Issues of fragmentation functions and medium effects in single inclusive production and two-particle correlations in p-p and A+A collisions

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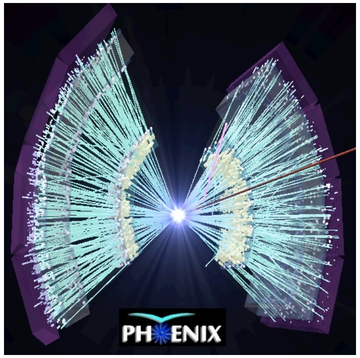

Workshop on Parton Fragmentation Processes: in the Vacuum and in the Medium
ECT* Trento, Italy
February 25-29, 2008

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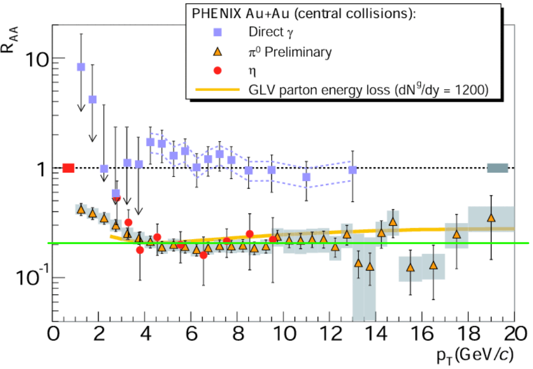

Workshop on Parton Fragmentation Processes: in the Vacuum and in the Medium
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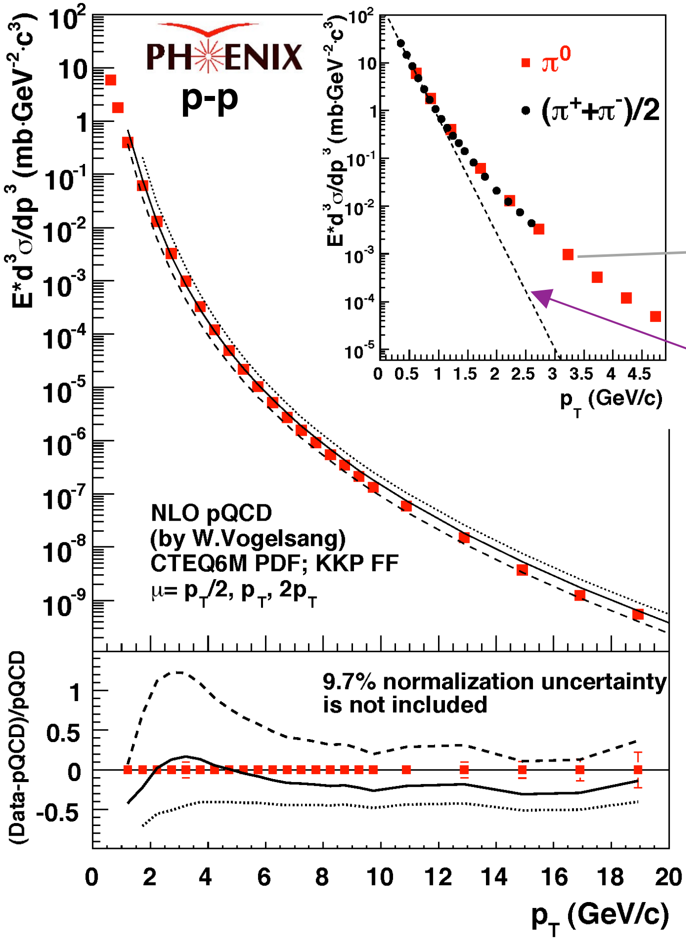
Inclusive invariant π^0 spectrum in p-p and AuAu is beautiful power law for $p_T \geq 3$ GeV/c n=8.1

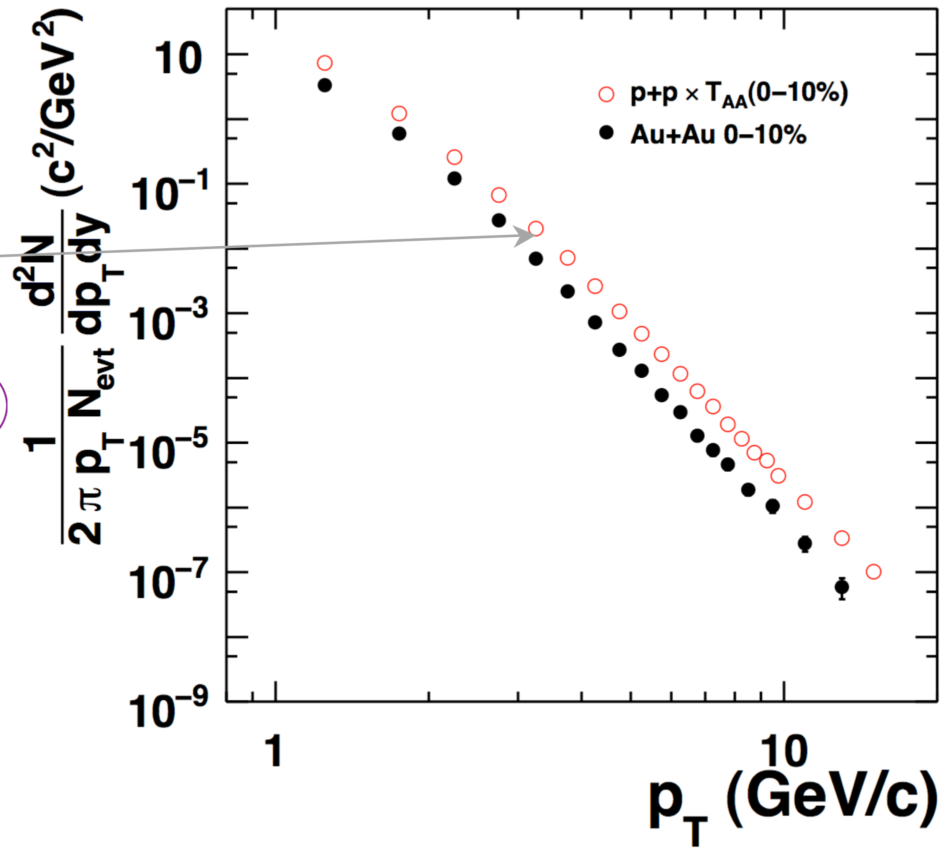
PHENIX, PRD**76**(2007)051006(R)

$\sqrt{s}$=200 GeV

PHENIX, PRC**76**(2007)034904



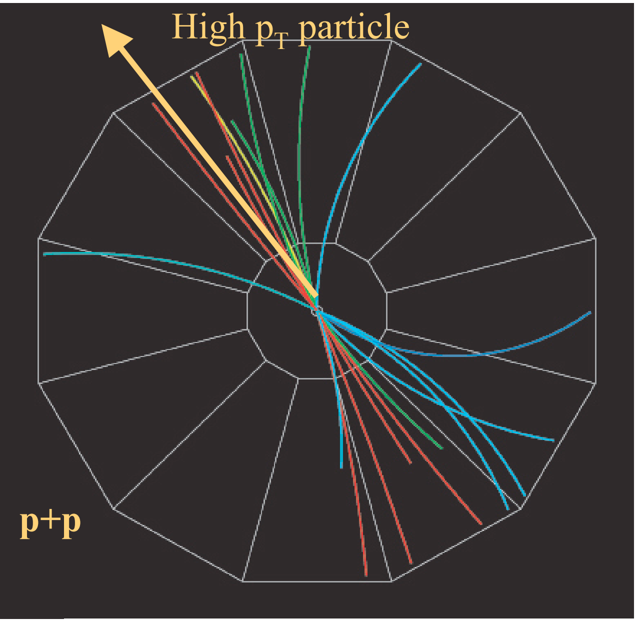
$$e^{-5.6\,p_T}$$



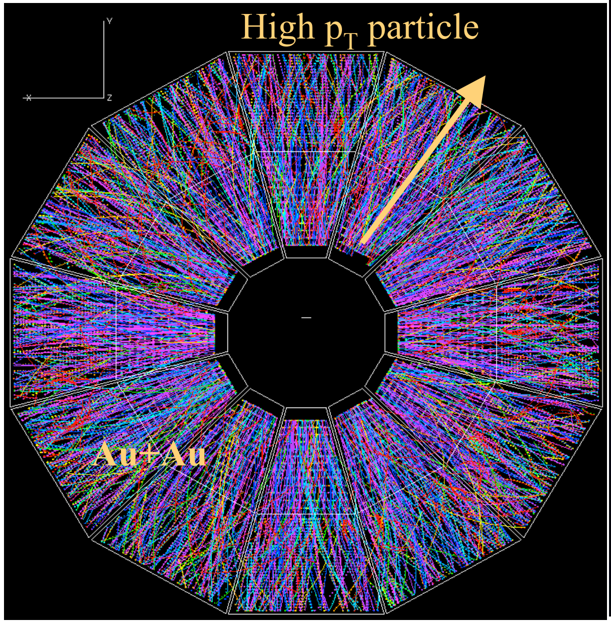
Power Law ⇒ Hard Scattering (pQCD)

Au+Au Central Collisions cf. p-p

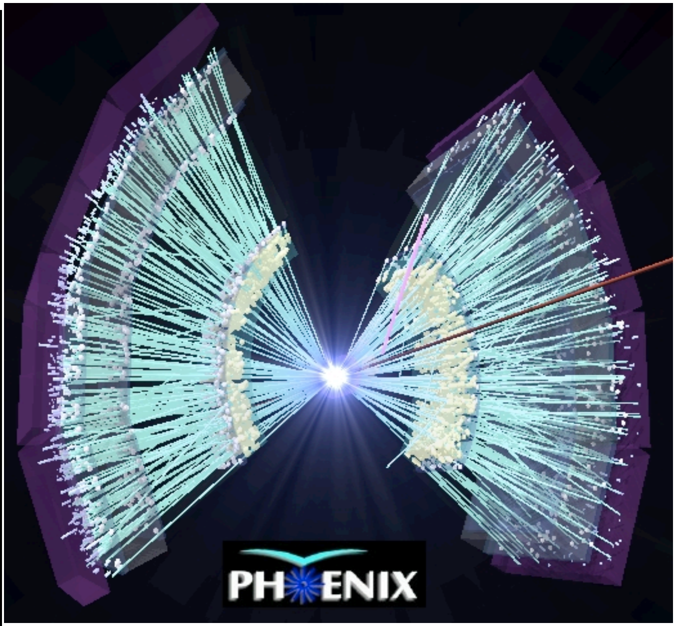
STAR-Jet event in pp

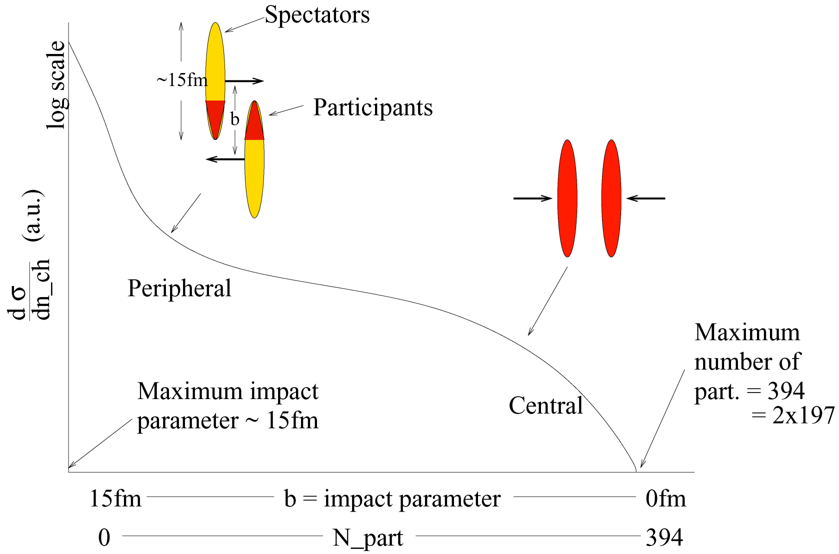


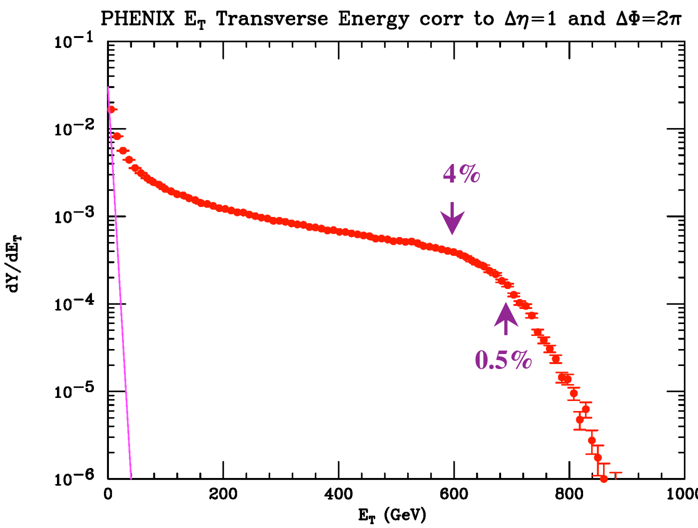
STAR Au+Au central



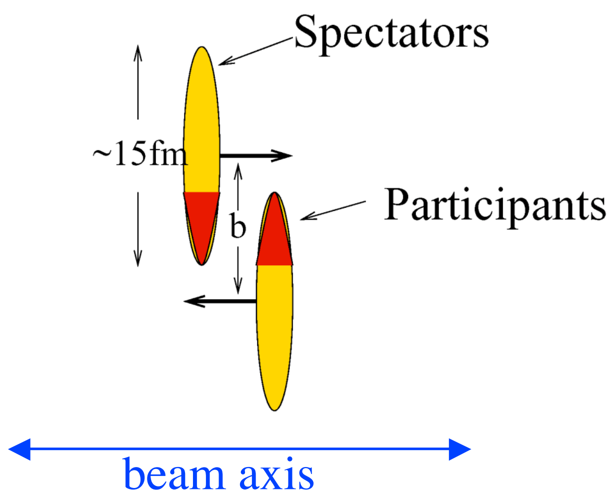
PHENIX Au+Au central

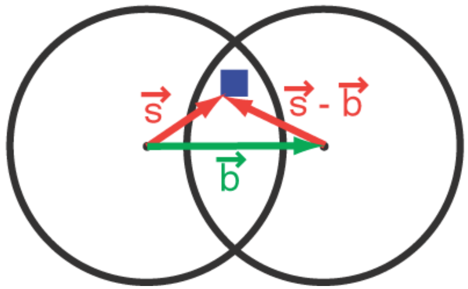






High p_T in A+B collisions---T_{AB} Scaling





view along beam axis

- For point-like processes, the cross section in p+A or A+B collisions compared to p-p is simply proportional to the relative number of pointlike encounters
  - ✓ A for p+A, AB for A+B for the total rate
  - ✓ T_{AB} the overlap integral of the nuclear profile functions, as a function of impact parameter b

Latest π^0 Au+Au arXiv:0801.4020

Power Law p_T>3GeV/c all centralities n=8.10±0.05

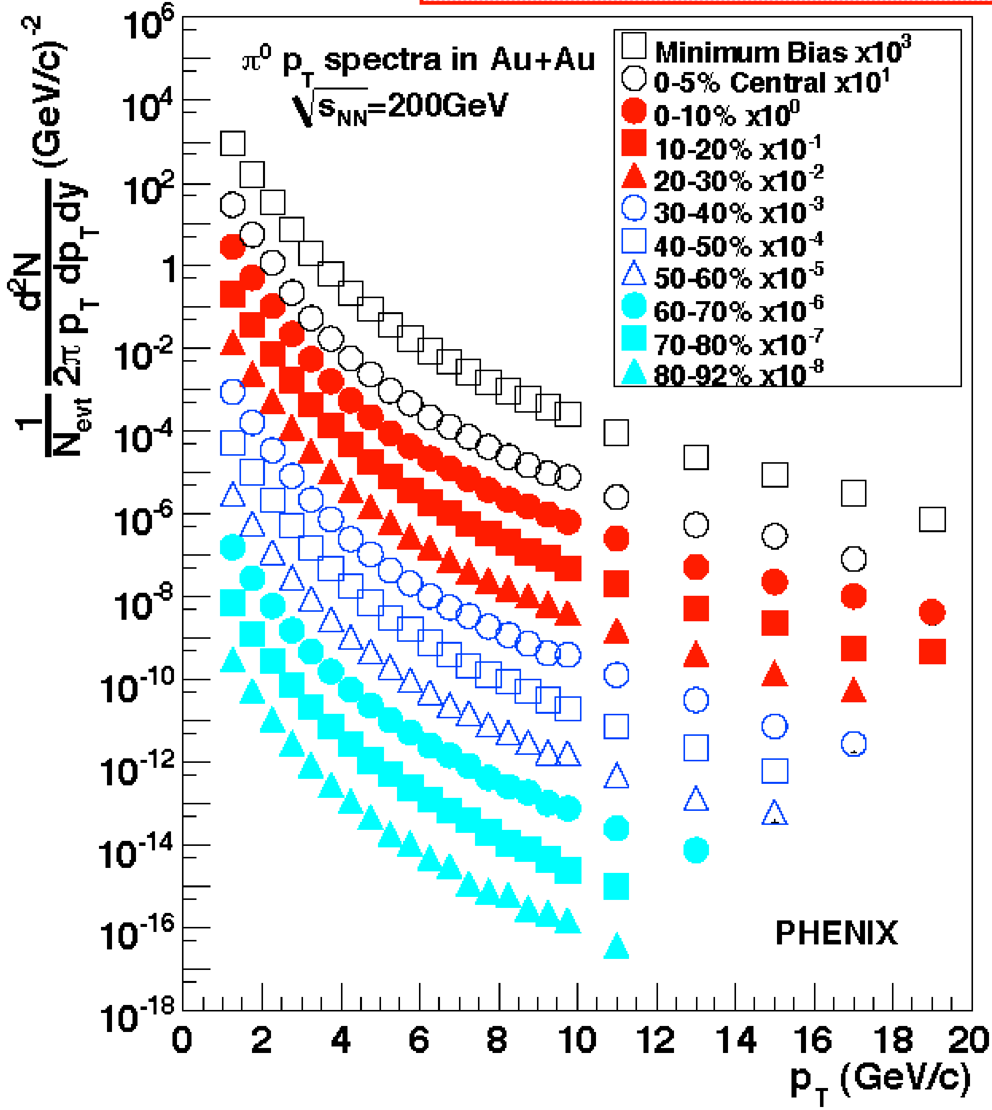
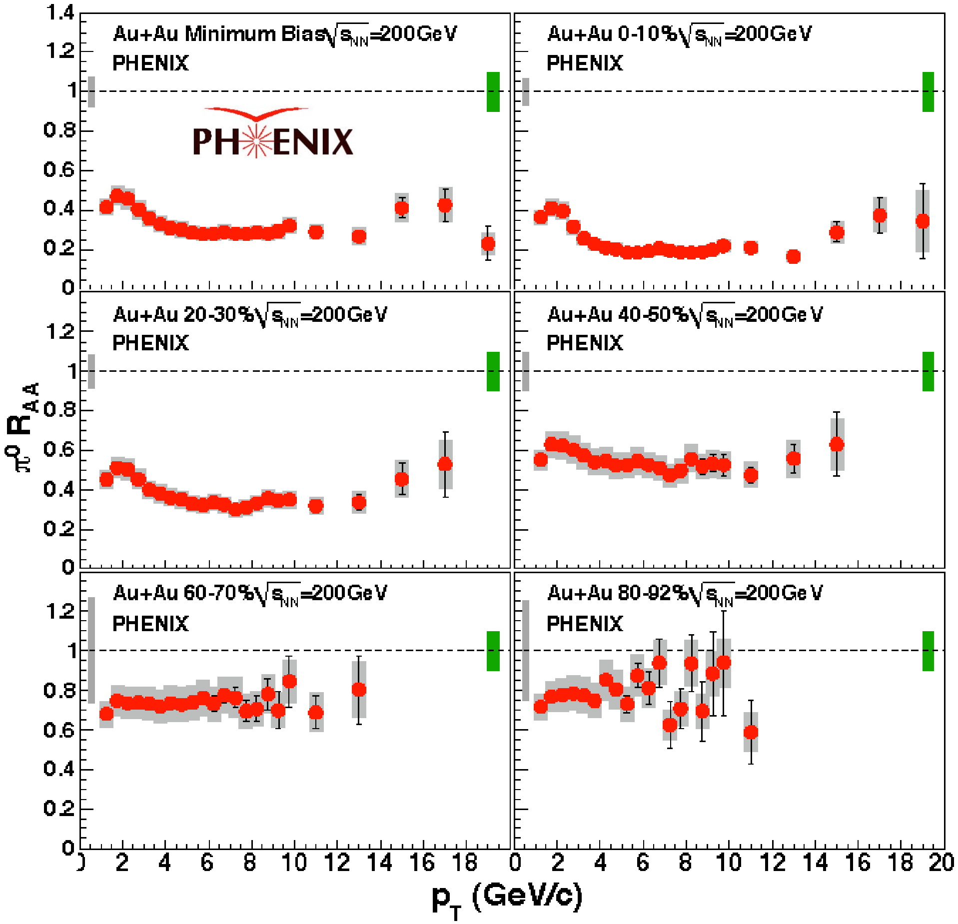


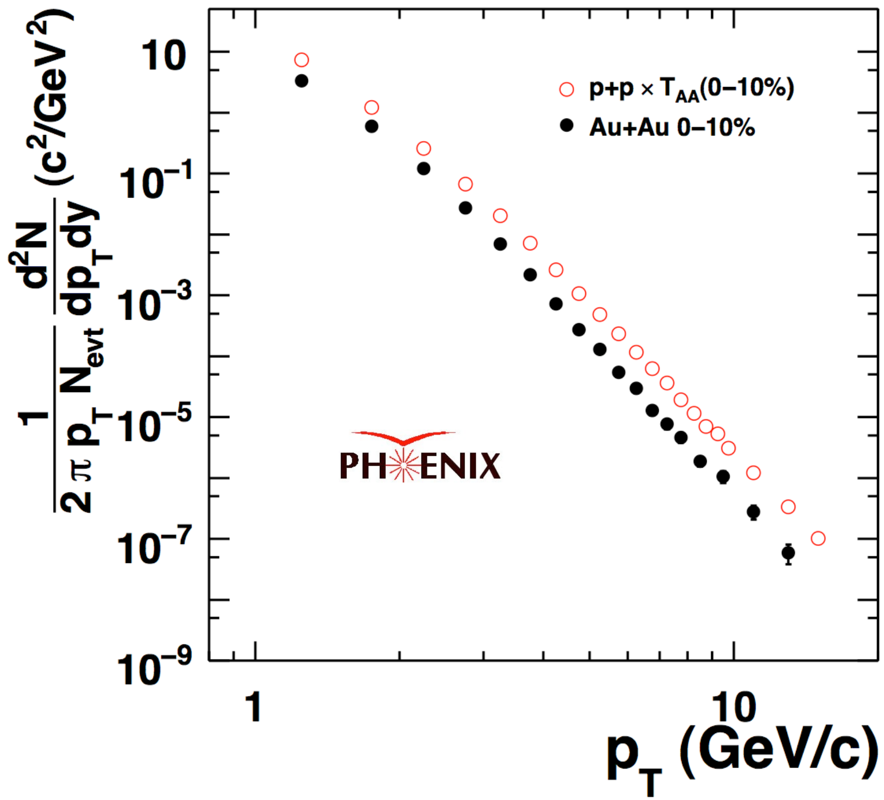
Table 5: Fit parameters for p_T>3 GeV/c

System	A	n	χ^2/NDF
p+p	14.61±1.45	8.12±0.05	5.68/17
Au+Au 0-5 %	81.18±10.30	8.20±0.07	9.66/16
Au+Au 0-10 %	75.28±8.89	8.18±0.06	10.62/17
Au+Au 10-20 %	64.62±7.64	8.19±0.06	10.04/17
Au+Au 20-30 %	49.33±5.78	8.18±0.06	6.63/16
Au+Au 30-40 %	30.85±3.53	8.10±0.06	10.63/16
Au+Au 40-50 %	22.58±2.61	8.13±0.06	3.50/15
Au+Au 50-60 %	12.40±1.48	8.06±0.07	8.09/15
Au+Au 60-70 %	6.25±0.78	8.03±0.07	2.89/14
Au+Au 70-80 %	3.38±0.45	8.12±0.08	8.42/13
Au+Au 80-92 %	1.19±0.18	8.03±0.09	9.84/13
Au+Au 0-92 %	29.31±3.07	8.17±0.05	6.83/17

Suppression of π^0 is arguably the major discovery at RHIC. Energy loss in medium?

Au Au $\sqrt{s_{NN}}$=200 GeV arXiv:0801.4020





Original π^0 discovery, PHENIX PRL **88** (2002)022301

$$R_{AA}(p_T) = \frac{d^2N^{\pi}_{AA}/dp_T dy N^{inel}_{AA}}{\langle T_{AA}\rangle d^2\sigma^{\pi}_{pp}/dp_T dy}$$

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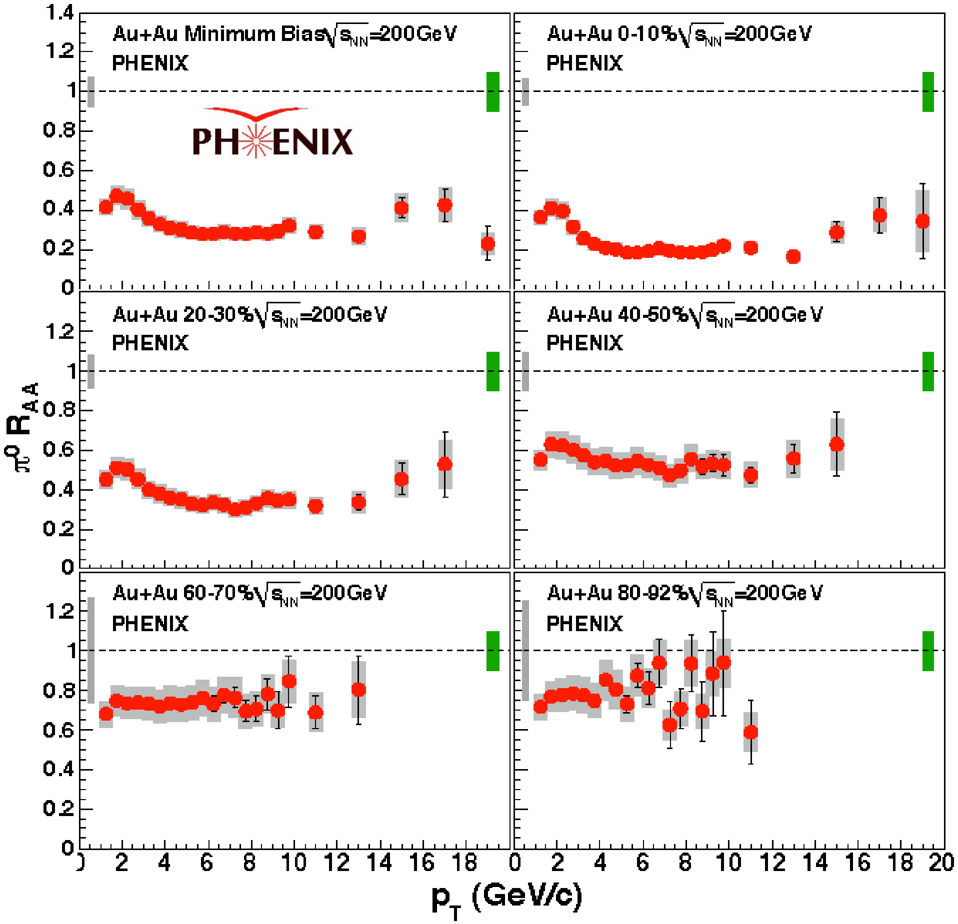

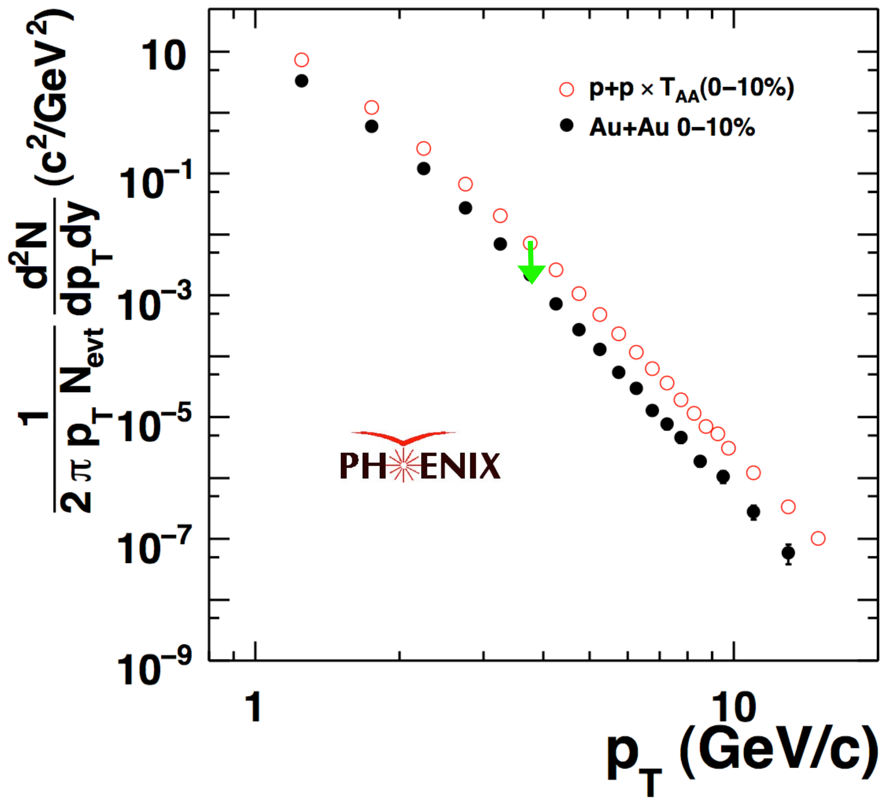

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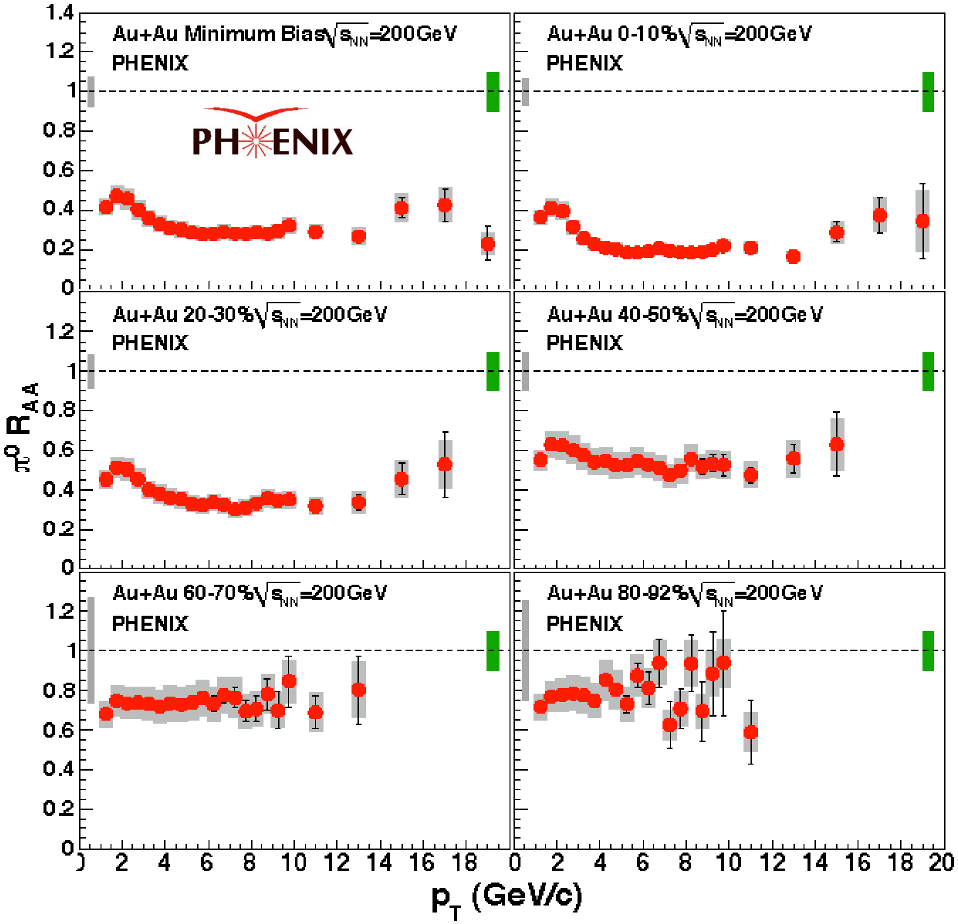

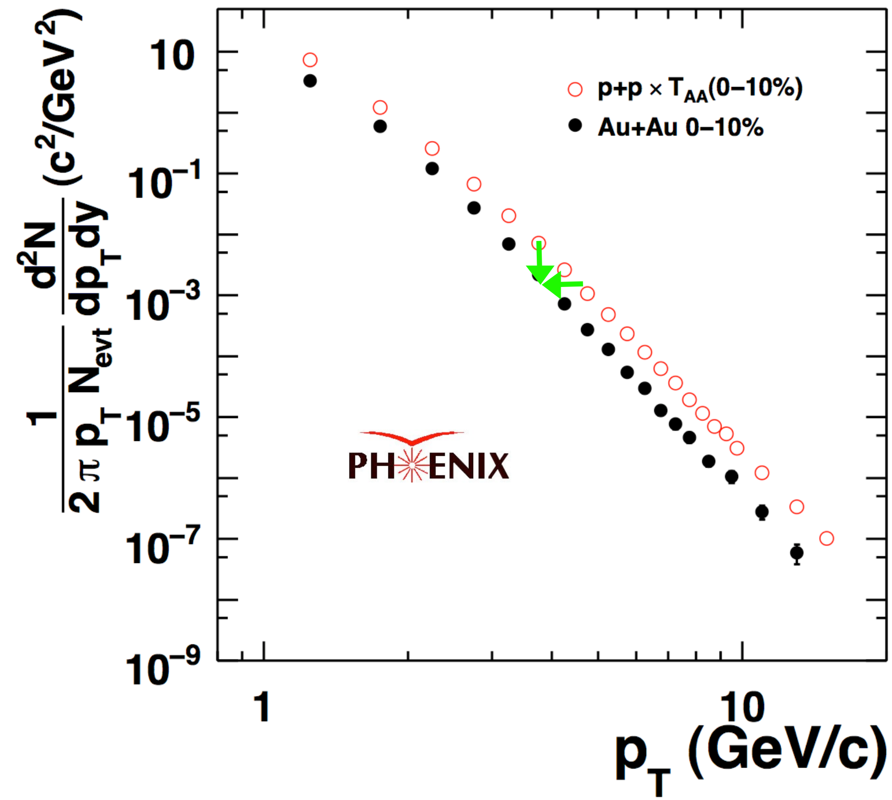

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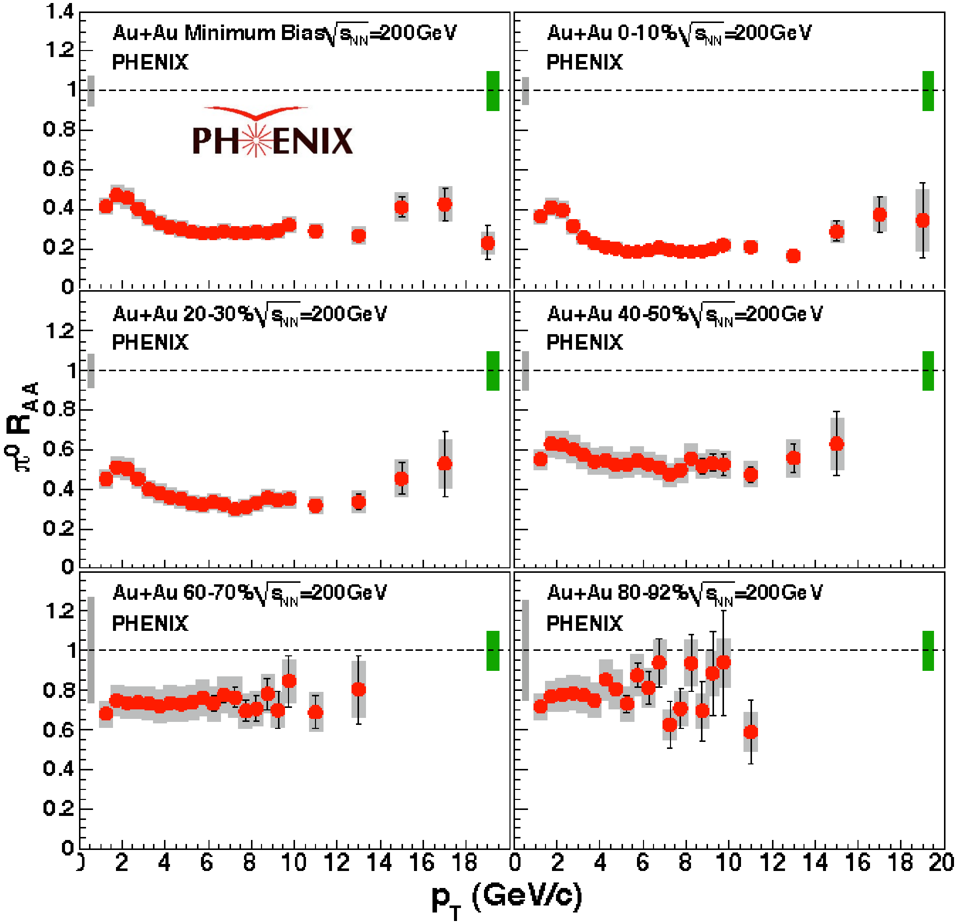
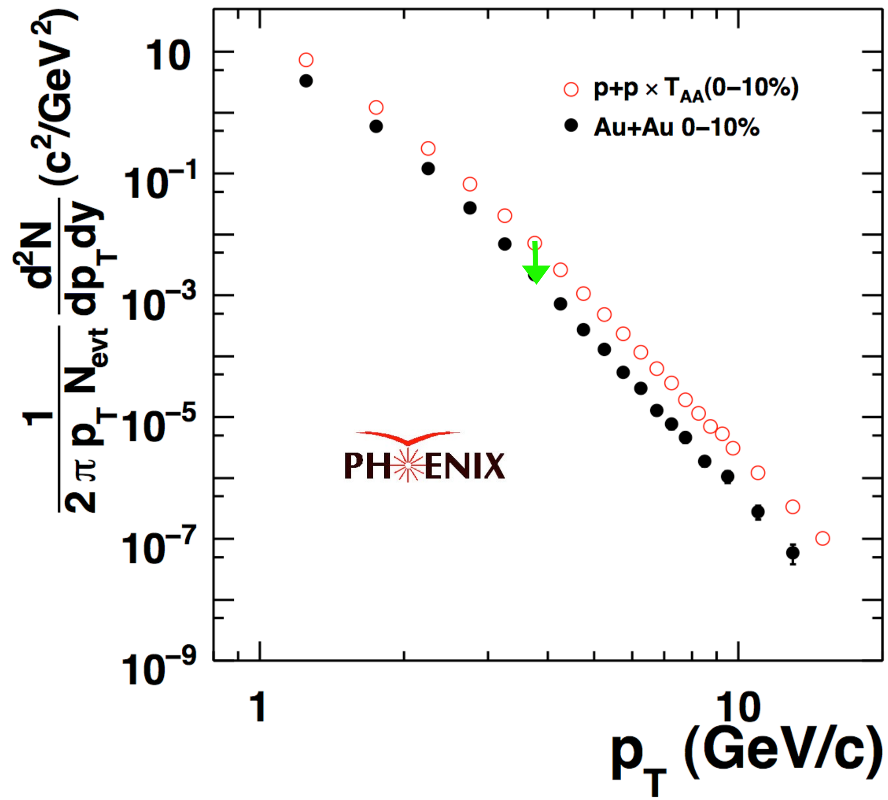




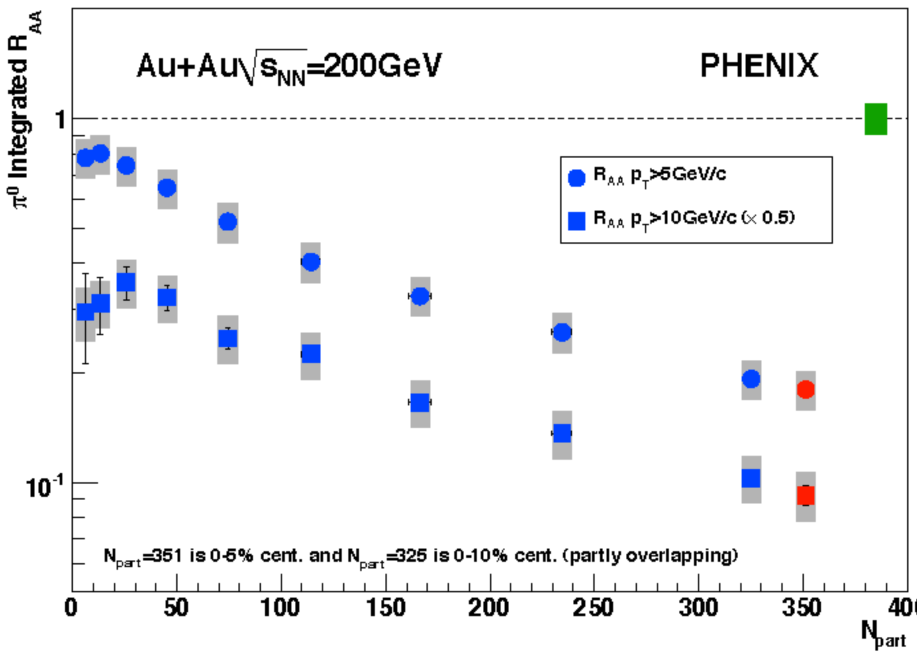
Nuclear Modification Factor

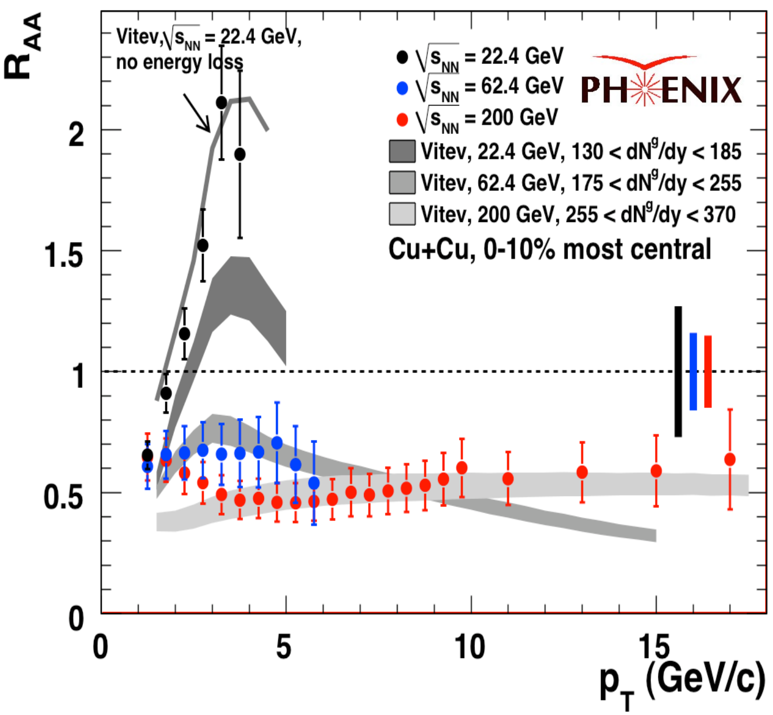
$$R_{AA}(p_T) = \frac{d^2N^{\pi}_{AA}/dp_T dy N^{inel}_{AA}}{\langle T_{AA}\rangle d^2\sigma^{\pi}_{pp}/dp_T dy}$$

Original π^0 discovery, PHENIX PRL **88** (2002)022301

AuAu $\sqrt{s_{NN}}$=200 GeV R_{AA} vs centrality

CuCu central 10% R_{AA} vs $\sqrt{s_{NN}}$





R_{AA} doesn't saturate with increasing centrality, continues to decrease to 5%, N_{part}=351

Suppression starts somewhere $22.4 < \sqrt{s_{NN}} \leq 62.4$ GeV

PHENIX, arXiv:0801.4555 [nucl-ex]

LO-QCD in 1 slide

Cross Section in p-p collisions c.m. energy $\sqrt{s}$

The overall p-p reaction cross section
is the sum over constituent reactions

$$a + b \rightarrow c + d$$

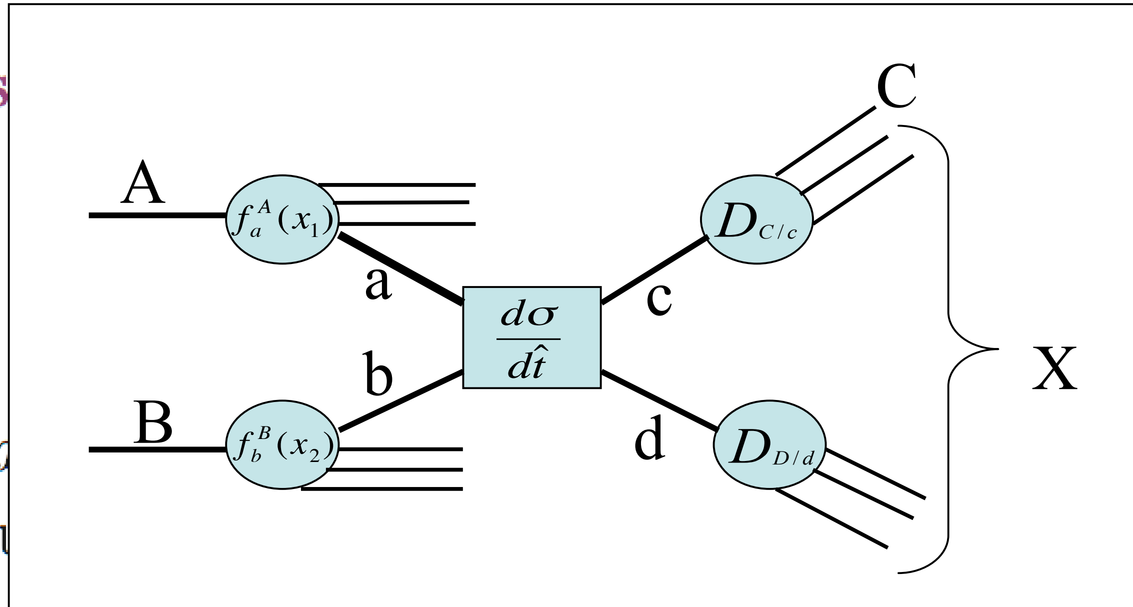
$f_a^A(x_1)$, $f_b^B(x_2)$, are structure functions, the differential probabilities for constituents a and b to carry momentum fractions x_1 and x_2 of their respective protons, e.g. $u(x_1)$,

$$\frac{d^3\sigma}{dx_1 dx_2 d\cos\theta^*} = \frac{1}{s}\sum_{ab} f_a^A(x_1) f_b^B(x_2) \frac{\pi\alpha_s^2(Q^2)}{2x_1x_2} \Sigma^{ab}(\cos\theta^*)$$

$\Sigma^{ab}(\cos\theta^*)$, the characteristic subprocess angular distributions and $\alpha_s(Q^2) = \frac{12\pi}{25\ln(Q^2/\Lambda^2)}$ are predicted by QCD

LO-QCD in 1 slide

Cross ... gy $\sqrt{s}$



$f_a^A(x_1)$, $f_b^B($ ... probabilities

for constitu ... x_1 and x_2

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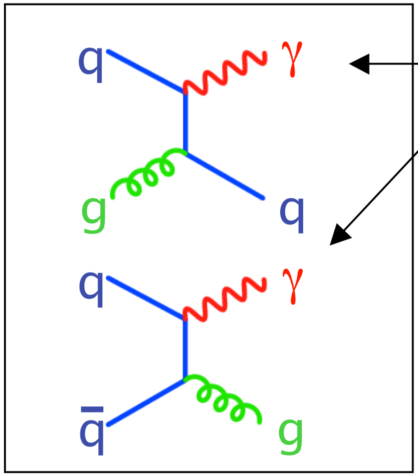
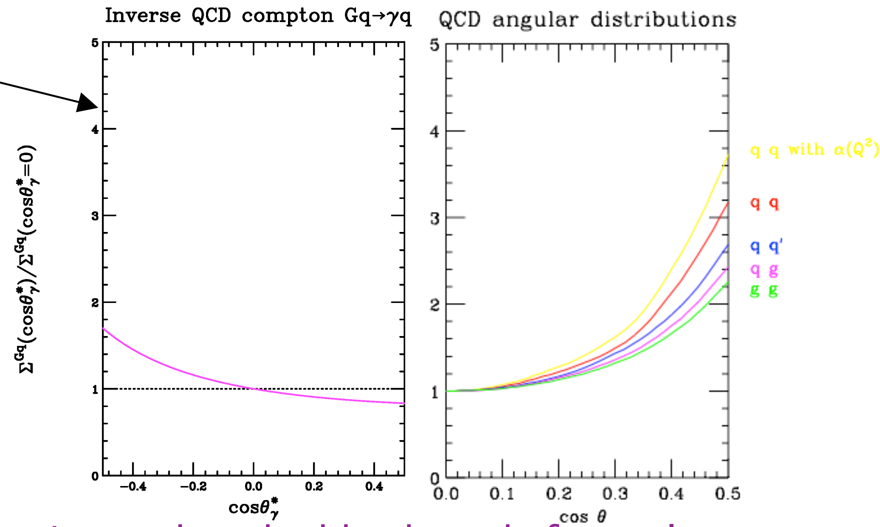
$\Sigma^{ab}(\cos\theta^*)$, the characteristic subprocess angular distributions and $\alpha_s(Q^2) = \frac{12\pi}{25\ln(Q^2/\Lambda^2)}$ are predicted by QCD

Direct photon production-simple theory hard experiment

See the classic paper of Fritzsch and Minkowski, PLB **69** (1977) 316-320





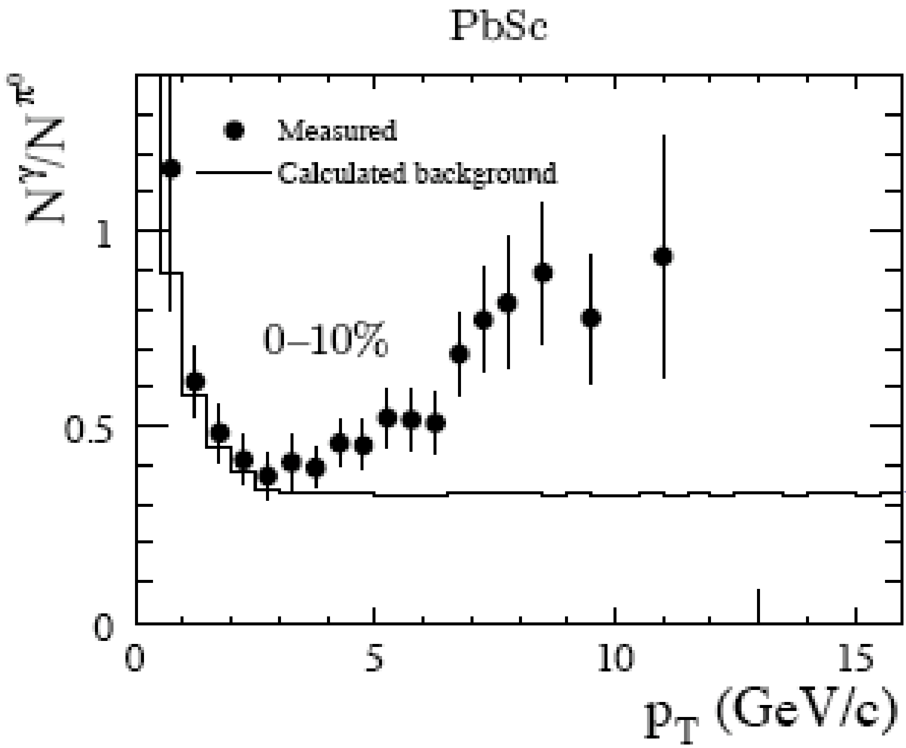
Compton distribution is much flatter than scattering and peaked backwards from gluon

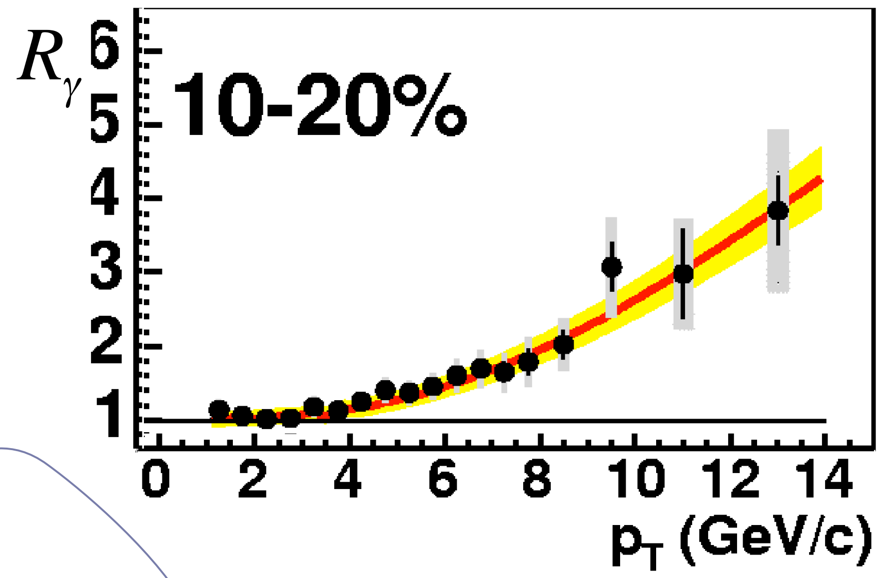
$$\Sigma^{gq}(\cos\theta^*) = \frac{\alpha\, e_q^2}{3\,\alpha_s}\left(\frac{1+\cos\theta^*}{2} + \frac{2}{1+\cos\theta^*}\right)$$

Substitution and Jacobean gives for a photon at p_T, y_c (and *parton* (jet) at p_T, y_d):

$$\frac{d^3\sigma}{dp_T^2\, dy_c\, dy_d} = x_1 f_g^A(x_1)\, F_{2B}(x_2, Q^2)\frac{\pi\alpha\alpha_s(Q^2)}{3\hat{s}^2}\left(\frac{1+\cos\theta^*}{2} + \frac{2}{1+\cos\theta^*}\right)$$
$$+\, F_{2A}(x_1, Q^2)\, x_2 f_g^B(x_2)\frac{\pi\alpha\alpha_s(Q^2)}{3\hat{s}^2}\left(\frac{1-\cos\theta^*}{2} + \frac{2}{1-\cos\theta^*}\right)$$

Experimental problem is HUGE background from $\pi^0 \rightarrow \gamma\gamma$, $\eta \rightarrow \gamma\gamma$, etc. But this is less of a problem in Au+Au due to suppression of π^0





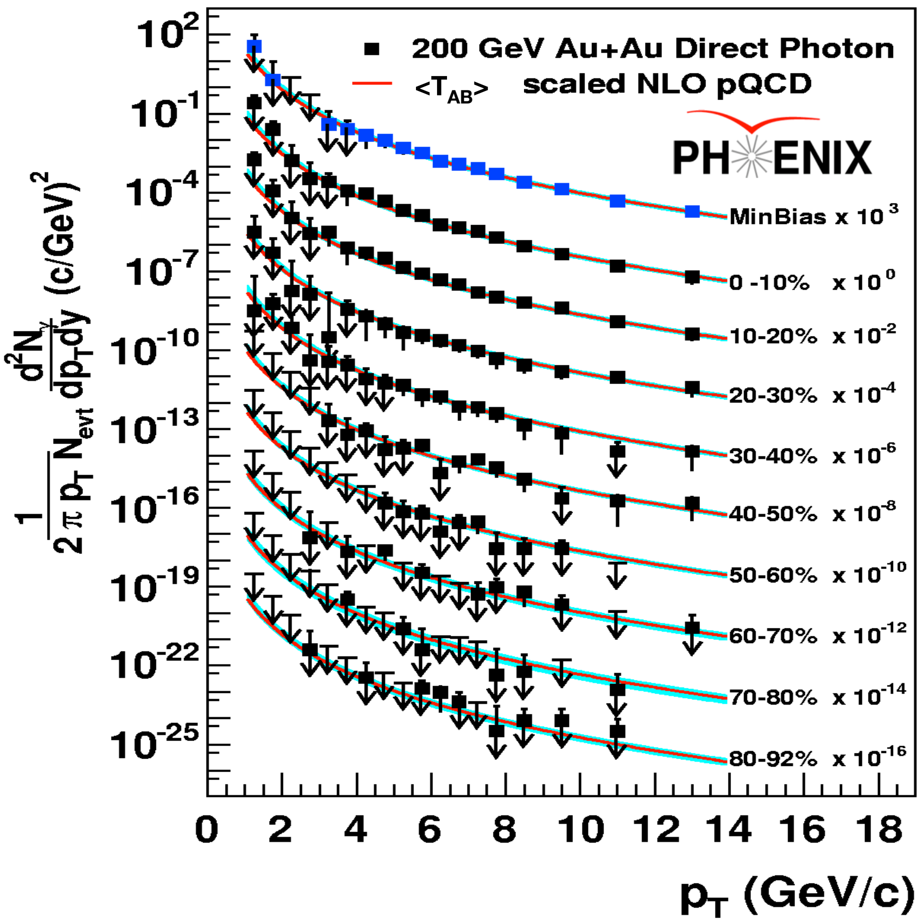
$$R_\gamma = \frac{(\gamma/\pi^0)_{\rm Measured}}{(\gamma/\pi^0)_{\rm Background}} \approx \frac{\gamma_{\rm Measured}}{\gamma_{\rm Background}}$$

If $\quad \dfrac{dn_{\pi^0}}{p_T dp_T} \propto p_T^{-n} \quad$ then $\quad \dfrac{\gamma}{\pi^0}\Big|_{\pi^0}(p_T) = 2/(n-1)$.

Since the the photons from $\pi^0 \rightarrow \gamma+\gamma$, $\eta \rightarrow \gamma+\gamma$ and similar decays are the principal background to direct photon production, the importance of a precise estimate of this background can not be overstated.

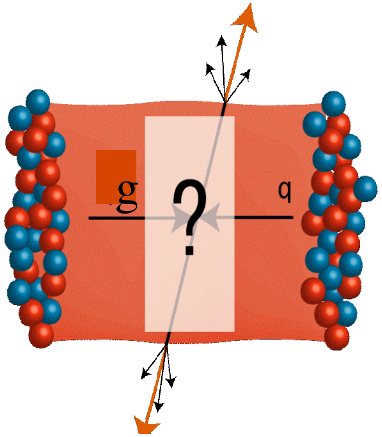
2004--Direct Photons in Au+Au 200 GeV: follow T_{AB} scaling from p-p for all centralities-no suppression

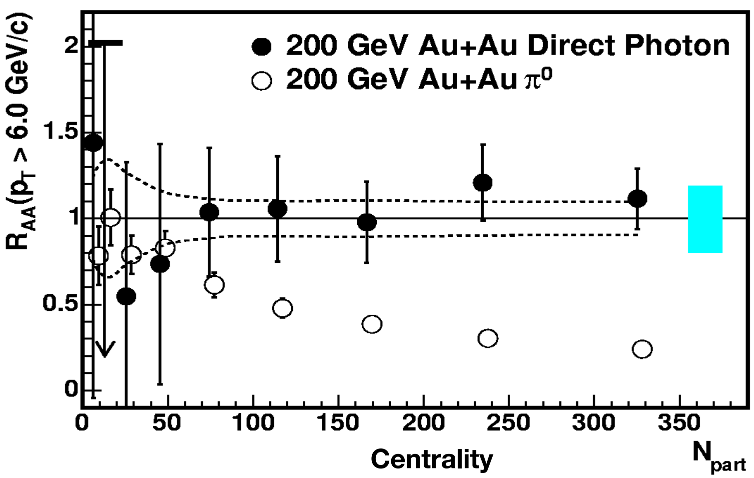
1) Proves that initial state Au structure function is simply a superposition of p-p structure functions including g(x).



PHENIX PRL**94** (2005) 232301

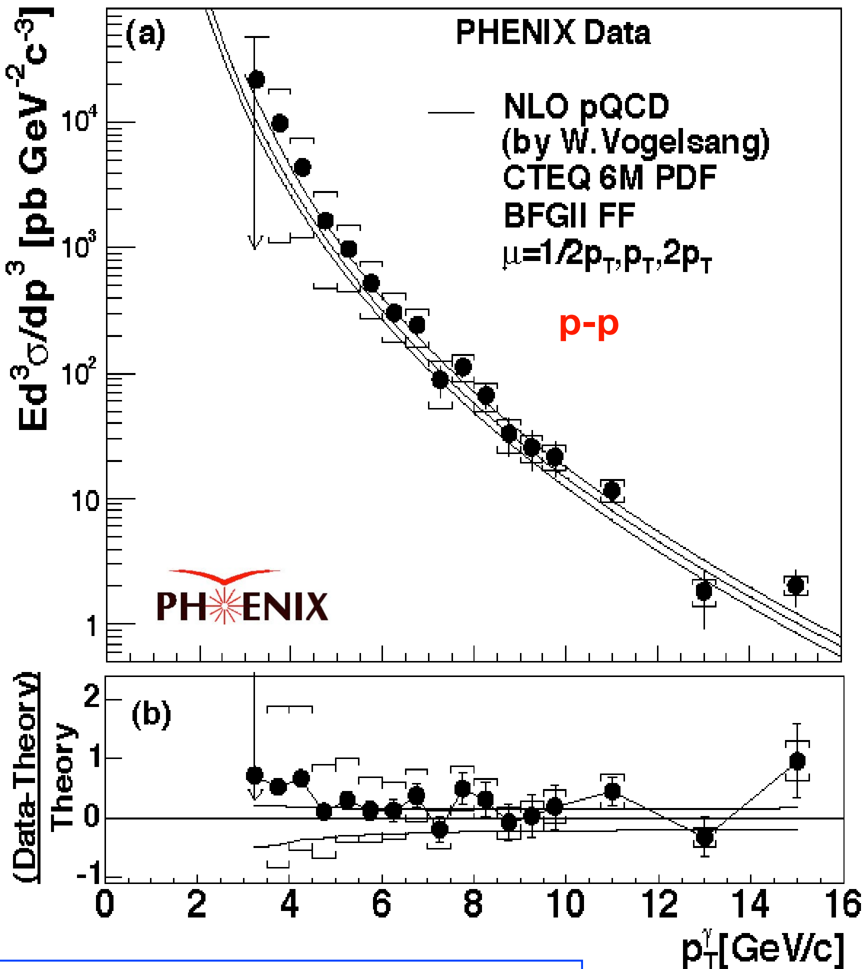
Au+Au

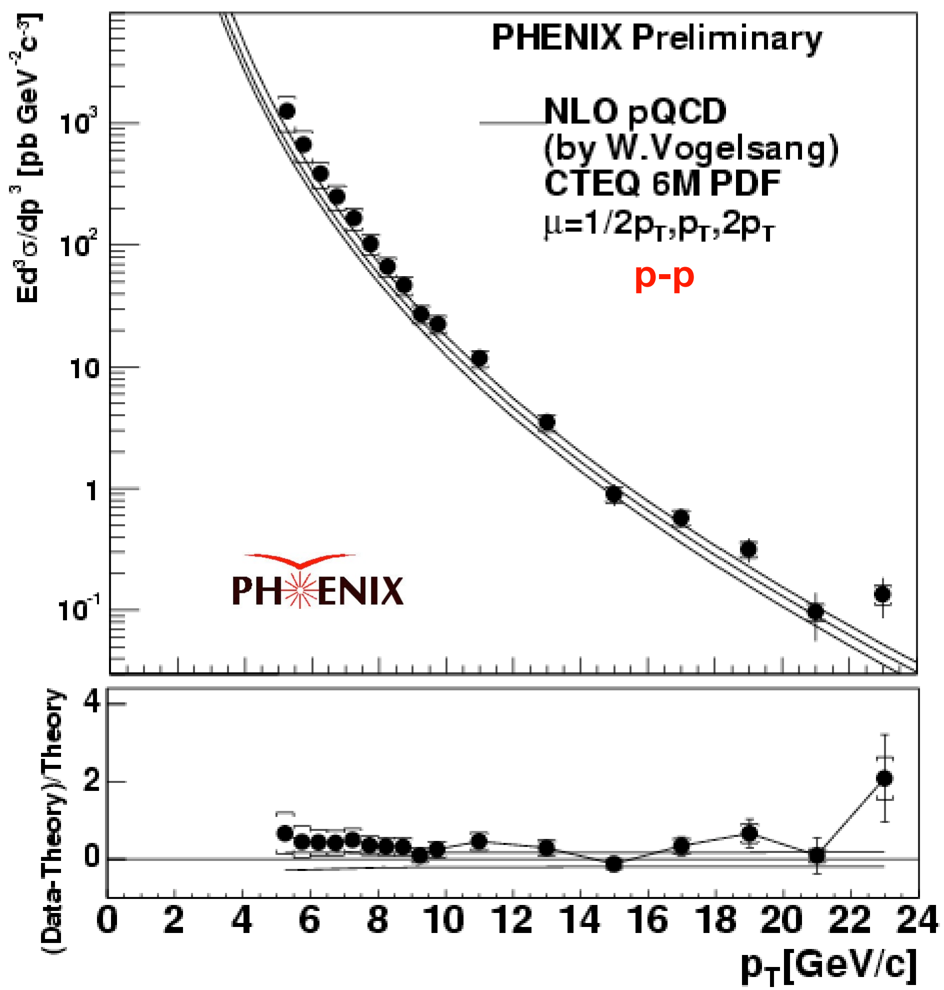




Direct photons unaffected by QCD medium in Au+Au →
π^0 suppression is medium effect

Direct γ's in p-p √s=200 GeV: Data vs. pQCD





PHENIX PRL **98** (2007) 012002

Published results $3<p_T<15$ GeV/c

Preliminary results for $5<p_T<24$ GeV/c

Direct γ's in p-p are Isolated

PHENIX PRL **98** (2007) 012002

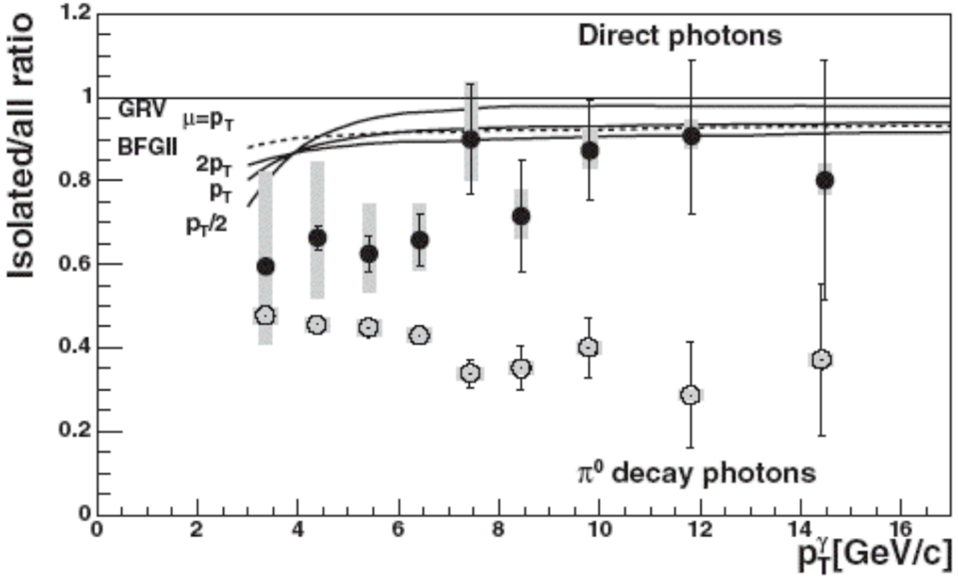
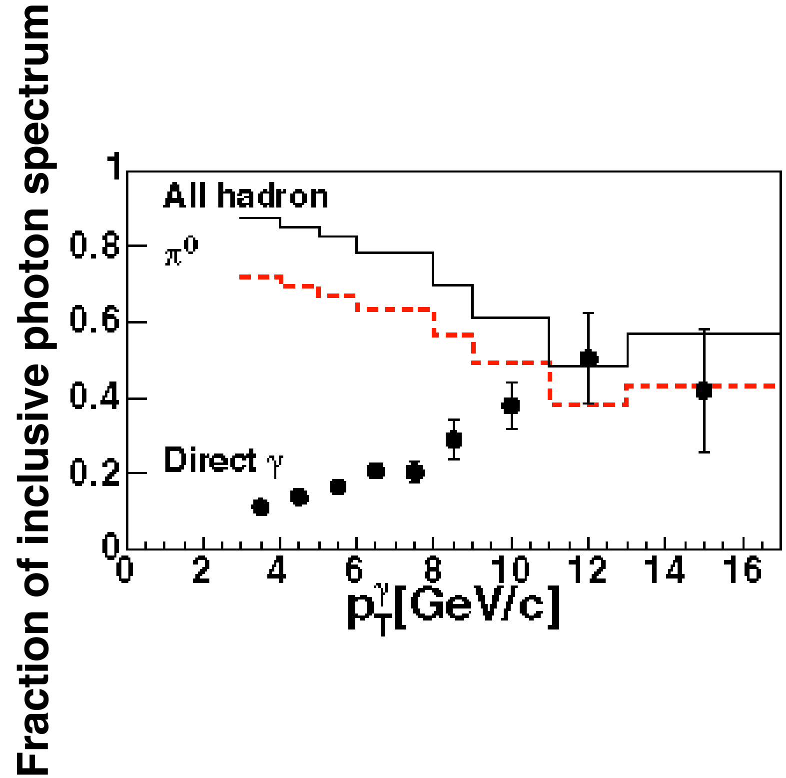


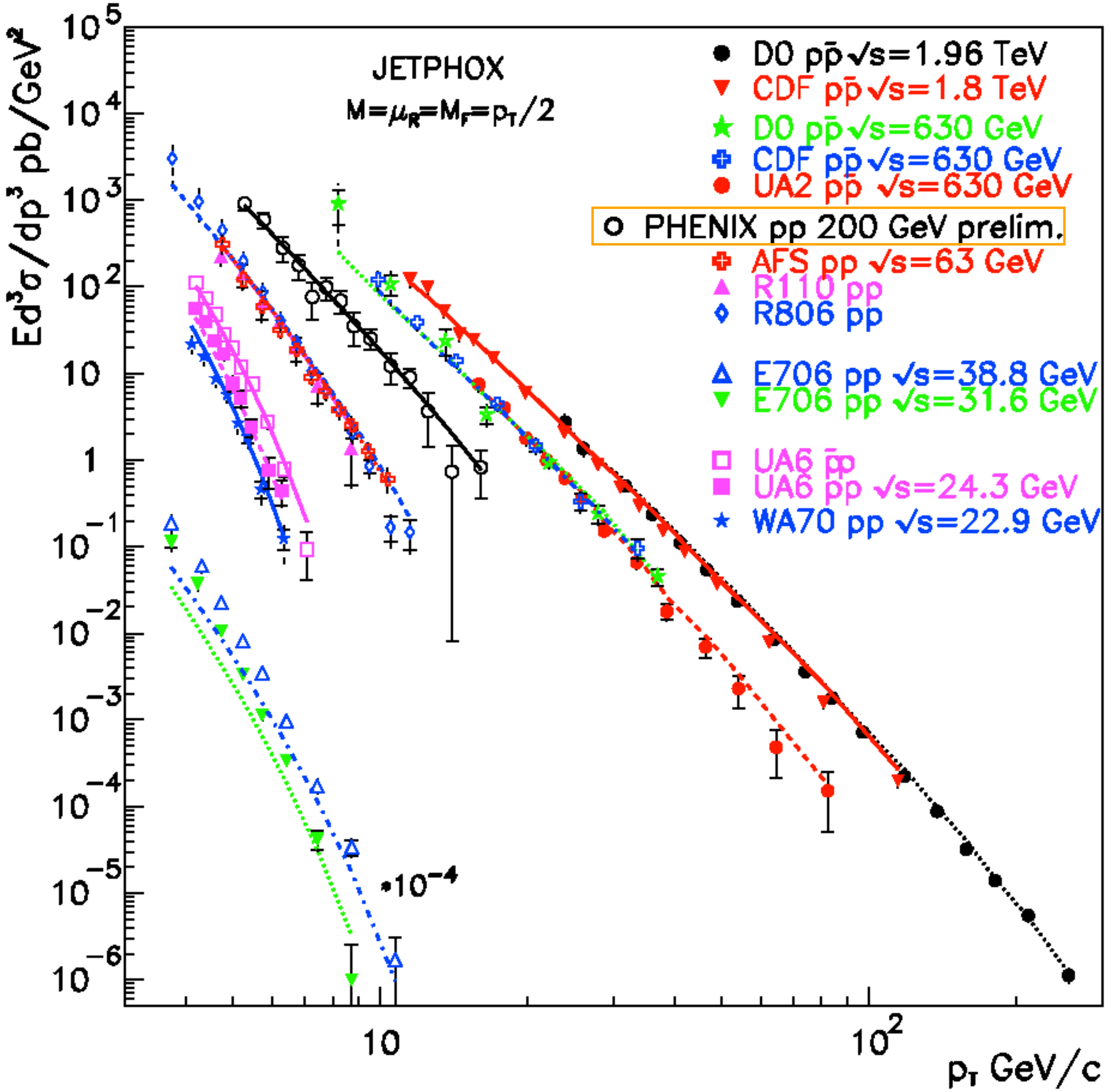
FIG. 3. Solid circles: Ratio of isolated direct photons to all direct photons from the π^0-tagging method. The statistical uncertainties are shown as black error bars and the systematic uncertainties are plotted as shaded bars. The solid and dashed curves are NLO pQCD calculations with three theory scales for BFGII [21] and one scale for GRV [25] parton to photon fragmentation functions. Open circles: Ratio of isolated photons from π^0 decays to all photons from π^0 decays.



Fragmentation photons <10% of direct γ for $p_T > 6$ GeV/c in agreement with GRV

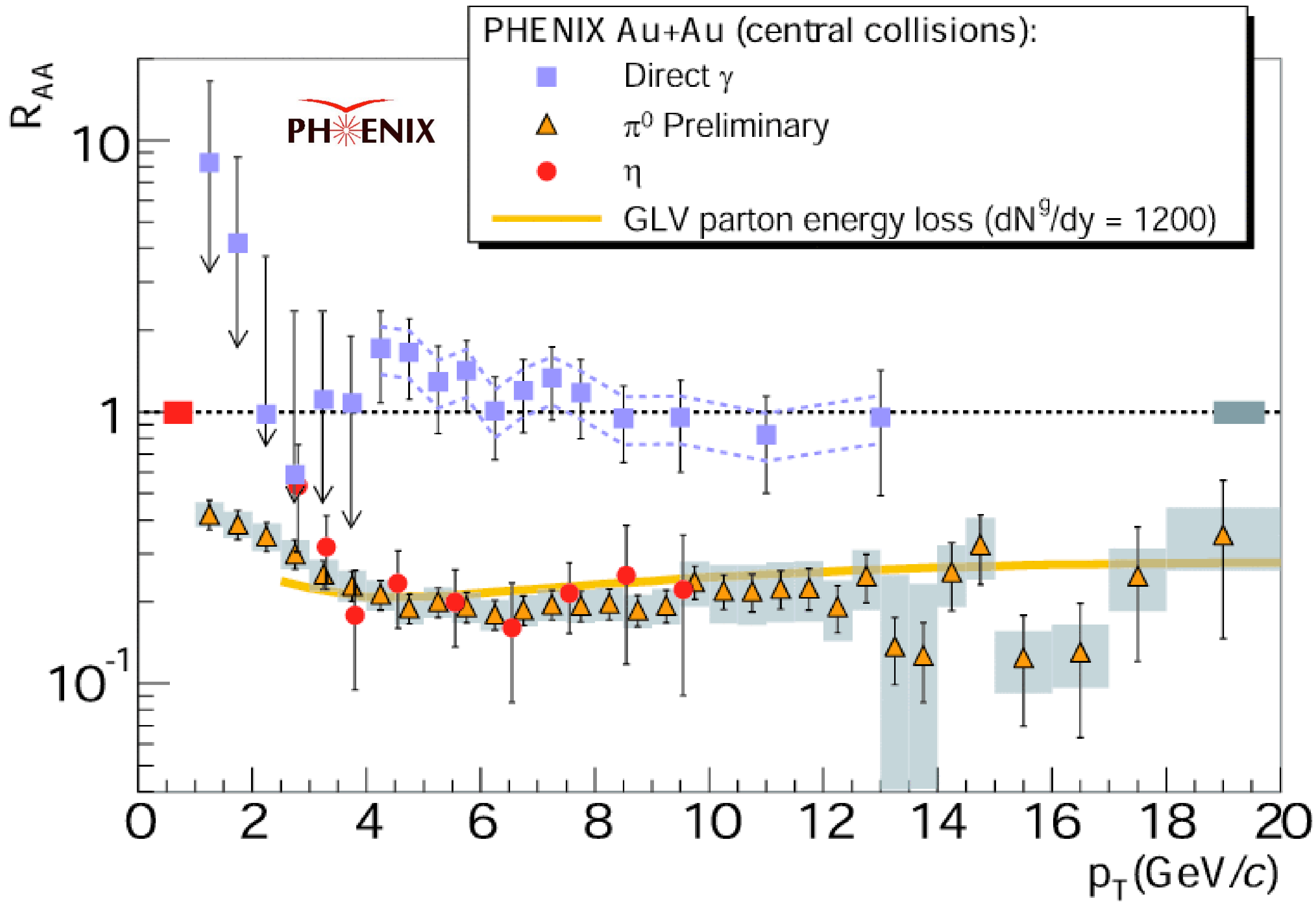
Comparison with other data and pQCD

P. Aurenche et al Phys. Rev. D **73**, 094007 (2006)



PHENIX data clarifies longstanding data/theory puzzle

Status of R_{AA} in AuAu at $\sqrt{s_{NN}}$=200 GeV QM05



Direct γ are not suppressed. π^0 and η suppressed even at high p_T
Implies a strong medium effect (energy loss) since γ not affected.
Suppression is flat at high p_T. Are data flatter than theory?

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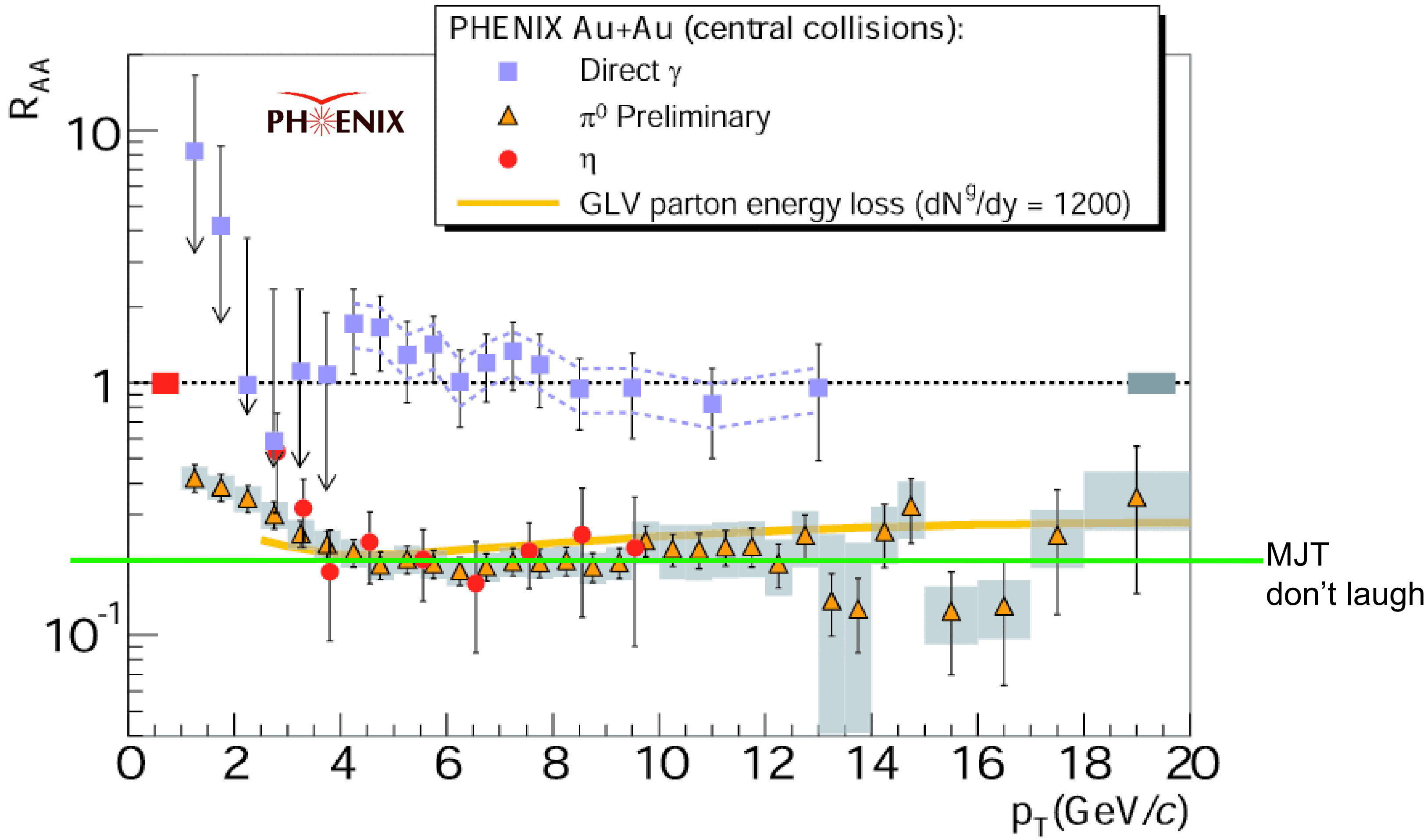

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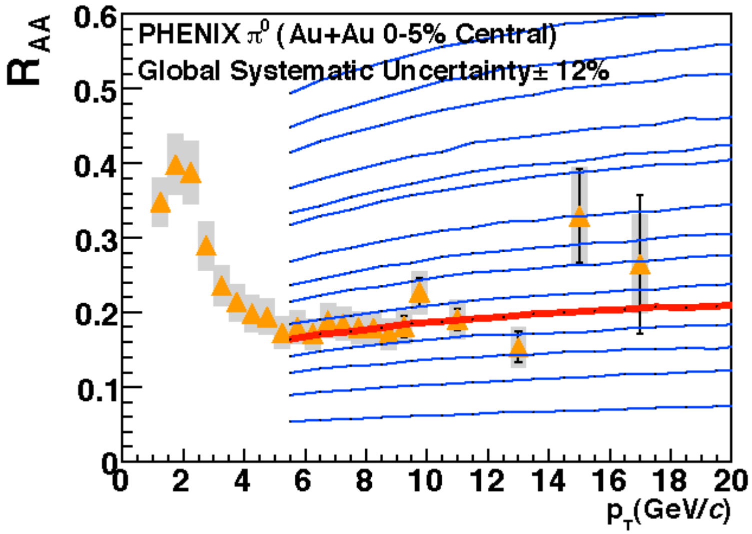
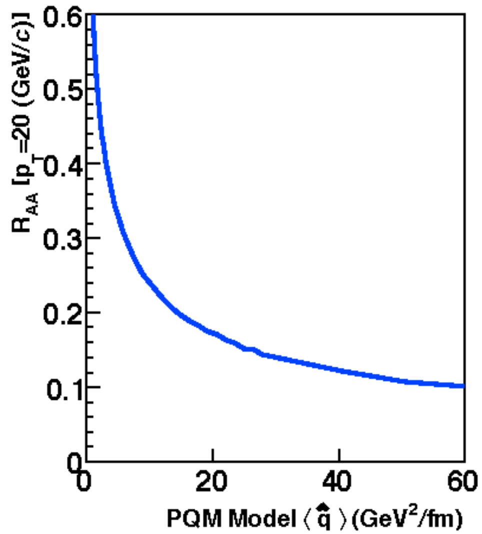
Comparison to Model(s) Including Systematic errors

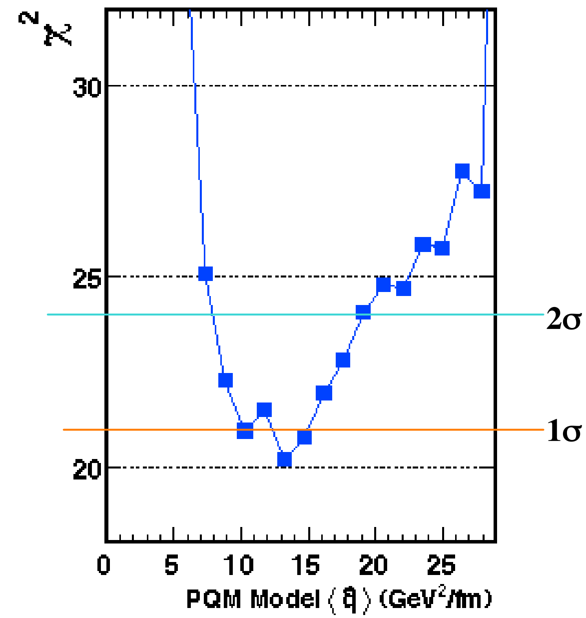
Many models explain R_{AA}. All have different assumptions about nuclear overlap geometry, medium expansion, parton propagation, etc, and use a parameter to characterize the medium. For example, we give a fit to the PQM model, Dainese, Loizides,Paic, EPJC**38**, 461 (2005)

The derived transport coefficient $\hat{q}$, the mean-4-momentum transfer²/mean free path, is strongly model dependent and under intense theoretical debate, e.g. see Baier,Schiff JHEP09(2006)059.

Fit by PHENIX including systematic errors arXiv:0801.1665







$$\tilde{\chi}^2 = \left[\sum_{i=1}^{n} \frac{(y_i + \epsilon_b \sigma_{b_i} + \epsilon_c y_i \sigma_c - \mu_i(p))^2}{\tilde{\sigma}_i^2} + \epsilon_b^2 + \epsilon_c^2\right]$$

$$\langle \hat{q} \rangle_{\rm pqm} = 13.2^{+3.1}_{-2.6}\ {\rm GeV}^2/{\rm fm}$$

$$\tilde{\sigma}_i = \sigma_i (y_i + \epsilon_b \sigma_{b_i} + \epsilon_c y_i \sigma_c)/y_i$$

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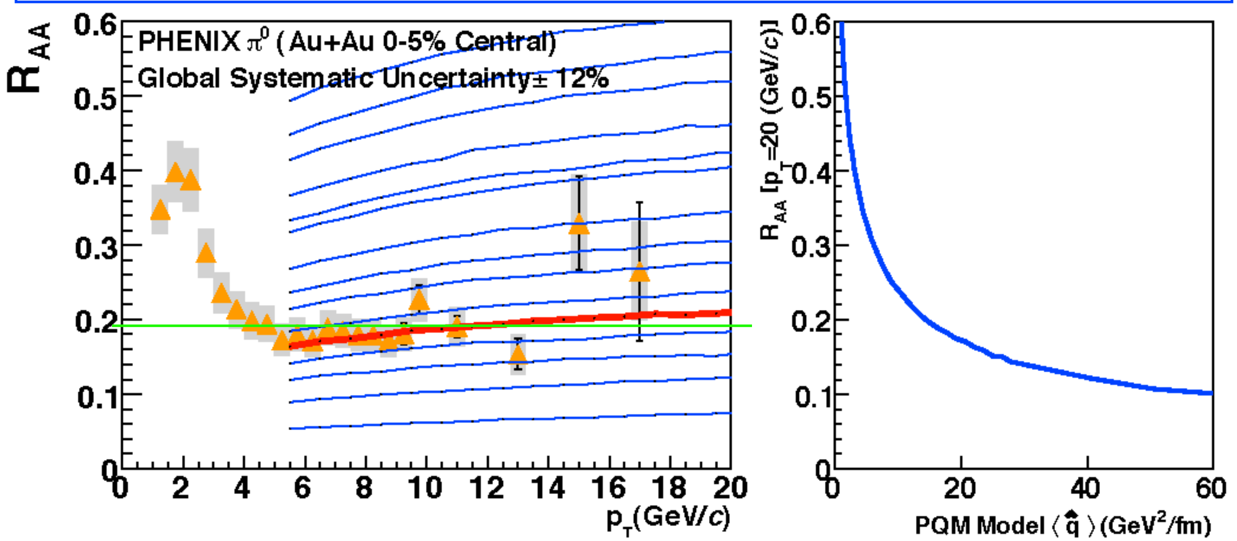
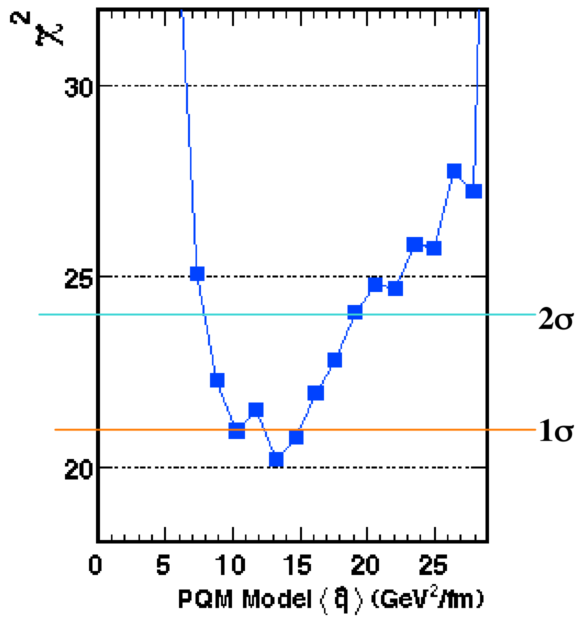




$$\tilde{\chi}^2 = \left[\sum_{i=1}^{n} \frac{(y_i + \epsilon_b \sigma_{b_i} + \epsilon_c y_i \sigma_c - \mu_i(p))^2}{\tilde{\sigma}_i^2} + \epsilon_b^2 + \epsilon_c^2\right]$$

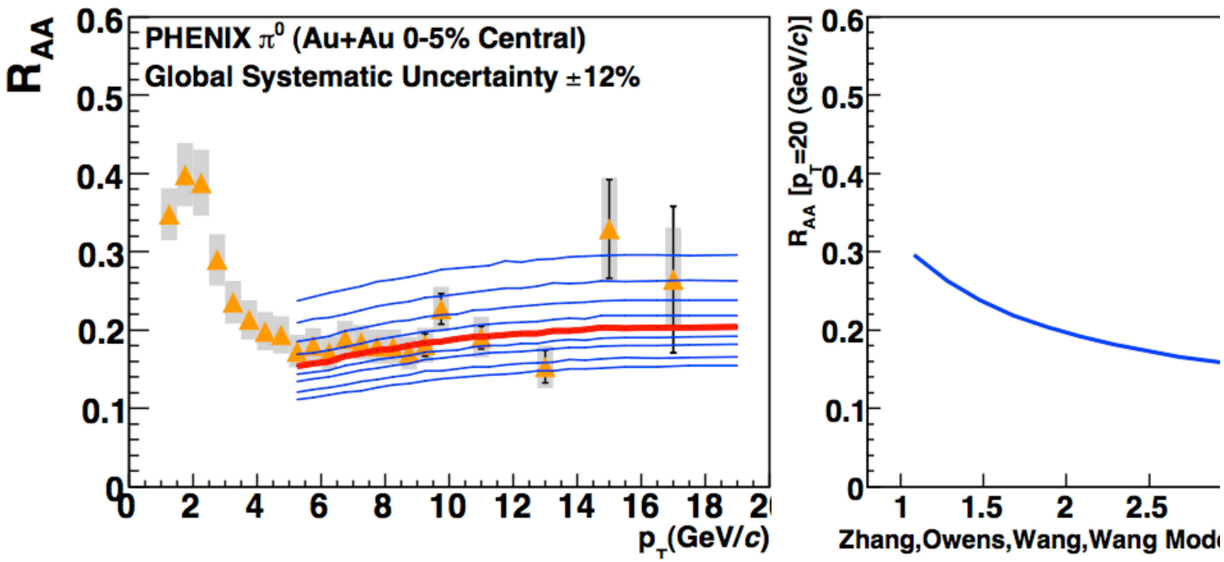
$$\tilde{\sigma}_i = \sigma_i(y_i + \epsilon_b \sigma_{b_i} + \epsilon_c y_i \sigma_c)/y_i$$

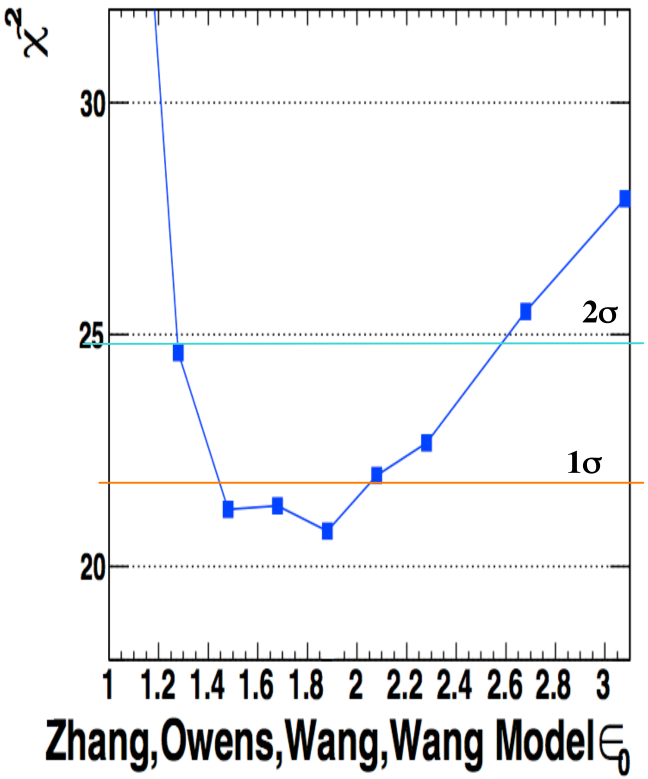
also consistent with:

$$R_{AA} = \text{const.} = 0.17 \pm 0.03$$

Zhang, Owens Wang, Wang Model

Zhang, Owens, Wang and Wang, PRL 98 (2007) 212301
found in their model, ε_0=1.6-2.1 GeV/fm





Fit by PHENIX including systematic errors arXiv:0801.1665

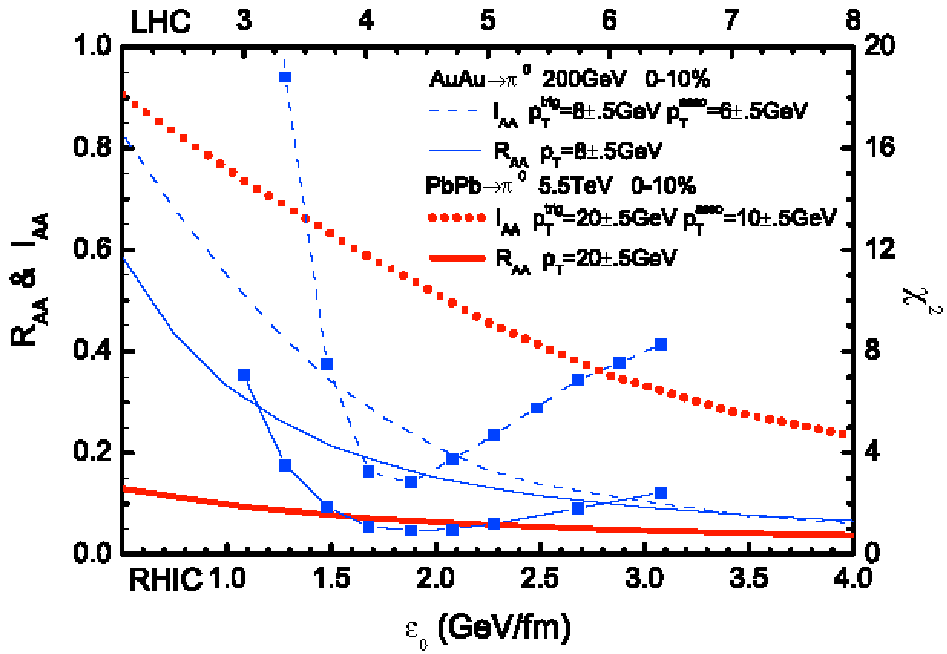
$$\tilde{\chi}^2 = \left[\sum_{i=1}^{n} \frac{(y_i + \epsilon_b \sigma_{b_i} + \epsilon_c y_i \sigma_c - \mu_i(p))^2}{\tilde{\sigma}_i^2} + \epsilon_b^2 + \epsilon_c^2\right]$$

$$\tilde{\sigma}_i = \sigma_i (y_i + \epsilon_b \sigma_{b_i} + \epsilon_c y_i \sigma_c)/y_i$$

$$\varepsilon_{0(\mathrm{ZOWW})} = 1.9^{+0.2}_{-0.5}\,\mathrm{GeV/fm}$$

Again a precision of 20-25% (1σ)

But-physics and statistics Issue

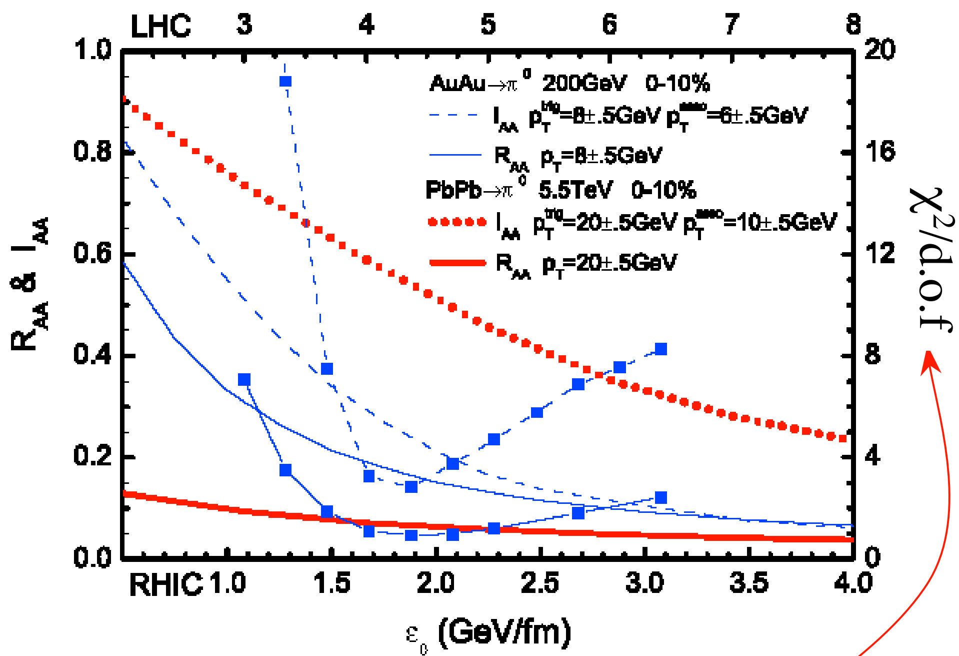


- can't get error on best fit from χ^2/d.o.f curves, need χ^2. N standard deviation errors on fit parameters are given by $\chi^2 = \chi^2_{min} + N^2$, so depending on d.o.f can't really tell from χ^2/d.o.f whether I_{AA} gives better constraint than R_{AA}
- However χ^2_{min}/d.o.f=2.8 for I_{AA} fit seems too large to be acceptable. (?)

FIG. 3 (color online). The suppression factors for single (R_{AA}), dihadron (I_{AA}) spectra at fixed transverse momentum and χ^2/d.o.f. (curves with filled squares) in fitting experimental data on single [16] ($p_T = 4$–20 GeV/c) and away-side spectra [17] ($p_T^{\mathrm{trig}} = 8$–15 GeV, $z_T = 0.45$–0.95) in central Au + Au

Zhang, Owens, Wang and Wang, PRL 98 (2007) 212301 found in their model, ε_0=1.6-2.1 GeV/fm. Good, but...

But-physics and statistics Issue



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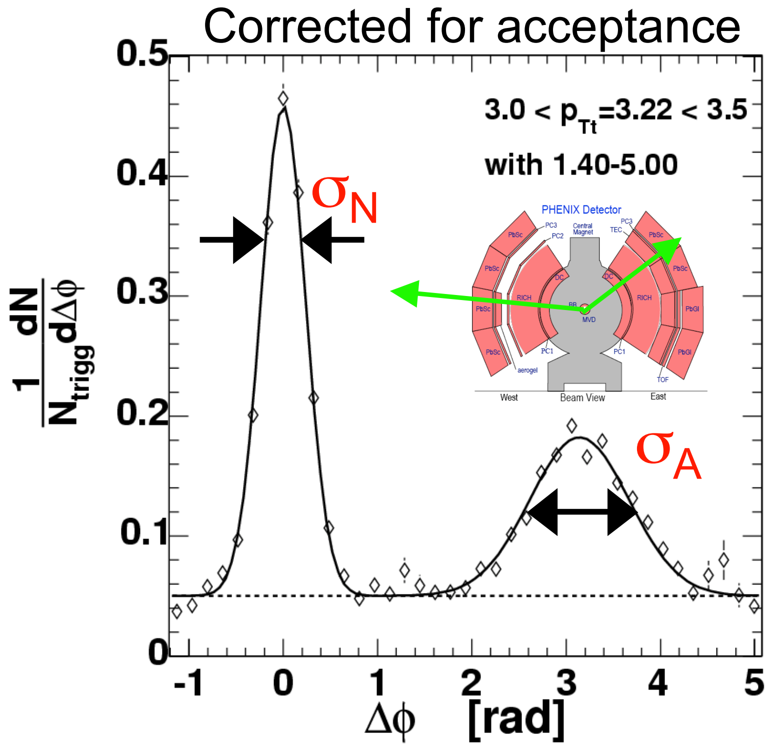
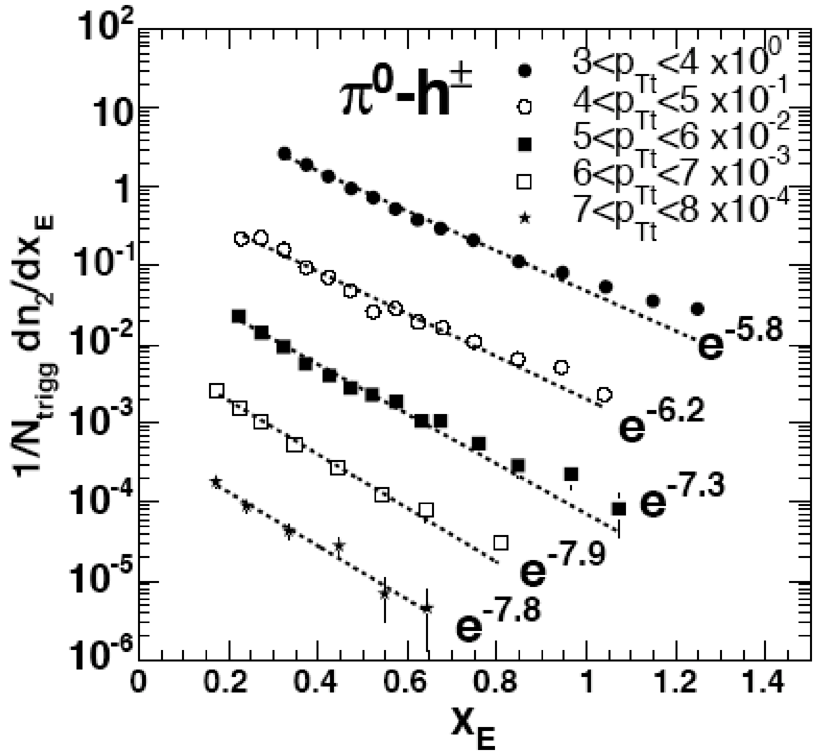
PHENIX π^o - $h^\pm$ correlation functions p+p $\sqrt{s}$=200 GeV: PRD **74**, 072002 (2006)

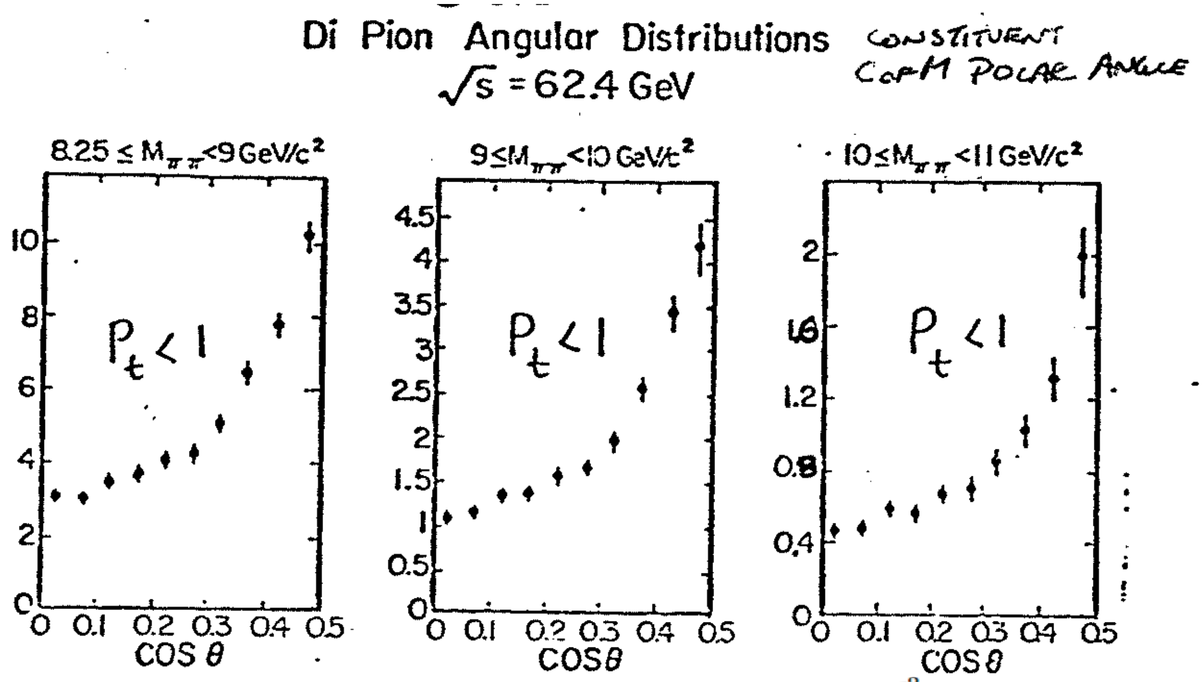




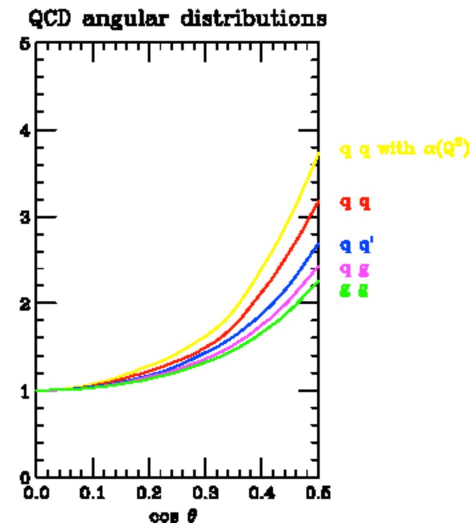
Trigger on a particle e.g. π^0 with transverse momentum p_{Tt}. Measure azimuthal angular distribution w.r.t the trigger azimuth of associated (charged) particles with transverse momentum p_{Ta}. The strong same and away side peaks in p-p collisions indicate di-jet origin from hard-scattering of partons. For the away distribution calculate the conditional yield in the peak as a function of $x_E \sim p_{Ta}/p_{Tt}$

ICHEP Paris1982: first unbiased jet (UA2) +first measurement of QCD subprocess angular distribution using π^0-π^0 correlations (CCOR)

DATA: CCOR NPB 209, 284 (1982)



QCD



$$\frac{d^3\sigma}{dx_1 dx_2 d\cos\theta^*} = \frac{1}{s}\sum_{ab} f_a^A(x_1) f_b^B(x_2) \frac{\pi\alpha_s^2(Q^2)}{2x_1x_2}\Sigma^{ab}(\cos\theta^*)$$

$\Sigma^{ab}(\cos\theta^*)$, the characteristic subprocess angular distributions and $\alpha_s(Q^2) = \frac{12\pi}{25\ln(Q^2/\Lambda^2)}$ are predicted by QCD

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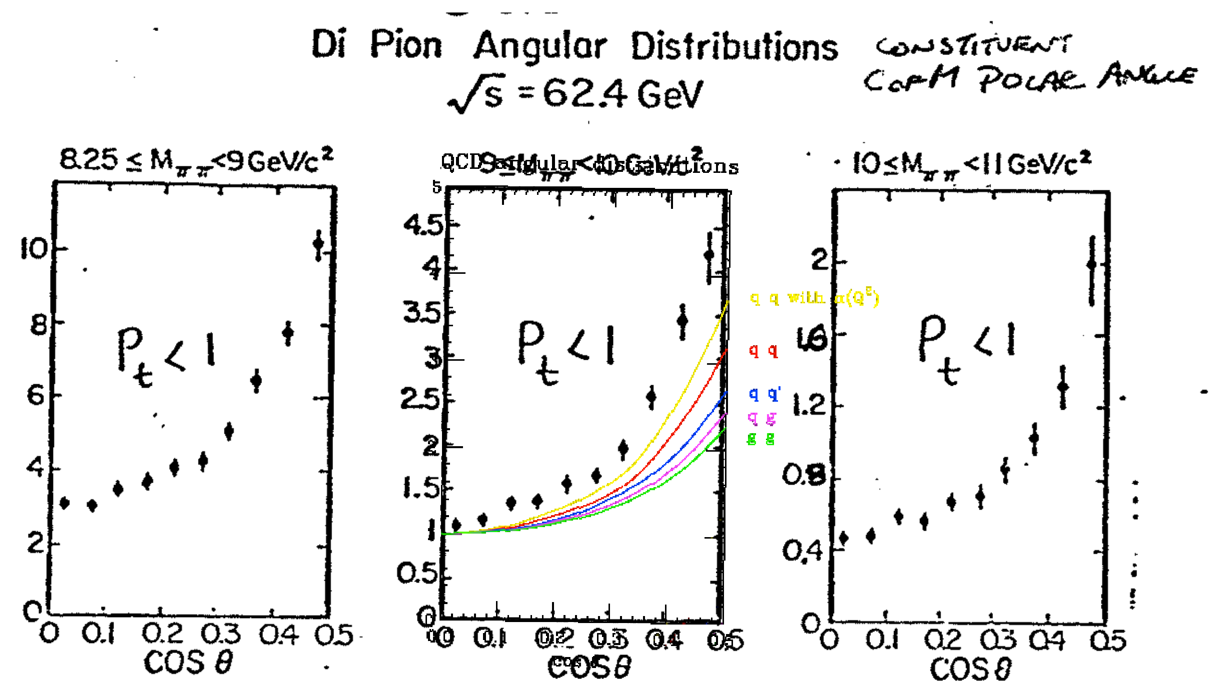
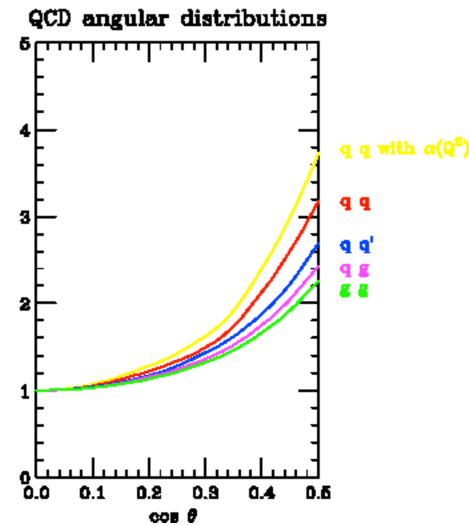


QCD



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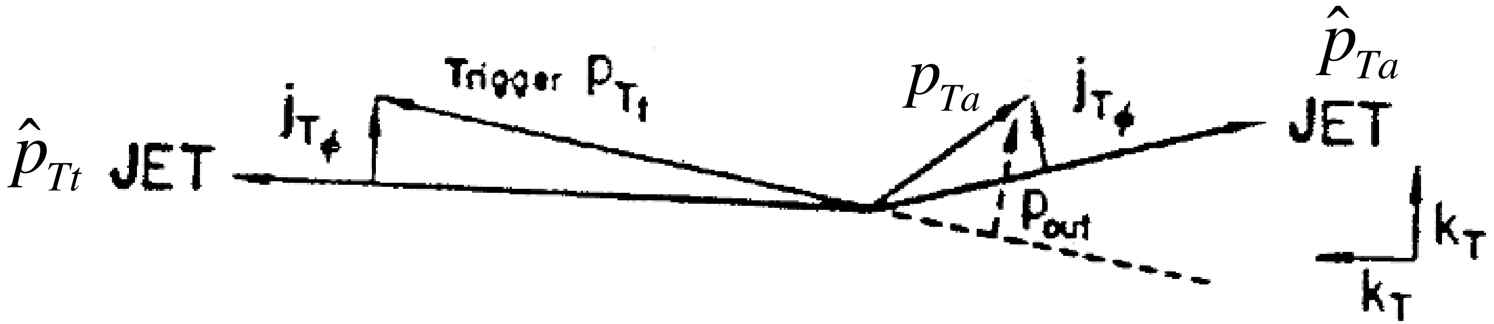
Kinematics



$z = p_T / \hat{p}_T$ is the jet fragmentation variable: z_t and z_a

$D_\pi^q(z) = Be^{-bz}$ is a typical Fragmentation Function, b~ 8-11 at RHIC

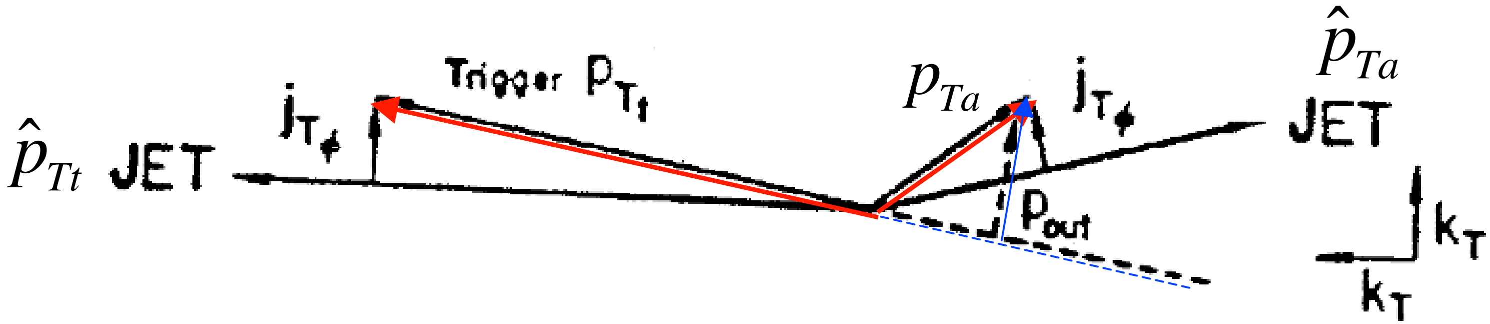
Due to the steeply falling spectrum, the trigger π^0 are biased towards large z_t, $\langle z_t \rangle \approx (n-1)/b$ while unbiased $\langle z \rangle \approx 1/b$

$$x_E = \left| \frac{\vec{p}_{T_a} \cdot \vec{p}_{T_t}}{p_{T_t}^2} \right| = \frac{-p_{T_a} \cos \Delta\phi}{p_{T_t}} \approx \frac{p_{T_a}}{p_{T_t}} = \frac{p_{T_a}/\hat{p}_{T_t}}{p_{T_t}/\hat{p}_{T_t}} \approx \frac{z_a}{\langle z_t \rangle}$$

From Feynman, Field and Fox: the x_E distribution corrected for $\langle z_t \rangle$ measures the unbiased fragmentation function

$$\frac{dP^{\mathrm{FFF}}}{dx_E} \approx \langle z_t \rangle \, B \exp -b \, \langle z_t \rangle \, x_E$$

Kinematics



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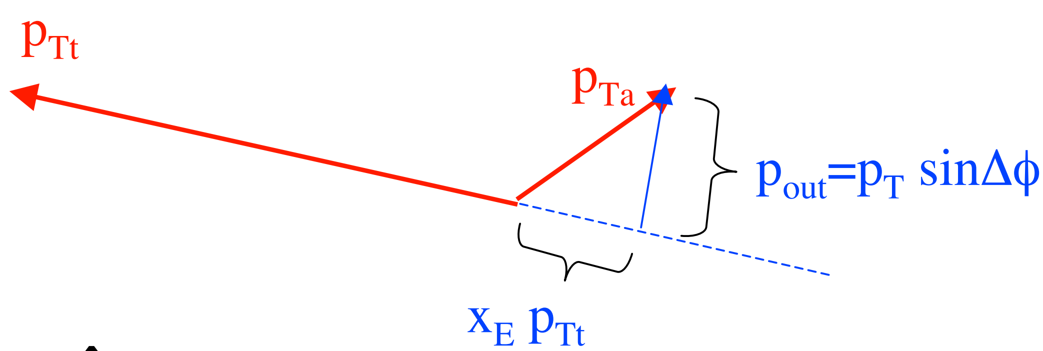

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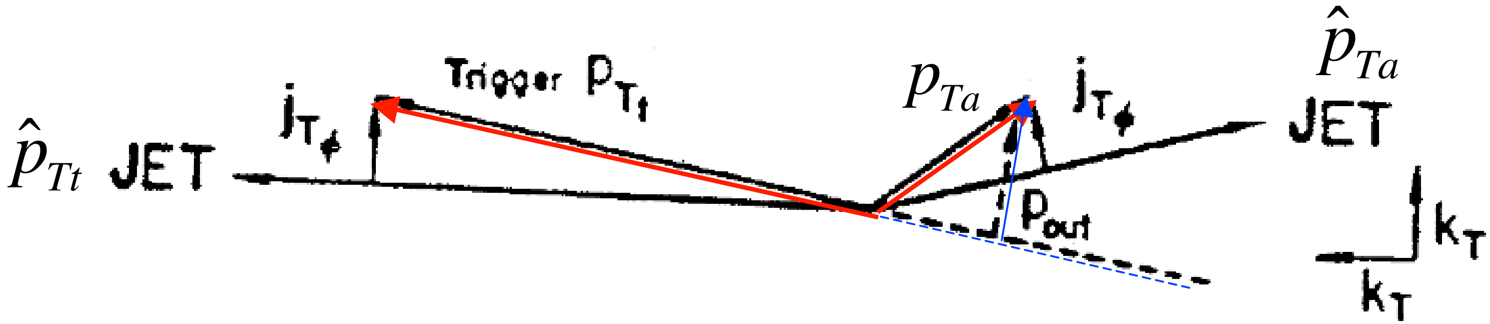

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From Feynman, Field and Fox
Nucl Phys B128 (1977) 1--65

38 *R.P Feynman et al. / Large transverse momenta*

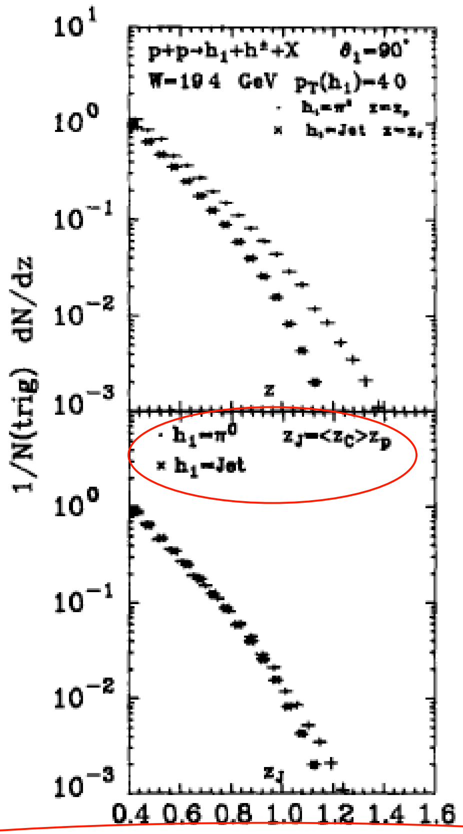
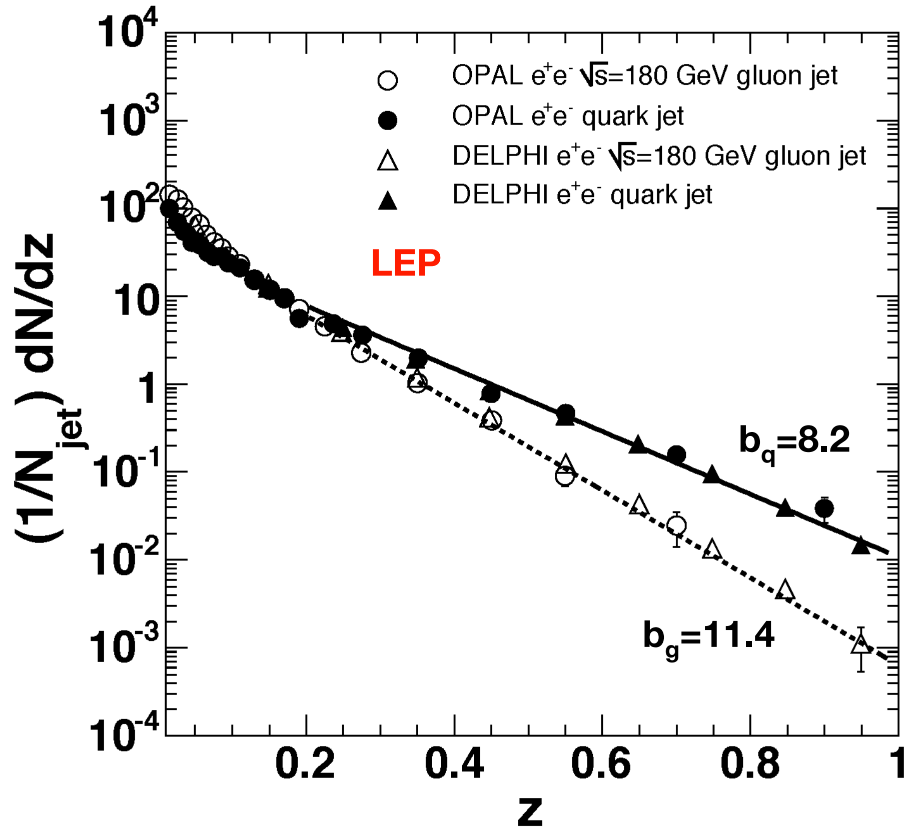
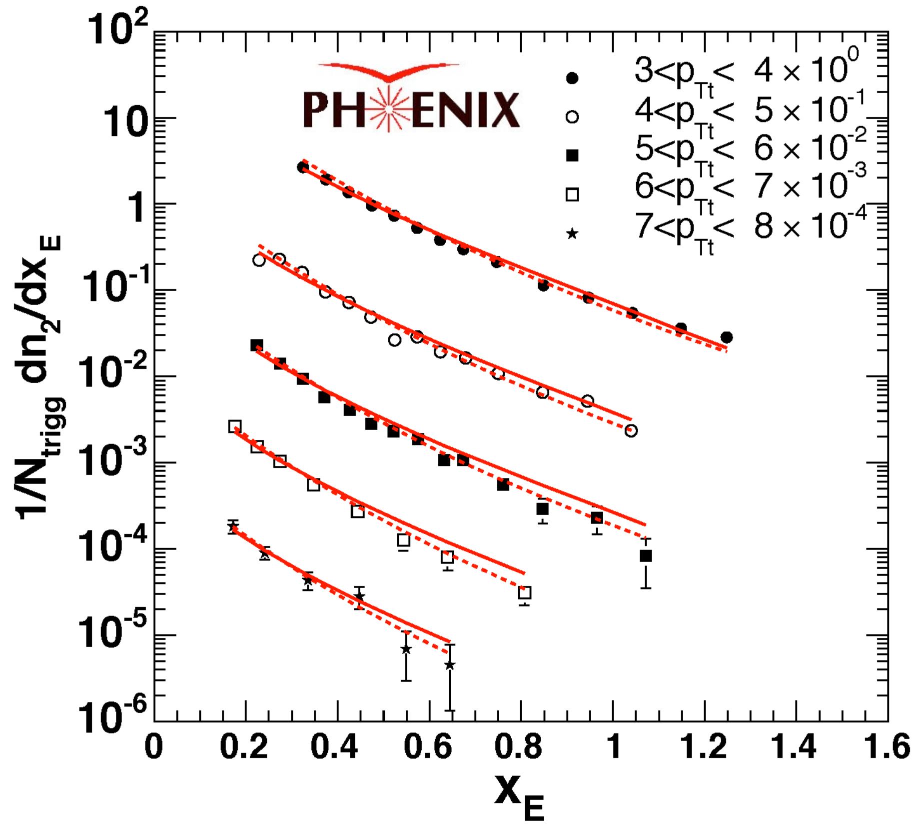


Fig. 23. Comparison of the π^0 and jet trigger away-side distribution of charged hadrons in pp collisions at W = 19.4 GeV, $\theta_1 = 90°$, and $p_\perp$ (trigger) = 4.0 GeV/c from the quark-quark scattering model. The upper figure shows the single-particle (π^0) trigger results plotted *versus* $z_p = -p_x(h^\pm)/p_\perp(\pi^0)$ and the jet trigger plotted *versus* $z_j = -p_x(h^\pm)/p_\perp(\text{jet})$ (see table 1). In the lower figure, we plot *both versus* z_J, where for the jet trigger $z_J = z_j$ but for the single-particle trigger $z_J = \langle z_c \rangle z_p$. The away hadrons are integrated over all rapidity Y and $|180° - \phi| \leq 45°$ and the theory is calculated using $\langle k_\perp \rangle_{h\to q}$ = 500 MeV. • $h_1 = \pi^0$, × h_1 = jet.

"There is a simple relationship between experiments done with single-particle triggers and those performed with jet triggers. The only difference in the opposite side correlation is due to the fact that the 'quark', from which a single-particle trigger came, always has a higher $p_\perp$ than the trigger (by factor $1/z_{trig}$). The away-side correlations for a single-particle trigger at $p_\perp$ should be roughly the same as the away side correlations for a jet trigger at $p_\perp$ (jet)= $p_\perp$ (single particle)/ $\langle z_{trig} \rangle$".

PHENIX-compared measured x_E distribution in p-p to numerical integral using LEP fragmentation functions

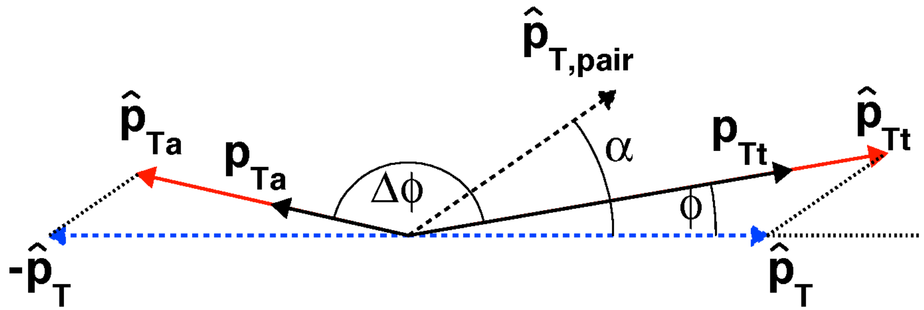




PHENIX PRD **74** (2006) 072002. The x_E distribution triggered by a leading fragment (π^0) is not sensitive to the shape of the fragmentation function!!! Disagrees with FFF!!

A very interesting new formula for the x_E distribution was derived by PHENIX in PRD74

$$\left.\frac{dP_\pi}{dx_E}\right|_{p_{T_t}} \approx \langle m \rangle (n-1) \frac{1}{\hat{x}_h} \frac{1}{(1+\frac{x_E}{\hat{x}_h})^n}$$



Relates ratio of particle p_T

$$x_E = \frac{-p_{T_a}\cos\Delta\phi}{p_{T_t}} \simeq \frac{p_{T_a}}{p_{T_t}}$$

measured

Ratio of jet transverse momenta

$$\hat{x}_h = \frac{\hat{p}_{T_a}}{\hat{p}_{T_t}}$$

Can be determined

If formula works, we can also use it in Au+Au to determine the relative energy loss of the away jet to the trigger jet (surface biased by large *n*)

Exponential Frag. Fn. and power law crucial

$$\frac{d^2\sigma_\pi(\hat{p}_{T_t}, z_t)}{d\hat{p}_{T_t} dz_t} = \frac{d\sigma_q}{d\hat{p}_{T_t}} \times D^q_\pi(z_t) = \boxed{\frac{A}{\hat{p}^{n-1}_{T_t}}} \times D^q_\pi(z_t)$$

Fragment spectrum given $\hat{p}_{T_t}$
Power law spectrum of parton $\hat{p}_{T_t}$

Let $\hat{p}_{T_t} = p_{T_t}/z_t \qquad d\hat{p}_{T_t}/dp_{T_t}|_{z_t} = 1/z_t$

$$\frac{d^2\sigma_\pi(p_{T_t}, z_t)}{dp_{T_t} dz_t} = \frac{A}{p^{n-1}_{T_t}} \times z_t^{n-2} D^q_\pi(z_t)$$

Fragment spectrum given p_{Tt} is weighted to high z_t by z_t^{n-2}

where $z_{t\min}|_{p_{T_t}} = x_{T_t}$

$$\boxed{D^q_\pi(z_t) = Be^{-bz_t}}$$

$$\boxed{\frac{1}{p_{T_t}} \frac{d\sigma_\pi}{dp_{T_t}} = \frac{AB}{p^n_{T_t}} \int_{x_{T_t}}^1 dz_t \; z_t^{n-2} \exp -bz_t}$$

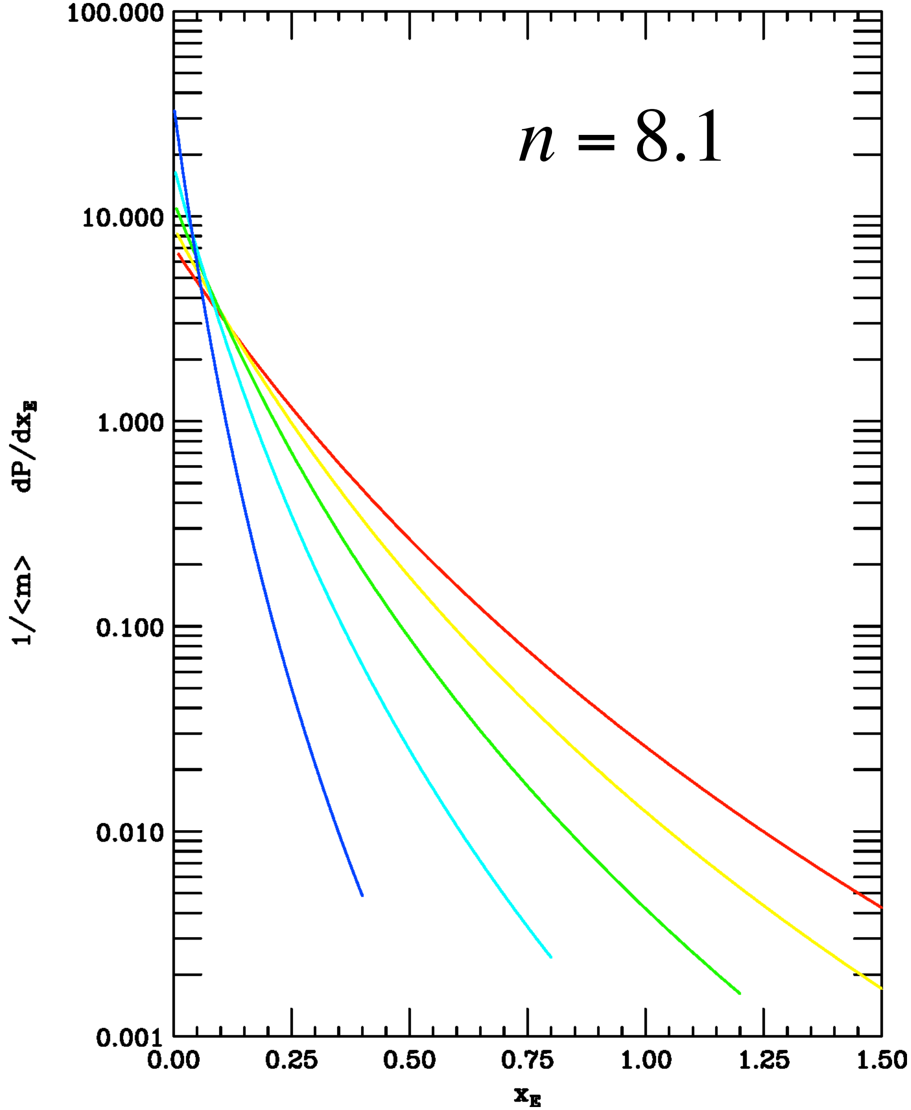
Incomplete gamma function

Good approximation $x_{T_t} \to 0$ upper limit $\to \infty$

$$\boxed{\frac{1}{p_{T_t}} \frac{d\sigma_\pi}{dp_{T_t}} \approx \frac{\Gamma(n-1)}{b^{n-1}} \frac{AB}{p^n_{T_t}}}$$

Bjorken parent-child relation: parton and particle invariant p_T spectra have same power *n*, etc.

Shape of x_E distribution depends on $\hat{x}_h$ and n but not on b



$$\left.\frac{dP_\pi}{dx_E}\right|_{p_{T_t}} \approx \langle m \rangle \, (n-1) \frac{1}{\hat{x}_h} \frac{1}{(1 + \frac{x_E}{\hat{x}_h})^n}$$

$\hat{x}_h$

1.0

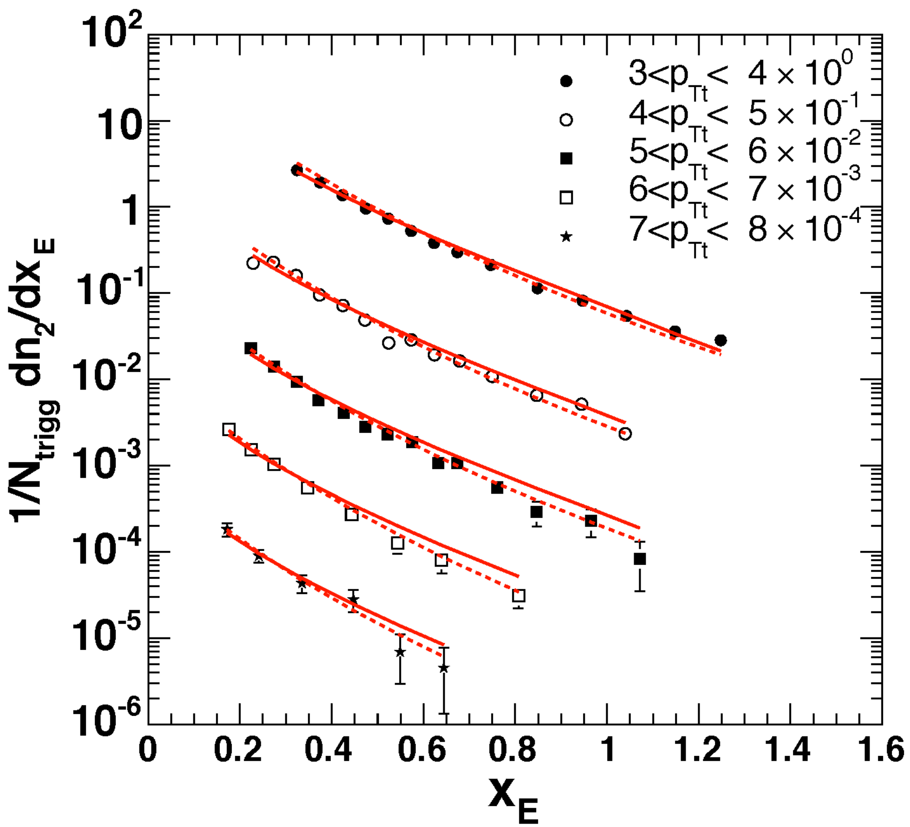
0.8

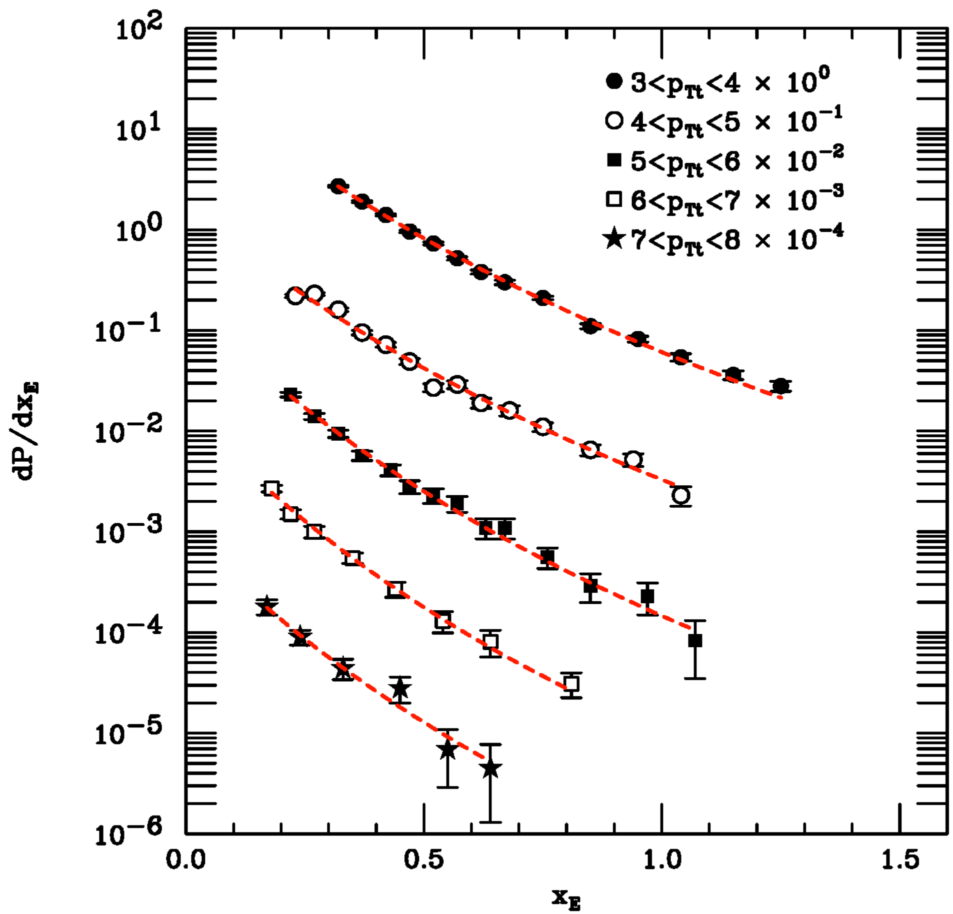
0.6

0.4

0.2

Fit works for PHENIX p+p PRD **74**, 072002

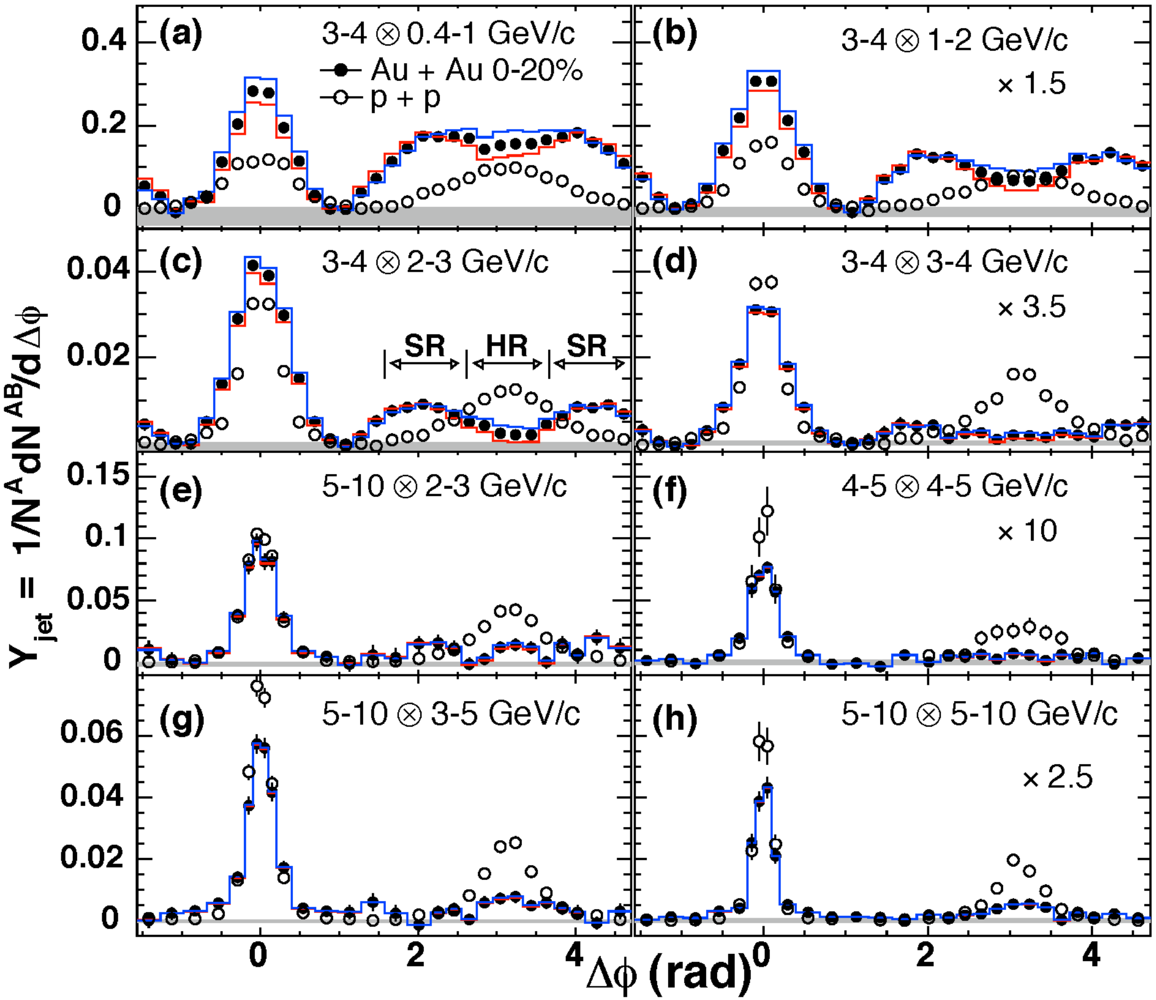




Calculation from Fragmentation Fn.

New fits. Very nice!

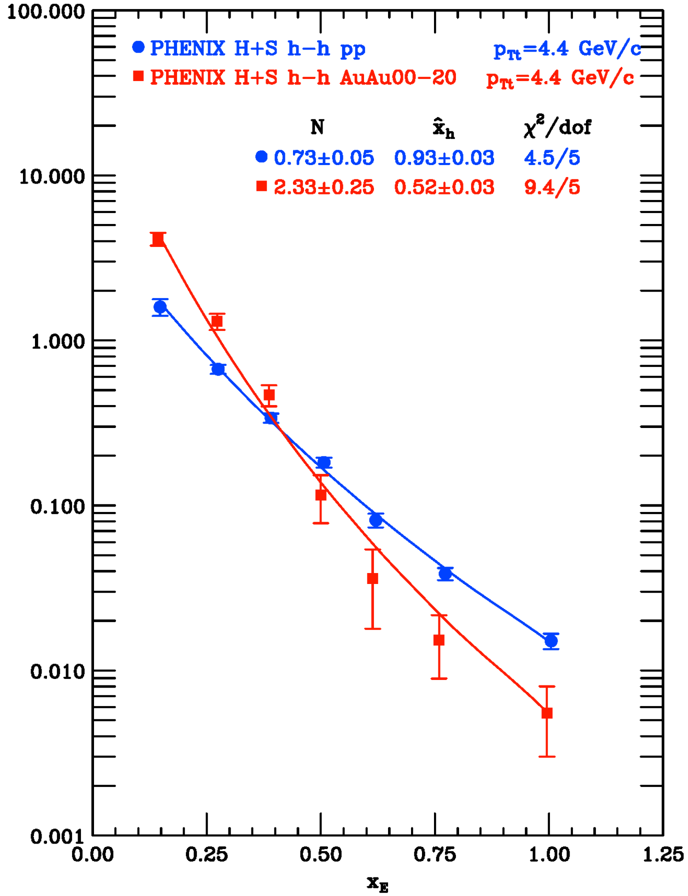
New PHENIX AuAu PRC 77,011901(R)(2008)



Away side correlation in Au+Au is generally wider than p-p with complicated structure

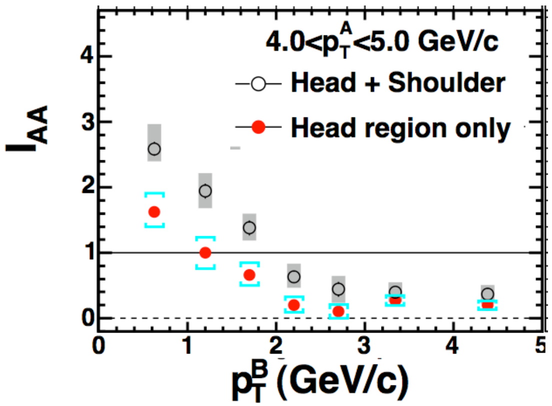
Define Head region (HR) and Shoulder regions (SR) for wide away side correlation.

Formula works for $h^{\pm}$-$h^{\pm}$ in Au+Au: Away-side yield vs p_{Ta}/p_{Tt} is steeper in Au+Au than p-p indicating energy loss



The away side $p_{Ta}/p_{Tt} \approx x_E$ distribution triggered by a leading particle with p_{Tt} was thought to be equal to the fragmentation function but we found that it is NOT sensitive to the shape of the fragmentation function but only to the shape of the inclusive p_{Tt} spectrum with power n (=8.1). Formula derived in PRD 74 (2006) 072002 works for pp and AA

$h^{\pm}(4<p_{Tt}<5$ GeV/c)--$h^{\pm}$



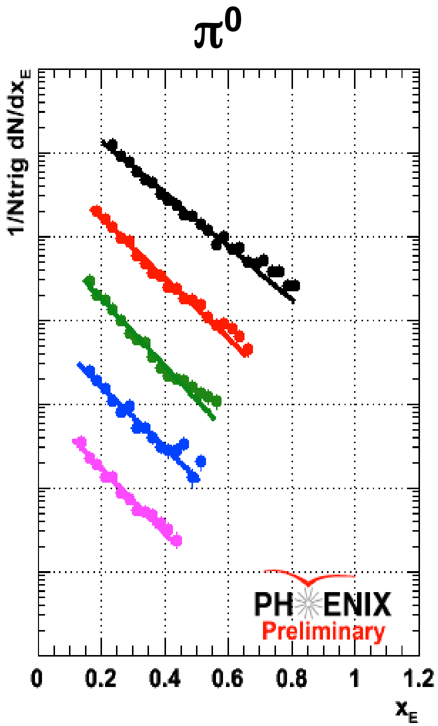
PHENIX AuAu PRC **77**, 011901(R)(2008)

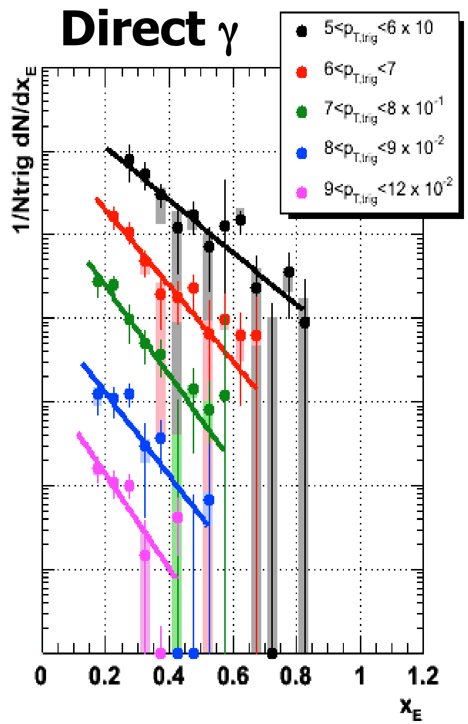
$$\left.\frac{dP_\pi}{dx_E}\right|_{p_{T_t}} \approx \langle m\rangle\,(n-1)\frac{1}{\hat{x}_h}\frac{1}{(1+\frac{x_E}{\hat{x}_h})^n}$$

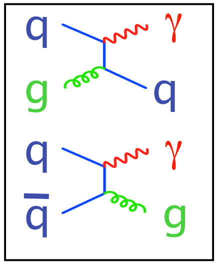
Measured ratio of particle $p_{Ta}/p_{Tt} \approx x_E \Rightarrow$ Ratio of jet transverse momenta $\hat{p}_{Ta}/\hat{p}_{Tt} \equiv \hat{x}_h$

$\hat{x}_h = 0.52 \pm 0.03$ in Au+Au indicates that away jet has lost energy relative to trigger jet.

Direct γ - h± correlations in p+p √s=200 GeV PHENIX preliminary result

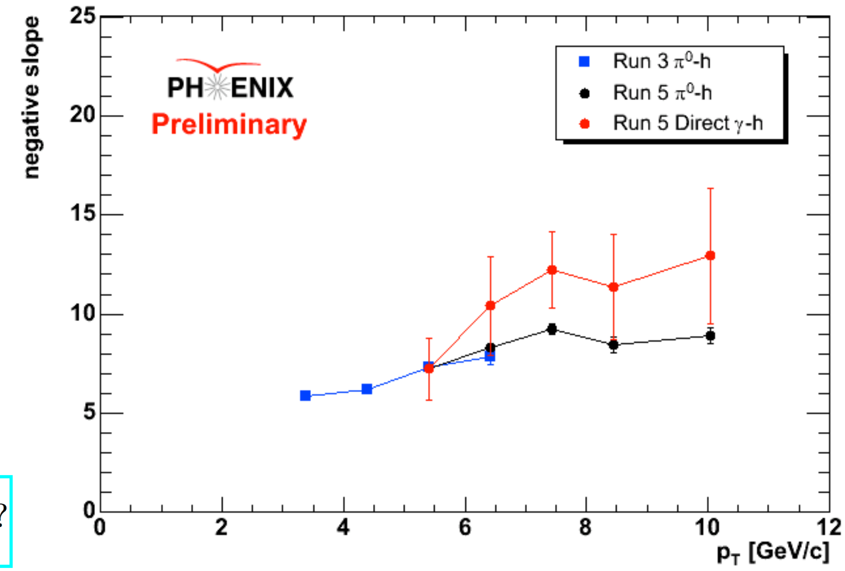






Compton

Annihilation

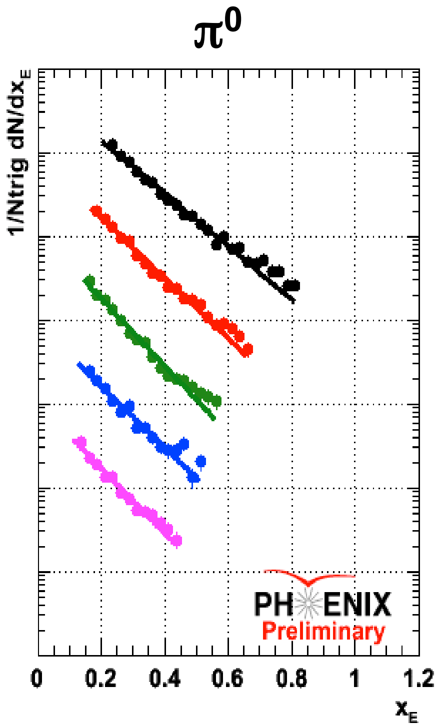


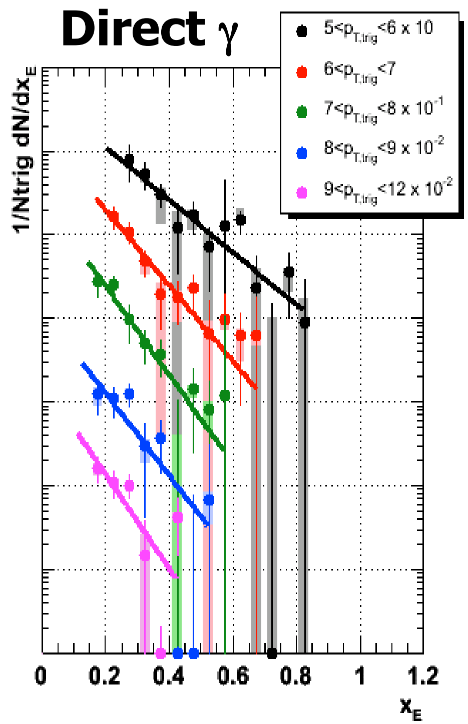
$$\left.\frac{dP_\pi}{dx_E}\right|_{p_{T_t}} \approx \langle m\rangle\,(n-1)\frac{1}{\hat{x}_h}\frac{1}{(1+\frac{x_E}{\hat{x}_h})^n}$$

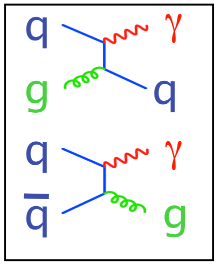
$$\left.\frac{dN}{dx_E}\right|_\gamma = D^q_h(z) \approx B\exp -bx_E \quad ?$$

In order to understand whether away side x_E distribution from a direct γ is the (quark) fragmentation function must understand why b≠ 8.2. Must also understand k_T smearing for direct γ.

Direct γ - h± correlations in p+p √s=200 GeV PHENIX preliminary result







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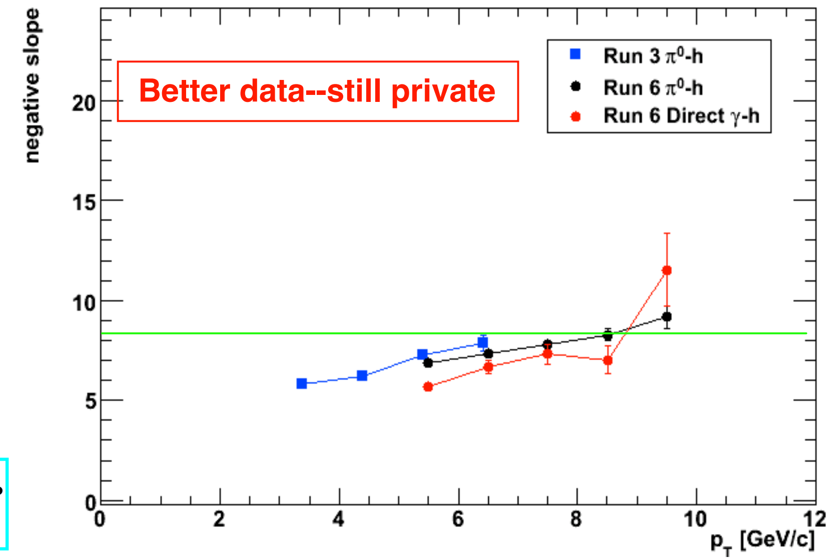
Annihilation



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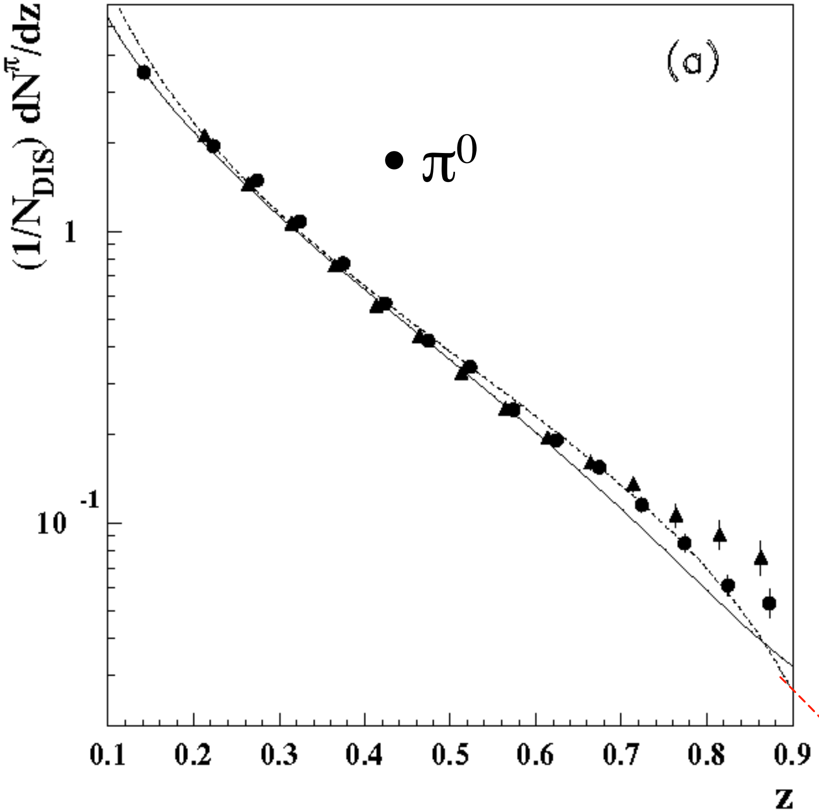
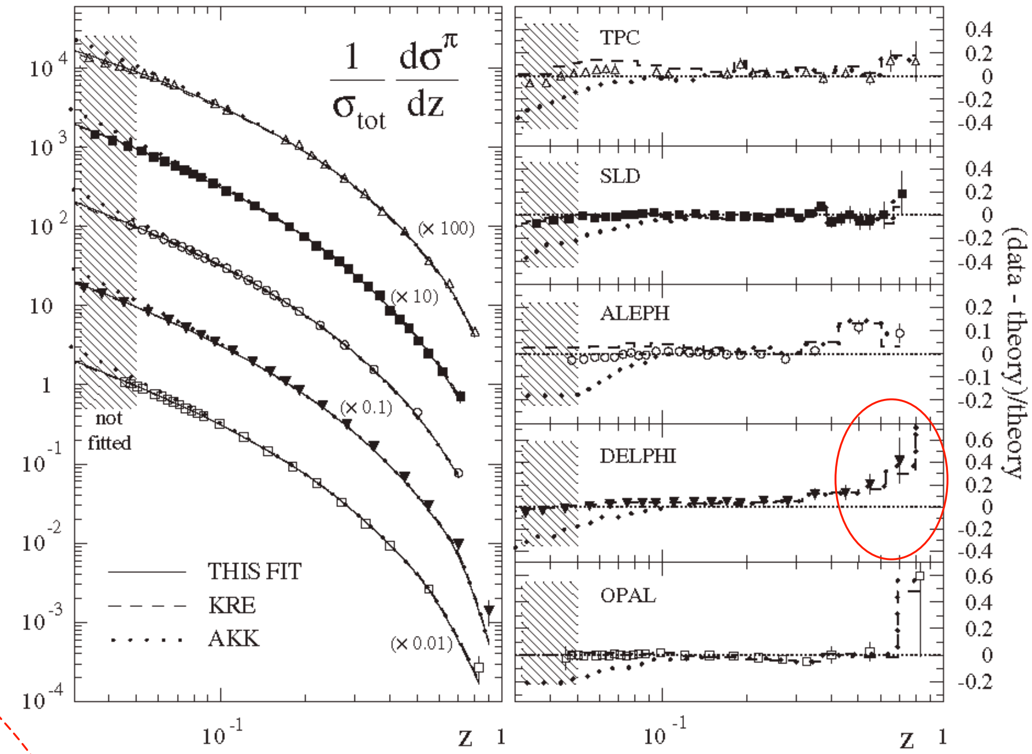
In order to understand whether away side x_E distribution from a direct γ is the (quark) fragmentation function must understand why b≠ 8.2. Must also understand k_T smearing for direct γ.

Are Frag Fns-Exponential: Hermes π^0; NLO?





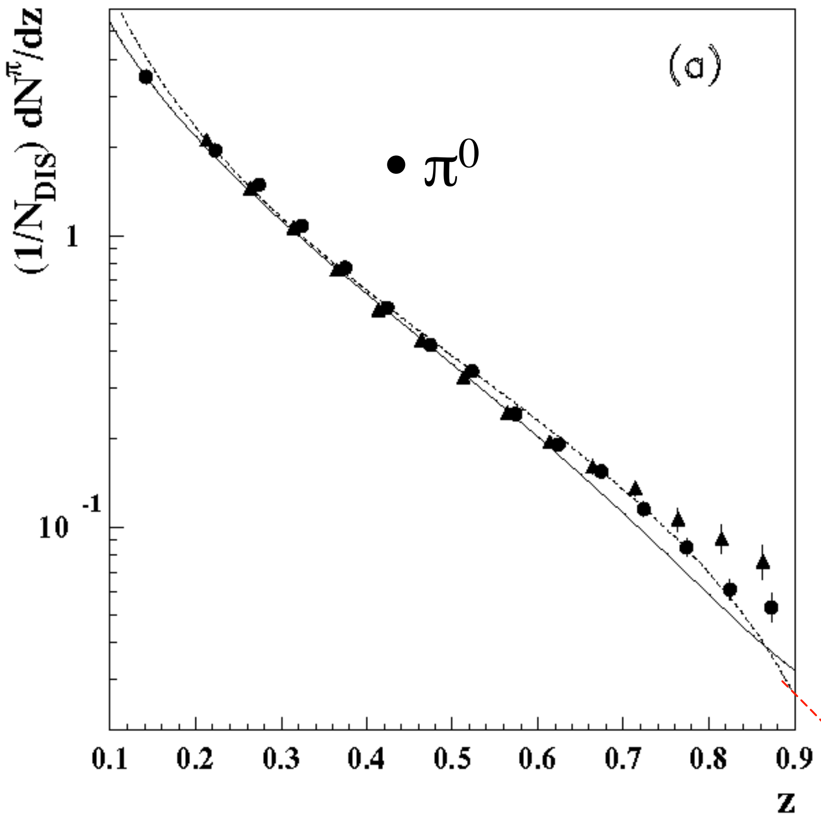
BKK inspired fit-doesn't fit large z

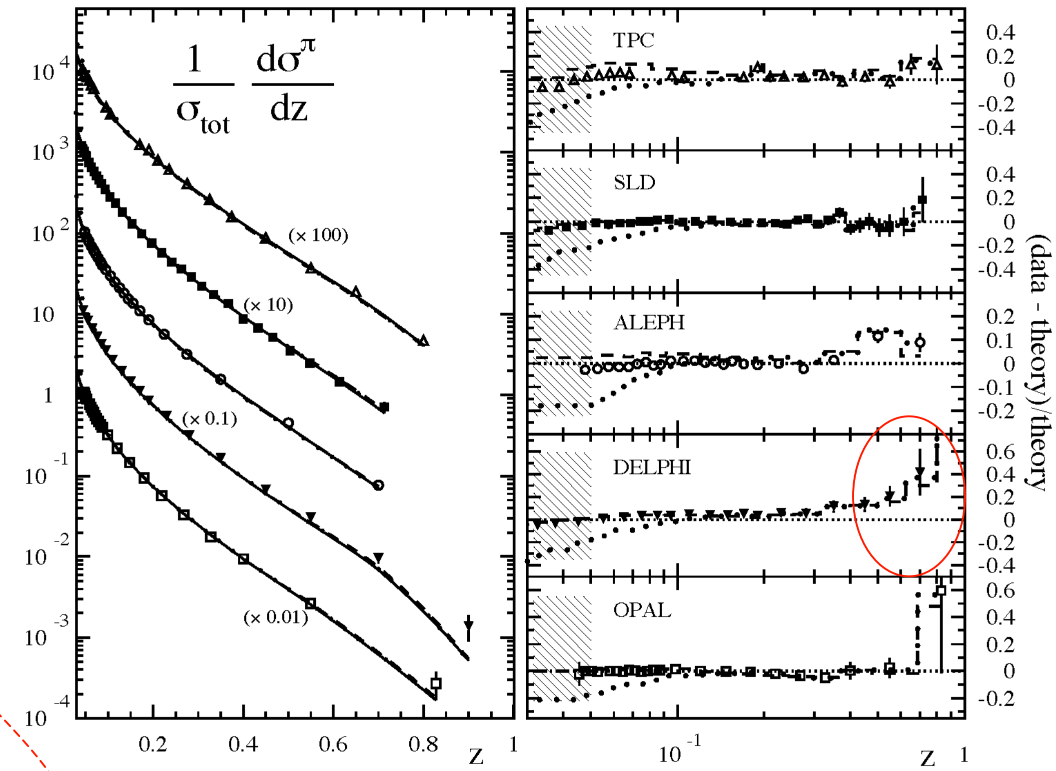
$$D_{\pi}^{q}(z) = 0.335 z^{-1.37}(1-z)^{1.17}$$

Hermes EPJC **21** (2001) 599-606

deFlorian,Sassot,Stratmann PRD **75**,114010(2007)
thanks to Marco via Werner V for this plot

Are Frag Fns-Exponential: Hermes π^0; NLO?





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Hermes EPJC **21** (2001) 599-606

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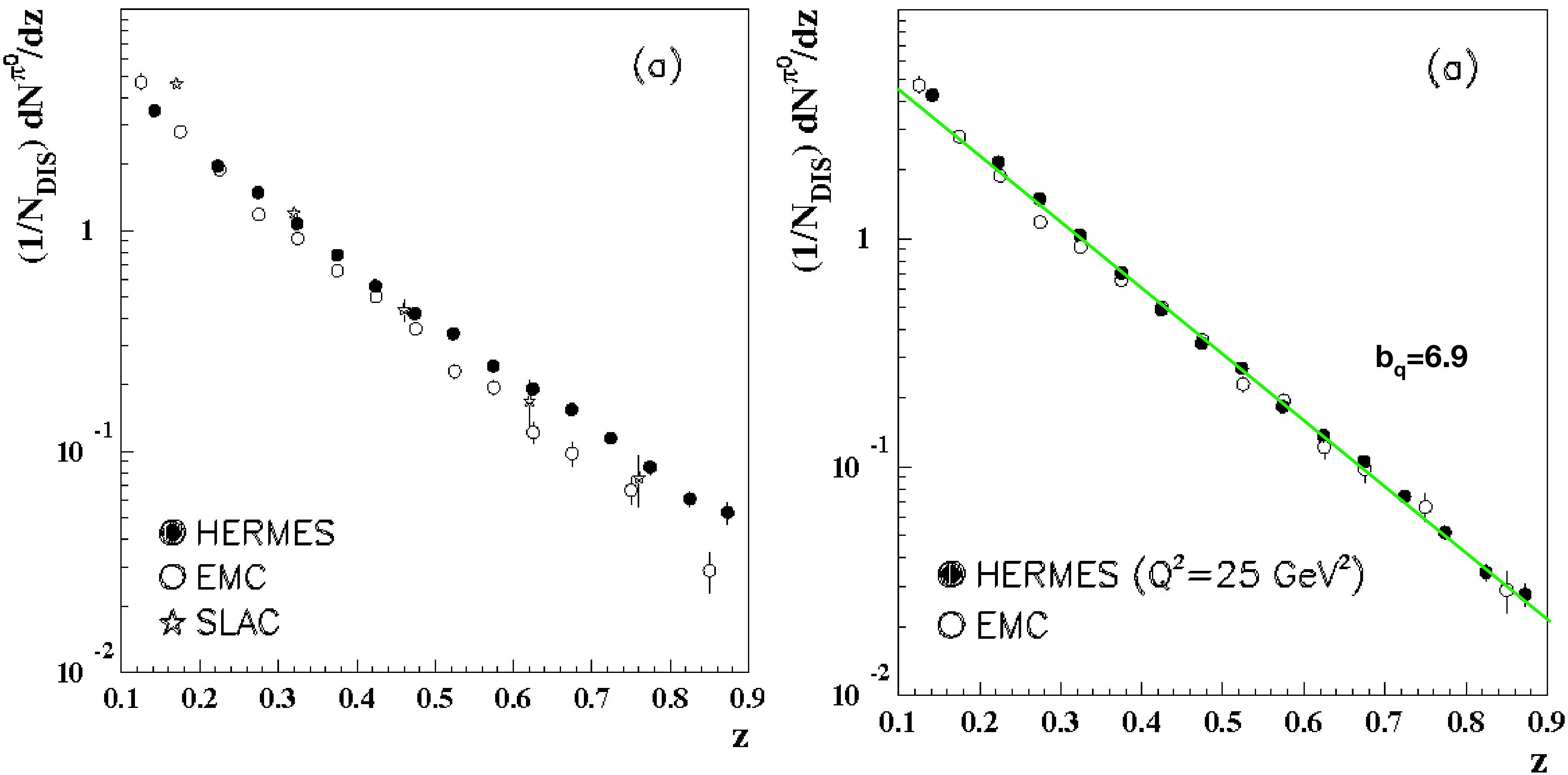
thanks to Marco via Werner V for this plot

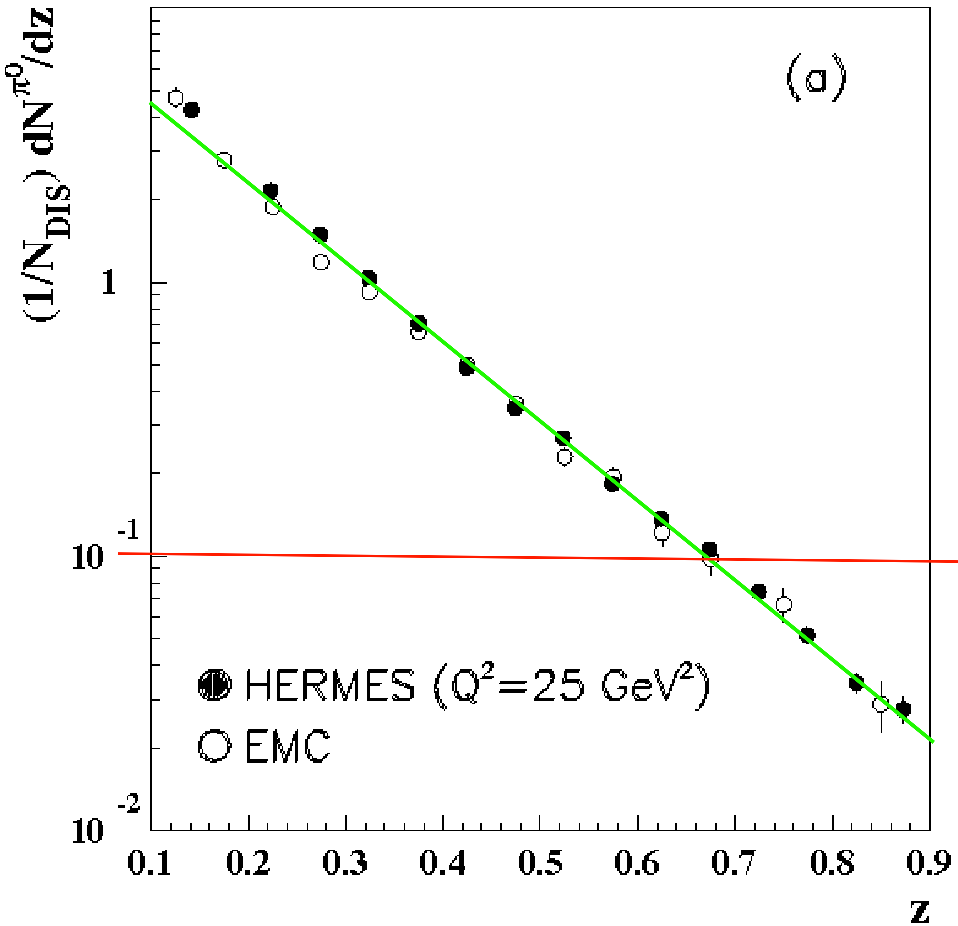
π Fragmentation Fns--very nice exponentials

Hermes EPJC **21** (2001) 599-606



Are there fragmentation photons? Must prove it!

DIS



e+ e- LEP

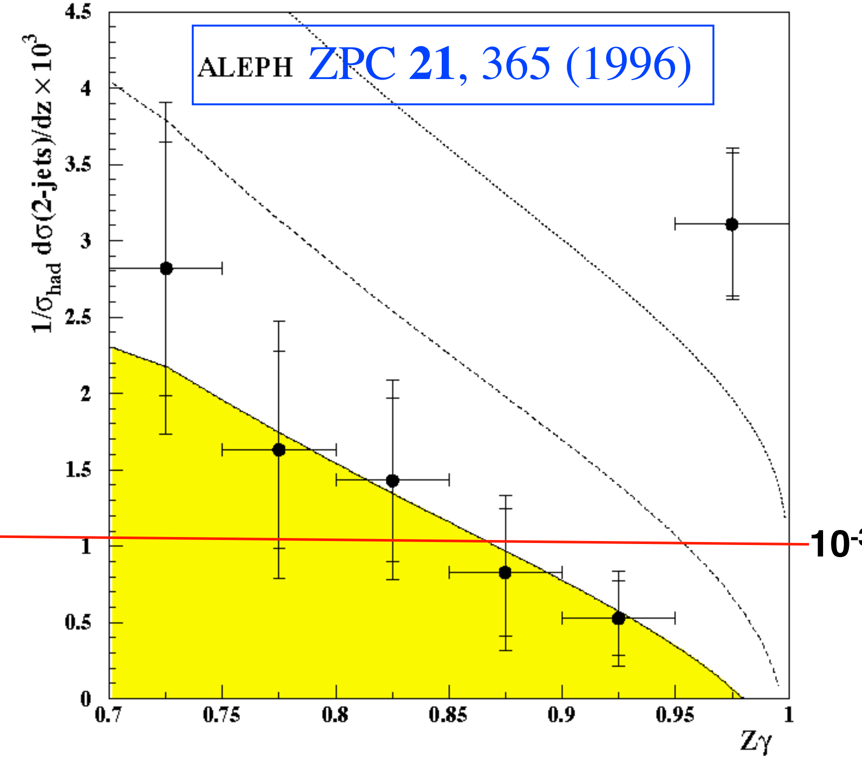


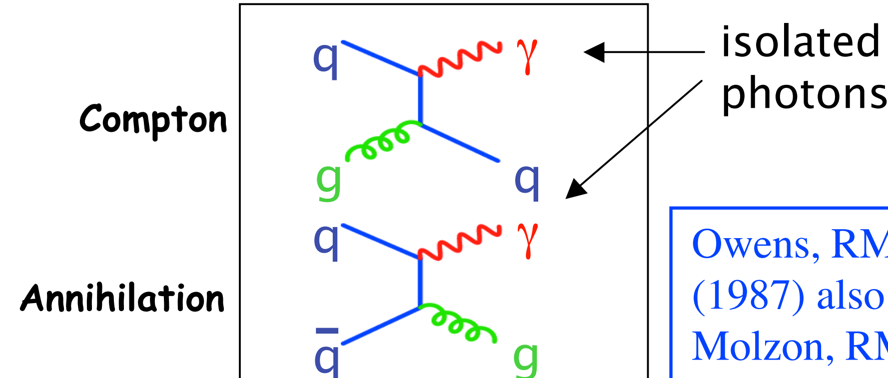
Fig. 4. Comparison of the measured $D(z_\gamma)$ function extracted from the 2-jet rates at $y_{cut} = 0.06$ to the Duke-Owens fragmentation function for $Q = p_T$ (dashed), and $Q = M_Z$ (dotted) with $\Lambda = 0.2$ GeV. The darkened area shows the result of a fit of the Duke-Owens function with $Q = p_T$ giving $\Lambda = 1.30^{+0.70}_{-0.45}$ GeV with $\chi^2/4 = 0.48$

$\gamma_{FRAG}/\pi^0 \sim 0.03$?

but, they didn't measure π^0; didn't show any γ/π

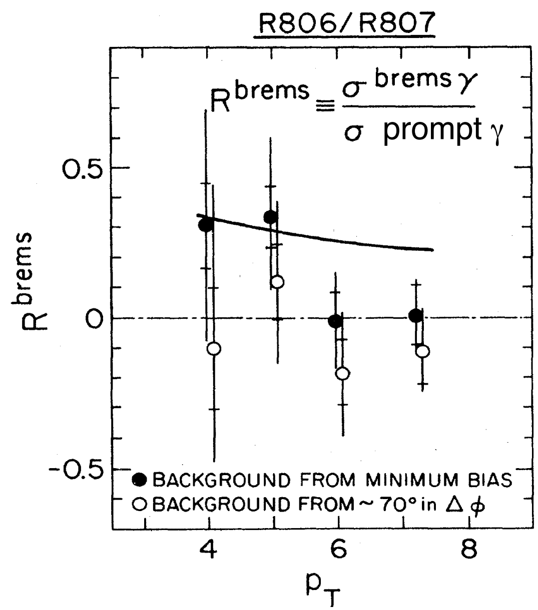
ISR direct photon production + correlations

See the classic paper of Fritzsch and Minkowski, PLB **69** (1977) 316-320



p-p

Owens, RMP**59**, 465 (1987) also see Ferbel, Molzon, RMP**56**, 181 (1984)



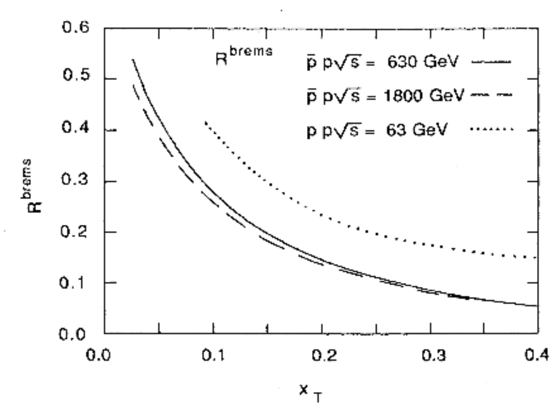
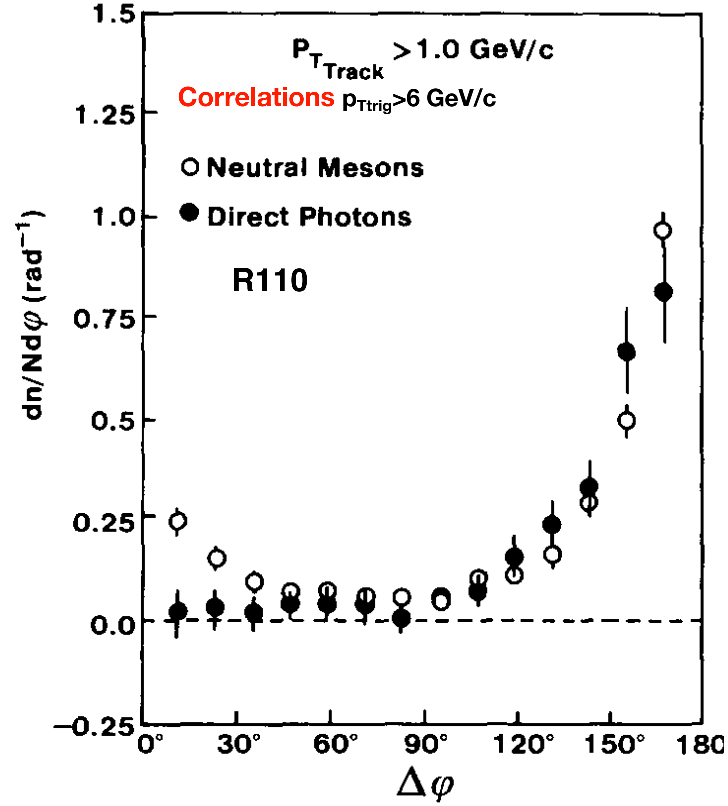


FIG. 27. Theoretical predictions for R^{brems} at various center-of-mass energies shown vs x_T.

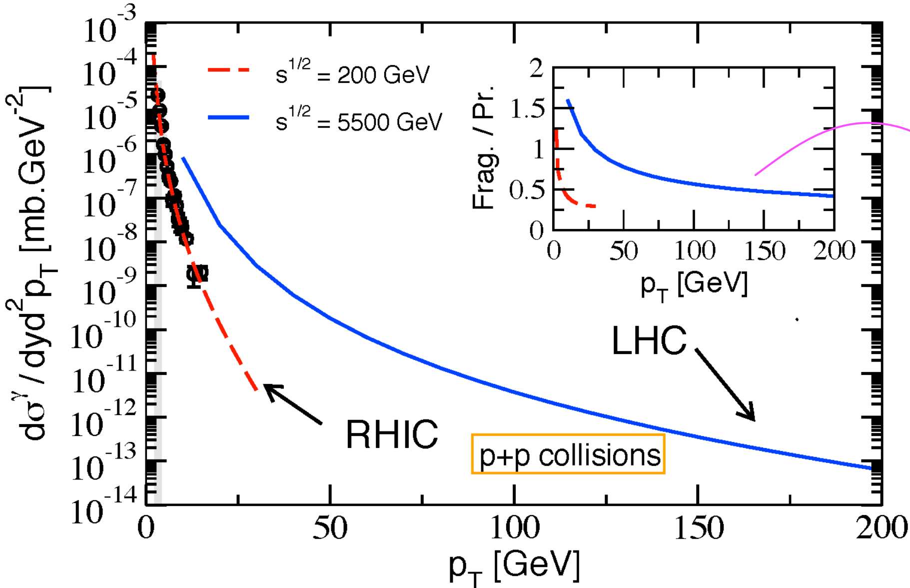


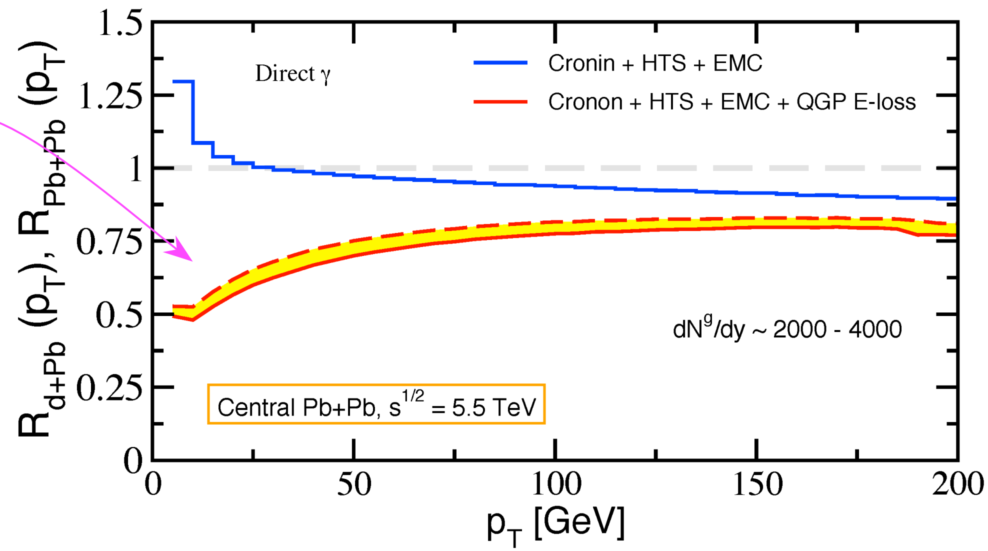
AFS (R806/7) PLB **118**, 178 (1982): R^{brems} <0.3 (2σ)

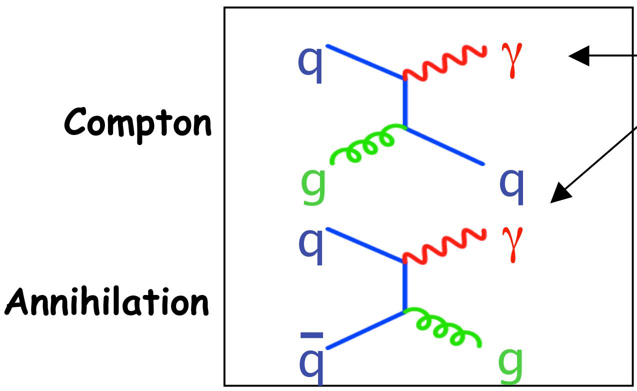
No evidence for brems. contribution to direct γ--same side correlation is zero--see CMOR (R110) NPB**327**, 541 (1989) for full list of references.

Fragmentation γ and possible direct γ suppression

e.g I. Vitev, pp 167-169 in Armesto, et al, arXiv:0711.0974 [hep-ph]





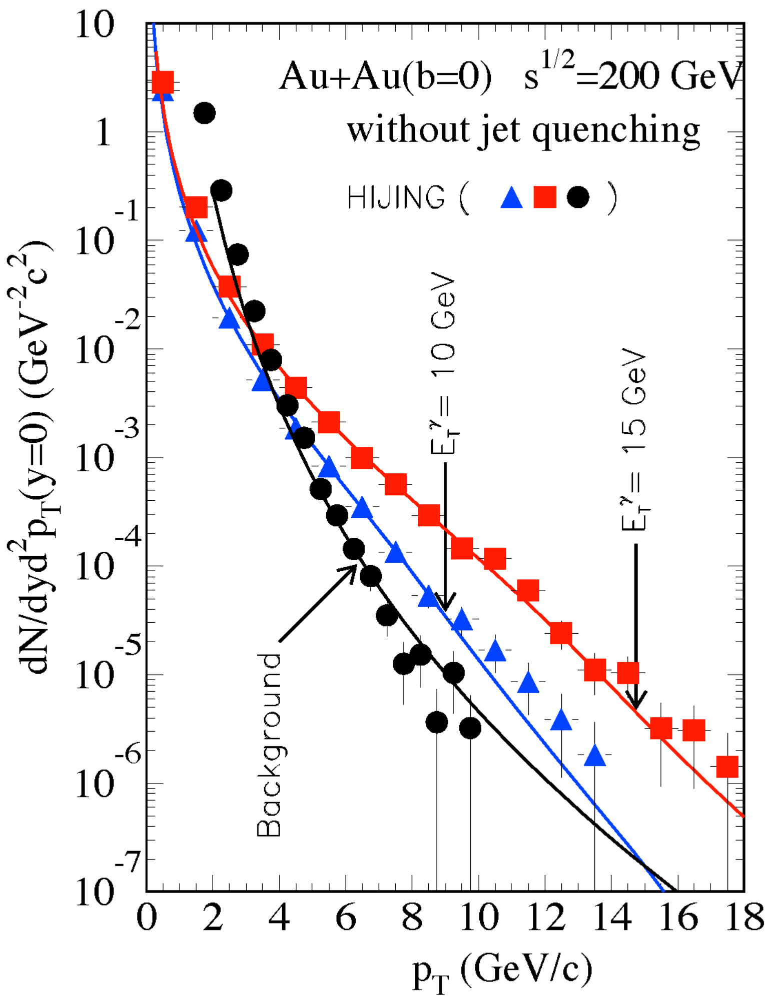


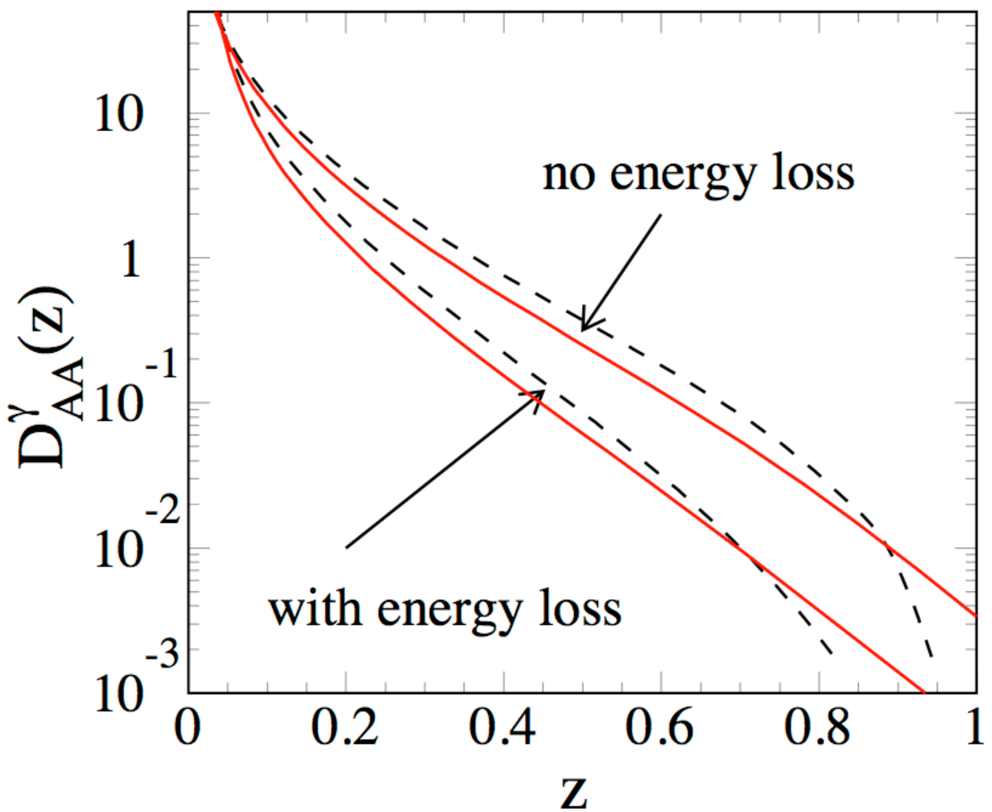
we want isolated photons from 2-2 Compton in p-p

No experimentalist would get this result because we want to use the Compton diagram in p+p as a baseline to measure the initial state effect in Pb+Pb for the case of an outgoing constituent from the 2-2 hard scattering, the photon, which does not interact with the medium.

The Holy Grail: γ-h correlations in Au+Au

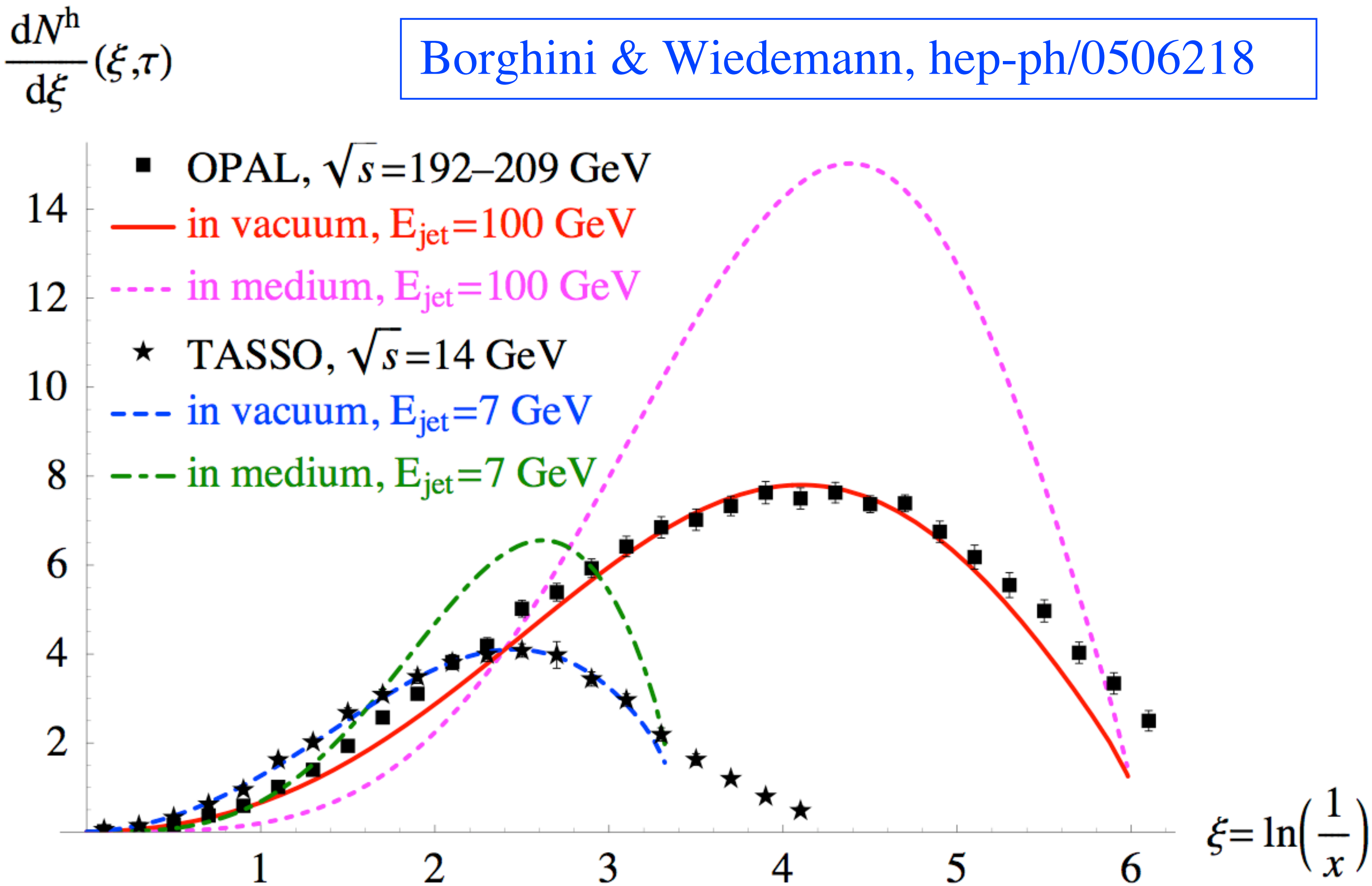
X-N. Wang and Z. Huang PRC 55, 3047 (1997)





MJT-should scale the z axis to see the energy loss (not take the ratio)

Prediction of Jet shape in vacuum and medium



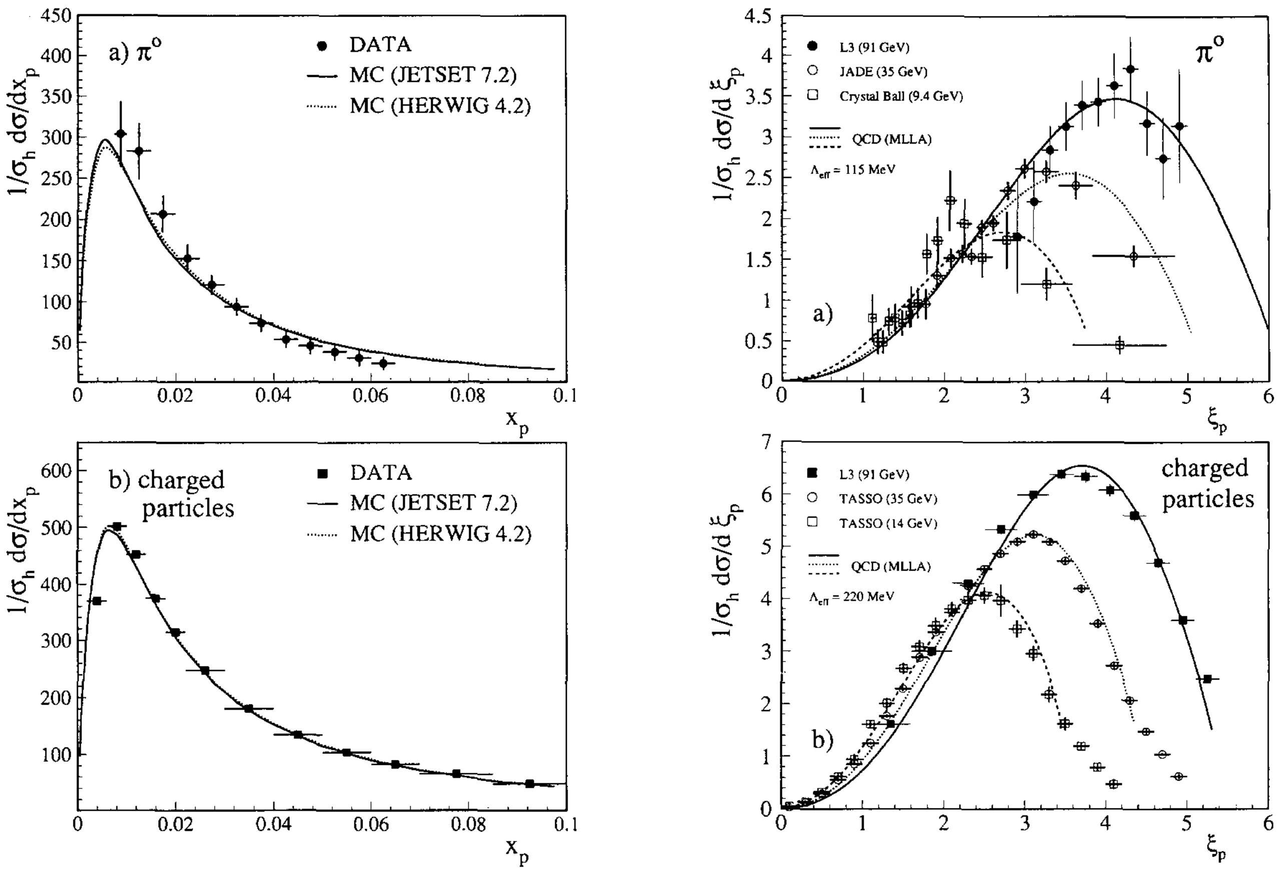
Would be much easier to understand if they also plotted z in addition to ξ=ln(1/z): e.g ξ=3.0→z=0.050

ξ from single inclusive π^0 at Z^0--L3. Thank you Sam Ting

L3, PLB 259 (1991)199-208

PHYSICS LETTERS B

18 April 1991



We should be able to do this from γ-π^0 γ-h away side correlations in p-p (almost there, convert xE plot to ξ) and Au+Au to get a Tannenbaum,Ting,Wang,Wiedemann (in alphabetical order) plot

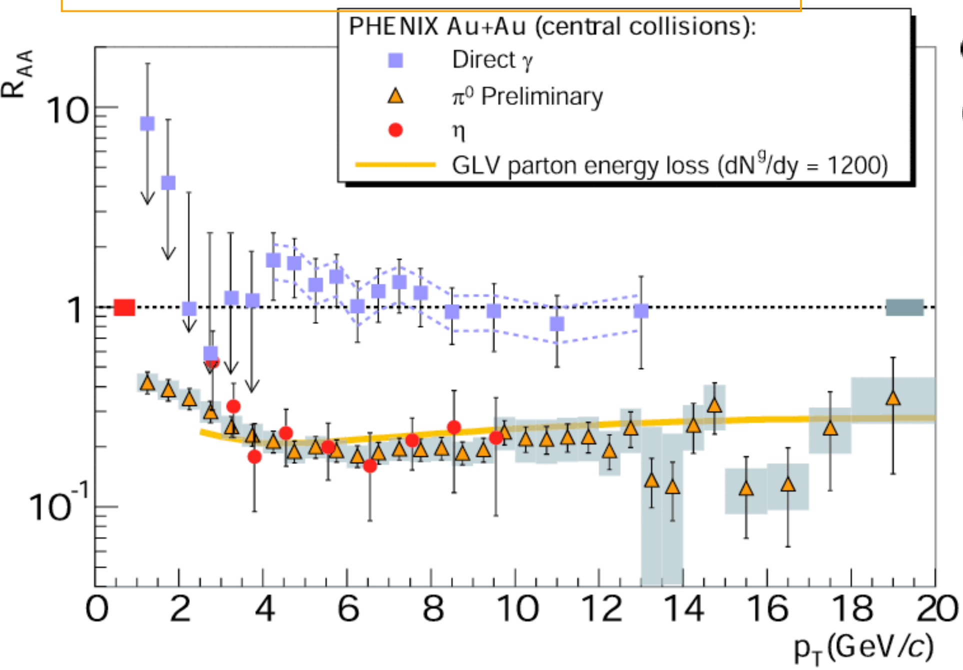
I could go on for hours, but

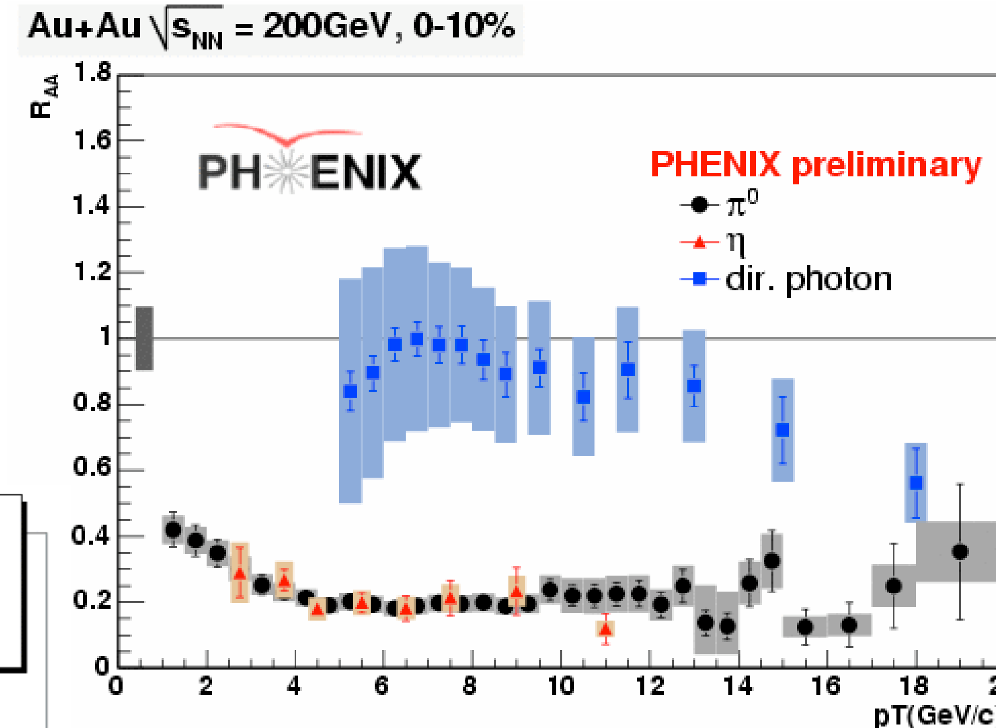
The End

The biggest result at QM2006??!

$$R_{AA}(p_T) = \frac{d^2N^{\pi}_{AA}/dp_T dy N^{inel}_{AA}}{\langle T_{AA}\rangle d^2\sigma^{\pi}_{pp}/dp_T dy}$$

QM2005
-I wanted to make a T-shirt
pp dir γ reference is pQCD





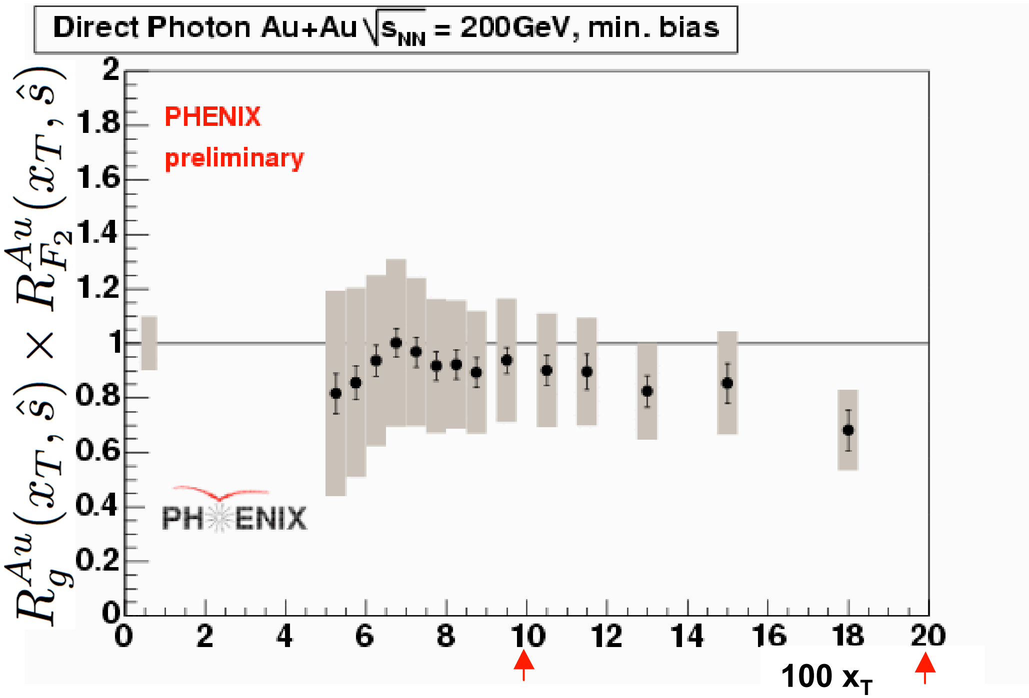
QM2006-
pp dir γ reference is run 5 msmt
If $R^{\pi}_{AA} = R^{\gamma}_{AA}$ the whole concept of energy loss changes: perhaps no effect for p_T>20 GeV

For Au+Au min bias direct γ R_{AA} is simple

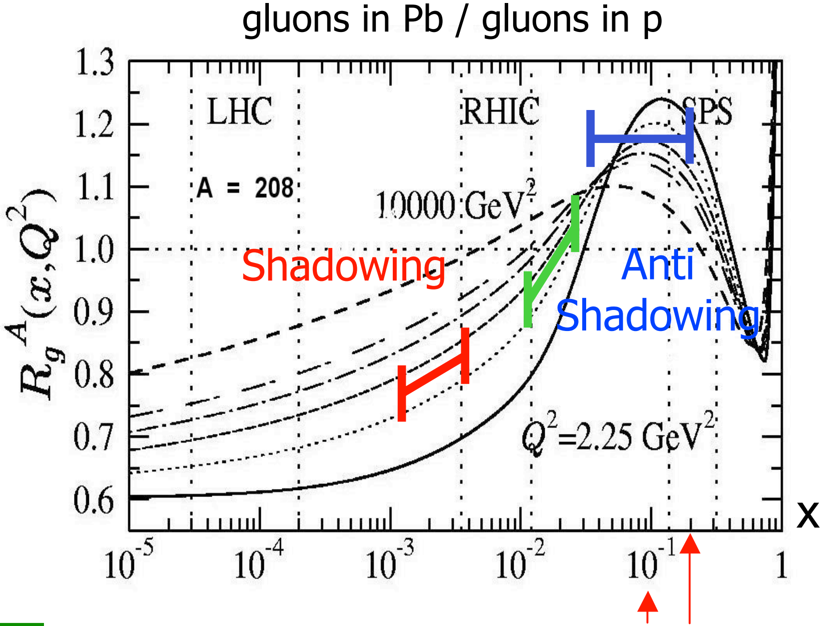
Au+Au minimum bias

Eskola,Kolhinen,Ruuskanen
Nucl. Phys. B**535**(1998)351

$$R_{AA} = \frac{d^2\sigma_\gamma^{AA}/dp_T^2\, dy_\gamma}{AA\, d^2\sigma_\gamma^{pp}/dp_T^2\, dy_\gamma} \approx \left(\frac{F_{2A}(x_T)}{AF_{2p}(x_T)} \times \frac{g_A(x_T)}{Ag_p(x_T)}\right)$$



Eskola et al. NPA**696** (2001) 729

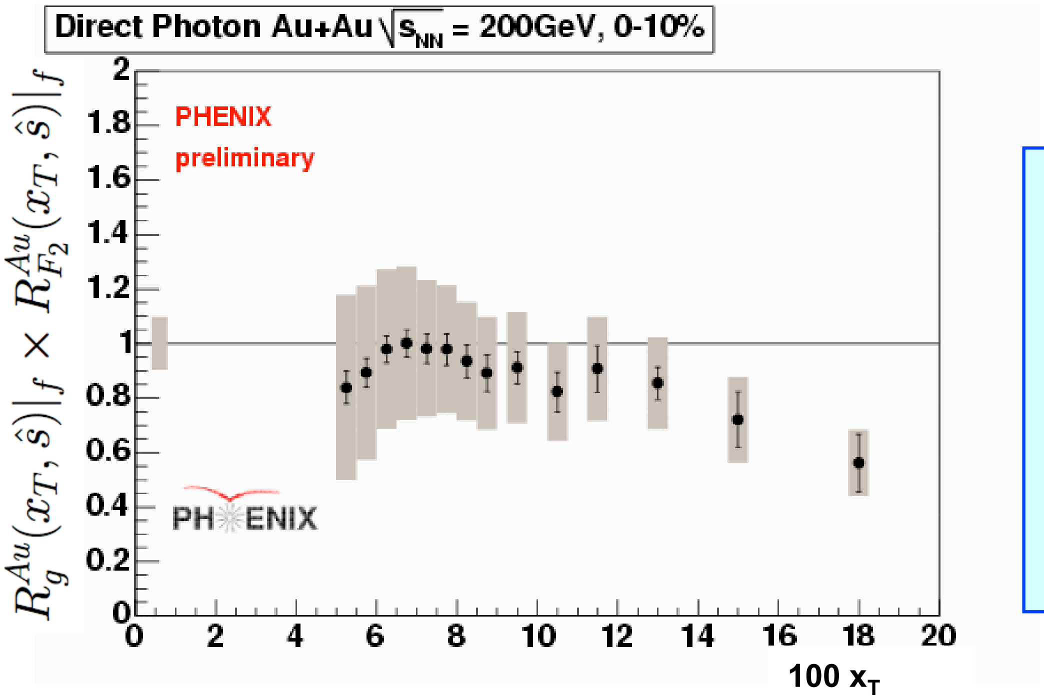


Do the structure function ratios actually drop by ~20% from x=0.1 to x=0.2?

Central Collisions---no theory counterpart-yet

Au+Au Central Collisions

$$R_{AA}=\frac{1}{A_f A_f}\frac{d^2\sigma_\gamma^{AA}|_f/dp_T^2\,dy_\gamma}{d^2\sigma_\gamma^{pp}/dp_T^2\,dy_\gamma}\approx\left(\frac{F_{2A}(x_T)|_f}{A_f F_{2p}(x_T)}\times\frac{g_A(x_T)|_f}{A_f g_p(x_T)}\right)=\quad ???$$



Theorists, HELP!

Very few attempts so far for structure function measurements or theory as a function of impact parameter:
E665, ZPC 65, 225 (1995)
Li and Wang, PLB 527, 85 (2002)
Klein and Vogt PRL 91, 142301 (2003)
Emel'yanov, et al. PRC 61, 044904 (2000)
and references therein.

Experimentalists:
RHIC p+A, eRHIC

Nobody has seriously measured nor calculated structure function ratios as a function of centrality!!!

CMOR, Nucl Phys B327 (1989) 541-568

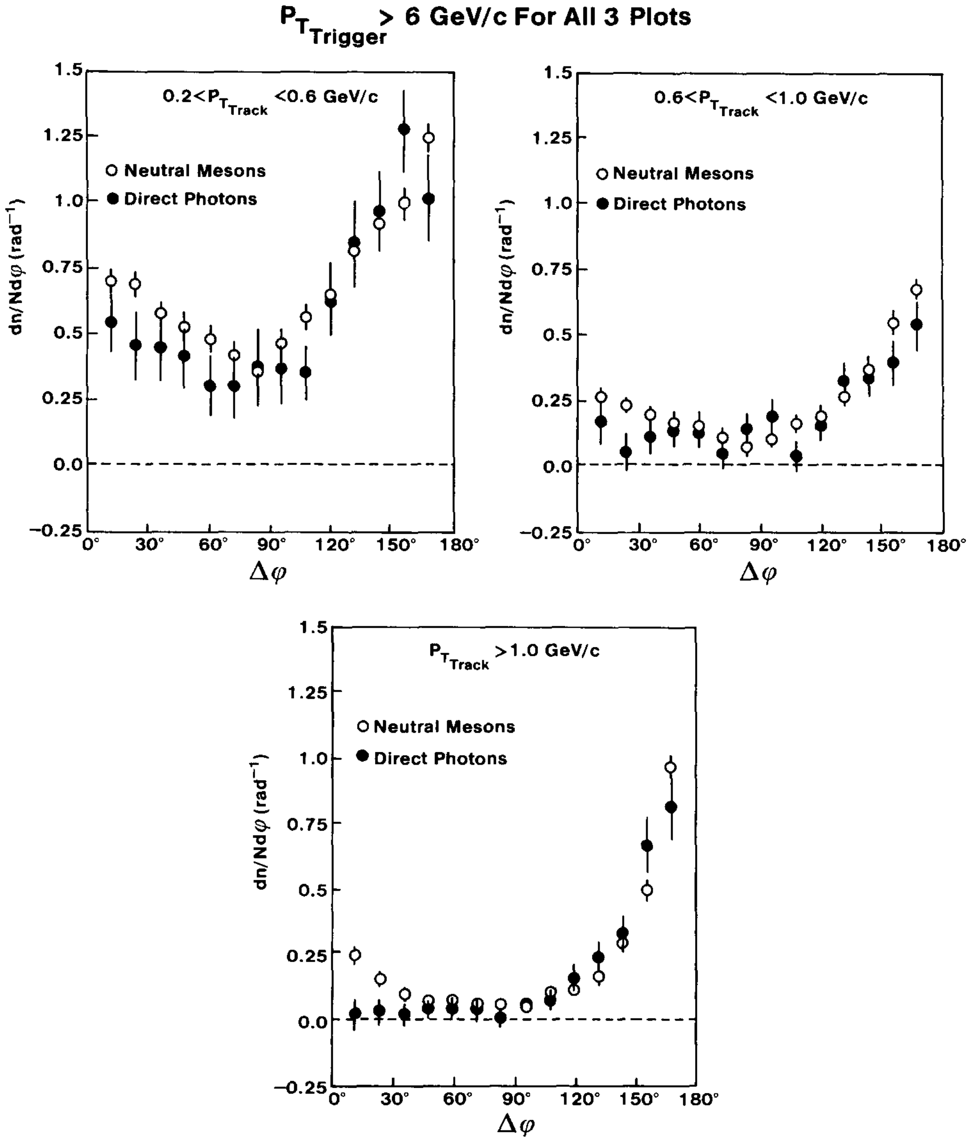
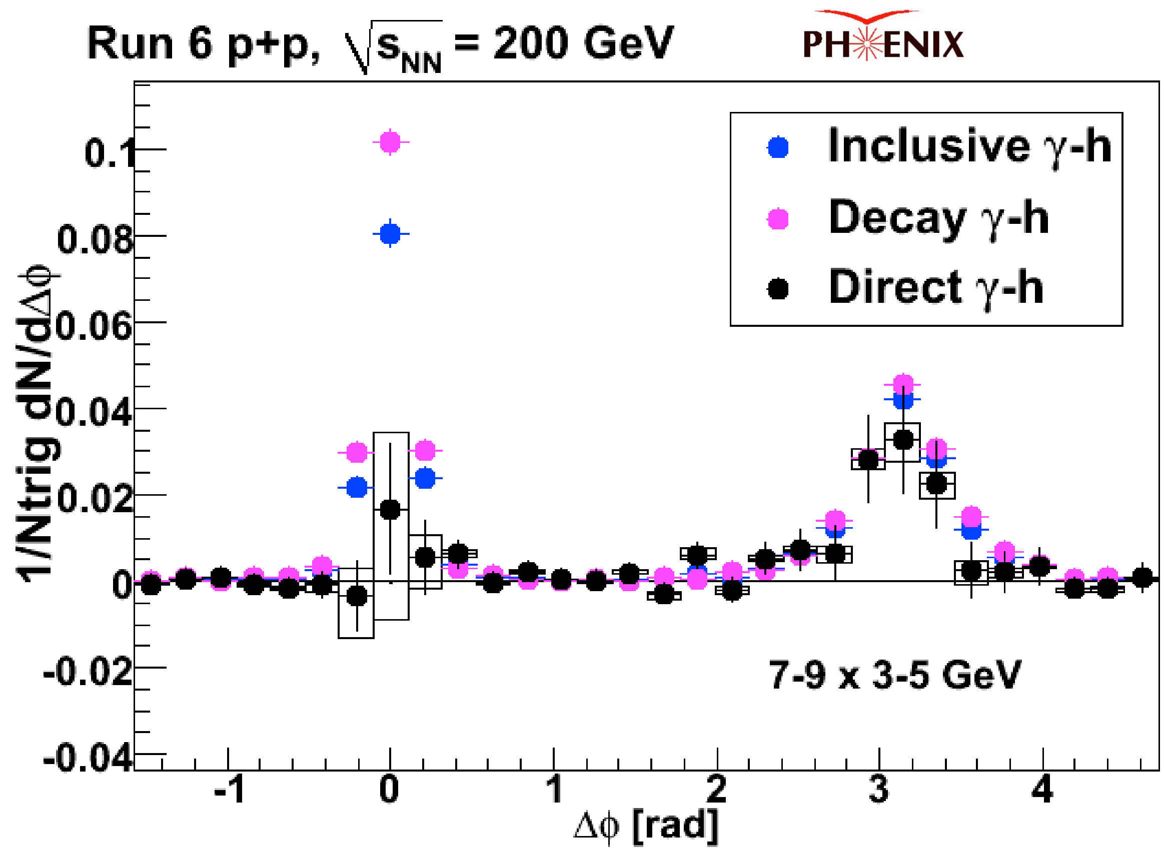


Fig. 15. Azimuthal distributions of charged associated particles for the direct photon and neutral meson samples (after background extraction) for $p_{T\,\mathrm{trigger}}$ greater than 6.0 GeV/c and for three different $p_{T\,\mathrm{track}}$ ranges.

QM2008-We're working on it



Source of formula on x_h

$$\frac{d^2\sigma_\pi}{dp_{\rm Tt}dp_{\rm Ta}} \approx \frac{\Gamma(n)}{b^n}\frac{B^2}{\hat{x}_{\rm h}}\frac{A}{p_{\rm T_t}^n}\frac{1}{(1+\frac{p_{\rm Ta}}{\hat{x}_{\rm h}p_{\rm Tt}})^n}, \qquad (40)$$

PHYSICAL REVIEW D 74, 072002 (2006)

$$\frac{d\sigma_\pi}{dp_{\rm Tt}} \approx \frac{\Gamma(n-1)}{b^{n-1}}\frac{AB}{p_{\rm T_t}^{n-1}}, \qquad (41)$$

The conditional probability is just the ratio of the joint probability Eq. (40) to the inclusive probability Eq. (41), or

$$\left.\frac{dP_\pi}{dp_{\rm Ta}}\right|_{p_{\rm Tt}} \approx \frac{B(n-1)}{bp_{\rm Tt}}\frac{1}{\hat{x}_{\rm h}}\frac{1}{(1+\frac{p_{\rm Ta}}{\hat{x}_{\rm h}p_{\rm Tt}})^n}. \qquad (42)$$

In the collinear limit, where $p_{\rm Ta} = x_{\rm E}p_{\rm Tt}$:

$$\left.\frac{dP_\pi}{dx_{\rm E}}\right|_{p_{\rm Tt}} \approx \frac{B(n-1)}{b}\frac{1}{\hat{x}_{\rm h}}\frac{1}{(1+\frac{x_{\rm E}}{\hat{x}_{\rm h}})^n}. \qquad (43)$$

The leading-particle effect a.k.a. trigger bias

- Due to the steeply falling power-law spectrum of the scattered partons, the inclusive particle p_T spectrum is dominated by fragments biased towards large z. This was unfortunately called trigger bias by M. Jacob and P. Landshoff, Phys. Rep. **48C**, 286 (1978) although it has nothing to do with a trigger.

$$\frac{d^2\sigma_\pi(\hat{p}_{T_t}, z_t)}{d\hat{p}_{T_t} dz_t} = \frac{d\sigma_q}{d\hat{p}_{T_t}} \times D^q_\pi(z_t)$$

Fragment spectrum given $\hat{p}_{T_t}$

$$= \frac{A}{\hat{p}^{n-1}_{T_t}} \times D^q_\pi(z_t)$$

Power law spectrum of parton $\hat{p}_{T_t}$

let $\hat{p}_{T_t} = p_{T_t}/z_t \qquad d\hat{p}_{T_t}/dp_{T_t}|_{z_t} = 1/z_t$

$$\frac{d^2\sigma_\pi(p_{T_t}, z_t)}{dp_{T_t} dz_t} = \frac{1}{z_t}\frac{A}{(p_{T_t}/z_t)^{n-1}} \times D^q_\pi(z_t)$$

$$= \frac{A}{p^{n-1}_{T_t}} \times z_t^{n-2} D^q_\pi(z_t)$$

Fragment spectrum given p_{Tt} is weighted to high z_t by z_t^{n-2}

where $z_{t\min}|_{p_{T_t}} = x_{T_t} \qquad D^q_\pi(z_t) = Be^{-bz_t}$ \qquad (<z>=1/b)

Continuing as in PRD **74**, 072002 (2006)

We can integrate over the trigger jet z_t and find the inclusive pion cross section:

$$\frac{1}{p_{T_t}}\frac{d\sigma_\pi}{dp_{T_t}} = \frac{AB}{p_{T_t}^n}\int_{x_{T_t}}^{1} dz_t z_t^{n-2} \exp -bz_t \qquad , \tag{8}$$

which can be written as:

$$\frac{1}{p_{T_t}}\frac{d\sigma_\pi}{dp_{T_t}} = \frac{AB}{p_{T_t}^n}\frac{1}{b^{n-1}}\left[\Gamma(n-1, bx_{T_t}) - \Gamma(n-1, b)\right] \qquad , \tag{9}$$

where

$$\Gamma(a,x) \equiv \int_x^\infty t^{a-1}\, e^{-t}\, dt \tag{10}$$

is the Complementary or upper Incomplete Gamma function, and $\Gamma(a,0) = \Gamma(a)$ is the Gamma function, where $\Gamma(a) = (a-1)!$ for a an integer.

A reasonable approximation for small x_T values is obtained by taking the lower limit of Eq. 8 to zero and the upper limit to infinity, with the result that:

$$\boxed{\frac{1}{p_{T_t}}\frac{d\sigma_\pi}{dp_{T_t}} \approx \frac{\Gamma(n-1)}{b^{n-1}}\frac{AB}{p_{T_t}^n}}$$

Bjorken parent-child relation: parton and particle invariant p_T spectra have same power *n*

$$\boxed{\langle z_t(p_{T_t})\rangle = \frac{\int_{x_{T_t}}^1 dz_t z_t^{n-1} \exp -bz_t}{\int_{x_{T_t}}^1 dz_t z_t^{n-2} \exp -bz_t} = \frac{1}{b}\frac{\left[\Gamma(n, bx_{T_t}) - \Gamma(n,b)\right]}{\left[\Gamma(n-1, bx_{T_t}) - \Gamma(n-1,b)\right]} \approx \frac{n-1}{b}}$$

Inclusive high p_T particle has n-1 times larger <z> than unbiased fragmentation, <z>=1/b

2 particle Correlations

$$\frac{d^2\sigma_\pi(\hat{p}_{T_t}, z_t)}{d\hat{p}_{T_t} dz_t} = \frac{d\sigma_q}{d\hat{p}_{T_t}} \times D^q_\pi(z_t)$$

Prob. that you make a jet with $\hat{p}_{Tt}$ which fragments to a π with $z_t = p_{Tt}/\hat{p}_{Tt}$

Also detect fragment with $z_a = p_{T_a}/\hat{p}_{T_a}$
from away jet with $\hat{p}_{T_a}/\hat{p}_{T_t} \equiv \hat{x}_h$

$$\frac{d^3\sigma_\pi(\hat{p}_{T_t}, z_t, z_a)}{d\hat{p}_{T_t} dz_t dz_a} = \frac{d\sigma_q}{d\hat{p}_{T_t}} \times D^q_\pi(z_t) \times D^q_\pi(z_a)$$

Prob. that away jet with $\hat{p}_{Ta}$ fragments to a π with $z_a = p_{Ta}/\hat{p}_{Ta}$

$$z_a = \frac{p_{T_a}}{\hat{p}_{T_a}} = \frac{p_{T_a}}{\hat{x}_{\rm h}\, \hat{p}_{T_t}} = \frac{z_t\, p_{T_a}}{\hat{x}_{\rm h}\, p_{T_t}}$$

(1) $$\boxed{\frac{d\sigma_\pi}{dp_{T_t} dz_t dp_{T_a}} = \frac{1}{\hat{x}_{\rm h}\, p_{T_t}} \frac{d\sigma_q}{d(p_{T_t}/z_t)} D^q_\pi(z_t) D^q_\pi\left(\frac{z_t p_{T_a}}{\hat{x}_{\rm h} p_{T_t}}\right)}$$

Appears to be sensitive to away jet Frag. Fn.

Amazingly, I got a neat analytical result

$$\boxed{\frac{d^3\sigma_\pi}{dp_{T_t}dz_t dp_{T_a}} = \frac{1}{\hat{x}_{\rm h}\, p_{T_t}} \frac{d\sigma_q}{d(p_{T_t}/z_t)} D^\pi_q(z_t) D^\pi_q(\frac{z_t p_{T_a}}{\hat{x}_{\rm h} p_{T_t}})} \qquad (1)$$

Take: $D(z) = B\exp(-bz)$ $\qquad \frac{d\sigma_q}{d\hat{p}_{T_t}} = \frac{A}{\hat{p}^{n-1}_{T_t}} = A\frac{{z_t}^{n-1}}{p^{n-1}_{T_t}}$

$$(2) \qquad \frac{d^2\sigma_\pi}{dp_{T_t}dp_{T_a}} = \frac{B^2}{\hat{x}_h}\frac{A}{p^n_{T_t}}\int_{x_{T_t}}^{\hat{x}_h\frac{p_{T_t}}{p_{T_a}}} dz_t z_t^{n-1} \exp[-bz_t(1+\frac{p_{T_a}}{\hat{x}_h p_{T_t}})]$$

$$\frac{d\sigma_\pi}{dp_{T_t}} = \frac{AB}{p^{n-1}_{T_t}}\int_{x_{T_t}}^1 dz_t z_t^{n-2} \exp -bz_t$$

Using: $\Gamma(a,x) \equiv \int_x^\infty t^{a-1}\, e^{-t}\, dt$ $\qquad$ Where $\Gamma(a,0) = \Gamma(a) = (a-1)\,\Gamma(a)$

The final result

$$\frac{d^2\sigma_\pi}{dp_{T_t}dp_{T_a}} \approx \frac{\Gamma(n)}{b^n}\frac{B^2}{\hat{x}_h}\frac{A}{p_{T_t}^n}\frac{1}{(1+\frac{p_{T_a}}{\hat{x}_h p_{T_t}})^n}$$

$$\frac{d\sigma_\pi}{dp_{T_t}} \approx \frac{\Gamma(n-1)}{b^{n-1}}\frac{AB}{p_{T_t}^{n-1}}$$

$$\left.\frac{dP_\pi}{dp_{T_a}}\right|_{p_{T_t}} \approx \frac{B(n-1)}{bp_{T_t}}\frac{1}{\hat{x}_h}\frac{1}{(1+\frac{p_{T_a}}{\hat{x}_h p_{T_t}})^n}$$

In the collinear limit, where $p_{T_a} = x_E p_{T_t}$:

$$\left.\frac{dP_\pi}{dx_E}\right|_{p_{T_t}} \approx \frac{B(n-1)}{b}\frac{1}{\hat{x}_h}\frac{1}{(1+\frac{x_E}{\hat{x}_h})^n}$$

Where B/b≈<m>≈b is the mean charged multiplicity in the jet

Normalization of Fragmentation Functions

For an exponential fragmentation function,

$$D(z) = Be^{-bz} \qquad ,$$

the mean multiplicity of fragments in the jet is:

$$\langle m \rangle = \int_0^1 D(z)dz = \frac{B}{b}(1 - e^{-b})$$

and these fragments carry the total momentum of the jet:

$$\int_0^1 zD(z)dz = \frac{B}{b^2}(1 - e^{-b}(1+b)) \equiv 1 \qquad ,$$

where the $\langle z \rangle$ per fragment is:

$$\langle z \rangle = \frac{\int_0^1 zD(z)dz}{\int_0^1 D(z)dz} = \frac{1}{\langle m \rangle} \qquad .$$

The results are:

$$B = \frac{b^2}{1 - e^{-b}(1+b)} \approx b^2$$

$$\langle m \rangle = \frac{b(1-e^{-b})}{1 - e^{-b}(1+b)} \approx b \qquad ,$$

$$\langle z \rangle = \frac{1 - e^{-b}(1+b)}{b(1-e^{-b})} \approx \frac{1}{b} \qquad .$$

I assumed b is the same for π^0 and all charged. Then using B/b=b which normalizes total momentum to 1, I get the correct jet cross section. Obviously the total momentum for π^0 is ~1/3 so B for pure π^0 ~1/3 b^2