Heavy-quark fragmentation with an effective coupling constant

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3. **Collinear and soft resummations**
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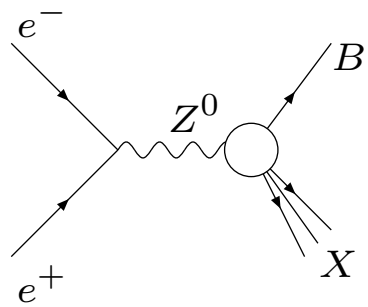
U. Aglietti, G.C. and G. Ferrera, NPB 775 (2007) 162;

G. C. and G. Ferrera, JHEP 0712 (2007) 029

Heavy-hadron production in e^+e^- annihilation



Perturbative QCD allows one to calculate the parton-level (b-quark) spectrum, but not the hadronization into a B

Phenomenological hadronization models are usually given in terms of non-perturbative fragmentation functions

$$\sigma(e^+e^- \to B) = \sigma(e^+e^- \to b\bar{b}) \otimes D_{\rm np}(b \to B)$$

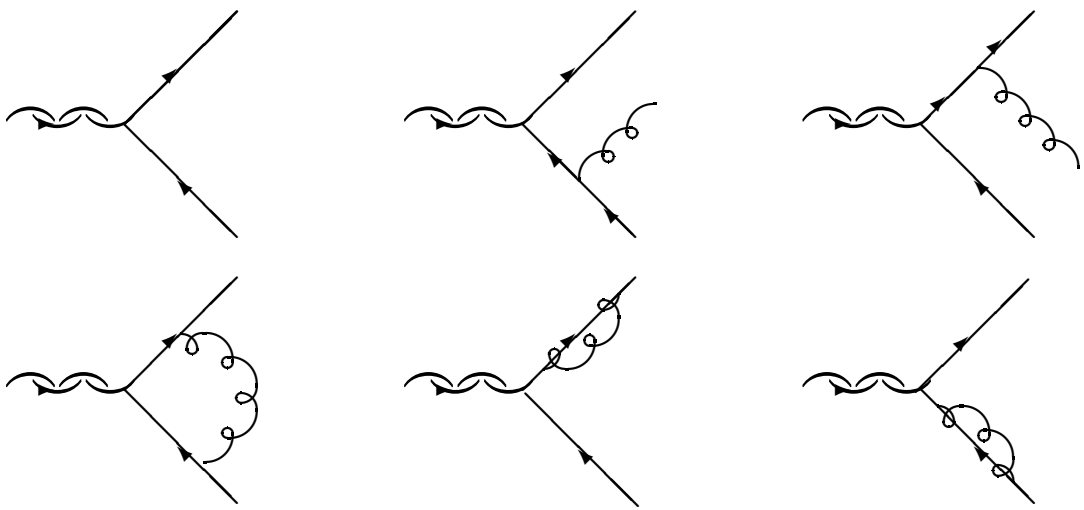
$D_{np}(b \to B)$ contains parameters to be fitted to experimental data

$$D_{\rm np,K}(x,\gamma) = (1+\gamma)(2+\gamma)x(1-x)^\gamma \qquad D_{\rm np,P}(x,\epsilon) = \frac{A}{x\left[1-1/x-\epsilon/(1-x)\right]^2}$$

Alternatively: non-perturbative corrections into an effective coupling constant

Heavy-quark production at NLO in e^+e^- annihilation at the Z^0 pole:

$$e^+e^- \to Z^0(q) \to b(p_b)\bar{b}(p_{\bar{b}})g(p_g)$$



$$x_b = \frac{2p_b \cdot q}{q^2} = \frac{2E_b}{m_Z} \quad ; \quad \sigma_0 = \sigma(e^+e^- \to b\bar{b})$$

$$\frac{1}{\sigma_0}\frac{d\sigma}{dx_b} = \delta(1-x_b) + \frac{\alpha_S(\mu)}{2\pi}\left[P_{qq}(x_b)\ln\frac{m_Z^2}{m_b^2} + A(x_b)\right] + \mathcal{O}\left[\left(\frac{m_b^2}{m_Z^2}\right)^p\right]$$

$$P_{qq}(x_b) = C_F\left(\frac{1+x_b^2}{1-x_b}\right)_+ \; ; \; \alpha_S(\mu)\ln\frac{m_Z^2}{m_b^2} \simeq \mathcal{O}(1) \; ; \; \int_0^1 dx_b f(x_b)[g(x_b)]_+ = \int_0^1 dx_b[f(x_b)-f(1)]g(x_b)$$

Perturbative fragmentation functions

B. Mele and P. Nason, NPB 361 (1991) 626

One further factorization at parton-level: $E_b \ll m_b \Rightarrow$ **the** b **is 'massless';** $E_b \simeq m_b \Rightarrow$ **the** b **is heavy**

$$\frac{1}{\sigma_0}\frac{d\sigma}{dx_b}(x_b, m_b \neq 0) = \frac{1}{\sigma_0}\sum_i \int_{x_b}^1 \frac{dz}{z}\frac{d\hat{\sigma}_i}{dz}(m_Z, m_i = 0, \mu_F) D_i\left(\frac{x_b}{z}, \mu_F, m_b\right) + \mathcal{O}\left[\left(\frac{m_b^2}{m_Z^2}\right)^p\right]$$

$D_i(x_b, \mu_F, m_b)$**: perturbative fragmentation function (PFF) for** $i \to b$

$$\frac{1}{\sigma_0}\frac{d\hat{\sigma}_b}{dz} = \delta(1-z) + \frac{\alpha_S(\mu)}{2\pi}\left[P_{qq}(z)\left(-\frac{1}{\epsilon} + \gamma_E - \ln 4\pi\right) + \hat{A}(z)\right] \qquad (d = 4 - 2\epsilon)$$

$\overline{\mathrm{MS}}$ **coefficient function** $(i = b)$**:**

$$\left(\frac{1}{\sigma_0}\frac{d\hat{\sigma}_b}{dz}\right)^{\overline{\mathrm{MS}}} = \delta(1-z) + \frac{\alpha_S(\mu)}{2\pi}\hat{A}(z)$$

$$\frac{1}{\sigma_0}\frac{d\sigma}{dx_b}(m_b) = \left(\frac{1}{\sigma_0}\frac{d\hat{\sigma}_b}{dx_b}(m_b = 0)\right)^{\overline{\mathrm{MS}}} \otimes D_b^{\overline{\mathrm{MS}}}(m_b)$$

DGLAP equations for PFFs (non-singlet approximation):

$$\frac{d}{d\ln\mu_F^2}D_i(x_b,\mu_F,m_b)=\sum_j\int_{x_b}^1\frac{dz}{z}P_{ij}\left(\frac{x_b}{z},\alpha_S(\mu_F)\right)D_j(z,\mu_F,m_b)$$

Initial condition $D(x_b,\mu_{0F})$ is process-independent (Mele–Nason, Cacciari–Catani)

$$D_b(x_b,\mu_{0F},m_b)=\delta(1-x_b)+\frac{\alpha_S(\mu_0)C_F}{2\pi}\left[\frac{1+x_b^2}{1-x_b}\left(\ln\frac{\mu_{0F}^2}{m_b^2}-2\ln(1-x_b)-1\right)\right]_+$$

$$\frac{dD_N(\mu_F,m_b)}{d\ln\mu_F^2}=\frac{\alpha_S(\mu_F)}{2\pi}\left[P_N^{(0)}+\frac{\alpha_S(\mu_F)}{2\pi}P_N^{(1)}\right]D_N(\mu_F,m_b)\qquad D_N=\int_0^1 dx x^{N-1}D(x)$$

$$D_N(\mu_F,m_b)=D_N(\mu_{0F},m_b)\exp\left\{\frac{P_N^{(0)}}{2\pi b_0}\ln\frac{\alpha_S(\mu_{0F})}{\alpha_S(\mu_F)}+\frac{\alpha_S(\mu_{0F})-\alpha_S(\mu_F)}{4\pi^2 b_0}\left[P_N^{(1)}-\frac{2\pi b_1}{b_0}P_N^{(0)}\right]\right\}$$

$$\begin{aligned}D_N(\mu_F,m_b)\quad&=\quad D_N(\mu_{0F},m_b)\exp\Big\{C_{1,0}\alpha_S(\mu_F)+C_{1,1}\alpha_S(\mu_F)\ln(\mu_F^2/\mu_{0F}^2)\ldots\\&+\quad C_{n,n-1}\alpha_S^n(\mu_F)\ln^{n-1}(\mu_F^2/\mu_{0F}^2)+C_{n,n}\alpha_S^n(\mu_F)\ln^n(\mu_F^2/\mu_{0F}^2)+\ldots\Big\}\end{aligned}$$

Resummation of LLs $\alpha_S^n\ln^n(\mu_F^2/\mu_{0F}^2)$ and NLLs $\alpha_S^n\ln^{n-1}(\mu_F^2/\mu_{0F}^2)$

$\mu_{0F}\simeq m_b$ **and** $\mu_F\simeq m_Z$: **resummation of NLL $\ln(m_Z^2/m_b^2)$ (collinear resummation)**

Coefficient function and initial condition present terms enhanced at large x_b, corresponding to soft or collinear radiation off the $b\bar{b}$ pair

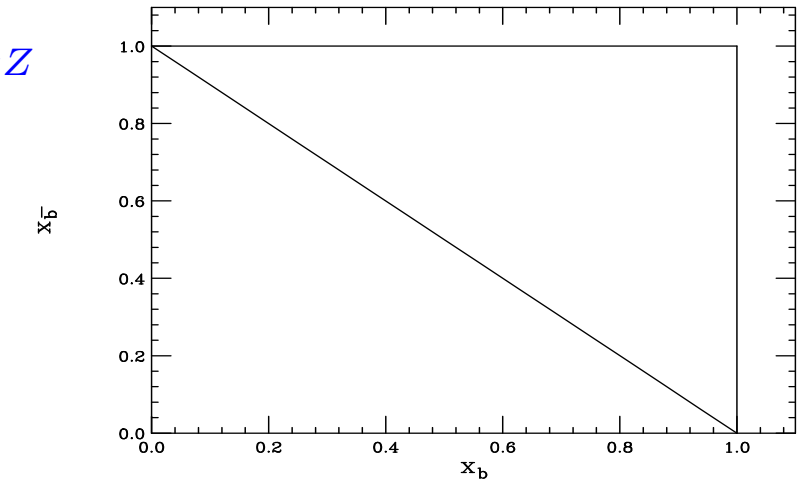
$$Z^0(q) \to b(p_b)\bar{b}(p_{\bar{b}})g(p_g) \quad ; \quad x_i = 2E_i/m_Z$$

$$1 - x_b \;=\; x_{\bar{b}}\; x_g\; (1 - \cos\theta_{\bar{b}g})/2$$

$$\bar{b} \parallel g \Rightarrow x_b = 1$$

$$b \parallel g \Rightarrow x_{\bar{b}} = 1$$

$$x_b = x_{\bar{b}} = 1 \Rightarrow x_g = 0 \quad (x_b + x_{\bar{b}} + x_g = 2)$$



$$\left[\frac{\ln(1-x)}{1-x}\right]_+ \to \ln^2 N \quad ; \quad \frac{1}{(1-x)_+} \to \ln N$$

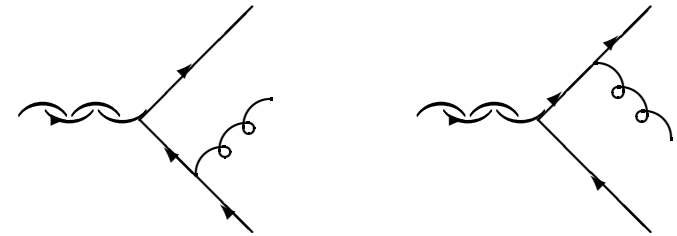
Large-N expressions:

$$C_N = 1 + \frac{\alpha_S C_F}{\pi}\left[\frac{1}{2}\ln^2 N + \left(\frac{3}{4} + \gamma_E - \ln\frac{m_Z^2}{\mu_F^2}\right)\ln N + K_C(\mu_F^2, m_Z^2) + \mathcal{O}\left(\frac{1}{N}\right)\right]$$

$$D_{\mathrm{ini},N} = 1 + \frac{\alpha_S C_F}{\pi}\left[-\ln^2 N + \left(1 - 2\gamma_E + \ln\frac{m_b^2}{\mu_{0F}^2}\right)\ln N + K_D(\mu_{0F}^2, m_b^2) + \mathcal{O}\left(\frac{1}{N}\right)\right]$$

Large-x resummation is process-dependent in the coefficient function and process-independent in the initial condition
(Cacciari–Catani)

Eikonal matrix element for $Z \to b(p_b)\bar{b}(p_{\bar{b}})g(p_g)$



$$\mathcal{M}^{a,\mu} = \mathcal{M}_0^\mu \, g_s T^a \epsilon_\nu \left(\frac{p_b^\nu}{p_b \cdot p_g} - \frac{p_{\bar{b}}^\nu}{p_{\bar{b}} \cdot p_g} \right) \quad ; \quad \sigma_N = \frac{C_F}{\pi} \int_0^1 dz \frac{z^{N-1}-1}{1-z} \int_0^{m_Z^2(1-z)} dk_T^2 \left[\frac{\alpha_S}{k_T^2} - \frac{3}{4}\alpha_S \right]$$

$z = 1 - x_g \;\; ; \;\; k_T^2 = (p_b + p_g)^2 (1-z) \simeq E_g^2 \sin^2\theta_{bg}$

$$\alpha_S \to \alpha_S(k_T^2) \;\; ; \;\; \frac{C_F}{\pi} \frac{\alpha_S(k_T^2)}{k_T^2} \to \frac{A\left[\alpha_S(k_T^2)\right]}{k_T^2} \quad \text{Catani} - \text{Trentadue (1989)}$$

$$\int_0^{m_Z^2(1-z)} dk_T^2 \alpha_S(k_T^2) \simeq \alpha_S\left(m_Z^2(1-z)\right) \to B\left[\alpha_S(m_Z^2(1-z))\right]$$

$$\Delta = \exp\left\{ \int_0^1 dz \frac{z^{N-1}-1}{1-z} \left\{ \int_{\mu_F^2}^{m_Z^2(1-z)} \frac{dk_T^2}{k_T^2} A\left[\alpha_S(k_T^2)\right] + B\left[\alpha_S\left(m_Z^2(1-z)\right)\right] \right\} \right\} = \exp(g_1 \ln N + g_2 + \alpha_S g_3)$$

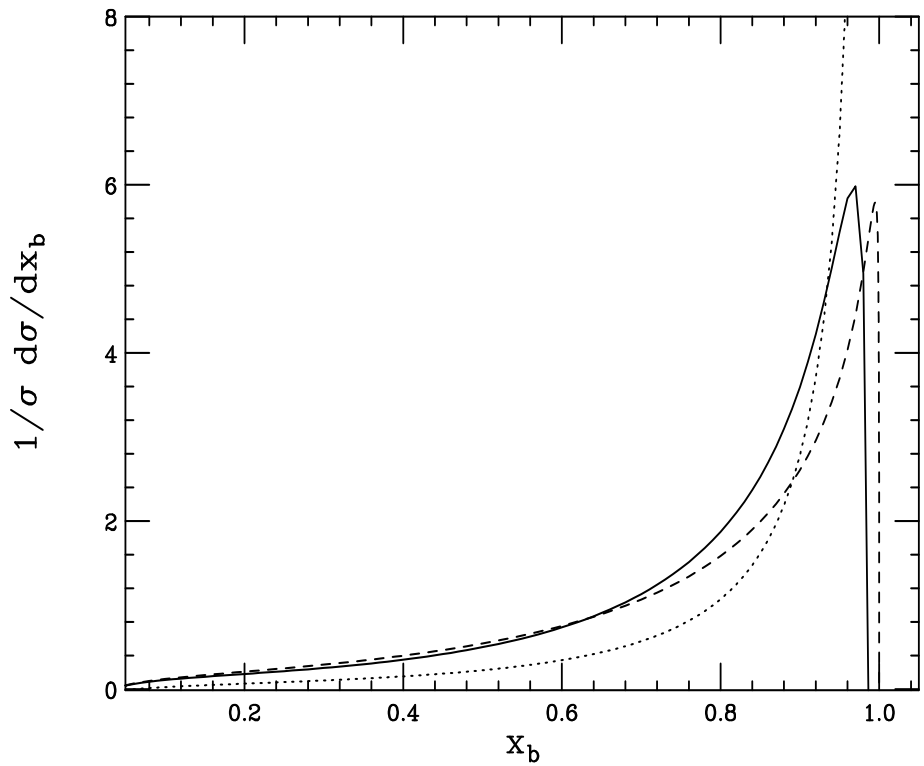
$$A(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n A^{(n)} \quad ; \quad B(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n B^{(n)}$$

$g_1 \log N \Rightarrow$ **LLs** $A^{(1)}$ **:** $\alpha_S \log^2 N, \ldots \alpha_S^n \log^{n+1} N$; $g_2 \Rightarrow$ **NLLs** $A^{(2)}, B^{(1)}$ **:** $\alpha_S \log N, \ldots \alpha_S^n \log^n N$

$\alpha_S \, g_3 \Rightarrow$ **NNLLs** $A^{(3)}, B^{(2)}$ **:** $\alpha_S^2 \ln N, \ldots \alpha_S^n \log^{n-1} N$

b-quark energy spectrum in e^+e^- annihilation

m_Z=91.19 GeV, m_b=5 GeV, $\mu_F = \mu_R = m_Z$, $\mu_{0R} = \mu_{0F} = m_b$, $\Lambda_{\overline{\rm MS}}$=200 GeV

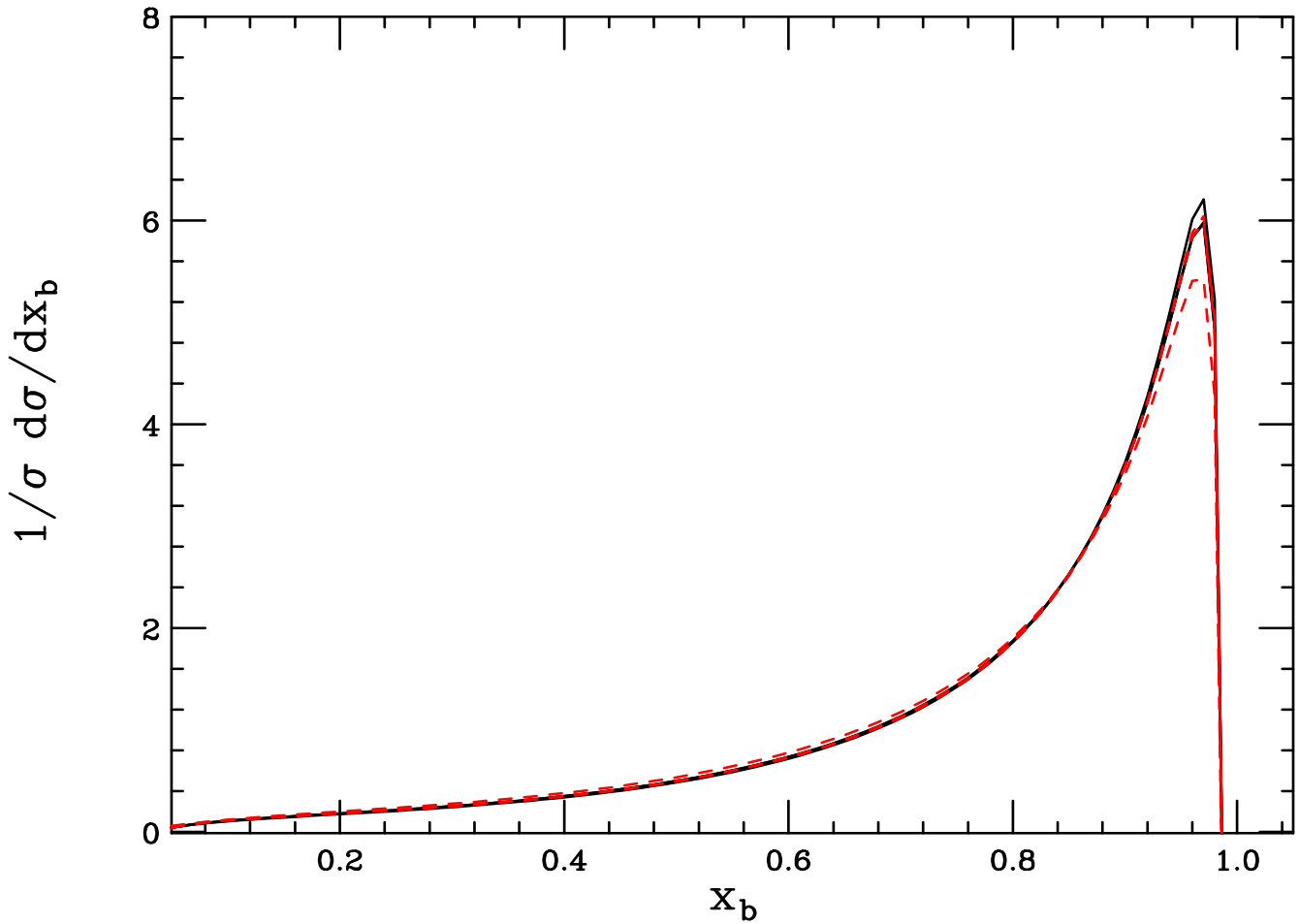


Solid: soft and collinear resummation Dashes: only collinear resummation

Dots: massive NLO without resummation

Remarkable impact of resummations

Very little dependence on factorization and renormalization scales

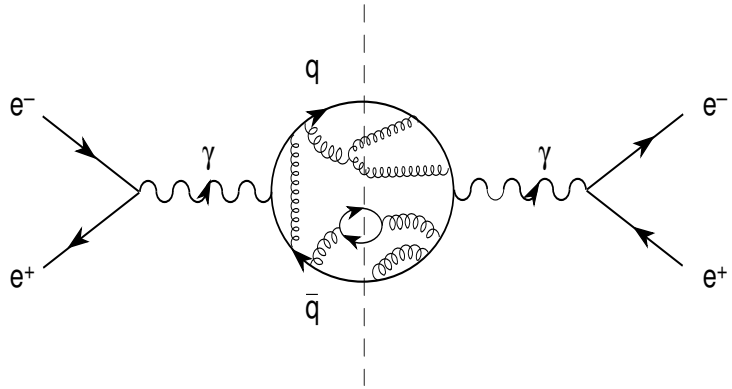


Solid: $\mu_F = m_Z/2, m_Z, 2m_Z$

Dashes: $\mu_{0F} = m_b/2, m_b, 2m_b$

Strong coupling constant in resummed calculations

Need to account for multiple emissions from the $q\bar{q}$ pair



D. Amati et al, NPB 173 (1980) 429:

$$\alpha_S \longrightarrow \frac{i}{2\pi}\int_0^{k_T^2} ds\,\mathrm{Disc}_s\,\frac{\alpha_S(-s)}{s} \simeq \alpha_S(k_T^2) \quad ; \quad \mathrm{Disc}\,f(s) = \lim_{\epsilon\to 0}\left[f(s+i\epsilon) - f(s-i\epsilon)\right]$$

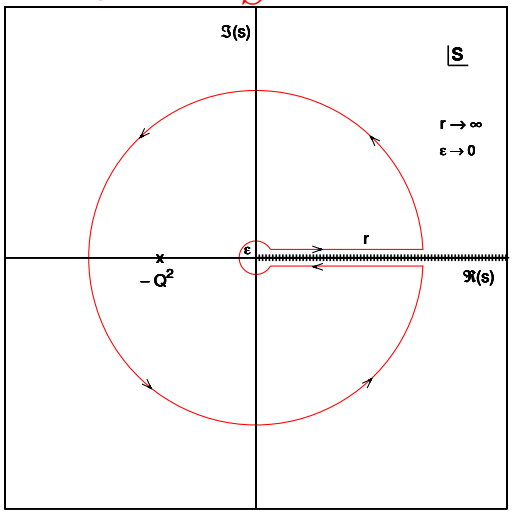
$$\alpha_{S,\mathrm{LO}}(-s) = \frac{1}{\beta_0 \ln(-s/\Lambda^2)} = \frac{1}{\beta_0[\ln(|s|/\Lambda^2) - i\pi\Theta(s)]} \quad ; \quad \ln\frac{|s|}{\Lambda^2} \gg \pi$$

Cut for $s > 0$; Landau pole for $|s| = \Lambda^2$

$$\alpha_S(k_T^2) = \frac{\alpha_S(m_Z^2)}{1 + \alpha_S(Q^2)\beta_0 \ln(m_Z^2/k_T^2)} \simeq \alpha_S(m_Z^2)\left[1 - \alpha_S(m_Z^2)\beta_0 \ln\frac{m_Z^2}{k_T^2} + \ldots\right] \qquad \beta_0 = \frac{33 - 12n_f}{12\pi} \quad \textbf{(LO)}$$

Resumming a class of soft/collinear enhanced terms $\sim \ln(m_Z^2/k_T^2)$

Analytic α'_S free from Landau pole with $\mathrm{Disc}(\alpha'_S) = \mathrm{Disc}(\alpha_S)$ D.Shirkov, PRL79(1997)1209



$$\alpha'_S(Q^2) = \frac{1}{2\pi i}\int_\Gamma ds\,\frac{\alpha'_S(-s)}{s+Q^2} = \frac{1}{2\pi i}\int_0^\infty \frac{ds}{s+Q^2}\mathrm{Disc}[\alpha'_S(-s)]$$

$$\alpha'_S(Q^2) = \frac{1}{2\pi i}\int_0^\infty \frac{ds}{s+Q^2}\mathrm{Disc}[\alpha_S(-s)] \qquad \alpha'_{S,\mathrm{LO}}(Q^2) = \frac{1}{\beta_0}\left[\frac{1}{\ln(Q^2/\Lambda^2)} - \frac{\Lambda^2}{Q^2-\Lambda^2}\right]$$

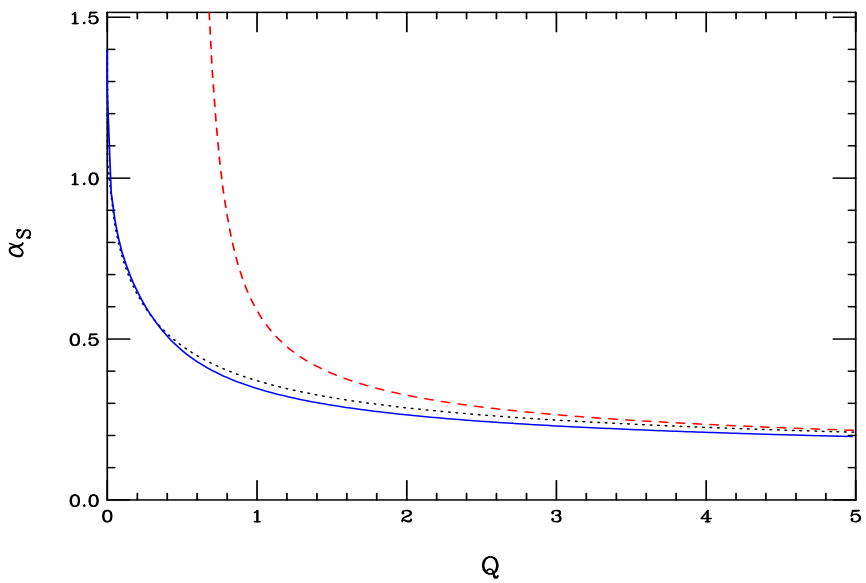
Our model : $$\tilde{\alpha}_S(k_T^2) = \frac{i}{2\pi}\int_0^{k_T^2} ds\ \mathrm{Disc}\frac{\alpha'_S(-s)}{s}$$

$$\mathrm{LO}:\ \tilde{\alpha}_S(k_T^2) = \frac{1}{2\pi i\beta_0}\left[\ln\left(\ln\frac{k_T^2}{\Lambda^2}+i\pi\right) - \ln\left(\ln\frac{k_T^2}{\Lambda^2}-i\pi\right)\right] = \frac{1}{\beta_0}\left\{\frac{1}{2} - \frac{1}{\pi}\arctan\left[\frac{\ln(k_T^2/\Lambda^2)}{\pi}\right]\right\}$$

$$k_T^2 \gg \Lambda^2:\quad \tilde{\alpha}_S(k_T^2) = \alpha_S(k_T^2) - \frac{(\pi\beta_0)^2}{3}\alpha_S^3(k_T^2) + \mathcal{O}(\alpha_S^4)$$

$\tilde{\alpha}_S(k_T^2)$ **does not exhibit the Landau pole and** $\to \alpha_S(k_T^2)$ **for** $k_T^2 \gg \Lambda^2$

Comparison of effective, analytic and standard coupling constants



Dashes: standard α_S

Dots: analytic α'_S

Solid: effective $\tilde{\alpha}_S$

Assumption of the effective-coupling model:

The transition $b \to B\ q$ is described by the effective coupling constant

$$\alpha_S(Q^2) \longrightarrow \tilde{\alpha}_S(Q^2) \quad \Longrightarrow \quad E_b \longrightarrow E_B$$

In NNLL large-x resummation, redefine $A^{(3)}$ in $A(\alpha_S) = \sum_n \alpha_S^n A^{(n)}$:

$$A^{(3)} \to \tilde{A}^{(3)} = A^{(3)} + \frac{(\pi\beta_0)^2}{3} A^{(1)}$$

The other coefficients $A^{(i)}$, $B^{(i)}$ remain the same

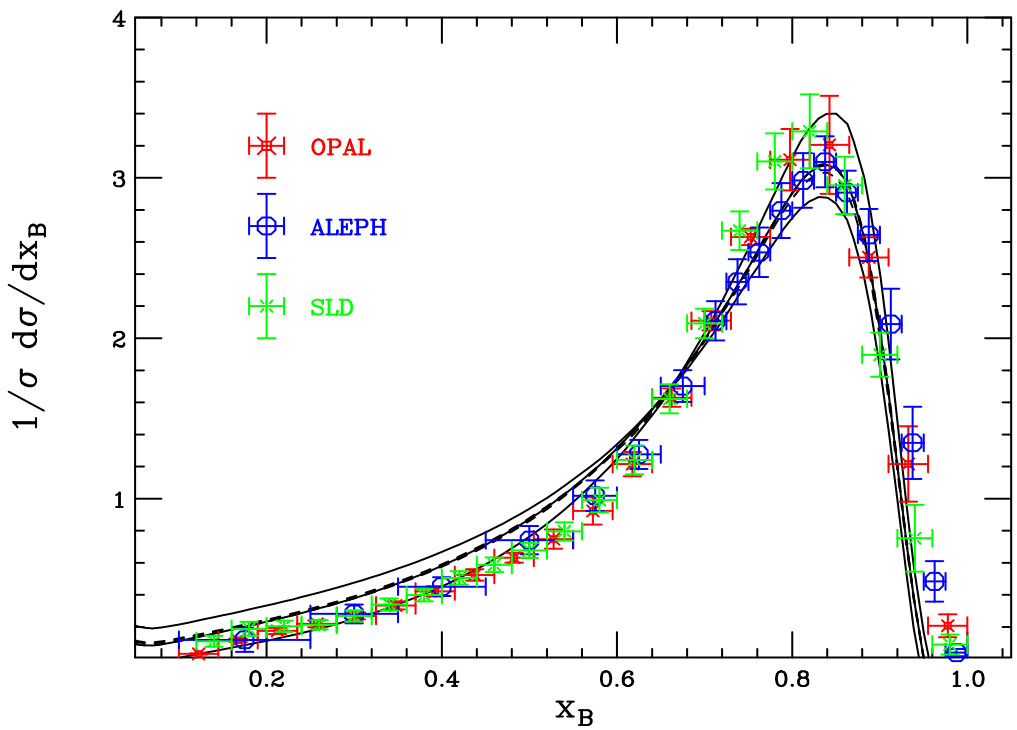
Bottom fragmentation with an effective coupling constant

In perturbative calculation (NLO hard scattering, NLL DGLAP, NNLL large-x resummation): $\alpha_S \longrightarrow \tilde{\alpha}_S \qquad x_b \longrightarrow x_B$ **(parton $\rightarrow$ hadron)**

Varying scales and masses (m_b or m_B?):

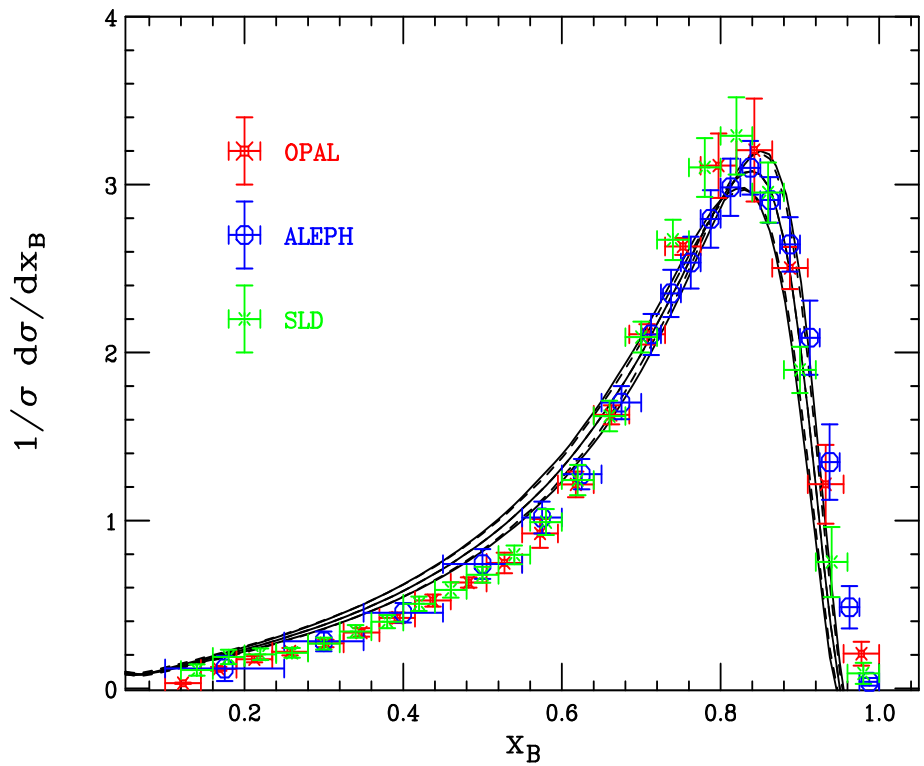
$m_Z/2 < \mu_{F,R} < 2m_Z \quad ; \quad m_b/2 < \mu_{0F,R} < 2m_b$

$4.7\ \mathrm{GeV} < m_b < 5.3\ \mathrm{GeV} \quad ; \quad 0.117 < \alpha_S(m_Z^2) < 0.121 \quad ; \quad 0.115 < \tilde{\alpha}_S(m_Z^2) < 0.119$



Solid: dependence on μ_{0F}; Dashes: dependence on μ_F

Dependence on $\alpha_S(m_Z^2)$ and m_b

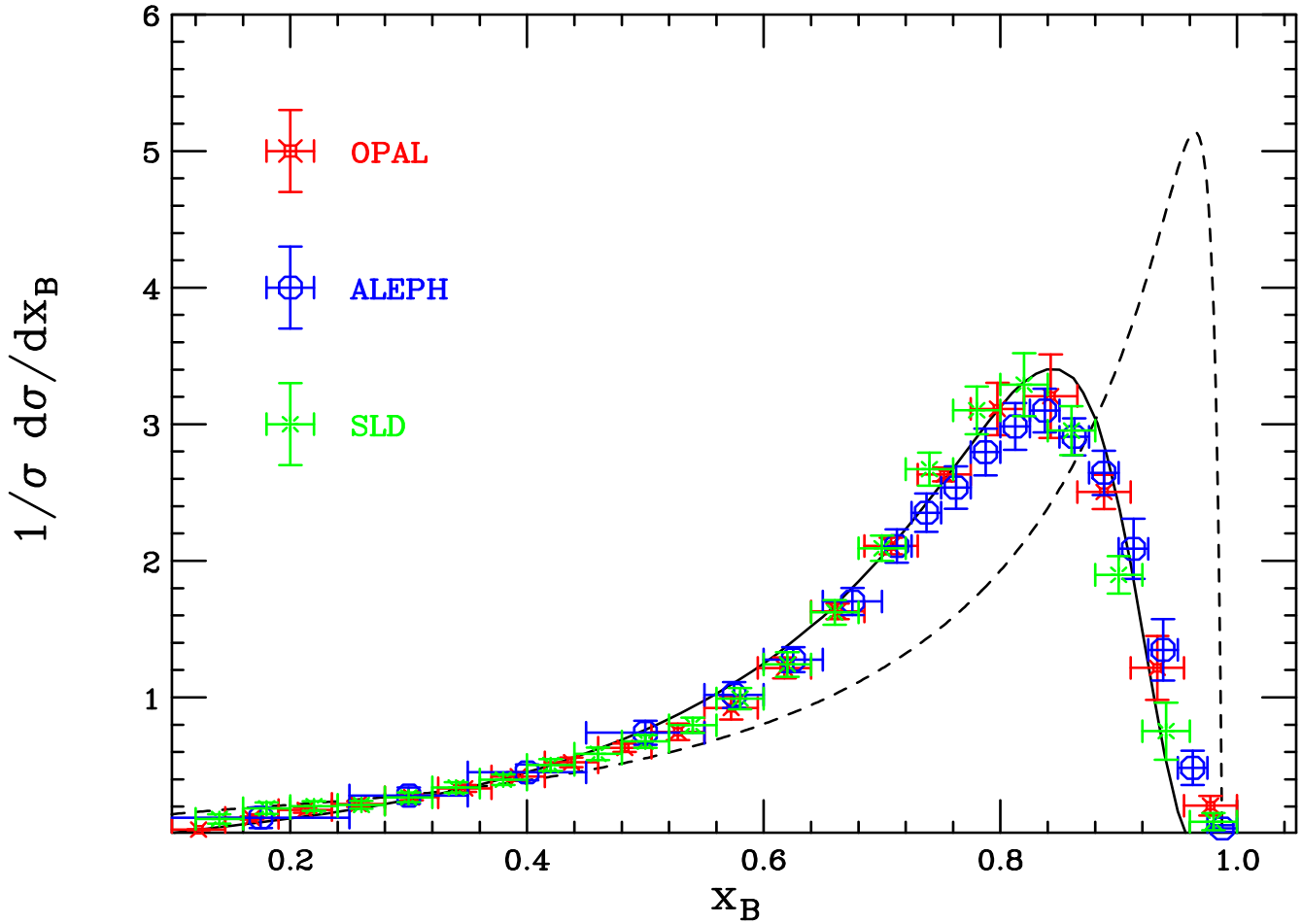


Solid: dependence on $\alpha_S(m_Z^2)$; Dashes: dependence on m_b

Main χ^2 values for $x_B \leq 0.92$

μ_{0F}	m_b	χ^2/dof (ALEPH)	χ^2/dof (OPAL)	χ^2/dof (SLD)	χ^2/dof (overall)
m_b	5 GeV	21.4/16	162.18/18	109.1/20	293.2/54
$m_b/2$	5 GeV	24.8/16	30.4/18	47.8/20	103.0/54
m_b	5.3 GeV	11.9/16	116.1/18	84.2/20	212.2/54

Overall impact of non-perturbative corrections via the effective coupling (best-fit parametrization)



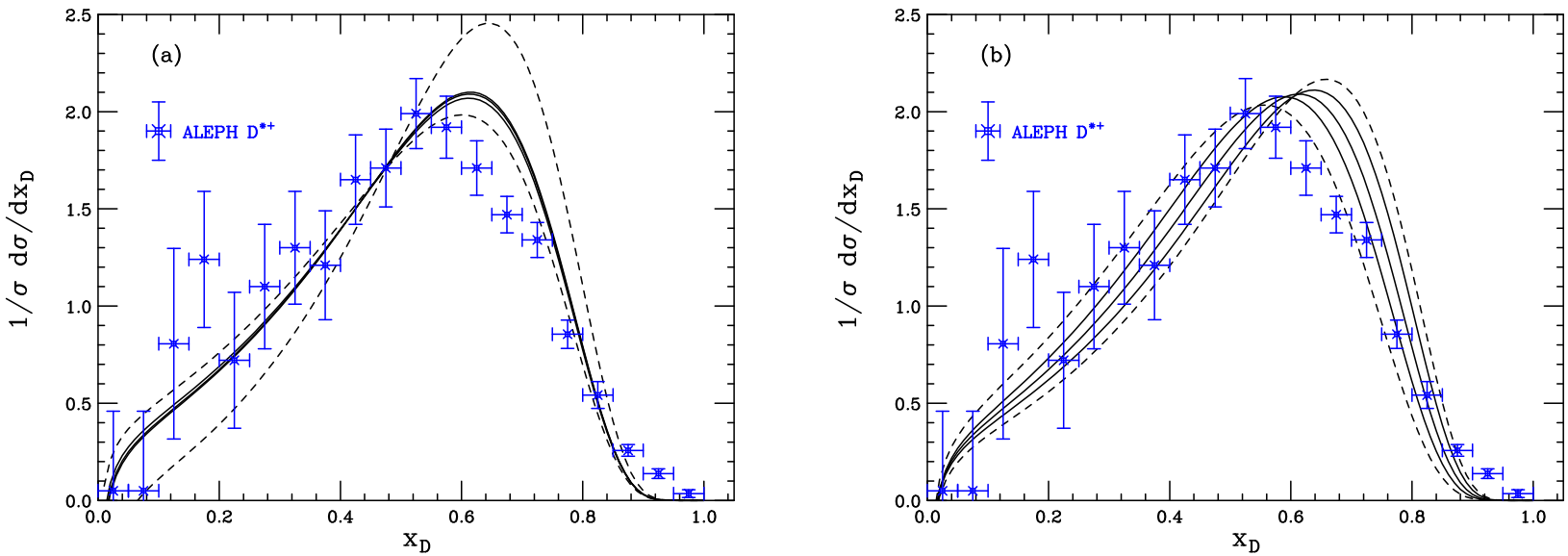
Mellin moments with the effective coupling

	$\langle x \rangle$	$\langle x^2 \rangle$	$\langle x^3 \rangle$	$\langle x^4 \rangle$
e^+e^- data σ_N^B	$\mathbf{0.7153 \pm 0.0052}$	$\mathbf{0.5401 \pm 0.0064}$	$\mathbf{0.4236 \pm 0.0065}$	$\mathbf{0.3406 \pm 0.0064}$
$[\sigma_N^B]_{\rm th}$	$\mathbf{0.6867 \pm 0.0403}$	$\mathbf{0.5019 \pm 0.0472}$	$\mathbf{0.3815 \pm 0.0465}$	$\mathbf{0.2976 \pm 0.0462}$
$\delta\sigma_N^B(\mu_R)$	0.0014	0.0011	0.0009	0.0007
$\delta\sigma_N^B(\mu_F)$	0.0066	0.0067	0.0059	0.0051
$\delta\sigma_N^B(\mu_{0R})$	0.0022	0.0028	0.0031	0.0033
$\delta\sigma_N^B(\mu_{0F})$	0.0364	0.0414	0.0398	0.0364
$\delta\sigma_N^B(m_b)$	0.0111	0.0145	0.0153	0.0150
$\delta\sigma_N^B(\bar{m}_b)$	0.0004	0.0005	0.0006	0.0006
$\delta\sigma_N^B(\bar{m}_c)$	0.0003	0.0005	0.0006	0.0006
$\delta\sigma_N^B(\bar{m}_s)$	0.0004	0.0007	0.0008	0.0008
$\delta\sigma_N^B(\alpha_S(m_Z^2))$	0.0113	0.0158	0.0173	0.0176
σ_N^b	0.7734 ± 0.0232	0.6333 ± 0.0311	0.5354 ± 0.0345	0.4617 ± 0.0346

Agreement with the DELPHI moments within the errors

Charm fragmentation with an effective coupling at LEP

$$e^+e^- \to Z^0(q) \to c\bar{c}(g) \to DX \quad ; \quad x_D = \frac{2E_D}{\sqrt{s}}$$



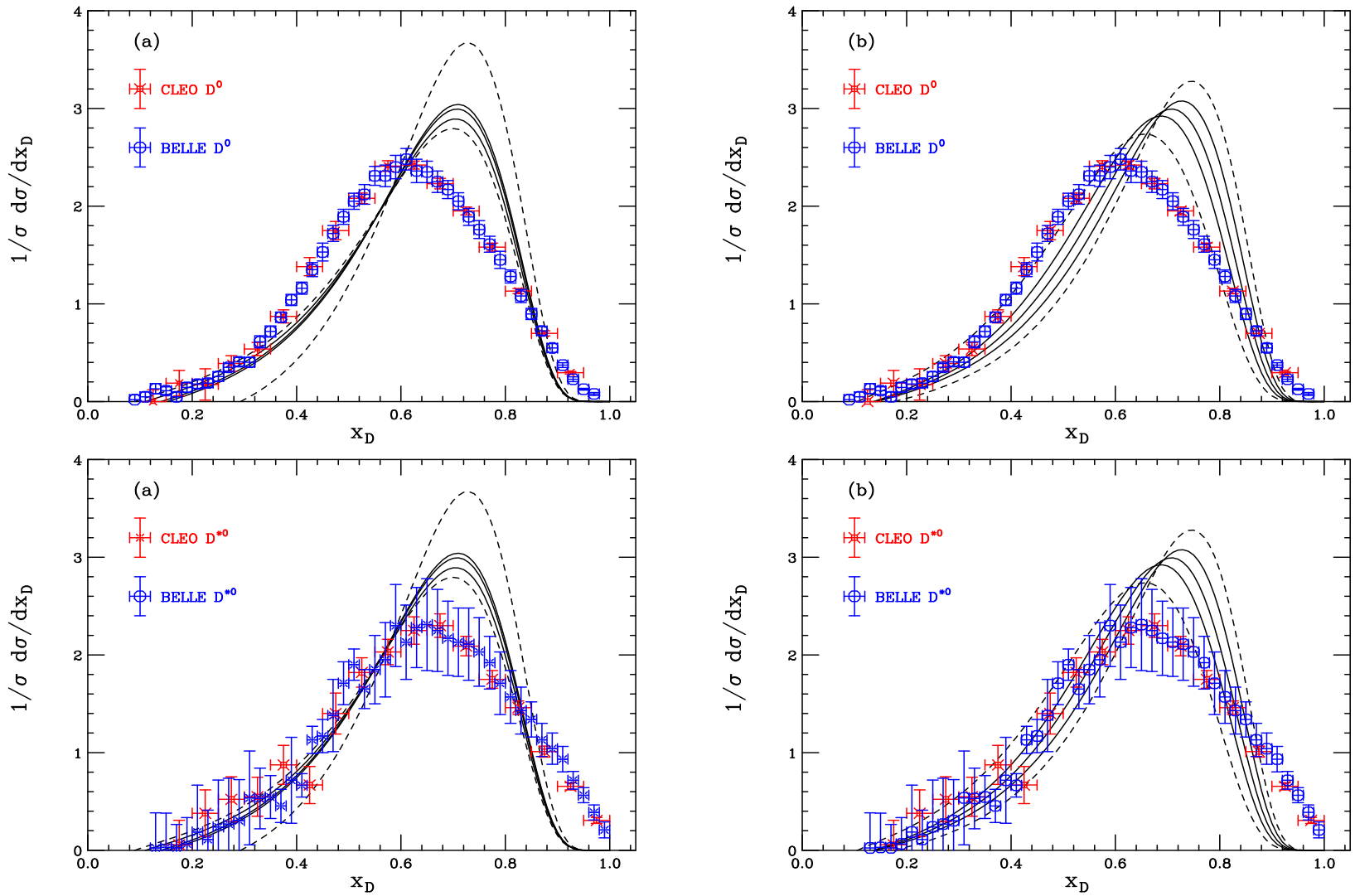
Left: **Solid: variation of μ_F; dashes: variation of μ_{0F}**

Right: **solid: dependence on $\alpha_S(m_Z^2)$; dashes: dependence on m_c**

$1.5\ \mathrm{GeV} < m_c < 2.1\ \mathrm{GeV}$

Best comparison for $\mu_{0F} = 2m_c$ ($\chi^2/\mathrm{dof} = 27.18/17$)

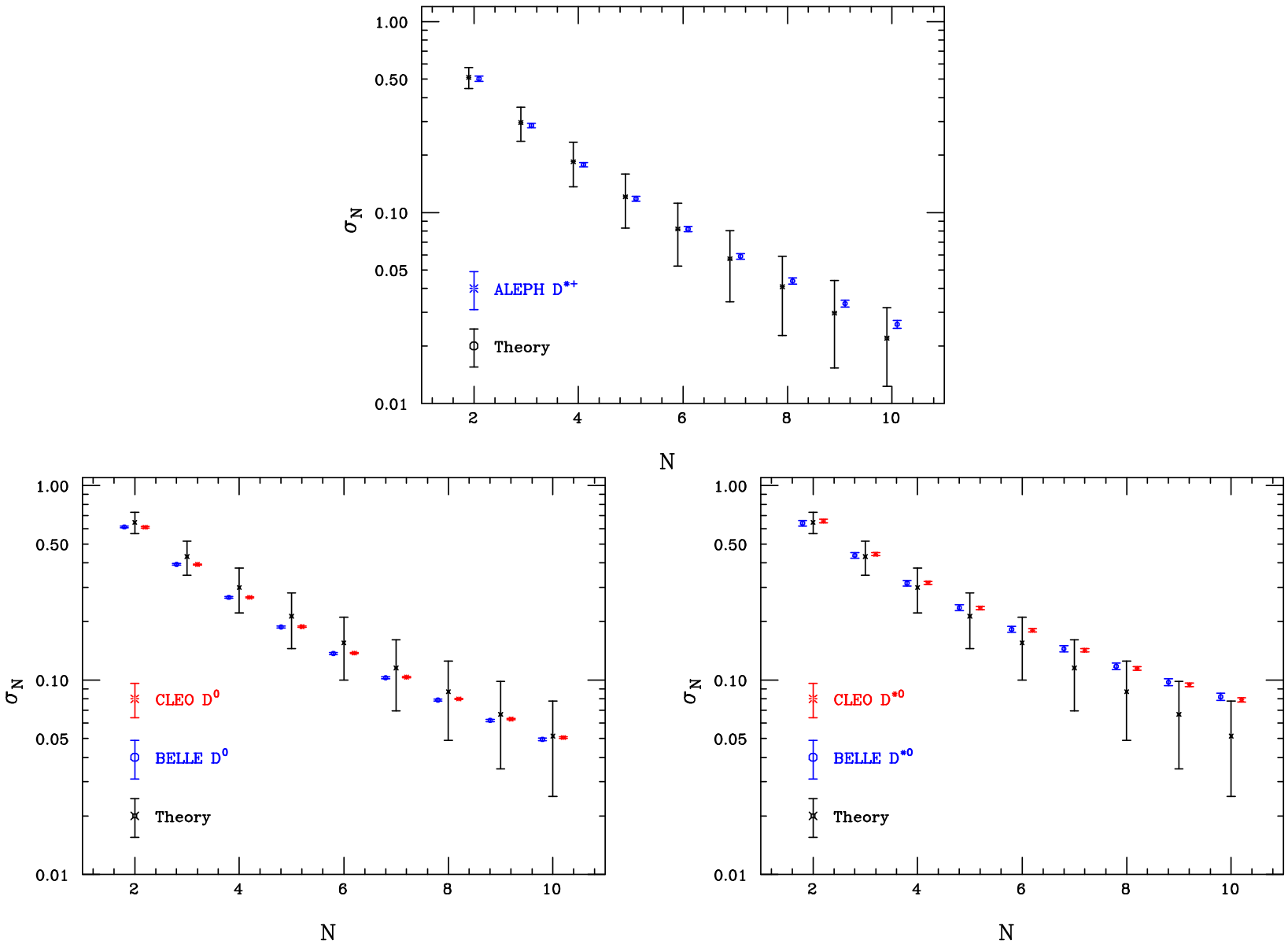
Charm fragmentation at B-factories: D and D^* data from CLEO and BELLE



Serious discrepancies with B-factory data

M. Cacciari, P. Nason and C.Oleari, JHEP 0604 (2006) 006: impossible to reproduce LEP and B-factory data with the same non-perturbative fragmentation function with 3 parameters (power corrections to the coefficient function)

Charm fragmentation in N-space



Acceptable description of all moments within the large uncertainties

Conclusions

Heavy-quark production in e^+e^- annihilation

A parameter-free non-perturbative model: effective coupling constant

Results for B and D spectra at LEP, SLD and B-factories

Reasonable agreement with B-hadron data at LEP and SLD and D^* at LEP

Disagreement with BELLE and CLEO

Good description of all data in moment space within the uncertainties

In progress:

Inclusion of NNLO corrections to to reduce the uncertainties

Investigation of disagreement with charm data, e.g. $\tilde{\alpha}_S \to \tilde{\alpha}_S + \delta\tilde{\alpha}_S$

Extension to hadron colliders: B and D spectra at the Tevatron and LHC with the effective coupling

Implementation of effective coupling constant in Monte Carlo (HERWIG) generators