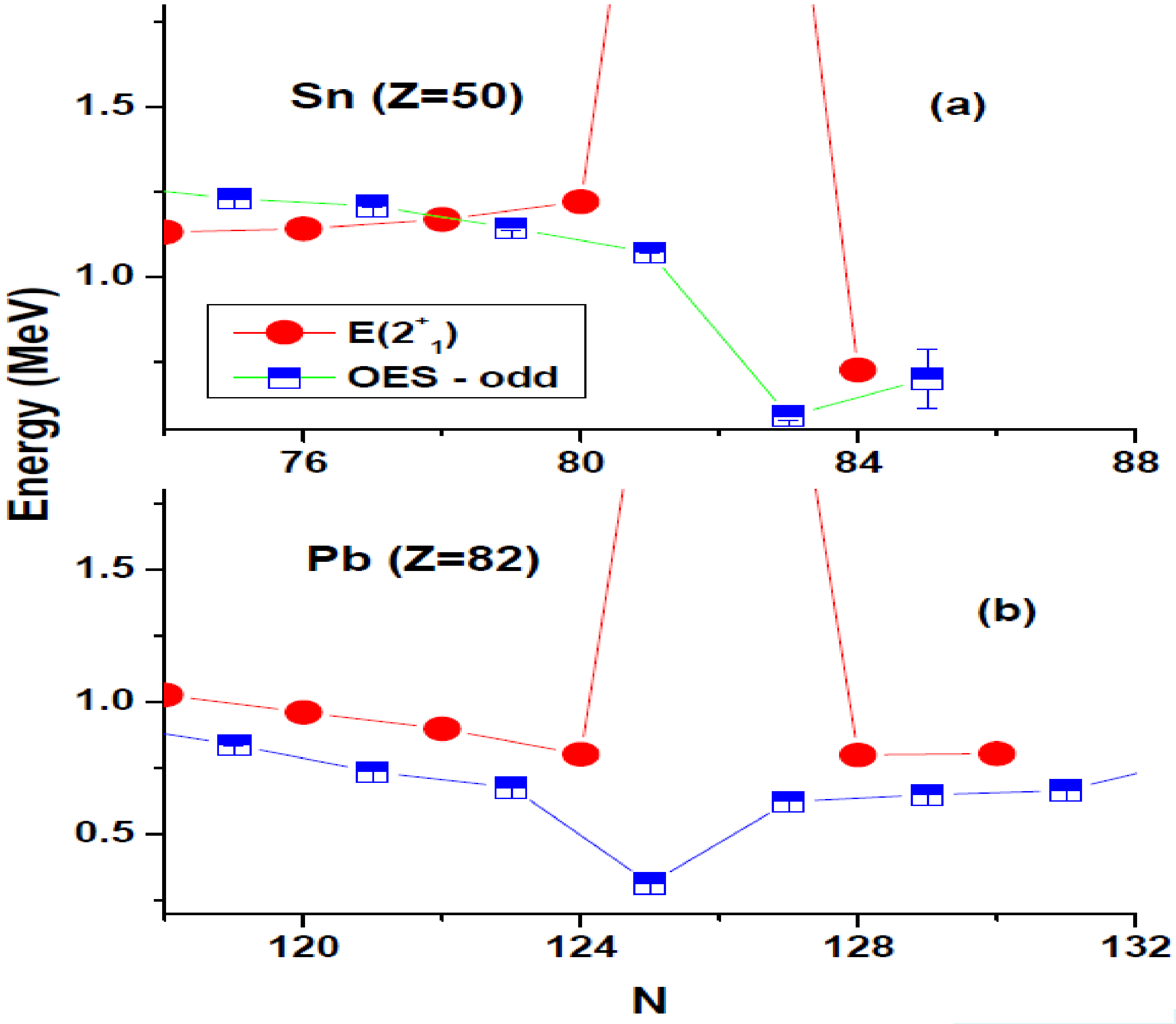
G. Audi, A.H. Wapstra and C. Thibault, Nucl. Phys. A 729, 337 (2003).

Odd-Even Staggering (OES) and pairing

$$\Delta_n(N,Z) = -(-)^{\pi_N}\frac{1}{2} * (S_n(N+1,Z) - S_n(N,Z))$$

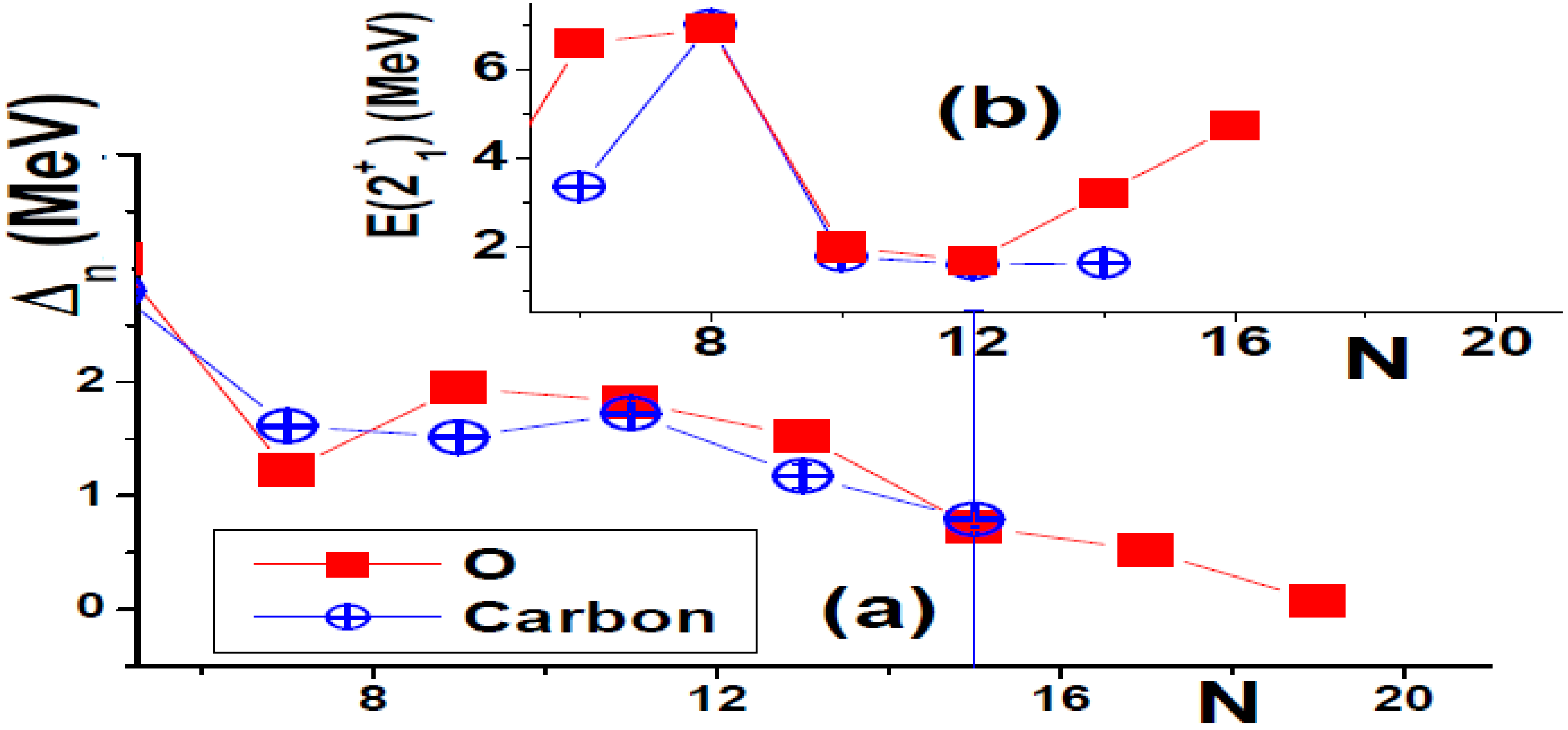
The empirical neutron and proton pairgaps are related to the odd-even staggering of separation energies and masses or binding energies

M. Saha Sarkar and S. Sarkar, Communicated



What is the difference between Sn and Pb isotopes?

Odd even staggering and E(2+) energies

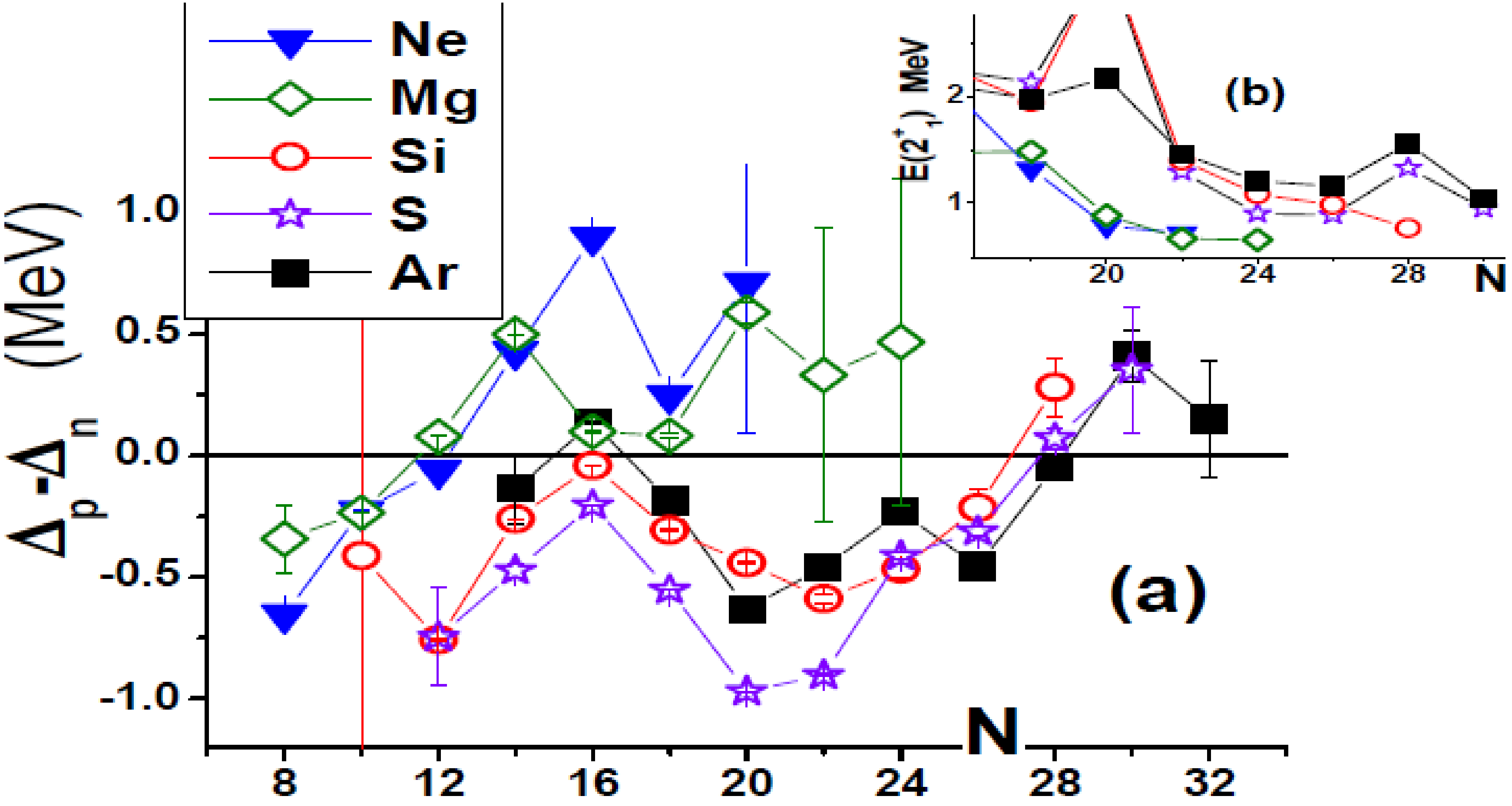


Variation of (a) Δ_n for odd isotopes, (b) $E(2_1^+)$ energies for even isotopes, of C and O with increasing neutron number (N).

Carbon & Oxygen

M. Saha Sarkar and S. Sarkar, Communicated

Pairing and E(2$^+$) in light nuclei



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Empirical observations of the features of neutron-rich nuclei

- **For proton - magic nuclei, reduction in neutron pairing will give rise to**

  – anomalous decrease in $E(2^+_1)$ with slower increase in deformation for isotopes with neutrons in unfilled subshells. For normal pairing observed near stability, similar value of low $E(2^+_1)$'s will indicate higher value of B(E2) or larger deformation.

  – appearance of new shell closures manifested through enhancement of $E(2^+_1)$ for isotopes with filled up sub-shells. The energy gap between the sub - shells must be substantially greater than the neutron pairing energy.

- **For nuclei with a few valence particles near shell closure having unfilled proton sub-shell, reduced neutron pairing will lead to**

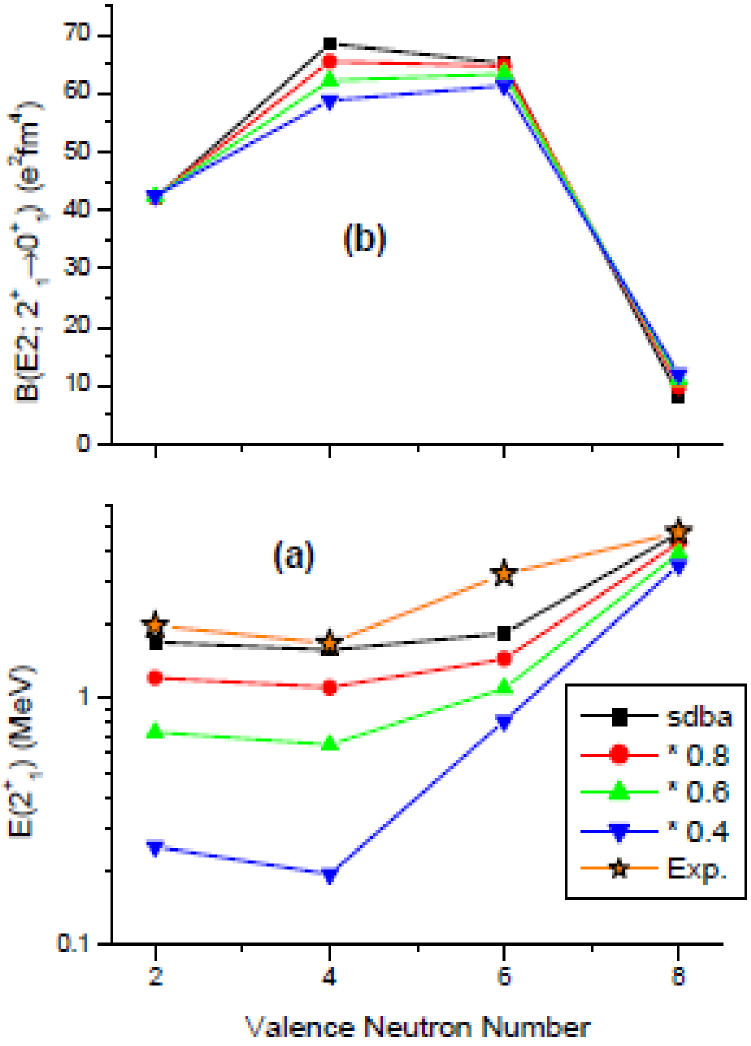
  – anomalous decrease in $E(2^+_1)$ with slower increase in deformation for isotopes with neutrons in unfilled sub-shells

  – onset of deformation for Nmagic nucleus, resulting in erosion of shell gap if proton pairing shows strong enhancement. This will be usually observed for lighter mass nuclei.

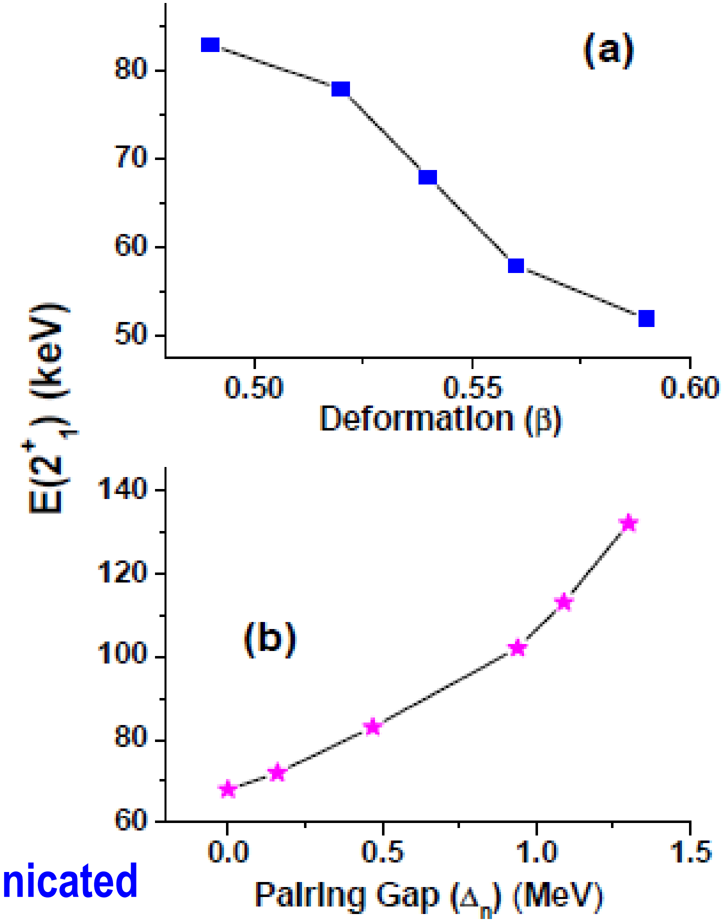
M. Saha Sarkar and S. Sarkar, Communicated

Calculations to show the effect of pairing on E(2+) and B(E2) values

Shell Model calculations



CHFB calculations



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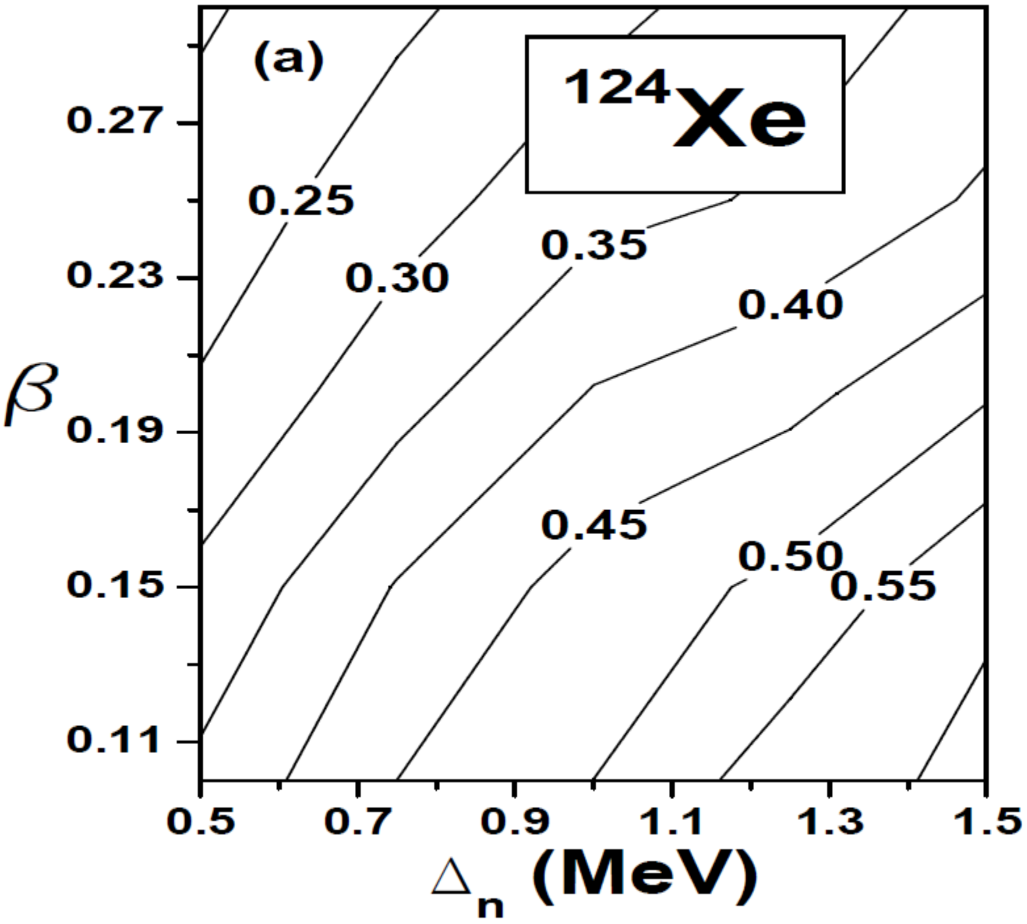
Pairing and deformation



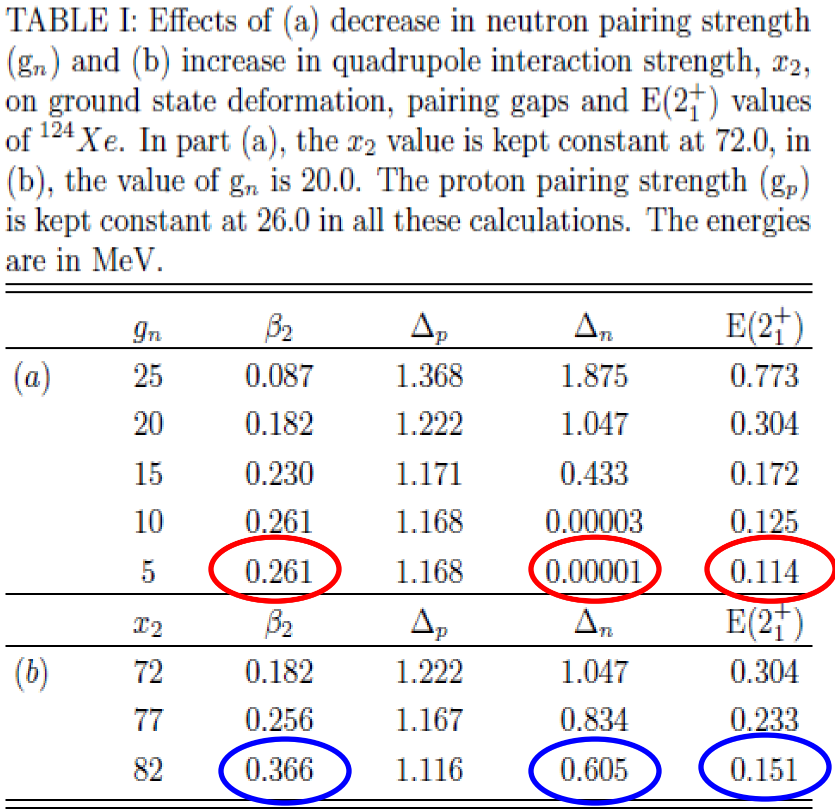


TABLE I: Effects of (a) decrease in neutron pairing strength (g_n) and (b) increase in quadrupole interaction strength, x_2, on ground state deformation, pairing gaps and $E(2_1^+)$ values of ^{124}Xe. In part (a), the x_2 value is kept constant at 72.0, in (b), the value of g_n is 20.0. The proton pairing strength (g_p) is kept constant at 26.0 in all these calculations. The energies are in MeV.

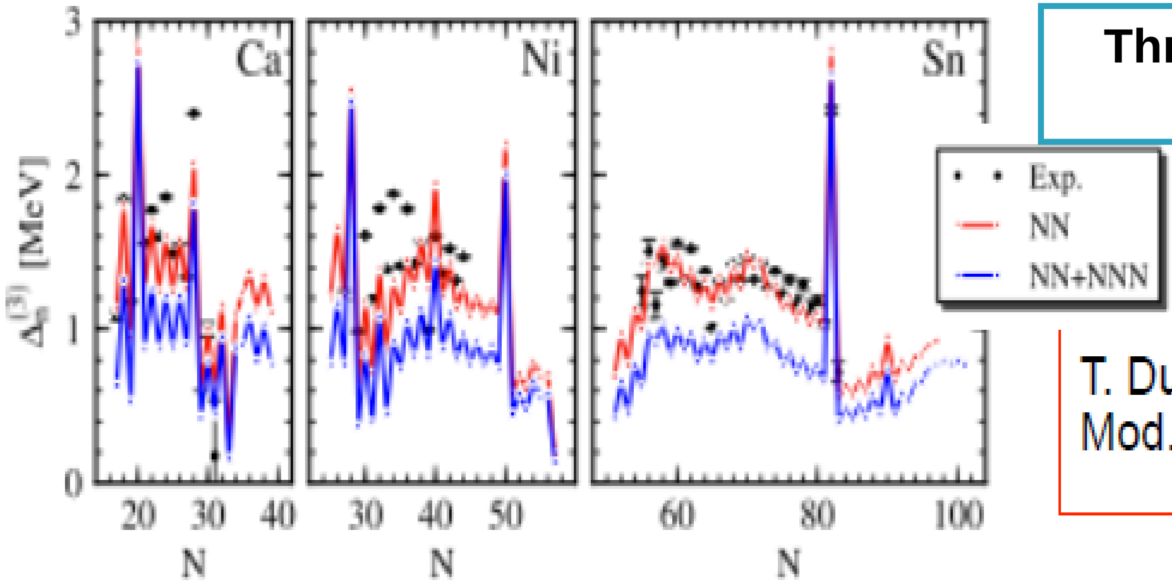
	g_n	β_2	Δ_p	Δ_n	$E(2_1^+)$
(a)	25	0.087	1.368	1.875	0.773
	20	0.182	1.222	1.047	0.304
	15	0.230	1.171	0.433	0.172
	10	0.261	1.168	0.00003	0.125
	5	0.261	1.168	0.00001	0.114
	x_2	β_2	Δ_p	Δ_n	$E(2_1^+)$
(b)	72	0.182	1.222	1.047	0.304
	77	0.256	1.167	0.834	0.233
	82	0.366	1.116	0.605	0.151

Self Consistent calculations with PPQ Hamiltonian

M. Saha Sarkar and S. Sarkar, Communicated

HFB

Mean field calculation with Vlow-k pairing force:
3-body force reduces the pairing gaps



Three body forces and pairing

T. Duguet, T. Lesinski, A.Schwenk,
Mod. Phys. Lett. (2010)

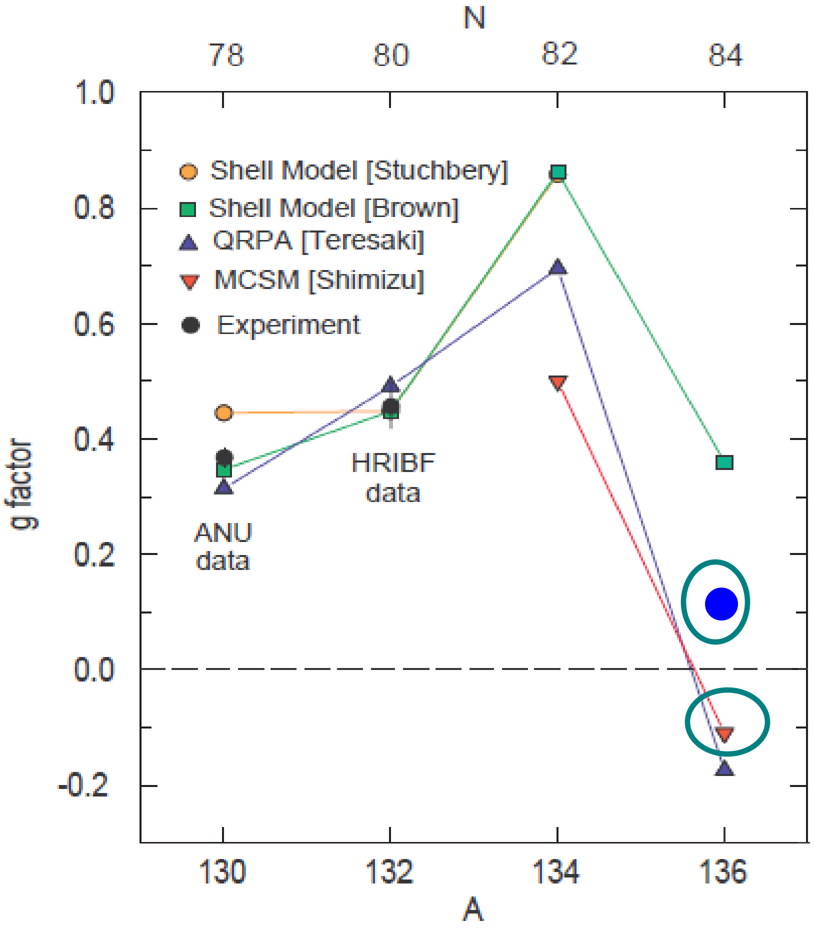


FIG. 11: Theoretical g factors in the Te isotopes compared with experiment. Shell model calculations are from [42, 43]. QRPA is from [44] and MCSM from [45].

g-factor (2+) in ^{136}Te and other Te isotopes

●	**SMPN**	**0.1042**
quenching 0.7	^{136}Te	**g(2+)**

○ **Expt. (Prelim.)**

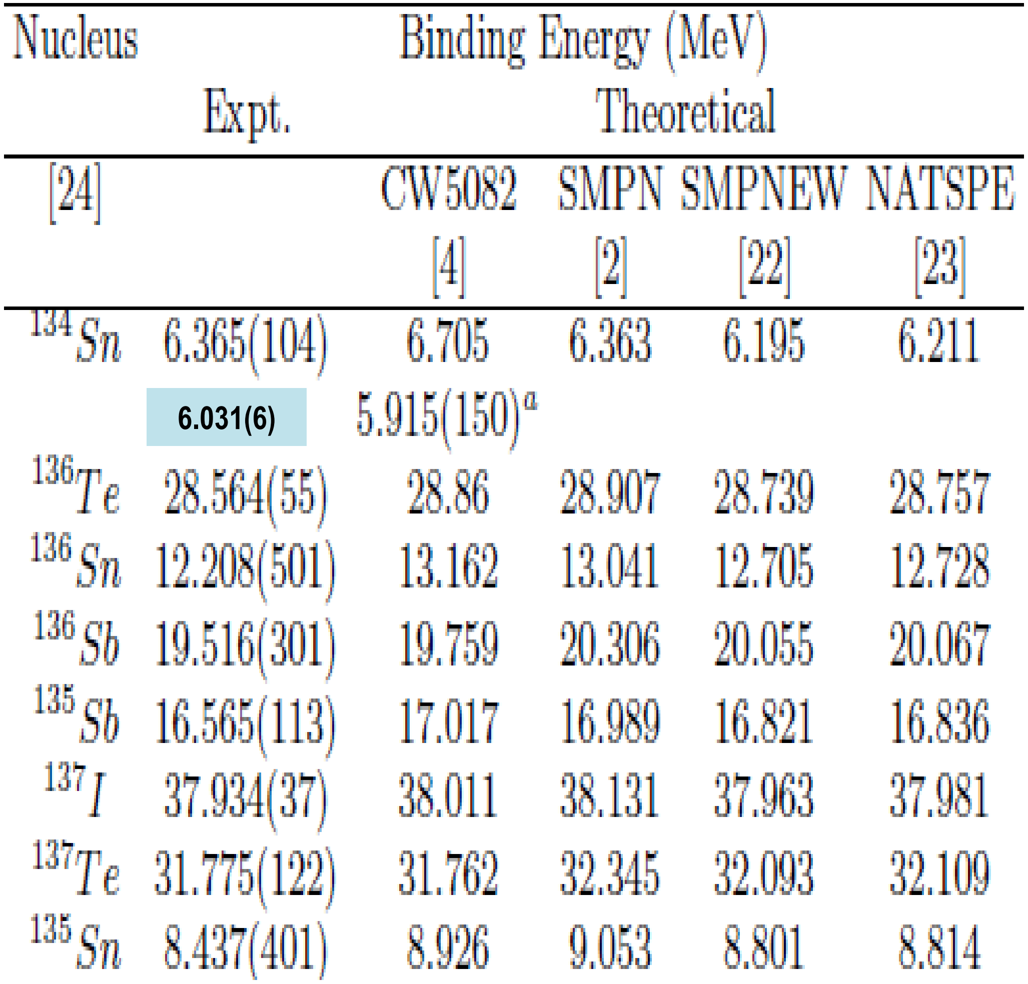
Andrew E. Stuchbery, ANU

Talk delivered at FIG12, 5-7 March 2012 at IUAC, Delhi

The new mass data and SMPN results

- **A high-precision direct Penning trap mass measurement $^{127,131-134}$Sn, M. Dworschak et al.,Phys. Rev. Lett. 100, 072501 (2008).**

- **The measurement of spectroscopic factors indicating purity of the measured single-particle states, K. L. Jones et al., Nature, 465, 454 (2010).**



Nucleus	Binding Energy (MeV)				
	Expt.	Theoretical			
[24]		CW5082 [4]	SMPN [2]	SMPNEW [22]	NATSPE [23]
^{134}Sn	6.365(104)	6.705	6.363	6.195	6.211
	6.031(6)	5.915(150)^a			
^{136}Te	28.564(55)	28.86	28.907	28.739	28.757
^{136}Sn	12.208(501)	13.162	13.041	12.705	12.728
^{136}Sb	19.516(301)	19.759	20.306	20.055	20.067
^{135}Sb	16.565(113)	17.017	16.989	16.821	16.836
^{137}I	37.934(37)	38.011	38.131	37.963	37.981
^{137}Te	31.775(122)	31.762	32.345	32.093	32.109
^{135}Sn	8.437(401)	8.926	9.053	8.801	8.814

J. Hakala et al., arXiv:1203.0958v2 [nucl-ex] 6 Mar 2012; J. Van Schelt et al., arXiv:1203.4470v1 [nucl-ex] 20 Mar 2012.

Proposal

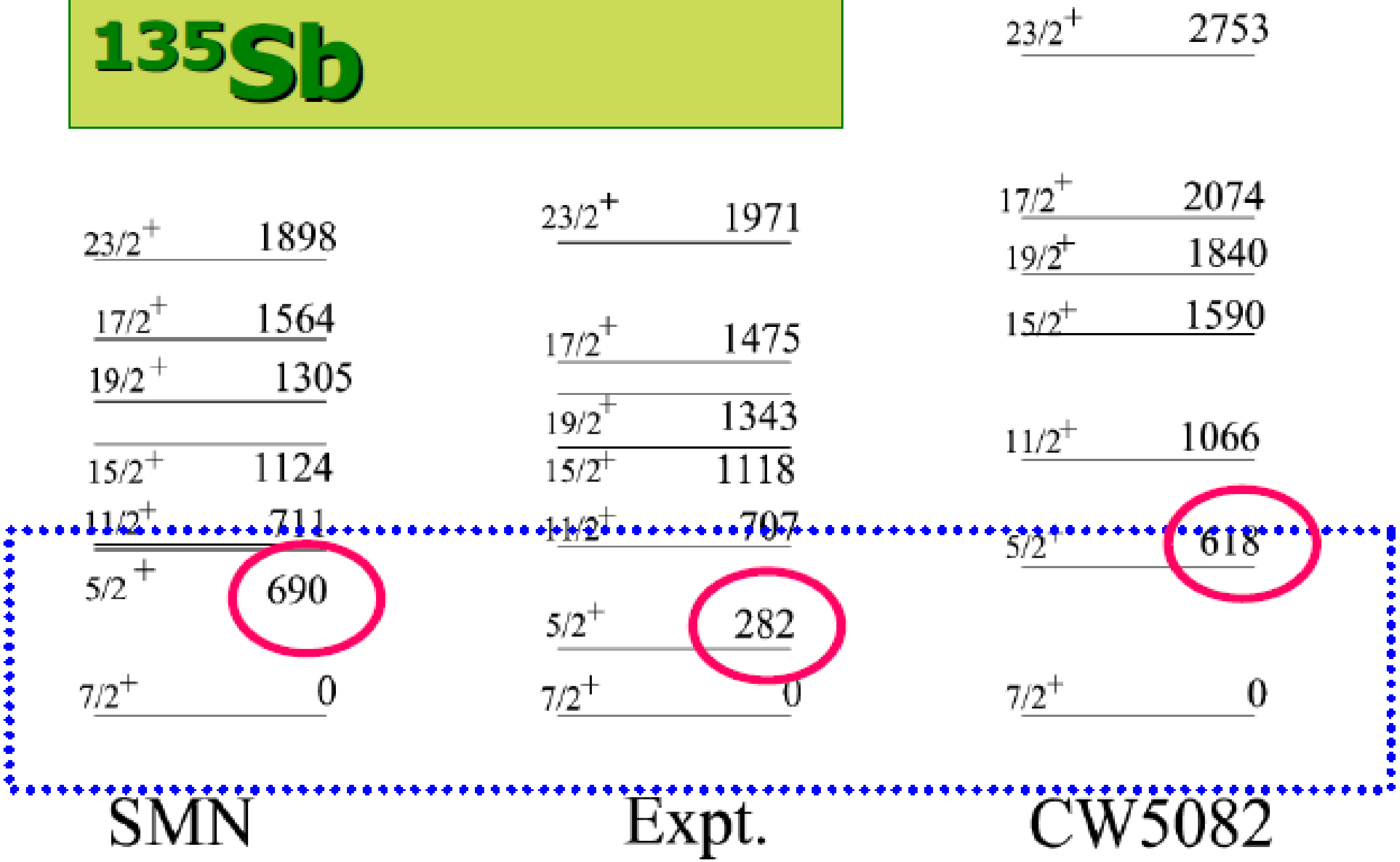
- **Measurement of 2^+ state energies of ^{136}Sn, ^{138}Sn**
- **Measurement of $B(E2, 0^+ \rightarrow 2^+)$ for these isotopes of Sn**

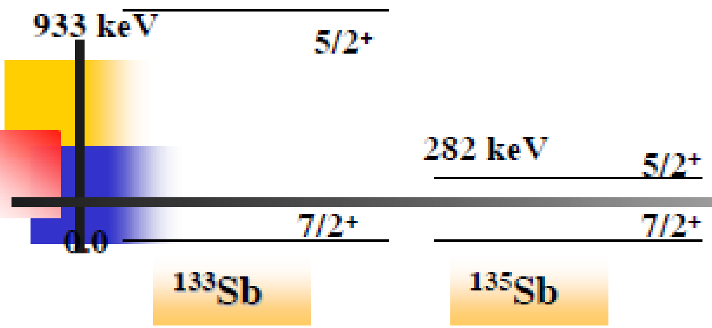
  - $T_{1/2}$ (**^{136}Sn**)= 0.25 s
  - $T_{1/2}$ (**^{138}Sn**)= **??**

- **Need for precise mass data for more neutron-rich species.**

- **Anomalous depression of 5/2$^+$ state in ^{135}Sb**

^{135}Sb


23/2+ 1898
17/2+ 1564
19/2+ 1305
15/2+ 1124
11/2+ 711
5/2+ 690
7/2+ 0
SMN
23/2+ 1971
17/2+ 1475
19/2+ 1343
15/2+ 1118
11/2+ 707
5/2+ 282
7/2+ 0
Expt.
23/2+ 2753
17/2+ 2074
19/2+ 1840
15/2+ 1590
11/2+ 1066
5/2+ 618
7/2+ 0
CW5082



Why such a reduction of $5/2^+_1$ excitation energy in ^{135}Sb?

This $5/2^+_1$ state in ^{135}Sb corresponds to a proton coupled to the ^{134} Sn core

- **arises mainly from $[(\pi 2d_{5/2}) \oplus (\nu 2f_{7/2}{}^2)^{0+}]^{5/2+}$ and**
- **partially from $[(\pi 1g_{7/2}) \oplus (\nu 2f_{7/2}{}^2)^{2+}]^{5/2+}$ configurations.**

Shell model calculations with different (1+2) body Hamiltonians also support this.

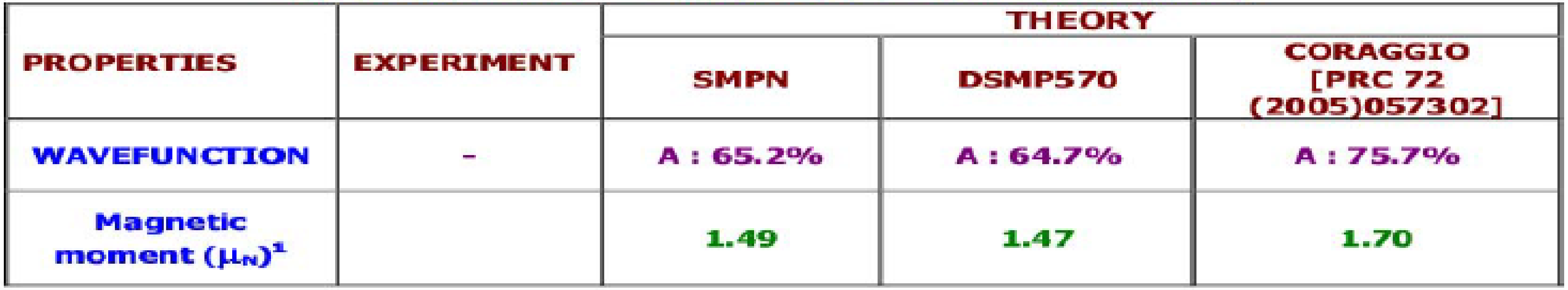
- The energy of this state seems anomalous ---likely due to a high neutron excess that decreases the relative separation energy between the $d_{5/2}$ and $g_{7/2}$ orbitals.

- **This idea can be examined via combined experimental and theoretical studies.**

For odd-odd nuclei the 6^- state arising from the coupling of proton $2d_{5/2}$ and neutron $2f_{7/2}$ carries the signature of $2d_{5/2}$ depression: already seen in

^{136}I, ^{138}I, ^{138}Cs

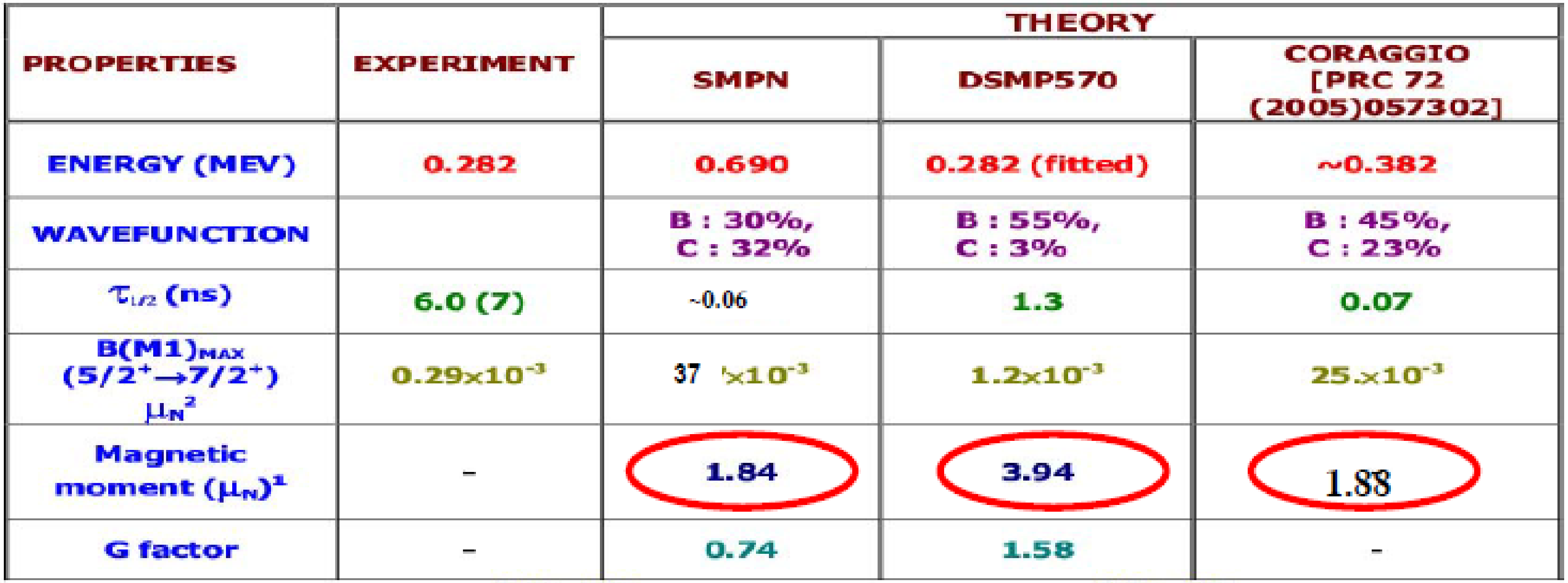
PROPERTY OF THE GROUND STATE ($7/2^+$)



PROPERTIES	EXPERIMENT	THEORY		
		SMPN	DSMP570	CORAGGIO [PRC 72 (2005)057302]
WAVEFUNCTION	-	A : 65.2%	A : 64.7%	A : 75.7%
Magnetic moment (μ_N)^1		1.49	1.47	1.70

A: $[(\pi 1g_{7/2})\otimes(\nu 2f_{7/2}^{\;2})^{0+}]^{7/2+}$

PROPERTY OF THE EXCITED STATE ($5/2^+_1$)



PROPERTIES	EXPERIMENT	THEORY		
		SMPN	DSMP570	CORAGGIO [PRC 72 (2005)057302]
ENERGY (MEV)	0.282	0.690	0.282 (fitted)	~0.382
WAVEFUNCTION		B : 30%, C : 32%	B : 55%, C : 3%	B : 45%, C : 23%
$\tau_{1/2}$ (ns)	6.0 (7)	~0.06	1.3	0.07
B(M1)$_{MAX}$ ($5/2^+\rightarrow 7/2^+$) μ_N^2	0.29×10^{-3}	37×10^{-3}	1.2×10^{-3}	$25.\times10^{-3}$
Magnetic moment (μ_N)^1	-	1.84	3.94	1.88
G factor	-	0.74	1.58	-

B: $[(\pi 2d_{5/2})\otimes(\nu 2f_{7/2}^{\;2})^{0+}]^{5/2+}$; C: $[(\pi 1g_{7/2})\otimes(\nu 2f_{7/2}^{\;2})^{2+}]^{5/2+}$

Proposal

- Measurement of Magnetic moment of the anomalously low first $5/2^+$ state of ^{135}Sb.
  - $T_{1/2}(5/2^+) = 6.1(4)$ ns.
  - Ground state spin $7/2^+$
  - $T_{1/2}$(g.s)= 1.679 s

THANK YOU

Construction of the new Hamiltonian

- Modification of the CW5082 [W.T. Chou and E.K. Warburton, Phys. Rev. C 45, 1720 (1992)] Hamiltonian in the light of recently available information on binding energies, low-lying spectra of A=134 Sn,Sb and Te isotopes.
- The spes of the single particle orbitals of the valence space above the ^{132}Sn core have been replaced by the recently measured ones.
- The details of this modification procedure have been given in [Sukhendusekhar Sarkar, M. Saha Sarkar, Eur. Phys. Jour. A 21 (2004) 61].
- The new Hamiltonians work remarkably well in predicting binding energies, low-lying spectra and electromagnetic transition probabilities for N=82,83 and even for N >= 84 isotones of Sn,Sb,Te,I,Xe and Cs nuclei.

Procedure

SINGLE PARTICLE ENERGIES (SPES)

modified the CW5082 interaction. The valence space consists of five proton orbitals, [$1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, $3s_{1/2}$ and $1h_{11/2}$] with energies [0.(-9.6629), 0.9624, 2.4396, 2.6972, 2.7915] respectively, and

[$1h_{9/2}$, $2f_{7/2}$, $2f_{5/2}$, $3p_{3/2}$, $3p_{1/2}$ and $1i_{13/2}$] for neutrons with energies in MeV, [1.5609, 0.0 (-2.4553), 2.0046, 0.8537, 1.6557, 2.6950], respectively with ^{132}Sn as the inert core.

CHANGE IN TWO BODY MATRIX ELEMENTS (TBMES)

In SMN

- change the neutron-neutron and proton-neutron tbmes keeping the proton-proton tbmes the same as those in CW5082.

In SMPN change the neutron-neutron, proton-neutron AND proton-proton tbmes .

Changes in two body matrix elements (tbmes)

- **Neutron-neutron tbmes**
  - **The six neutron-neutron diagonal tbmes with I = 0^+ were multiplied by a factor of 0.48. This factor is obtained by reproducing the binding energy of ^{134}Sn (-6.365 MeV). All the binding energies in MeV are with respect to ^{132}Sn.**
  - **Three excited states in ^{134}Sn, predominantly from the neutron $(2f_{7/2})^2$, at energies 725.6, 1073.4, and 1247.4 keV are used to modify the**

$< (2f_{7/2})^2 |V| (2f_{7/2})^2 >^{\{2+,4+,6+}$ tbmes for neutrons.

$< (1h_{9/2}\, 2f_{7/2}) |V| (1h_{9/2}\, 2f_{7/2}) >^{\{8+}$ changed to reproduce the energy of 8^+ level at 2508.9keV.

- **neutron – proton tbmes**
  - **Similarly, using binding energy (-12.952 MeV) and 1^-, 2^-, 3^-, 4^-, 7^-, 8^-, 10^+, 9^+, 10^-, 11^- and 12^- excited levels at energies 13.0, 330.7, 383.5, 554.8, 283.0, 1073, 2434, 2126, 4094, 4425 and 4517 keV respectively, of ^{134}Sb, we have modified 12 dominant proton-neutron tbmes.**

The modified two body matrix elements (tbmes) of SMPN

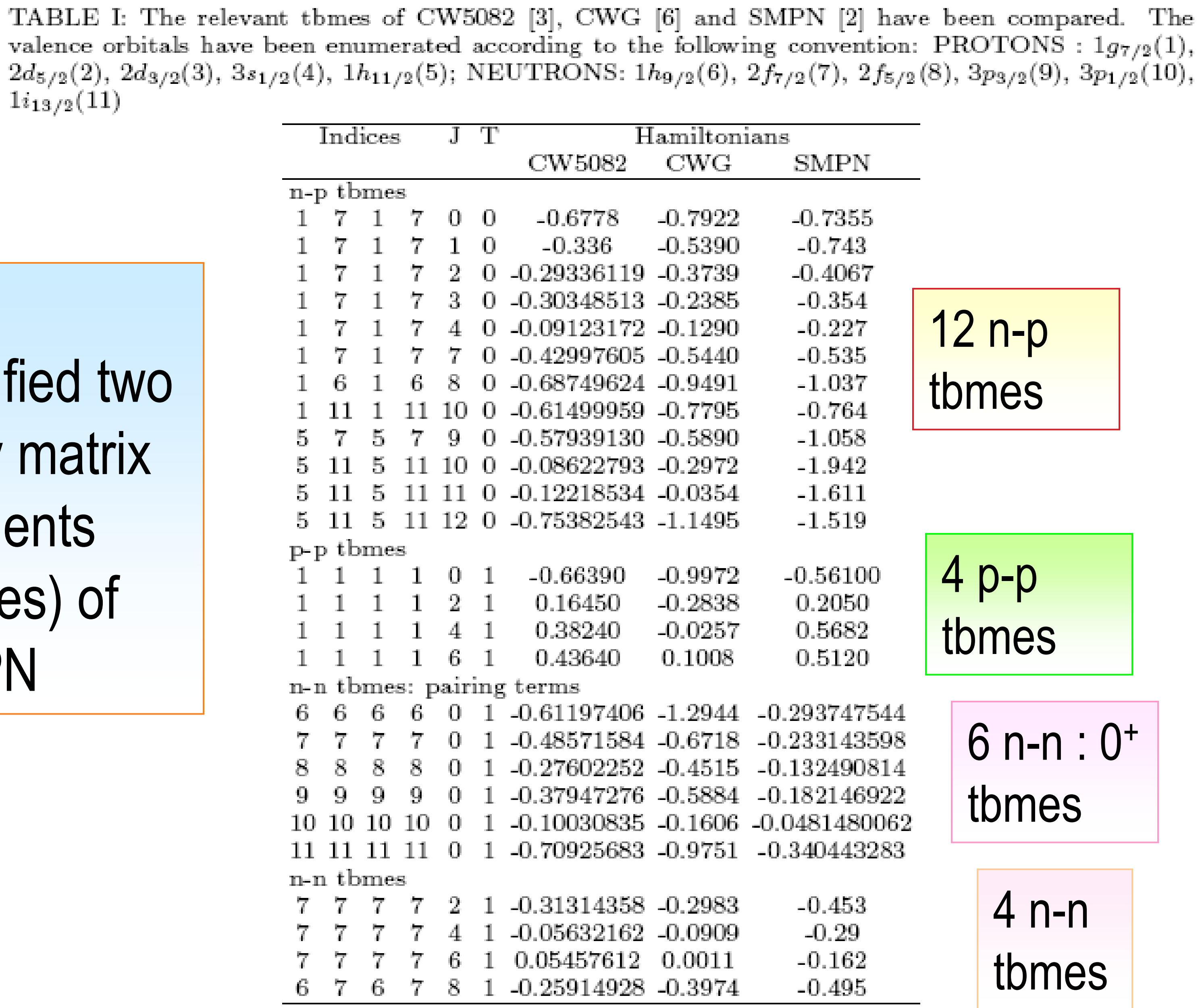


TABLE I: The relevant tbmes of CW5082 [3], CWG [6] and SMPN [2] have been compared. The valence orbitals have been enumerated according to the following convention: PROTONS : $1g_{7/2}(1)$, $2d_{5/2}(2)$, $2d_{3/2}(3)$, $3s_{1/2}(4)$, $1h_{11/2}(5)$; NEUTRONS: $1h_{9/2}(6)$, $2f_{7/2}(7)$, $2f_{5/2}(8)$, $3p_{3/2}(9)$, $3p_{1/2}(10)$, $1i_{13/2}(11)$

Indices				J	T	Hamiltonians		
						CW5082	CWG	SMPN
n-p tbmes								
1	7	1	7	0	0	-0.6778	-0.7922	-0.7355
1	7	1	7	1	0	-0.336	-0.5390	-0.743
1	7	1	7	2	0	-0.29336119	-0.3739	-0.4067
1	7	1	7	3	0	-0.30348513	-0.2385	-0.354
1	7	1	7	4	0	-0.09123172	-0.1290	-0.227
1	7	1	7	7	0	-0.42997605	-0.5440	-0.535
1	6	1	6	8	0	-0.68749624	-0.9491	-1.037
1	11	1	11	10	0	-0.61499959	-0.7795	-0.764
5	7	5	7	9	0	-0.57939130	-0.5890	-1.058
5	11	5	11	10	0	-0.08622793	-0.2972	-1.942
5	11	5	11	11	0	-0.12218534	-0.0354	-1.611
5	11	5	11	12	0	-0.75382543	-1.1495	-1.519
p-p tbmes								
1	1	1	1	0	1	-0.66390	-0.9972	-0.56100
1	1	1	1	2	1	0.16450	-0.2838	0.2050
1	1	1	1	4	1	0.38240	-0.0257	0.5682
1	1	1	1	6	1	0.43640	0.1008	0.5120
n-n tbmes: pairing terms								
6	6	6	6	0	1	-0.61197406	-1.2944	-0.293747544
7	7	7	7	0	1	-0.48571584	-0.6718	-0.233143598
8	8	8	8	0	1	-0.27602252	-0.4515	-0.132490814
9	9	9	9	0	1	-0.37947276	-0.5884	-0.182146922
10	10	10	10	0	1	-0.10030835	-0.1606	-0.0481480062
11	11	11	11	0	1	-0.70925683	-0.9751	-0.340443283
n-n tbmes								
7	7	7	7	2	1	-0.31314358	-0.2983	-0.453
7	7	7	7	4	1	-0.05632162	-0.0909	-0.29
7	7	7	7	6	1	0.05457612	0.0011	-0.162
6	7	6	7	8	1	-0.25914928	-0.3974	-0.495

12 n-p tbmes

4 p-p tbmes

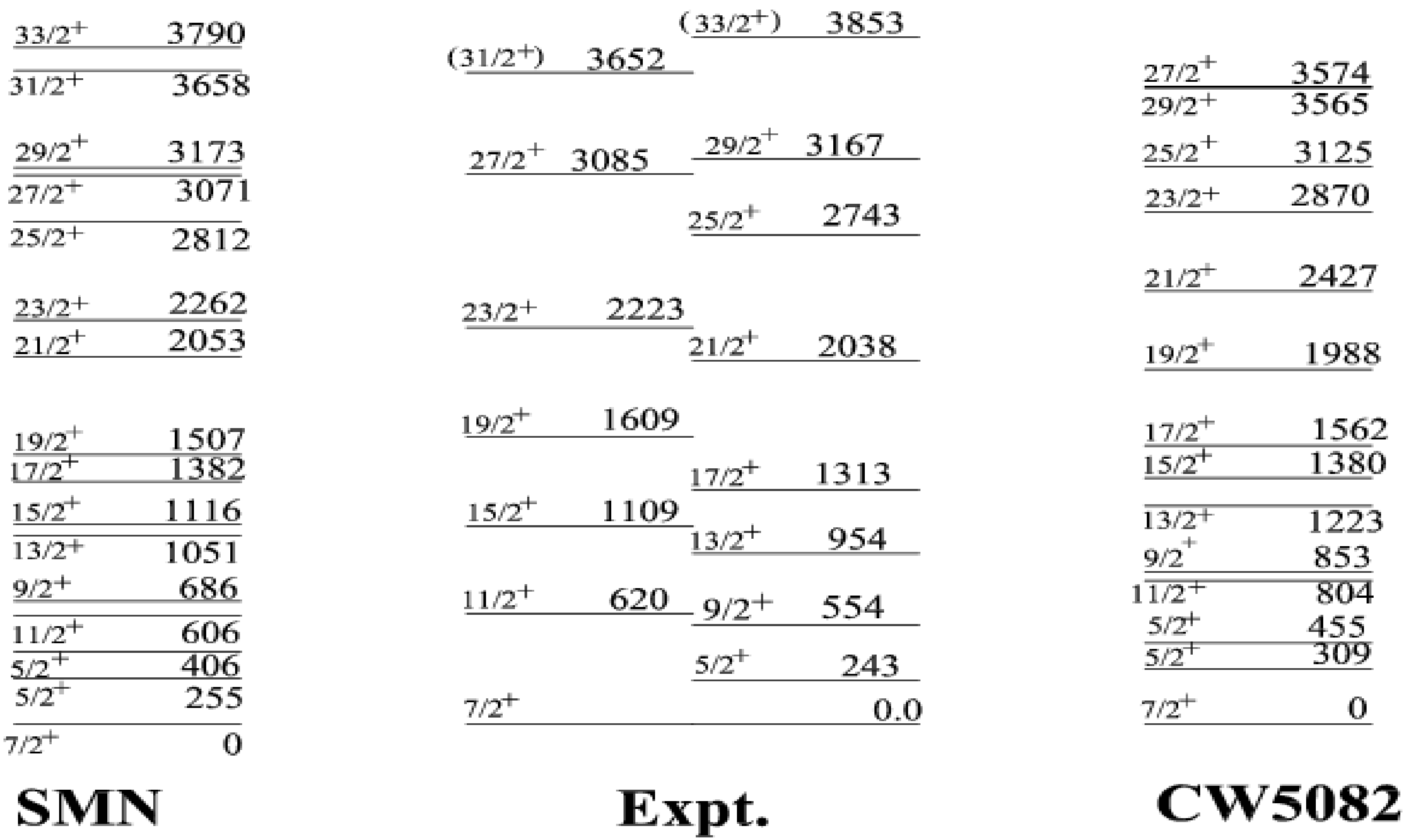
6 n-n : 0^+ tbmes

4 n-n tbmes

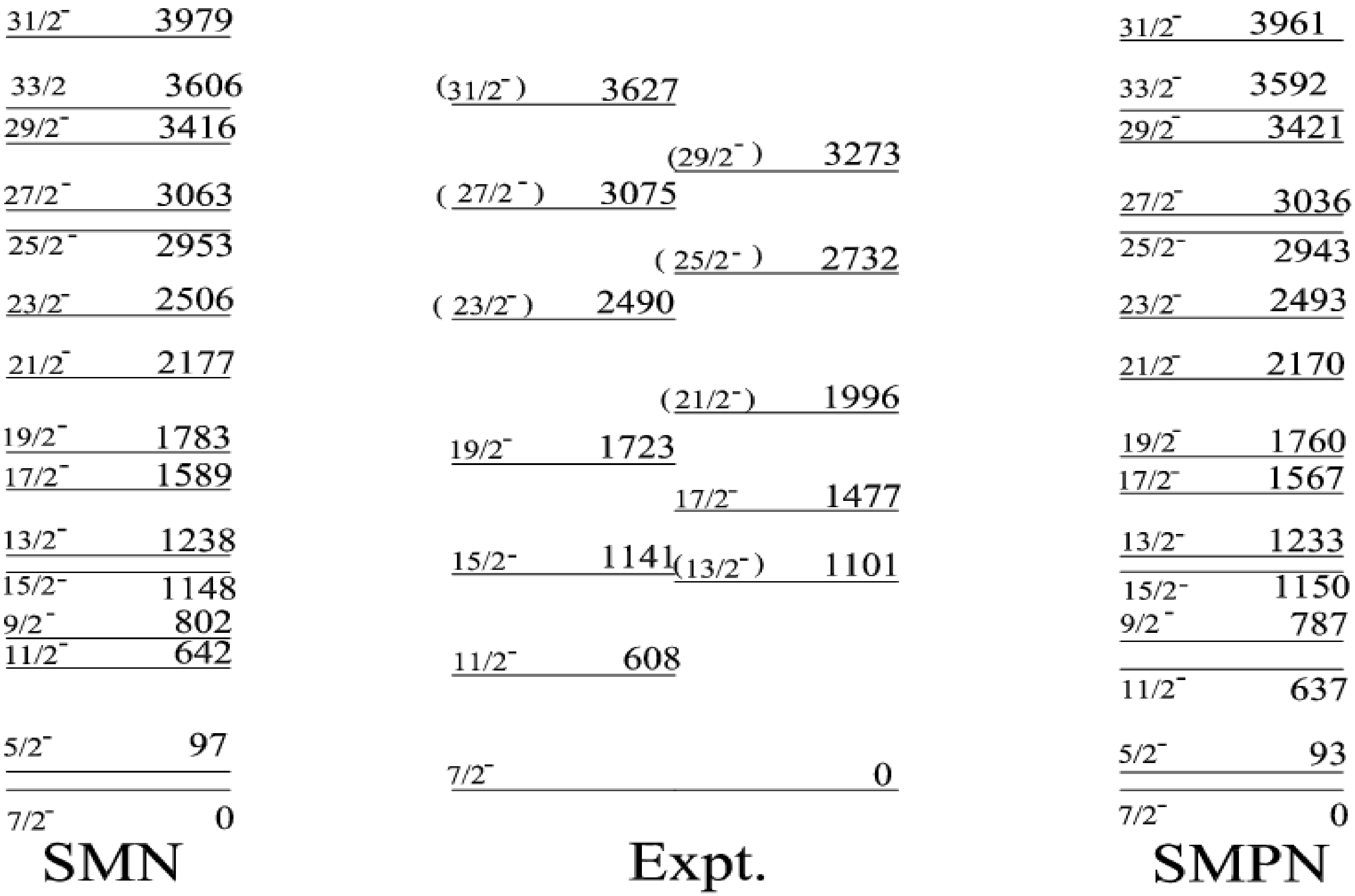
The implication of ALS term?

- **ALS component in the tbmes** corresponds to those
- **LS-coupled matrix elements which have S≠S′**, i.e., terms non-diagonal in S (spin). **Do not conserve total spin** of the matrix elements.
- But the interactions which are parity conserving and isospin conserving must also conserve the total spin.
- **Bare nucleon- nucleon force contains no ALS term.**
- But effective interaction is not simply related to bare nucleon-nucleon force. **Core polarisation corrections to the G-matrix give rise to non-zero but small ALS** matrix elements.
- A characteristic feature common to many **empirical effective interactions is strong ALS components** in the tbmes.
  - It usually arises from **inadequate constraint by the data**.
  - It indicates the important contributions from **higher order renormalisation or many body forces** to the effective interactions.
  - In empirical SMPN such **many - body effects might have been included** in some way through the modification of important tbmes.

137I


33/2+ 3790
31/2+ 3658
29/2+ 3173
27/2+ 3071
25/2+ 2812
23/2+ 2262
21/2+ 2053
19/2+ 1507
17/2+ 1382
15/2+ 1116
13/2+ 1051
9/2+ 686
11/2+ 606
5/2+ 406
5/2+ 255
7/2+ 0
SMN
(33/2+) 3853
(31/2+) 3652
29/2+ 3167
27/2+ 3085
25/2+ 2743
23/2+ 2223
21/2+ 2038
19/2+ 1609
17/2+ 1313
15/2+ 1109
13/2+ 954
11/2+ 620
9/2+ 554
5/2+ 243
7/2+ 0.0
Expt.
33/2+ 4588
31/2+ 4510
27/2+ 3574
29/2+ 3565
25/2+ 3125
23/2+ 2870
21/2+ 2427
19/2+ 1988
17/2+ 1562
15/2+ 1380
13/2+ 1223
9/2+ 853
11/2+ 804
5/2+ 455
5/2+ 309
7/2+ 0
CW5082

^{137}Te

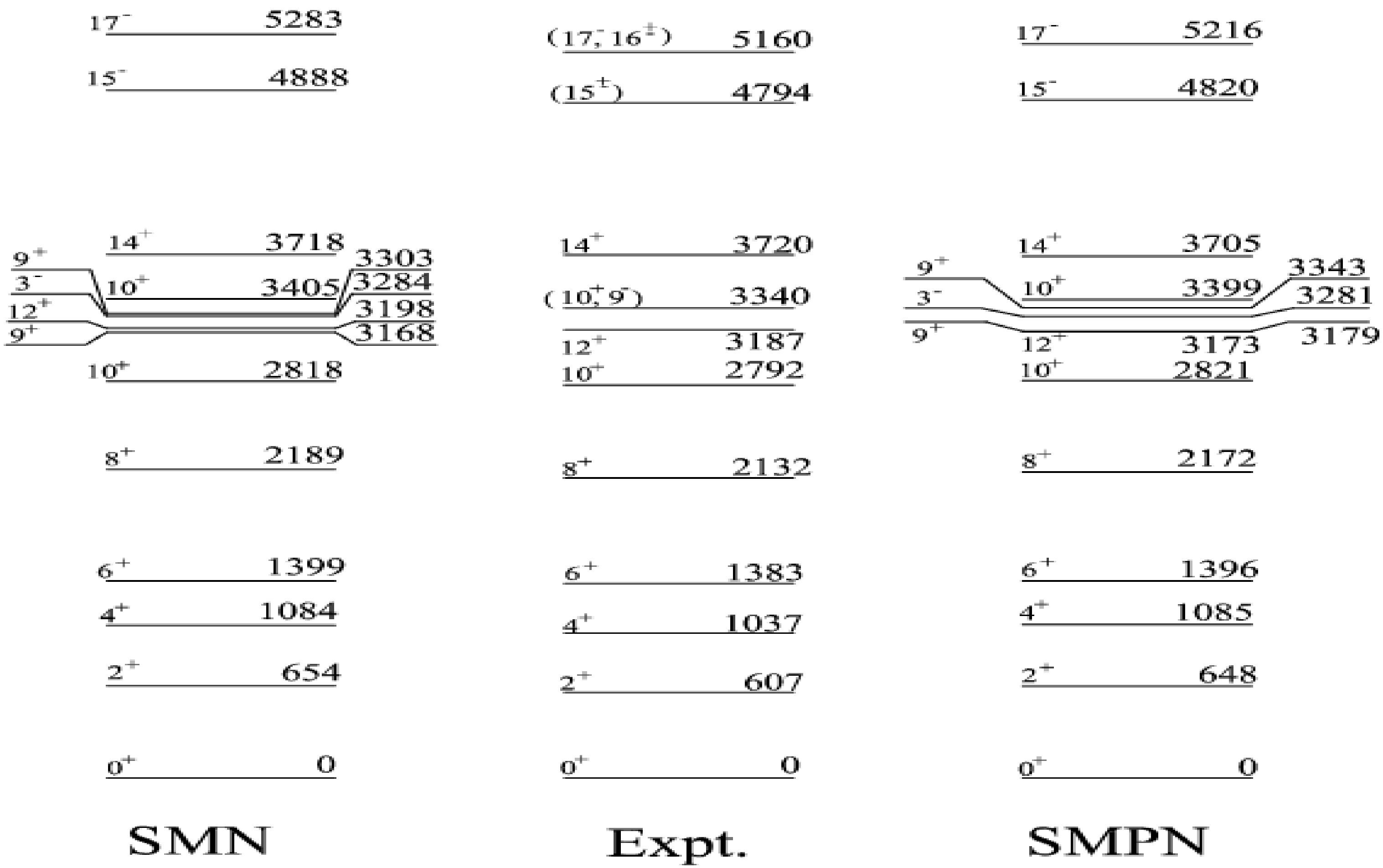


31/2$^-$ 3979
33/2 3606
29/2$^-$ 3416
27/2$^-$ 3063
25/2$^-$ 2953
23/2$^-$ 2506
21/2$^-$ 2177
19/2$^-$ 1783
17/2$^-$ 1589
13/2$^-$ 1238
15/2$^-$ 1148
9/2$^-$ 802
11/2$^-$ 642
5/2$^-$ 97
7/2$^-$ 0
SMN

(31/2$^-$) 3627
(29/2$^-$) 3273
(27/2$^-$) 3075
(25/2$^-$) 2732
(23/2$^-$) 2490
(21/2$^-$) 1996
19/2$^-$ 1723
17/2$^-$ 1477
15/2$^-$ 1141
(13/2$^-$) 1101
11/2$^-$ 608
7/2$^-$ 0
Expt.

31/2$^-$ 3961
33/2$^-$ 3592
29/2$^-$ 3421
27/2$^-$ 3036
25/2$^-$ 2943
23/2$^-$ 2493
21/2$^-$ 2170
19/2$^-$ 1760
17/2$^-$ 1567
13/2$^-$ 1233
15/2$^-$ 1150
9/2$^-$ 787
11/2$^-$ 637
5/2$^-$ 93
7/2$^-$ 0
SMPN

^{136}Te



17⁻ 5283
15⁻ 4888
14⁺ 3718
9⁺ 3⁻ 12⁺ 9⁺
10⁺ 3405
3303
3284
3198
3168
10⁺ 2818
8⁺ 2189
6⁺ 1399
4⁺ 1084
2⁺ 654
0⁺ 0
SMN
(17⁻, 16⁺) 5160
(15⁺) 4794
14⁺ 3720
(10⁺, 9⁻) 3340
12⁺ 3187
10⁺ 2792
8⁺ 2132
6⁺ 1383
4⁺ 1037
2⁺ 607
0⁺ 0
Expt.
17⁻ 5216
15⁻ 4820
14⁺ 3705
9⁺ 3⁻ 9⁺
10⁺ 3399
3343
3281
12⁺ 3173
3179
10⁺ 2821
8⁺ 2172
6⁺ 1396
4⁺ 1085
2⁺ 648
0⁺ 0
SMPN

Pairing and Neutron –rich Nuclei

- Terasaki et al. traced the origin of this anomalous behaviour in ^{136}Te isotope to a **reduced neutron pairing** above the N=82 magic gap. **(PRC 66, 054313 (2002))**

- Possible **quenching of the neutron pairing gap** while approaching the drip line for neutron-rich nuclei to the nuclear systems found in the crust of a neutron star **(PRC 81, 045804 (2010)).**

- The primary difference of improved empirical interaction SMPN with realistic CWG . Much weaker six neutron-neutron diagonal tbmes with $I^{\pi}= 0^{+}$, in SMPN
- Interestingly, the SMPN interaction predicted the "anomalous" behaviour of the neutron -rich Sn nuclei, viz., decrease in both the $E(2^{+}_{1})$ and B(E2) values with increasing neutron number.

- Signoracci et al. discussed that for neutron -rich nuclei in the sd-fp space - a reduction in the size of matrix elements - most notable for pairing matrix elements **(PRC 83, 024315 (2011).**

- In the SDPF-U interaction Nowacki and Poves used different neutron-neutron pairing matrix elements for $Z \geq 15$ and for $Z \leq 14$ (reduced) to account for 2p-2h excitations of the core correctly **(PRC 79, 014310 (2009)).**

Nuclear deformation in neutron-rich nuclei

- On stability, for E2 transitions, experiments and theoretical calculations show a general trend that even–even nuclei with a small 2^+ excitation energy $E(2^+{}_1)$ have a large $B(\mathrm{E2};\, 0^+{}_{\mathrm{g.s.}} \rightarrow 2^+{}_1)$ value .
- Recently, compared to the systematics, the neutron-rich ^{16}C, ^{18}C and ^{20}C isotopes, which have 2–6 more neutrons than the magic number $N = 8$, were found to have small $B(\mathrm{E2})$ values .
- In heavier nuclei, after observation of reduction of both $E(2^+{}_1)$ and B(E2) in ^{136}Te, Terasaki et al. from QRPA calculations, traced the origin of this anomalous behaviour in ^{136}Te isotope to a reduced neutron pairing above the N=82 magic gap.
- Around this period, new empirical interactions proposed for neutron rich isotopes above the ^{132}Sn core and later those for neutron rich nuclei in the sd − fp shell also included reduction of pairing matrix elements for better reproduction of data. So these studies indicate an important role of pairing in the evolution of the structure of exotic nuclei.