HKNS fragmentation functions and proposal for exotic-hadron search

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M. Oka (Tokyo Tech), K. Sudoh (KEK)

Workshop on parton fragmentation processes
in the vacuum and in the medium
ECT*, Trento, Italy, February 25-29, 2008

February 25, 2008

References on our works

Part 1: **Determination of fragmentation function and their uncertainties**
M. Hirai, SK, T.-H. Nagai, K. Sudoh
(hep-ph/0702250) Phys. Rev. D75 (2007) 094009, 1-17.

Determination of FFs for *π, K, p* in LO and NLO
with their uncertainties.

Part 2: **Proposal for exotic-hadron search by fragmentation functions**
M. Hirai, S. Kumano, M. Oka, and K. Sudoh
(arXiv:0708.1816) Phys. Rev. D77 (2008) 017504, 1-4.

Determination of FFs for $f_0(980)$ in NLO
with their uncertainties → Internal quark structure of $f_0(980)$

$f_0(980)$: ordinanary $q\bar{q}$, $s\bar{s}$, $qq\bar{q}\bar{q}$, $K\bar{K}$?

Part I

HKNS Fragmentation Functions

For π, K, $p/\bar{p}$

Ref. M. Hirai, SK, T.-H. Nagai, K. Sudoh
Phys. Rev. D75 (2007) 094009, 1-17.

Code for calculating the fragmentation functions is available at
http://research.kek.jp/people/kumanos/ffs.html .

Contents

(1) Introduction to fragmentation functions (FFs)

- **Definition of FFs**
- **Motivation for determining FFs**

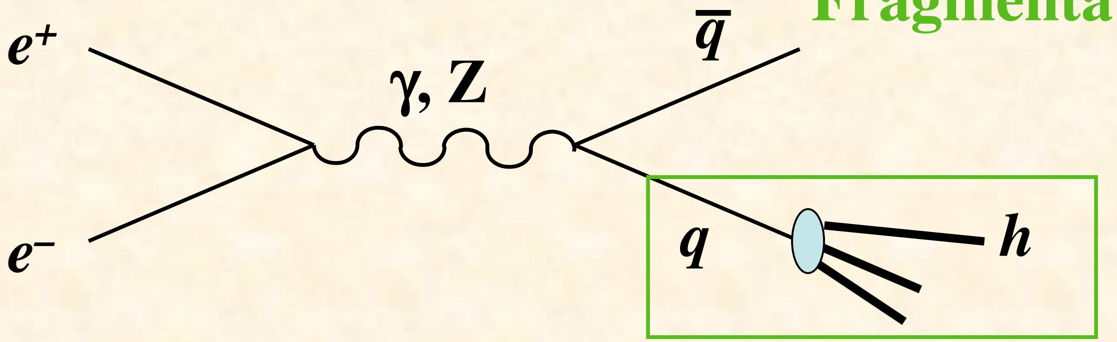
(2) Determination of FFs

- **Analysis method**
- **Results**
- **Comparison with other parameterizations**

(3) Summary for part I

Introduction

Fragmentation Function



Fragmentation: **hadron production from a quark, antiquark, or gluon**

$$z \equiv \frac{E_h}{\sqrt{s}/2} = \frac{2E_h}{Q} = \frac{E_h}{E_q}, \quad s = Q^2$$

Fragmentation function is defined by

$$F^h(z,Q^2) = \frac{1}{\sigma_{tot}} \frac{d\sigma(e^+e^- \to hX)}{dz}$$

σ_{tot} = total hadronic cross section

Variable *z*
- **Hadron energy / Beam energy**
- **Hadron energy / Primary quark energy**

A fragmentation process occurs from quarks, antiquarks, and gluons, so that F^h is expressed by their individual contributions:

$$F^h(z,Q^2) = \sum_i \int_z^1 \frac{dy}{y} \underbrace{C_i\left(\frac{z}{y},Q^2\right)}_{\text{Calculated in perturbative QCD}} \underbrace{D_i^h(y,Q^2)}_{\text{Non-perturbative (determined from experiments)}}$$

Calculated in perturbative QCD

Non-perturbative (determined from experiments)

$C_i(z,Q^2)$ = coefficient function

$D_i^h(z,Q^2)$ = fragmentation function of hadron h from a parton i

Momentum (energy) sum rule

$D_i^h(z, Q^2) =$ probability to find the hadron h from a parton i with the energy fraction z

Energy conservation: $\sum_h \int_0^1 dz\, z\, D_i^h(z, Q^2) = 1$

$$h = \pi^+,\ \pi^0,\ \pi^-,\ K^+,\ K^0,\ \bar{K}^0,\ K^-,\ p,\ \bar{p},\ n,\ \bar{n},\ \cdots$$

Favored and disfavored fragmentation functions

Simple quark model: $\pi^+(u\bar{d}),\ K^+(u\bar{s}),\ p(uud),\ \cdots$

Favored fragmentation: $D_u^{\pi^+},\ D_{\bar{d}}^{\pi^+}, \ldots$

(from a quark which exists in a naive quark model)

Disfavored fragmentation: $D_d^{\pi^+},\ D_{\bar{u}}^{\pi^+},\ D_s^{\pi^+}, \ldots$

(from a quark which does not exist in a naive quark model)

Purposes of investigating fragmentation functions

Semi-inclusive reactions have been used for investigating

- **origin of proton spin**

  $\vec{e} + \vec{p} \to e' + h + X$ (e.g. HERMES), $\vec{p} + \vec{p} \to h + X$ (RHIC-Spin)

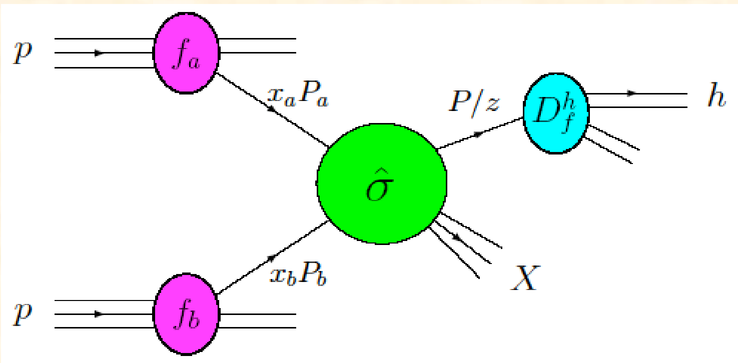
  Quark, antiquark, and gluon contributions to proton spin

  (flavor separation, gluon polarization)

- **properties of quark-hadron matters** $A + A' \to h + X$ (RHIC, LHC)

  Nuclear modification

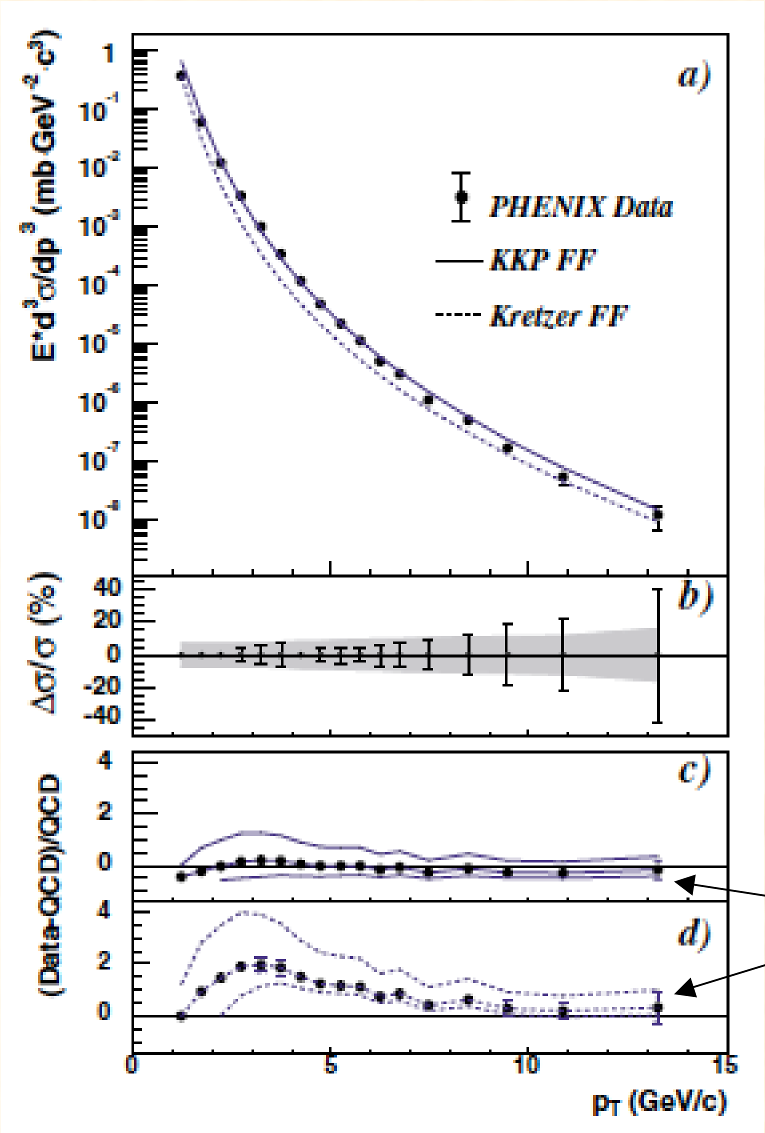
  (recombination, energy loss, …)

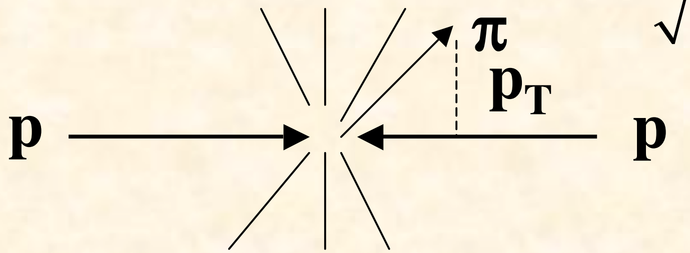


$$\sigma = \sum_{a,b,c} f_a(x_a, Q^2) \otimes f_b(x_b, Q^2) \otimes \hat{\sigma}(ab \to cX) \otimes D_c^{\pi}(z, Q^2)$$

Pion production at RHIC: p + p$\rightarrow \pi^0$ + X

S. S. Adler et al. (PHENIX), PRL 91 (2003) 241803





$\sqrt{s} = 200$ GeV

- **Consistent with NLO QCD calculation up to 10^{-8}**
- **Data agree with NLO pQCD + KKP**
- **Large differences between Kretzer and KKP calculations at small p_T**

$\rightarrow$ Importance of accurate fragmentation functions

Blue band indicates the scale uncertainty by taking Q=2p_T and p_T/2.

Situation of fragmentation functions (before 2007)

There are two widely used fragmentation functions by Kretzer and KKP.

An updated version of KKP is AKK.

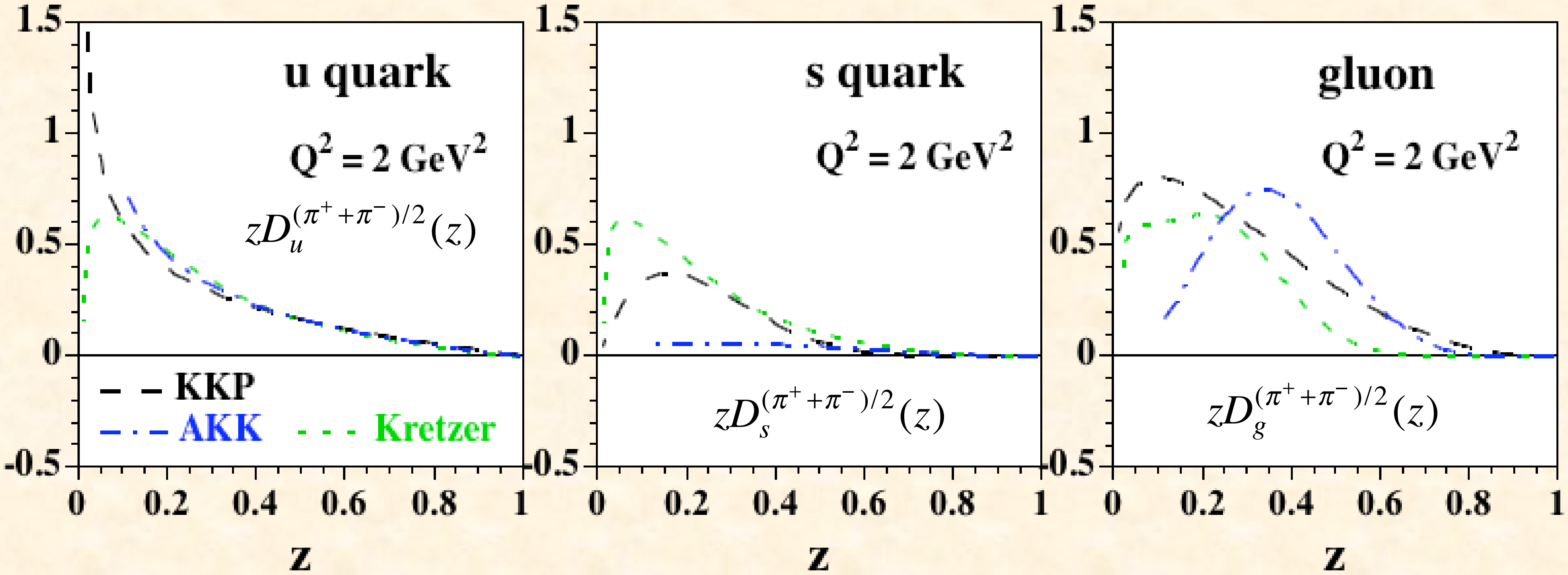
See also Bourhis-Fontannaz-Guillet-Werlen (2001) for FFs without hadron separation.

(Kretzer) S. Kretzer, PRD 62 (2000) 054001

(KKP) B. A. Kniehl, G. Kramer, B. Pötter, NPB 582 (2000) 514

(AKK) S. Albino, B.A. Kniehl, G. Kramer, NPB 725 (2005) 181

The functions of Kretzer and KKP (AKK) are very different.



Status of determining fragmentation functions (before 2007)

Parton Distribution Functions (PDFs), Fragmentation Functions (FFs)

	Nulceonic PDFs	Polarized PDFs	Nuclear PDFs	FFs
Determination	****	**	**	**
Uncertainty	○	○	○	×
Comments	Accurate determination from small x to large x	Gluon & antiquark polarization? Flavor separation?	Gluon? Antiquark at medium x? Flavor separation?	Large differences between Kretzer and KKP (AKK)

Uncertainty ranges of determined fragmentation functions were not estimated, although there are such studies in nucleonic and nuclear PDFs.

The large differences indicate that the determined FFs have much ambiguities.

Determination of Fragmentation Functions

New aspects in our analysis (compared with Kretzer, KKP, AKK)

- Determination of fragmentation functions (FFs) and their uncertainties in LO and NLO.
- Discuss NLO improvement in comparison with LO by considering the uncertainties. (Namely, roles of NLO terms in the determination of FFs)
- Comparison with other parametrizations
- Avoid assumptions on parameters as much as we can, Avoid contradiction to the momentum sum rule
- SLD (2004) data are included.

Comparison with other NLO analyses in π

	HKNS (Ours)	Kretzer	KKP (AKK)
Function form	$N_i^{\pi^+} z^{\alpha_i^{\pi^+}} (1-z)^{\beta_i^{\pi^+}}$	$N_i^{\pi^+} z^{\alpha_i^{\pi^+}} (1-z)^{\beta_i^{\pi^+}}$	$N_i^{\pi^\pm} z^{\alpha_i^{\pi^\pm}} (1-z)^{\beta_i^{\pi^\pm}}$
# of parameters	14	11	15 (18)
Initial scale $Q_0{}^2$ (NLO)	1.0 GeV^2	0.4 GeV^2	2.0 GeV^2
Major ansatz Comment	One constraint: A gluon parameter is fixed.	Four constraints: $D_{\bar{u}}^{\pi^+} = (1-z) D_u^{\pi^+}$ $M_g = \frac{M_u + M_{\bar{u}}}{2}$	$M_i^h \equiv \int_{0.05}^1 z D_i^h(z,Q^2) dz$ (momentum sum?) No π^+, π^- separation

comments on DSS later

Initial functions for pion

Note: constituent-quark composition $\pi^+ = u\bar{d}$, $\pi^- = \bar{u}d$

$$D_q^{\pi^-} = D_{\bar{q}}^{\pi^+}$$

$$D_u^{\pi^+}(z,Q_0^2) = N_u^{\pi^+} z^{\alpha_u^{\pi^+}} (1-z)^{\beta_u^{\pi^+}} = D_{\bar{d}}^{\pi^+}(z,Q_0^2)$$

$$D_{\bar{u}}^{\pi^+}(z,Q_0^2) = N_{\bar{u}}^{\pi^+} z^{\alpha_{\bar{u}}^{\pi^+}} (1-z)^{\beta_{\bar{u}}^{\pi^+}} = D_d^{\pi^+}(z,Q_0^2) = D_s^{\pi^+}(z,Q_0^2) = D_{\bar{s}}^{\pi^+}(z,Q_0^2)$$

$$D_c^{\pi^+}(z,m_c^2) = N_c^{\pi^+} z^{\alpha_c^{\pi^+}} (1-z)^{\beta_c^{\pi^+}} = D_{\bar{c}}^{\pi^+}(z,m_c^2)$$

$$D_b^{\pi^+}(z,m_b^2) = N_b^{\pi^+} z^{\alpha_b^{\pi^+}} (1-z)^{\beta_b^{\pi^+}} = D_{\bar{b}}^{\pi^+}(z,m_b^2)$$

$$D_g^{\pi^+}(z,Q_0^2) = N_g^{\pi^+} z^{\alpha_g^{\pi^+}} (1-z)^{\beta_g^{\pi^+}}$$

Constraint: 2nd moment should be finite and less than 1

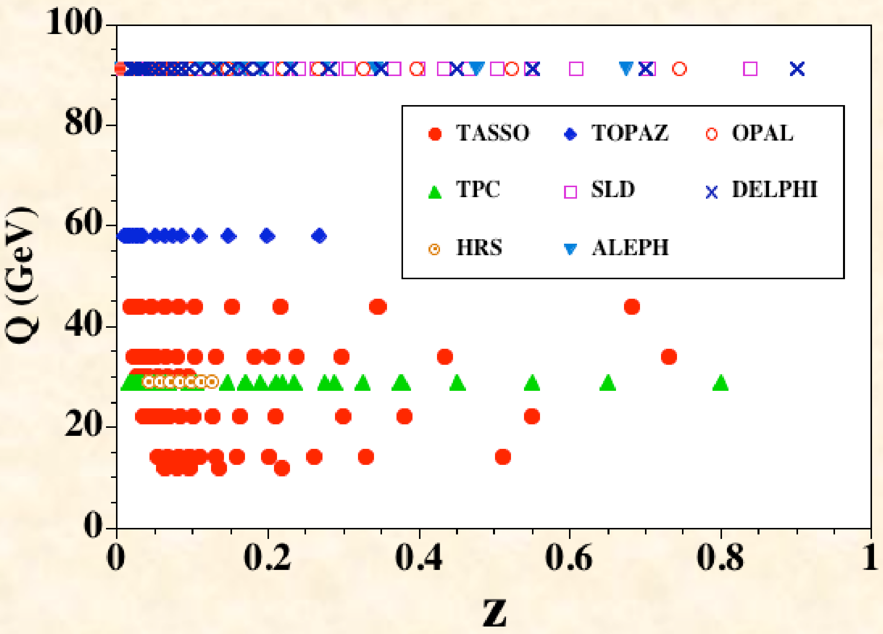
$N = \dfrac{M}{B(\alpha+2,\beta+1)}$, $\quad M \equiv \int_0^1 zD(z)dz$ **(2nd moment)**, $\quad B(\alpha+2,\beta+1) =$ **beta function**

$0 < M_i^h < 1$ **because of the sum rule** $\sum_h M_i^h = 1$

Experimental data for pion

Total number of data: 264

	$\sqrt{s}$ (GeV)	# of data
TASSO	12,14,22,30,34,44	29
TCP	29	18
HRS	29	2
TOPAZ	58	4
SLD	91.2	29
SLD [light quark]		29
SLD [c quark]		29
SLD [b quark]		29
ALEPH	91.2	22
OPAL	91.2	22
DELPHI	91.2	17
DELPHI [light quark]		17
DELPHI [b quark]		17



Analysis

Initial scale: $Q_0^2 = 1$ **GeV**^{2}

Scale parameter: $\Lambda_{QCD}^{n_f=4} = 0.220$ **(LO)**, 0.323 **(NLO)**

α_s **varies with** n_f

Heavy-quark masses: $m_c = 1.43$ **GeV,** $m_b = 4.3$ **GeV**

Results for the pion χ^2**/d.o.f.** $= 1.81$ **(LO),** 1.73 **(NLO)**

Uncertainty estimation: Hessian method

$$\Delta\chi^2 \equiv \chi^2(\hat{a}+\delta a) - \chi^2(\hat{a}) = \sum_{i,j} H_{ij}\delta a_i \delta a_j, \qquad H_{ij} = \frac{\partial^2 \chi^2(\hat{a})}{\partial a_i \partial a_j}$$

$$\left[\delta D(z)\right]^2 = \Delta\chi^2 \sum_{i,j} \frac{\partial D(z,\hat{a})}{\partial a_i} H_{ij}^{-1} \frac{\partial D(z,\hat{a})}{\partial a_j}$$

Error estimation

Hessian method

$\chi^2(\xi)$ is expanded around its minimum ξ_0 (ξ =parameter)

$$\chi^2(\xi_0+\delta\xi) = \chi^2(\xi_0) + \sum_i \frac{\partial \chi^2(\xi_0)}{\partial \xi_i}\delta\xi_i + \frac{1}{2}\sum_{i,j}\frac{\partial^2\chi^2(\xi_0)}{\partial\xi_i\partial\xi_j}\delta\xi_i\delta\xi_j + \cdots$$

where the Hessian matrix is defined by $H_{ij} = \frac{1}{2}\frac{\partial^2 \chi^2(a_0)}{\partial \xi_i \partial \xi_j}$

In the χ^2 analysis, 1σ standard error is

$$\Delta\chi^2 = \chi^2(\xi_0+\delta\xi) - \chi^2(\xi_0) = \sum_{i,j}\delta\xi_i H_{ij}\delta\xi_j$$

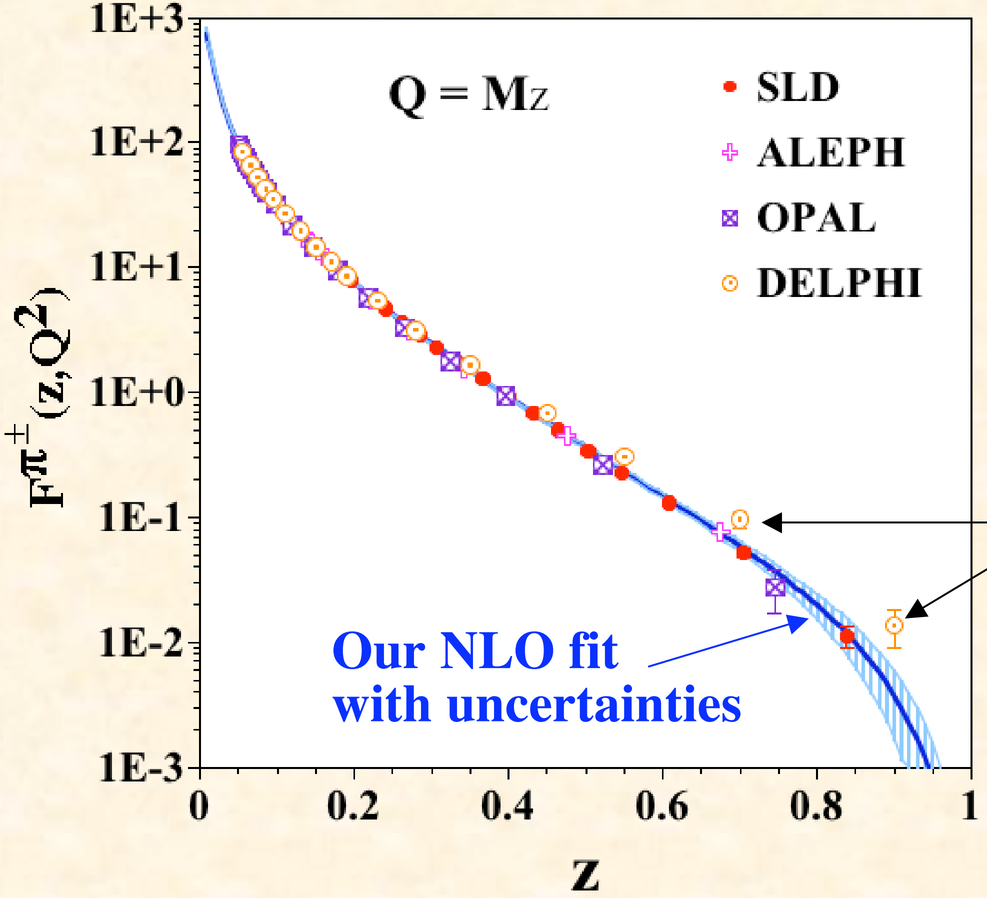
$P(s)_N = \chi^2$ distribution with N degrees of freedom

$$\int_0^{\Delta\chi^2} ds P(s)_{N=14} = 0.6826 \rightarrow \Delta\chi^2 = 15.94 \quad (\text{N=1 case, } \Delta\chi^2 = 1)$$

The error of a distribution $F(x)$ is given by

$$\left[\delta D(z)\right]^2 = \Delta\chi^2 \sum_{i,j}\frac{\partial D(z,\hat{\xi})}{\partial\xi_i}H_{ij}^{-1}\frac{\partial D(z,\hat{\xi})}{\partial\xi_j}$$

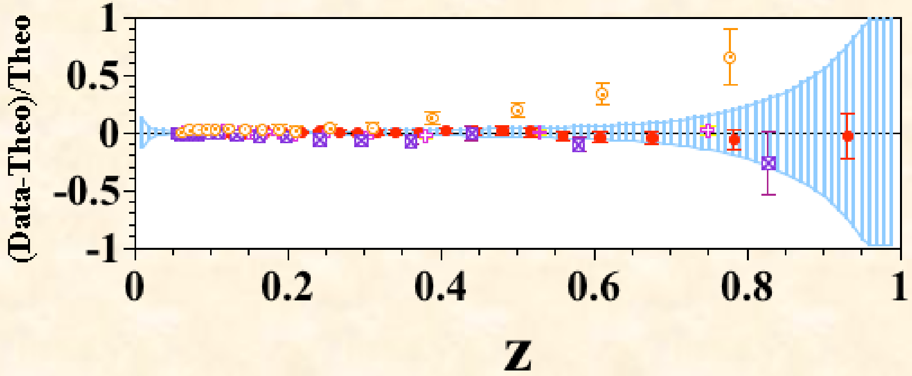
Comparison with pion data



$$F^{\pi^\pm}(z,Q^2) = \frac{1}{\sigma_{tot}} \frac{d\sigma(e^+e^- \to \pi^\pm X)}{dz}$$

Our fit is successful to reproduce the pion data.

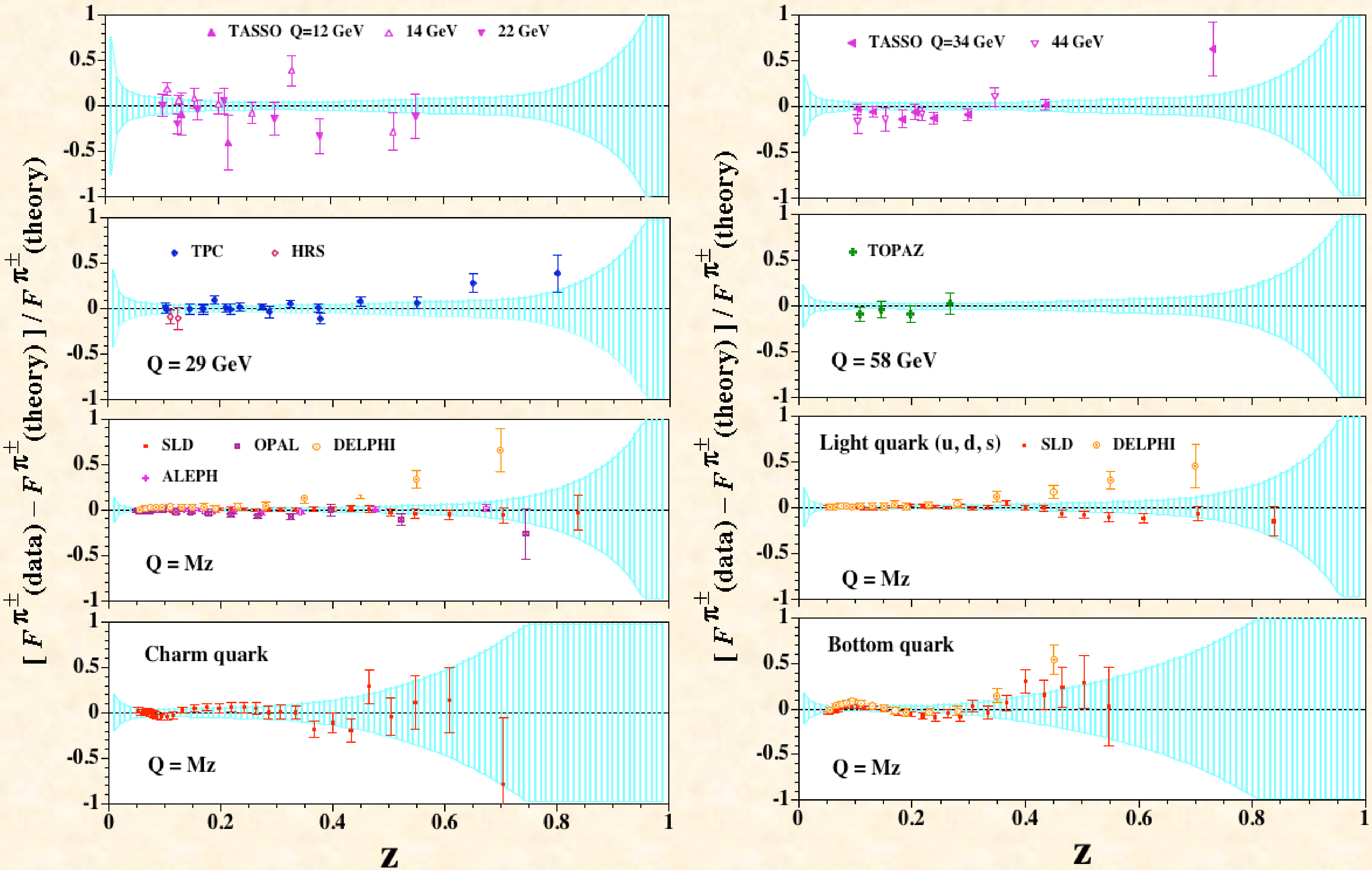
The DELPHI data deviate from our fit at large z.



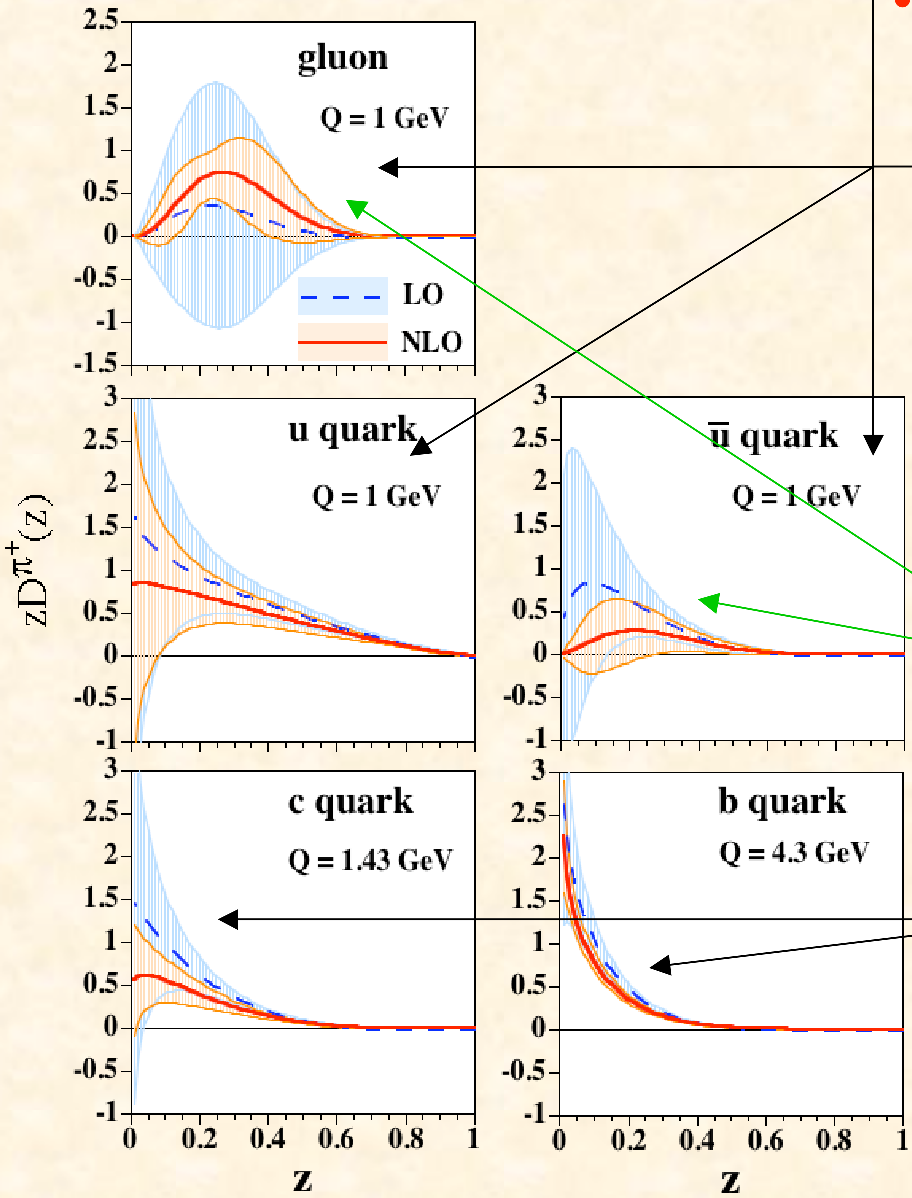
Rational difference between data and theory

$$\frac{F^{\pi^\pm}(z,Q^2)_{\text{data}} - F^{\pi^\pm}(z,Q^2)_{\text{theory}}}{F^{\pi^\pm}(z,Q^2)_{\text{theory}}}$$

Comparison with pion data: (data-theory)/theory



Determined fragmentation functions for pion



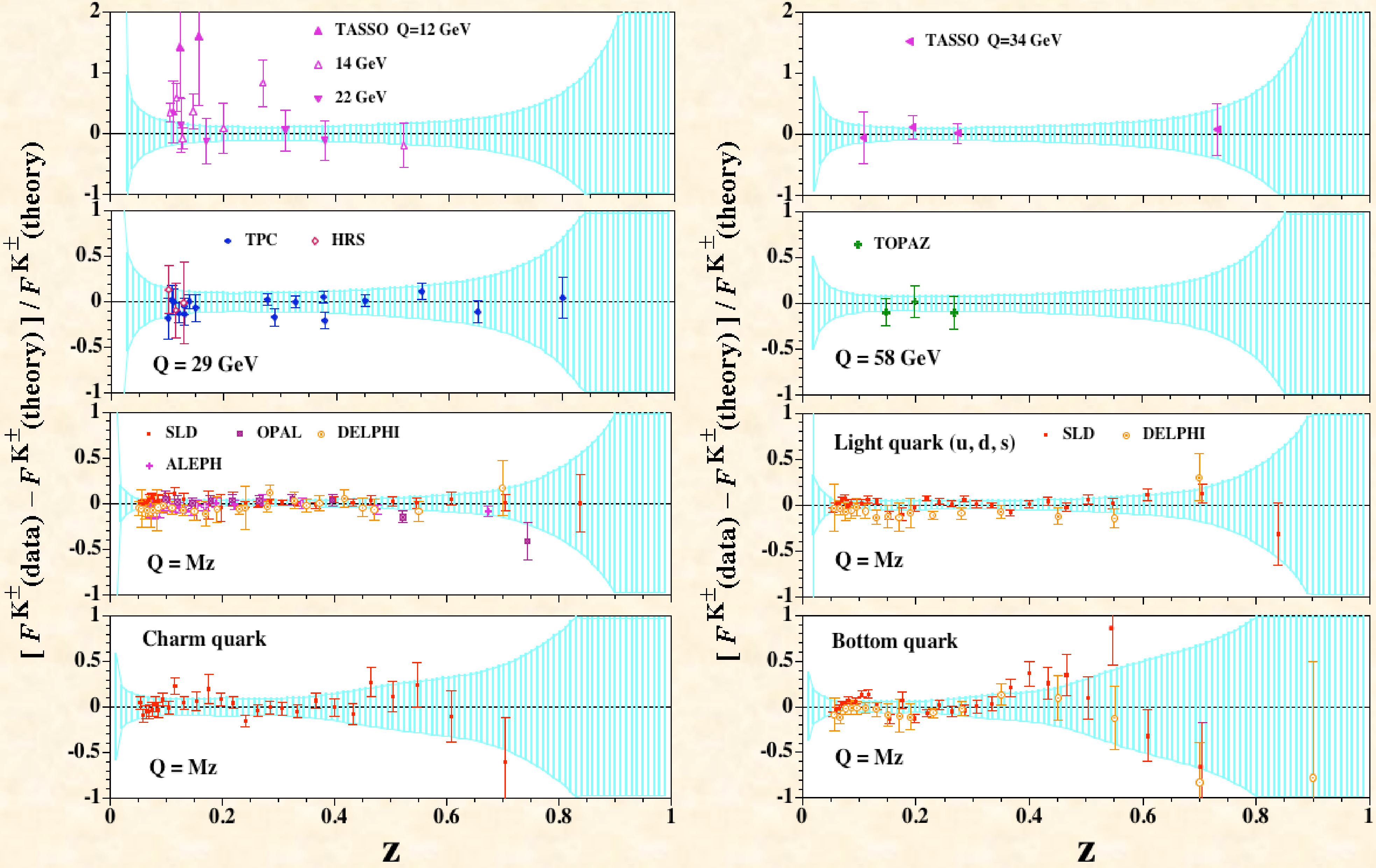
- Gluon and light-quark fragmentation functions have large uncertainties.

- Uncertainty bands become smaller in NLO in comparison with LO.
  → The data are sensitive to NLO effects.

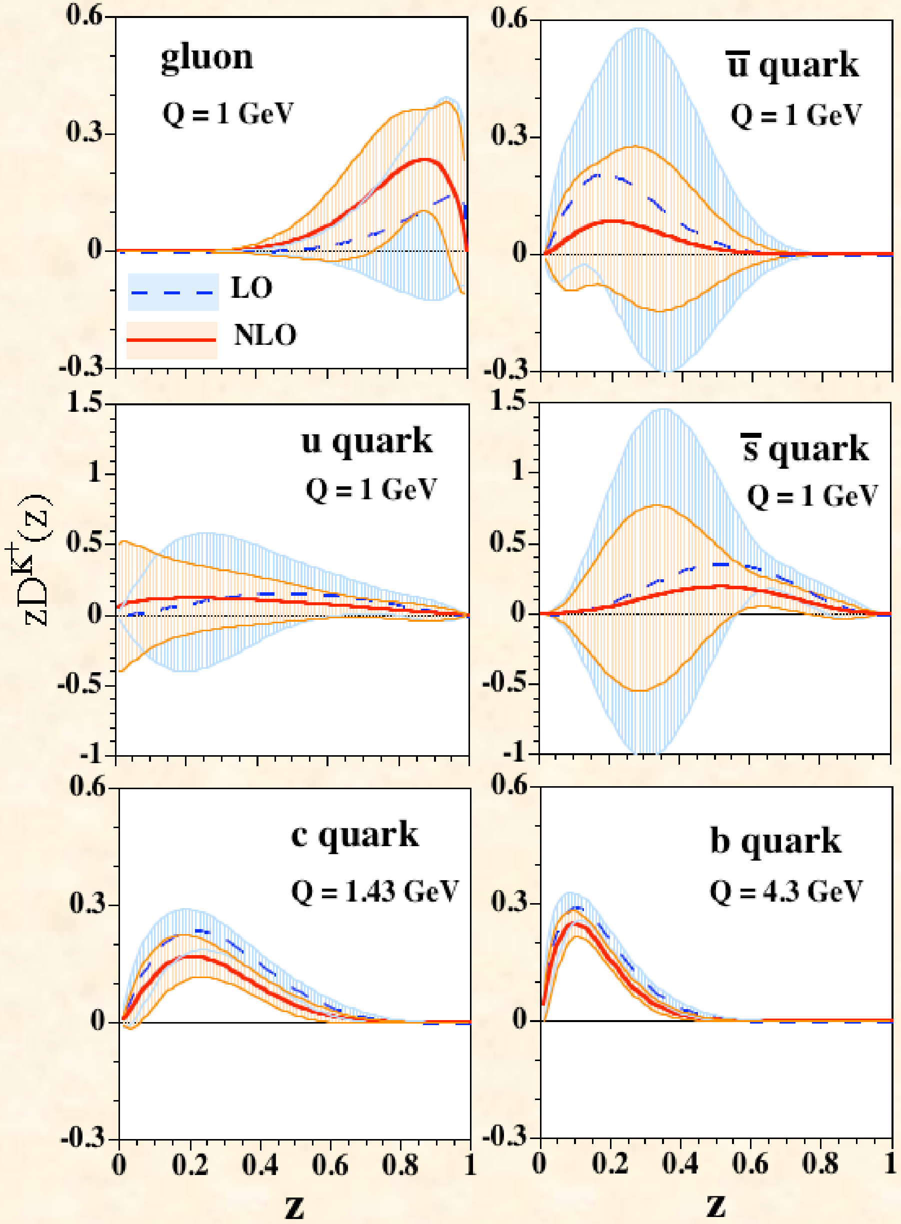
- The NLO improvement is clear especially in gluon and disfavored functions.

- Heavy-quark functions are relatively well determined.

Comparison with kaon data



Determined functions for kaon



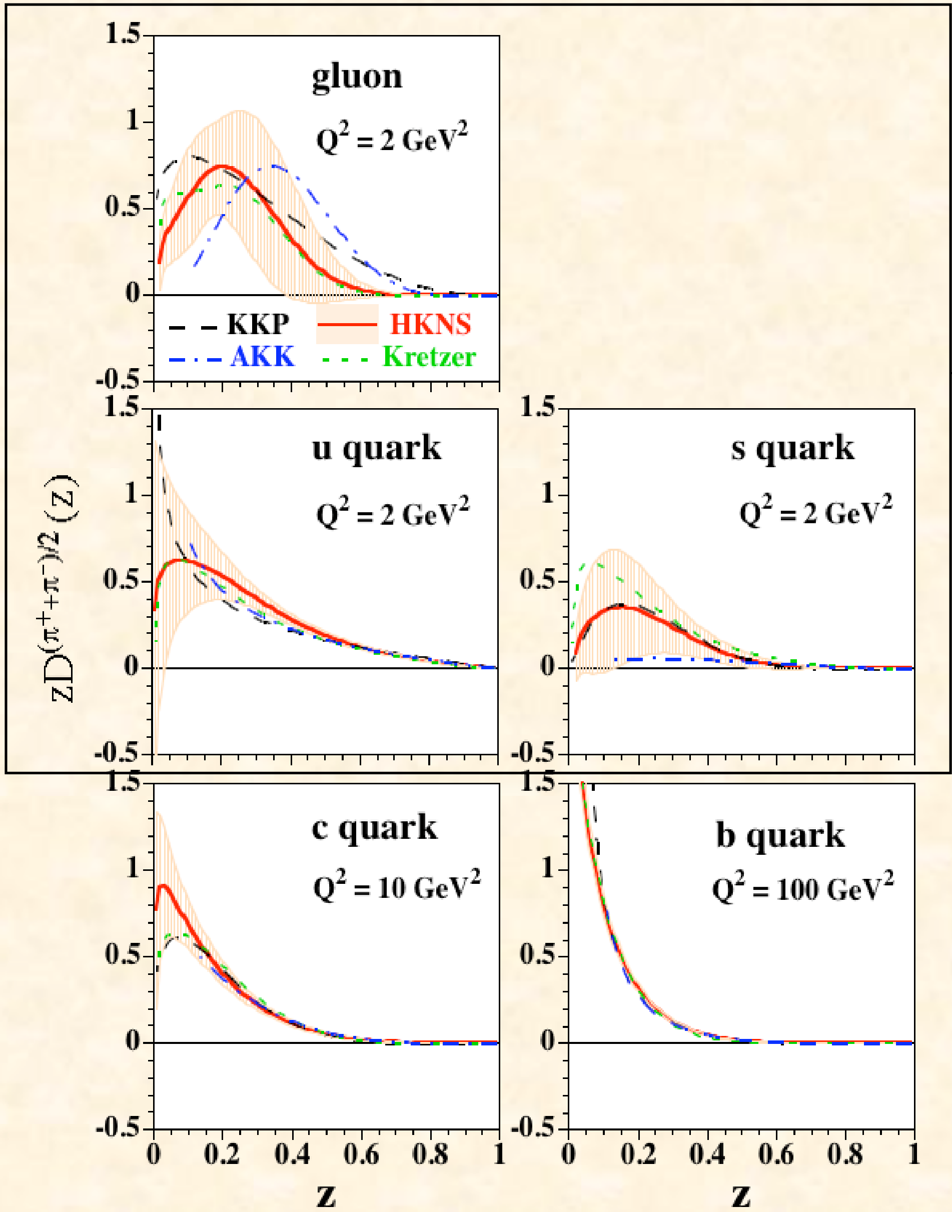
The situation is similar to the pion functions.

- Gluon and light-quark fragmentation functions have large uncertainties, which are larger than the pionic ones.

- Uncertainty bands become smaller in NLO in comparison with LO.

- Heavy-quark functions are relatively well determined.

Comparison with other parametrizations in pion



(KKP) Kniehl, Kramer, Pötter

(AKK) Albino, Kniehl, Kramer

(HKNS) Hirai, Kumano, Nagai, Sudoh

- Gluon and light-quark disfavored fragmentation functions have large differences, but they are within the uncertainty bands.

  → The functions of KKP, Kretzer, AKK, and HKNS are consistent with each other.

All the parametrizations agree in charm and bottom functions.

→ DSS (next page)

D. De Florian, R. Sassot, M. Stratmann,

Phys. Rev. D 75 (2007) 114010; 76 (2007) 074033.

Comparison with DSS in pion

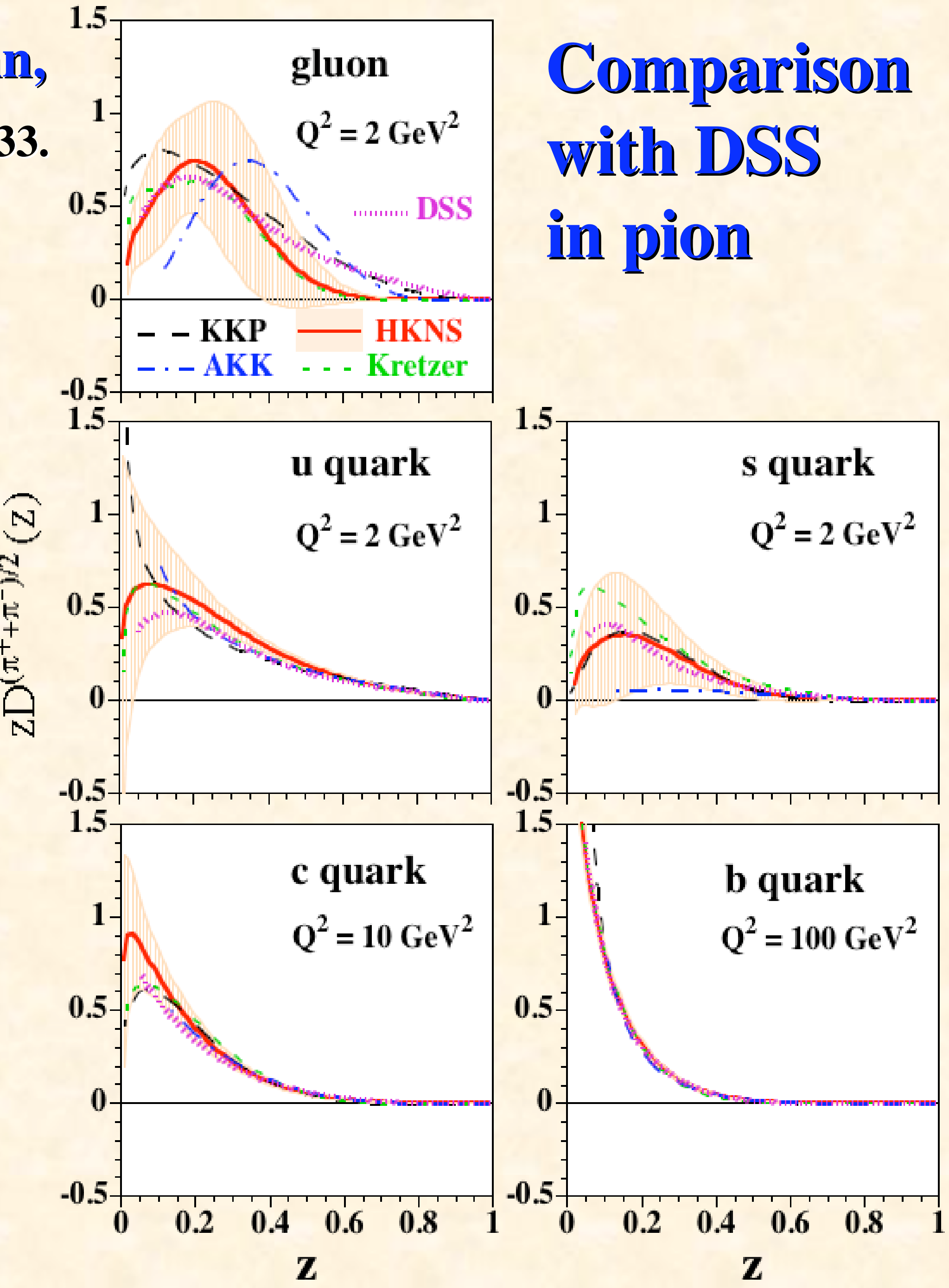
- Analysis for π, K, $p/\bar{p}$, and $h^{\pm}$
- Functional form is different

$$D_i^h(z,Q_0^2) = N_i z^{\alpha_i}(1-z)^{\beta_i}\left[1+\gamma_i(1-z)^{\delta_i}\right]$$

$$(Q_0^2 = 1\ \mathrm{GeV}^2)$$

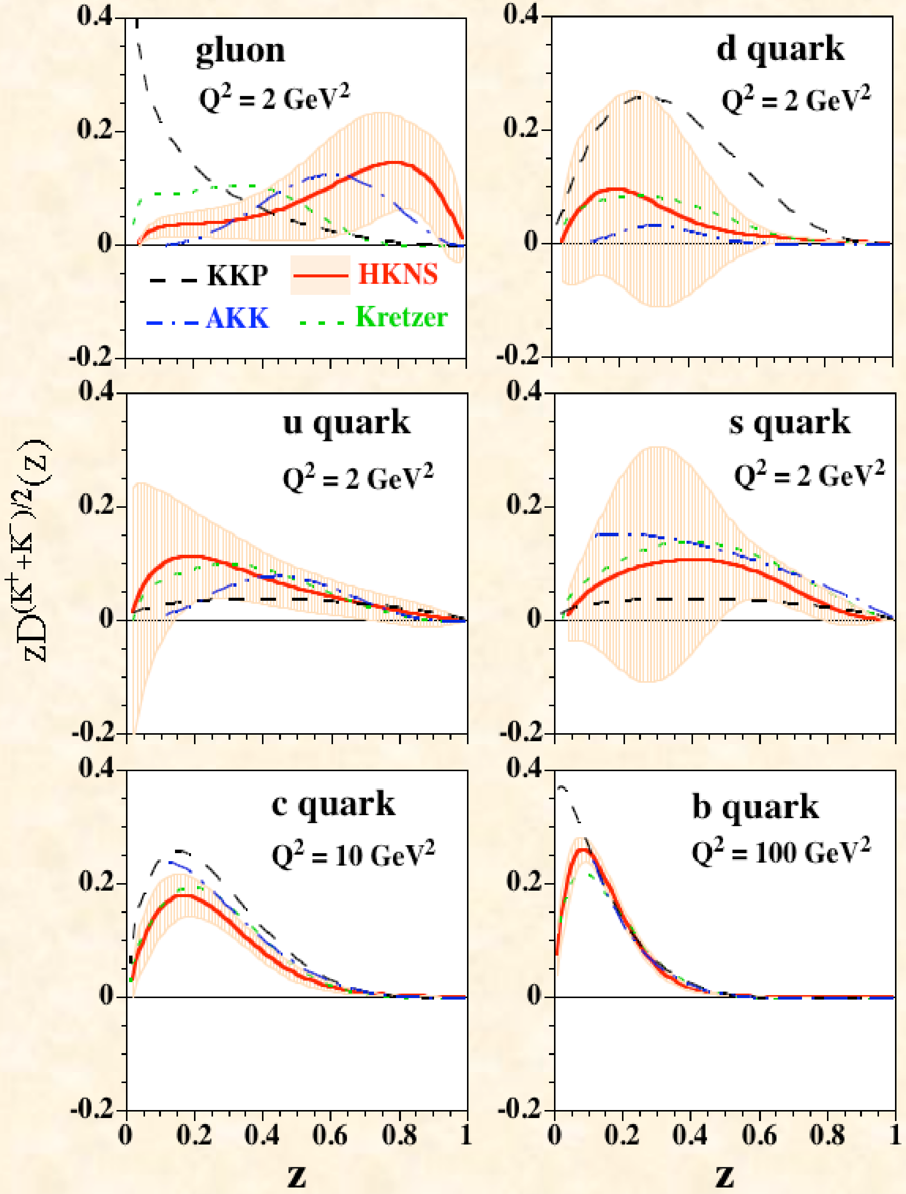
- Semi-inclusive HERMES, PHENIX, STAR, BRAHMS are included in the analysis.

DSS functions generally agree well with HKNS in the pion.

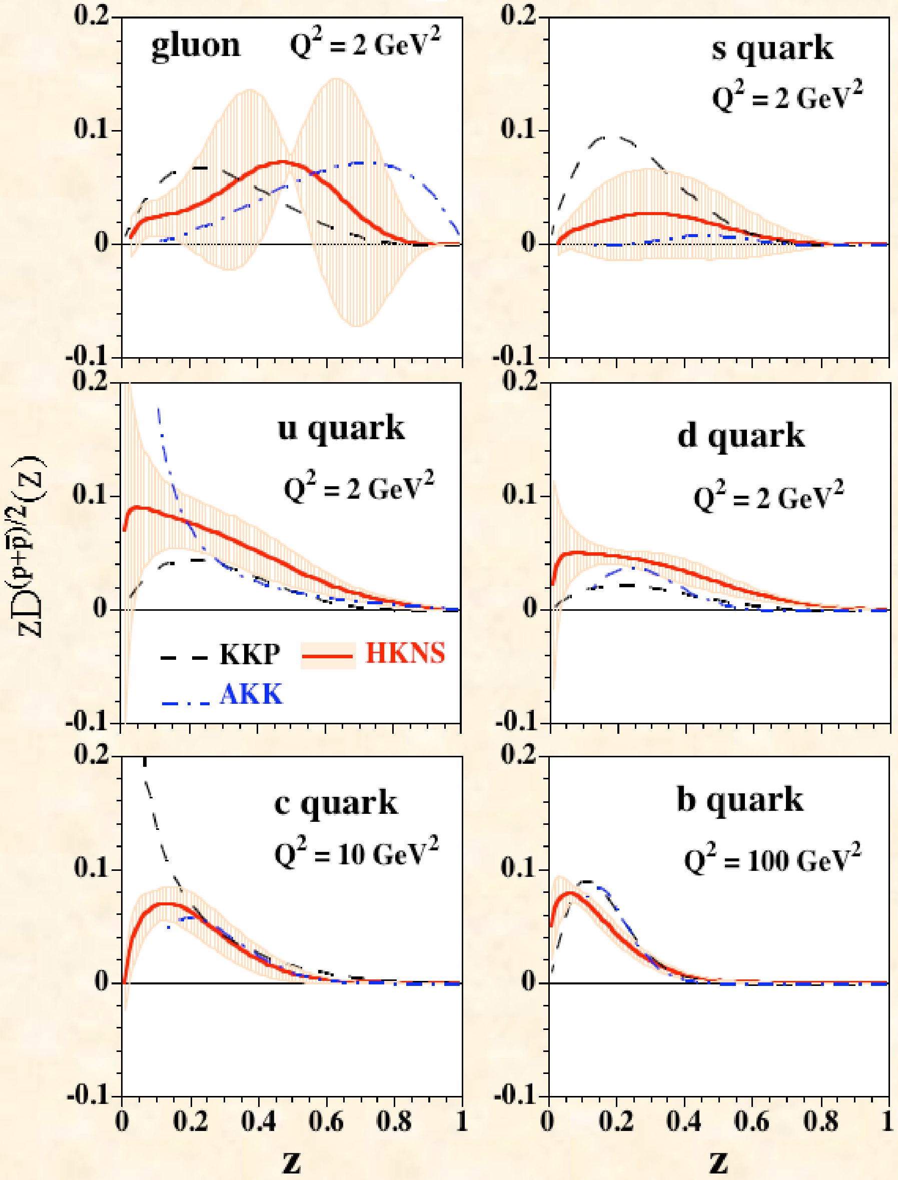


Comparison with other parametrizations in kaon and proton

kaon

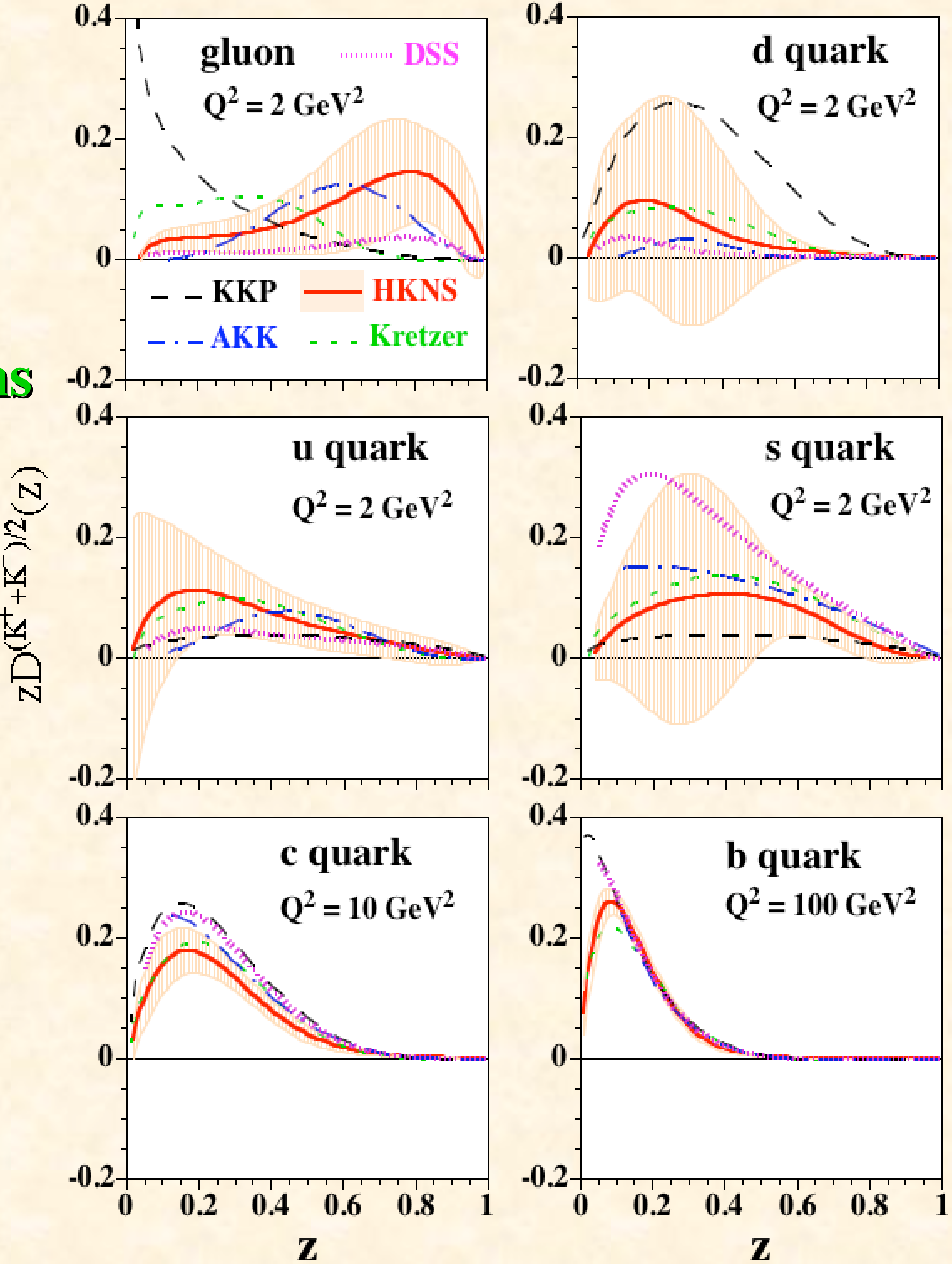


proton



Comparison with DSS in kaon

DSS functions differ from HKNS in light-quark and gluon functions of the kaon; however, they are within the uncertainties.



Comments on "low-energy" experiments, Belle & BaBar

Gluon fragmentation function is very important for hadron production at small p_T at RHIC (heavy ion, spin) and LHC,

(see the next transparency)

and it is "not determined" as shown in this analysis.

$\rightarrow$ Need to determine it accurately.

$\rightarrow$ Gluon function is a NLO effect with the coefficient function and in Q^2 evolution.

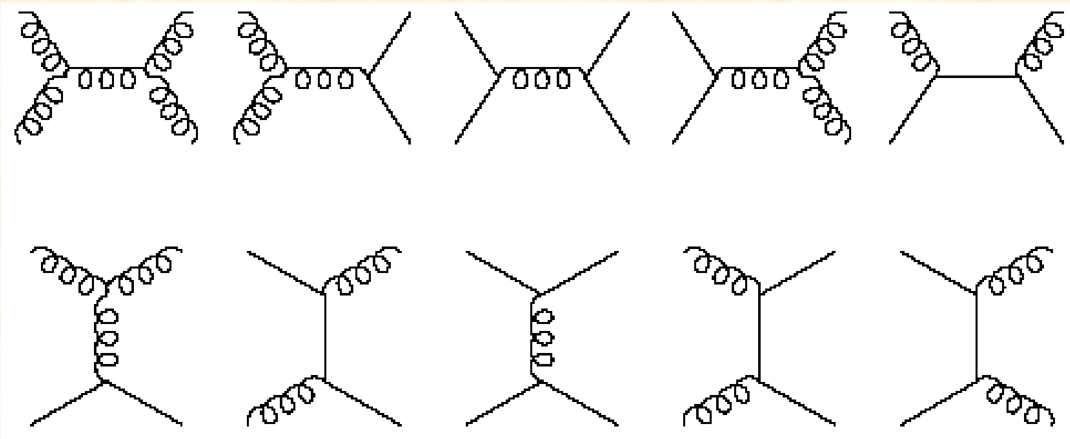
We have precise data such as the SLD ones at Q=Mz, so that accurate small-Q^2 data are needed for probing the Q^2 evolution, namely the gluon fragmentation functions.

(Belle, BaBar ?)

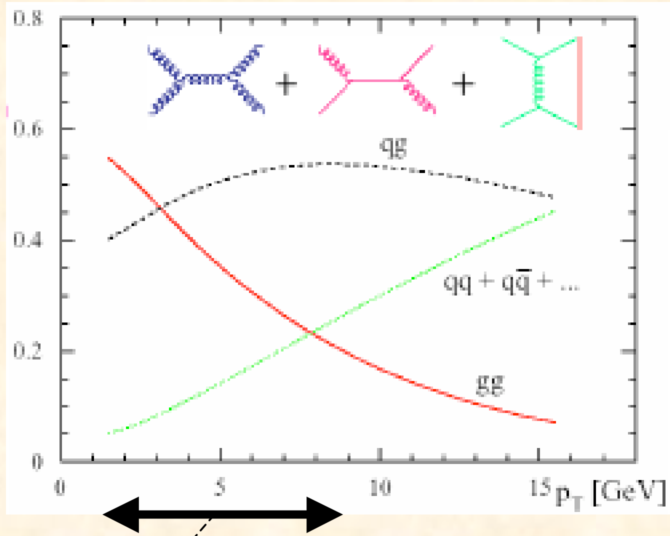
Pion production at RHIC: $\vec{p}+\vec{p}\rightarrow\pi^0+X$

Subprocesses

$$gg\rightarrow q(g)X,\quad qg\rightarrow q(g)X,\quad qq\rightarrow qX,$$
$$q\bar{q}\rightarrow q(g,q')X,\quad qq'\rightarrow qX,\quad q\bar{q}'\rightarrow qX$$



(from Torii's talk at Pacific-Spin05)



$g+g\rightarrow q(g)+X$ processes are dominant at small p_T

$q+g\rightarrow q(g)+X$ at medium p_T

Gluon polarization Δg at small p_T

$\rightarrow$ Gluon fragmentation function plays a major role

Summary on Part I

Determination of the optimum fragmentation functions for π, K, p in LO and NLO by a global analysis of $e^+ + e^- \rightarrow h + X$ data.

- **This is the first time that uncertainties of the fragmentation functions are estimated.**
- **Gluon and disfavored light-quark functions have large uncertainties.**
  - → **The uncertainties could be important for discussing physics in**
    $\vec{p} + \vec{p} \rightarrow \pi^0 + X, \;\; A + A' \rightarrow h + X$ (RHIC, LHC), HERMES, JLab, ...
  - → **Need accurate data at low energies (Belle and BaBar).**
- **For the pion and kaon, the uncertainties are reduced in NLO in comparison with LO.**
  For the proton, such improvement is not obvious.
- **Heavy-quark functions are well determined.**
- **Code for calculating the fragmentation functions is available at**
  http://research.kek.jp/people/kumanos/ffs.html .

Part II

Fragmentation Functions For Exotic-Hadron Search: $f_0(980)$ as an example

Ref. M. Hirai, S. Kumano, M. Oka, and K. Sudoh
Phys. Rev. D77 (2008) 017504, 1-4.

Contents

(1) Introduction to exotic hadrons

- Recent discoveries
- Exotic hadrons at $M \sim 1$ GeV, especially $f_0(980)$
- FFs in heavy-ion collisions

(2) Criteria for determining quark configurations by fragmentation functions

- Functional forms, Second moments

(3) Analysis of $e^+ + e^- \rightarrow f_0 + X$ data for determining fragmentation functions for $f_0(980)$

- Analysis method, Results, Discussions

(4) Summary for part II

Introduction

Recent progress in exotic hadrons

$q\bar{q}$ Meson
q^3 Baryon

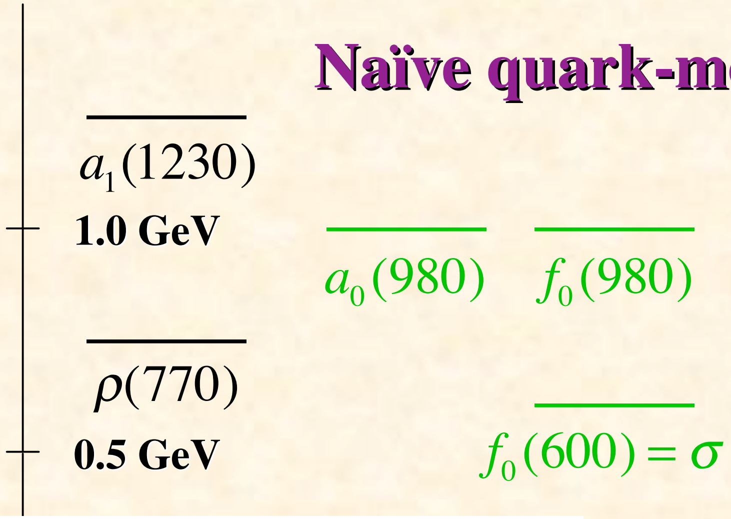
$q^2\bar{q}^2$ Tetraquark
$q^4\bar{q}$ Pentaquark
q^6 Dibaryon
…
$q^{10}\bar{q}$ e.g. Strange tribaryon
…
gg Glueball
…

(Japanese ?) Exotics

- $\Theta^+(1540)$?: LEPS
  Pentaquark?
  $\boxed{uudd\bar{s}\ ?}$
- $S^0(3115)$, $S^+(3140)$: KEK-PS
  Strange tribaryons?
  $\boxed{K^- pnn \quad K^- ppn\ ?}$
- X (3872), Y(3940): Belle
  Tetraquark, $D\bar{D}$ molecule
  $c\bar{c}$
  $D^0(c\bar{u})\bar{D}^0(\bar{c}u)$
  $D^+(c\bar{d})D^-(\bar{c}d)$?
- $D_{sJ}(2317)$, $D_{sJ}(2460)$: BaBar, CLEO, Belle
  Tetraquark, DK molecule
  $c\bar{s}$
  $D^0(c\bar{u})K^+(u\bar{s})$
  $D^+(c\bar{d})K^0(d\bar{s})$?
- Z (4430): Belle
  Tetraquark, …
  Note: $Z(4430) \neq q\bar{q}$
  $\boxed{c\bar{c}u\bar{d},\ D \text{ molecule?}}$

Scalar mesons $J^P=0^+$ at $M\sim 1$ GeV

Naïve quark-model



$$\sigma = f_0(600) = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$

$$f_0(980) = s\bar{s} \quad \rightarrow \text{denote } f_0 \text{ in this talk}$$

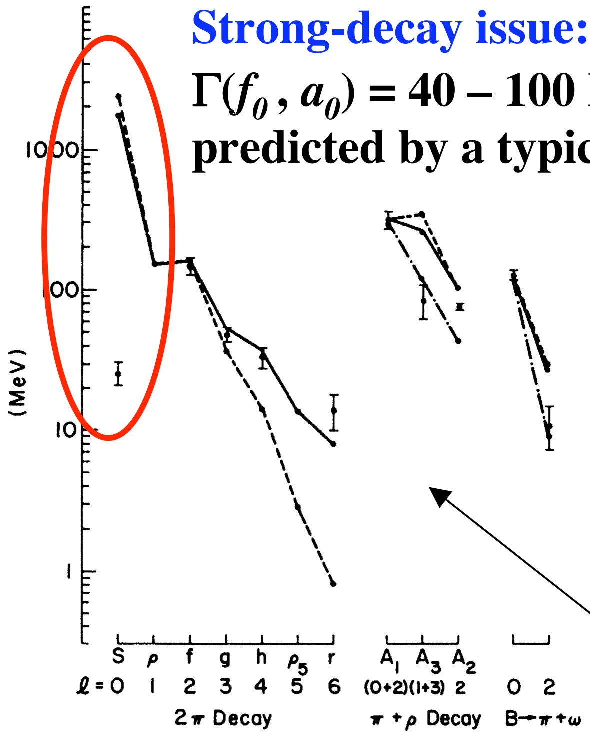
$$a_0(980) = u\bar{d}, \quad \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}), \quad d\bar{u}$$

Naive model: $m(\sigma) \sim m(a_0) < m(f_0)$

$\Updownarrow$ **contradiction**

Experiment: $m(\sigma) < m(a_0) \sim m(f_0)$

Strong-decay issue: The experimental widths $\Gamma(f_0, a_0) = 40 - 100$ MeV are too small to be predicted by a typical quark model.



These issues could be resolved

if f_0 is a tetraquark ($qq\bar{q}\bar{q}$) or a $K\bar{K}$ molecule,

namely an "exotic" hadron.

R. Kokoski and N. Isgur, Phys. Rev. D35 (1987) 907;
SK and V. R. Pandharipande, Phys. Rev. D38 (1988) 146.

Determination of $f_0(980)$ structure by electromagnetic decays

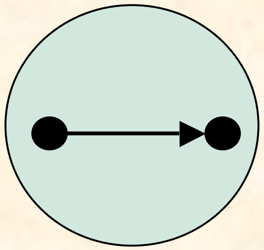
F. E. Close, N. Isgur, and SK, Nucl. Phys. B389 (1993) 513.

Radiative decay: $\phi \to S\gamma$

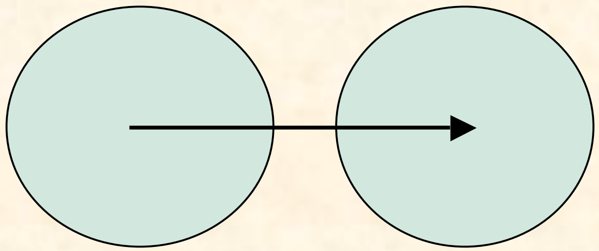
$S = f_0(980)$, $a_0(980)$

$J^p = 1^- \to 0^+$ **E1 transition**

Electric dipole:
$e\vec{r}$ **(distance!)**



$q\bar{q}$ model:
Γ **= small**



$K\bar{K}$ molecule
or $qq\bar{q}\bar{q}$: Γ = large

Experimental results of VEPP-2M and DAΦNE suggest that f_0 is a tetraquark state (or a $K\bar{K}$ molecule?).

CMD-2 (1999):	$B(\phi \to f_0\gamma) = (1.93 \pm 0.46 \pm 0.50) \times 10^{-4}$
SND (2000):	$(3.5 \pm 0.3^{+1.3}_{-0.5}) \times 10^{-4}$
KLOE (2002):	$(4.47 \pm 0.21_{\text{stat+syst}}) \times 10^{-4}$

For recent discussions,
N. N. Achasov and A. V. Kiselev, PRD 73 (2006) 054029;
D74 (2006) 059902(E); D76 (2007) 077501;
Y. S. Kalashnikova *et al.*, Eur. Phys. J. A24 (2005) 437.

See also Belle (2007)

$\Gamma(f_0 \to \gamma\gamma) = 0.205^{+0.095}_{-0.083}(\text{stat})^{+0.147}_{-0.117}(\text{syst})$ keV

$f_0(980)$ in heavy-ion collisions

L. Maiani, A. D. Polosa, V. Riquer, and C.A. Salgado, Phys. Lett. B645 (2007) 138.
(See also C. Nonaka *et al.*, Phys. Rev. C69 (2004) 031902.)

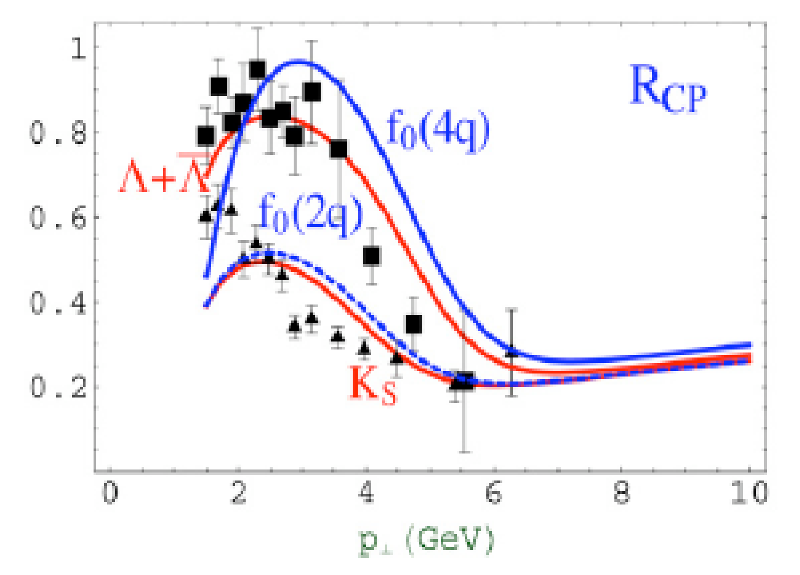
Recombination / Fragmentation

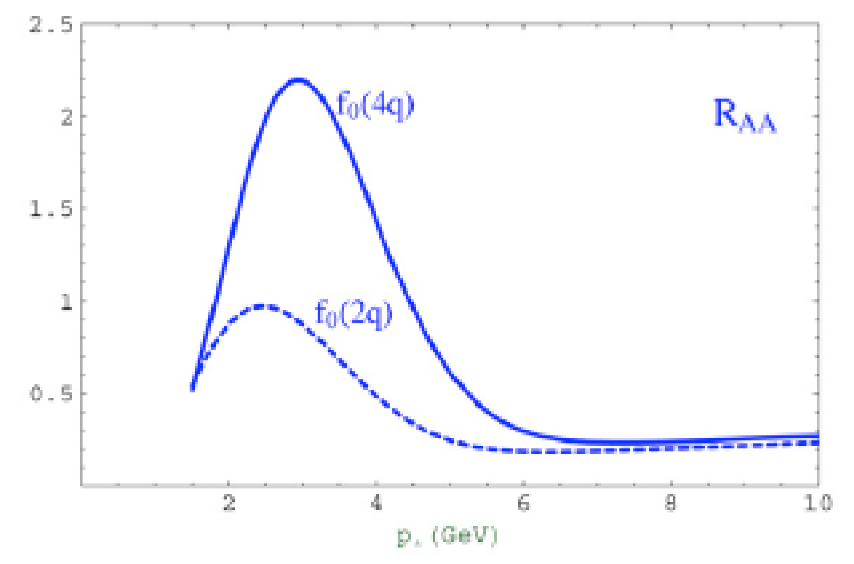
Central-to-Peripheral (CP) nuclear modification factor (R_{CP})

$$R_{CP}(p_T) = \frac{\text{Central}}{\text{Peripheral}} = \frac{N_{coll}(b)}{N_{coll}(b=0)} \cdot \frac{dN_{A+A}(b=0)/dp_\perp^2}{dN_{A+A}(b)/dp_\perp^2}$$

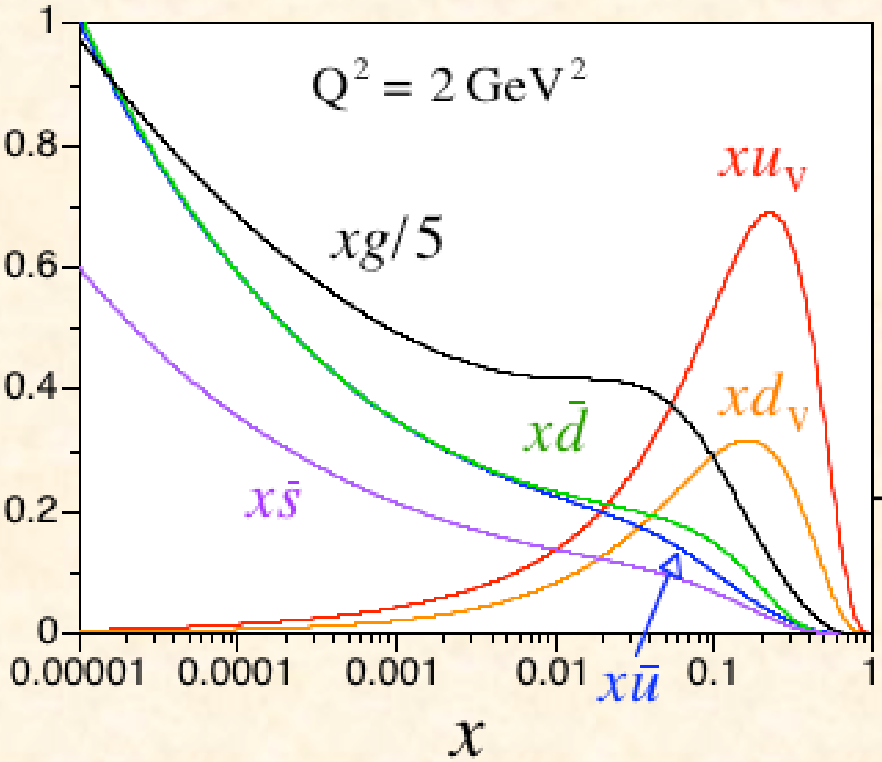
Nuclear modification factor (R_{AA})

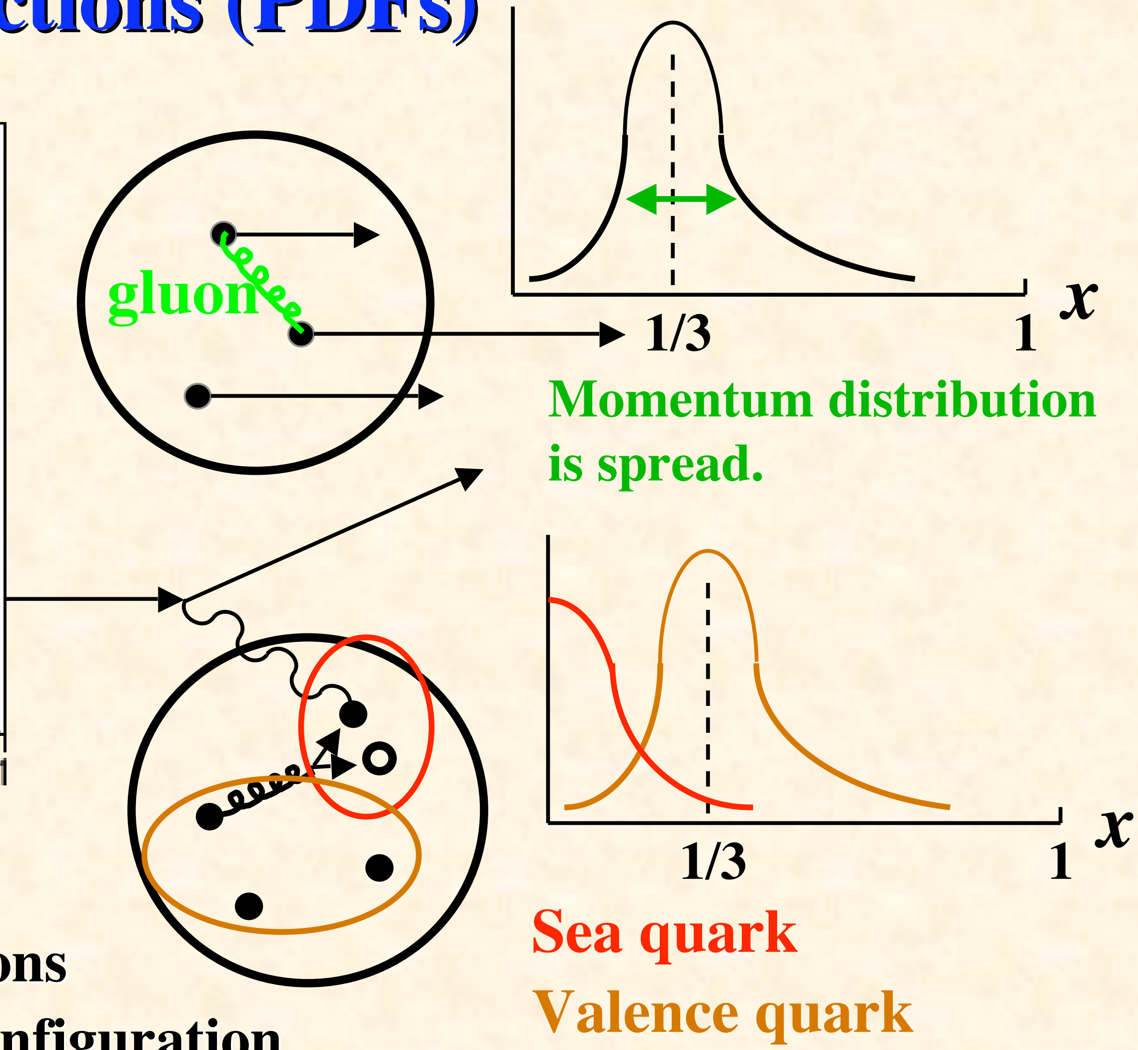
$$R_{AA}(p_T) = \frac{\text{Nucleus}}{\text{Nucleon}} = \frac{1}{N_{coll}(b=0)} \cdot \frac{dN_{A+A}(b=0)/dp_\perp^2}{dN_{p+p}/dp_\perp^2}$$





Parton distribution functions (PDFs)





Valence- and sea-quark distributions are different. → Internal quark configuration
However, PDFs of exotic hadrons could not be measured because lifetimes are too short.
→ Possible in fragmentation functions ?!

Criteria for determining internal structure of $f_0(980)$ by fragmentation functions

Criteria for determining f_0 structure by its fragmentation functions

Possible configurations of $f_0(980)$

(1) ordinary u, d - meson $\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})$

Contradicts with experimental widths

$\Gamma_{\text{theo}}(f_0 \to \pi\pi) = 500-1000 \text{ MeV}$
$\gg \Gamma_{\text{exp}} = 40-100 \text{ MeV}$

$\Gamma_{\text{theo}}(f_0 \to \gamma\gamma) = 1.3-1.8 \text{ keV}$
$\gg \Gamma_{\text{exp}} = 0.205 \text{ keV}$

(2) strange meson, $s\bar{s}$

(3) tetraquark ($K\bar{K}$), $\frac{1}{\sqrt{2}}(u\bar{u}s\bar{s}+d\bar{d}s\bar{s})$

(4) glueball gg

Contradicts with lattice-QCD estimate

$m_{\text{lattice}}(f_0) = 1600 \text{ MeV}$
$\gg m_{\text{exp}} = 980 \text{ MeV}$

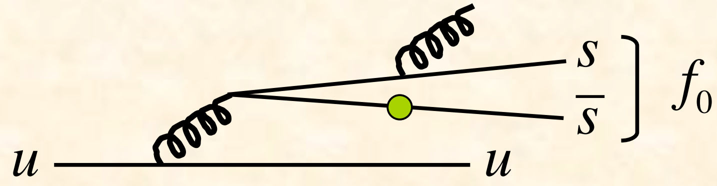
Discuss 2nd moments and functional forms (peak positions) of the fragmentation functions for f_0 by assuming the above configurations, (1), (2), (3), and (4).

$s\bar{s}$ picture for $f_0(980)$

2nd moment: $M(u) < M(s) \lesssim M(g)$

Peak of function: $z_{\max}(u) < z_{\max}(s) \simeq z_{\max}(g)$

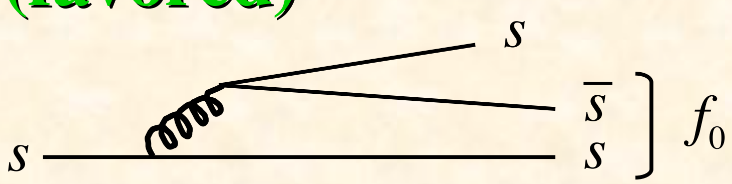
u (disfavored)



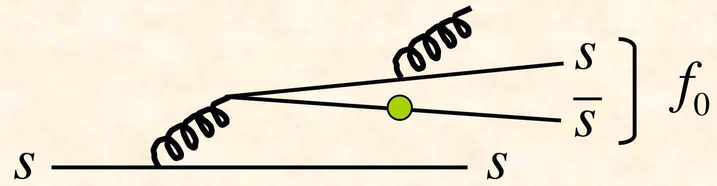
O (g^3)

+ one O (g^3) term of gluon radiation from the antiquark

s (favored)



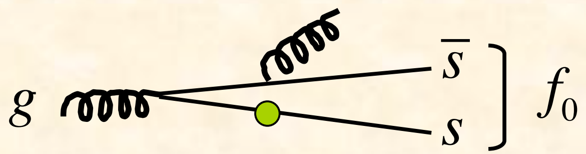
O (g^2)



O (g^3)

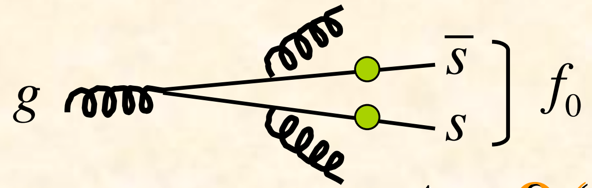
+ one O (g^3) term of gluon radiation from the antiquark

g



O (g^2)

+ one O (g^2) term of gluon radiation from the quark



O (g^3)

+ two O (g^3) terms of gluon radiation from the quark or antiquark

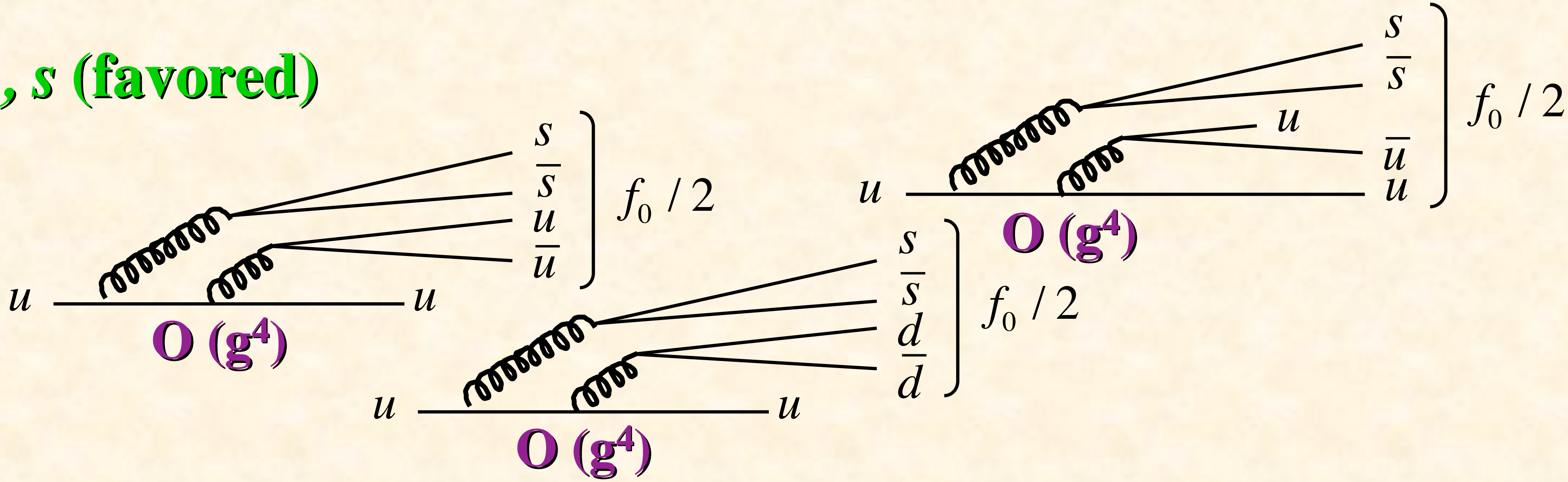
$n\bar{n}s\bar{s}$ picture for $f_0(980)$

$$f_0 = (u\bar{u}s\bar{s} + d\bar{d}s\bar{s})/\sqrt{2}$$

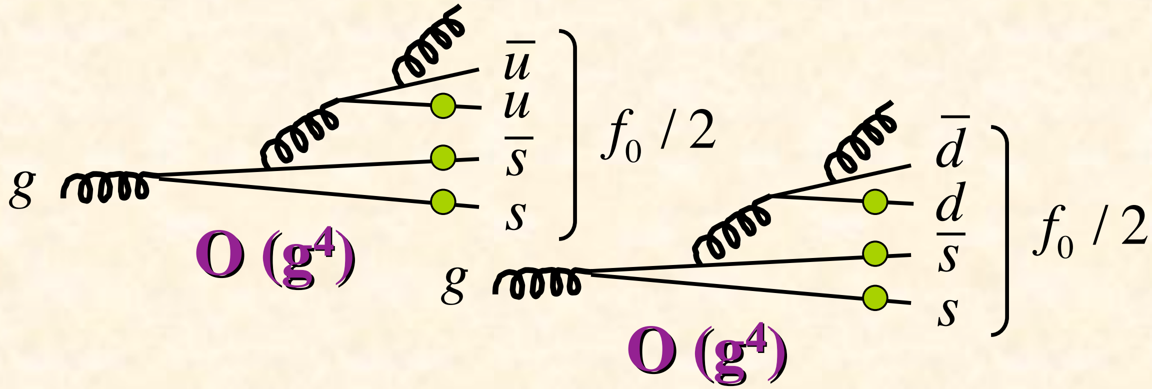
$K\bar{K}$ picture for $f_0(980)$

$$f_0 = \left[K^+(u\bar{s})K^-(\bar{u}s) + K^0(d\bar{s})\bar{K}^0(\bar{d}s)\right]/\sqrt{2}$$

u, s (favored)



g



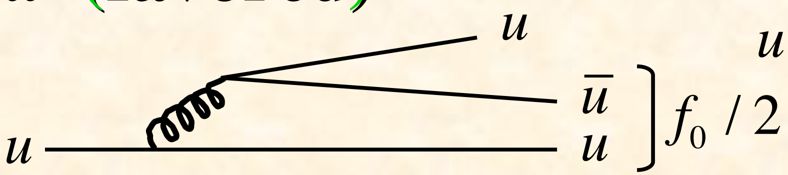
+ six O (g^4) terms of gluon radiation from other quarks

2nd moment:	$M(u) = M(s) \lesssim M(g)$
Peak of function:	$z_{max}(u) = z_{max}(s) \simeq z_{max}(g)$

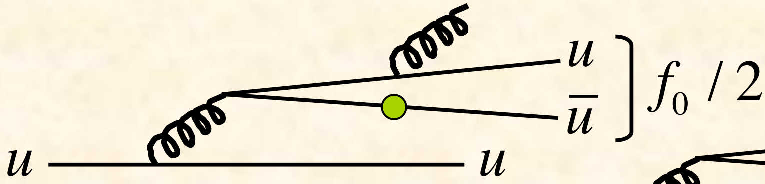
$n\bar{n}$ picture for $f_0(980)$

$$f_0 = (u\bar{u} + d\bar{d})/\sqrt{2}$$

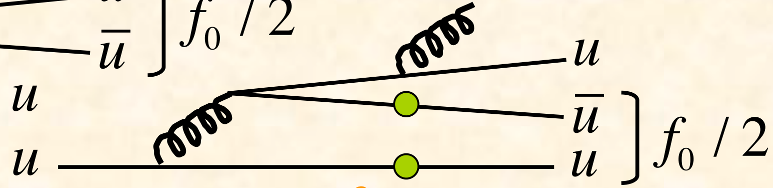
u (favored)



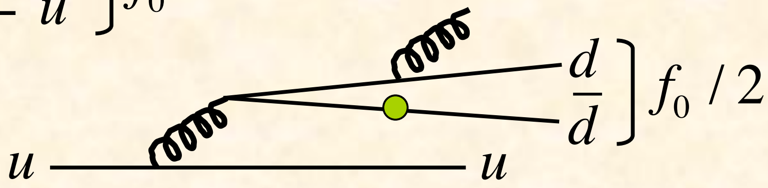
O (g^2)



O (g^3)



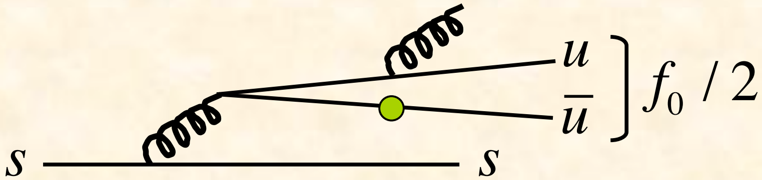
O (g^3)



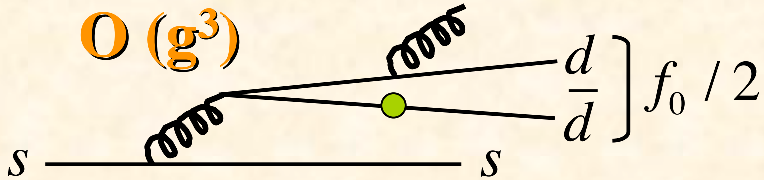
O (g^3)

+ four O (g^3) terms of gluon radiation from other quarks

s (disfavored)



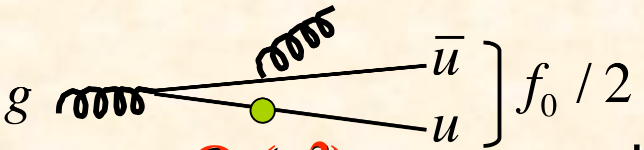
O (g^3)



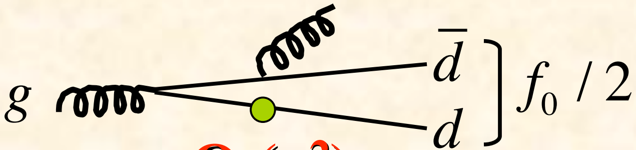
O (g^3)

+ two O (g^3) terms of gluon radiation from other quarks

g



O (g^2)



O (g^2)

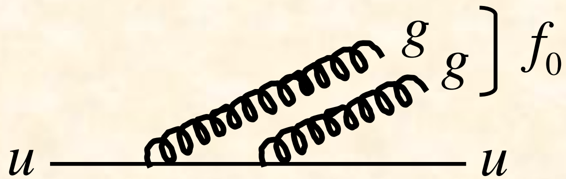
+ Two O (g^2) terms of gluon radiation from other quarks

2nd moment: $M(s) < M(u) < M(g)$

Peak of function: $z_{\max}(s) < z_{\max}(u) \simeq z_{\max}(g)$

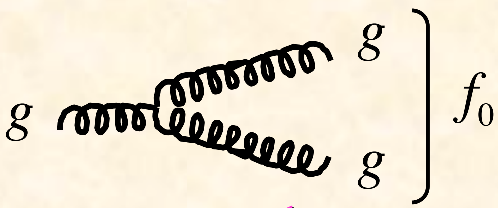
gg picture for $f_0(980)$

u, s (disfavored)



O (g^2)

g (favored)



O (g^1)

2nd moment: $M(u) = M(s) < M(g)$

Peak of function: $z_{max}(u) = z_{max}(s) < z_{max}(g)$

Judgment

Type	Configuration	2nd Moment	Peak z
Nonstrange $q\bar{q}$	$(u\bar{u}+d\bar{d})/\sqrt{2}$	$M(s)<M(u)<M(g)$	$z_{\max}(s)<z_{\max}(u)\simeq z_{\max}(g)$
Strange $q\bar{q}$	$s\bar{s}$	$M(u)<M(s)\lesssim M(g)$	$z_{\max}(u)<z_{\max}(s)\simeq z_{\max}(g)$
Tetraquark	$(u\bar{u}s\bar{s}+d\bar{d}s\bar{s})/\sqrt{2}$	$M(u)=M(s)\lesssim M(g)$	$z_{\max}(u)=z_{\max}(s)\simeq z_{\max}(g)$
$K\bar{K}$ **Molecule**	$(K^+K^-+K^0\bar{K}^0)/\sqrt{2}$	$M(u)=M(s)\lesssim M(g)$	$z_{\max}(u)=z_{\max}(s)\simeq z_{\max}(g)$
Glueball	gg	$M(u)=M(s)<M(g)$	$z_{\max}(u)=z_{\max}(s)<z_{\max}(g)$

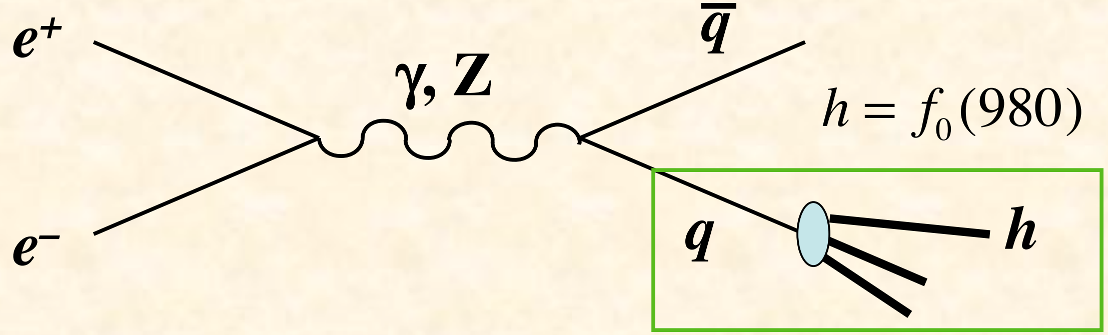
Since there is no difference between $D_u^{f_0}$ and $D_d^{f_0}$ in the models, they are assumed to be equal. On the other hand, $D_s^{f_0}$ and $D_g^{f_0}$ are generally different from them, so that they should be used for finding the internal structure. Therefore, simple and "model-independent" initial functions are

$$D_u^{f_0}(z,Q_0^2)=D_{\bar{u}}^{f_0}(z,Q_0^2)=D_d^{f_0}(z,Q_0^2)=D_{\bar{d}}^{f_0}(z,Q_0^2),\quad D_s^{f_0}(z,Q_0^2)=D_{\bar{s}}^{f_0}(z,Q_0^2),$$

$$D_g^{f_0}(z,Q_0^2),\qquad D_c^{f_0}(z,m_c^2)=D_{\bar{c}}^{f_0}(z,m_c^2),\qquad D_b^{f_0}(z,m_b^2)=D_{\bar{b}}^{f_0}(z,m_b^2).$$

Global analysis for fragmentation functions of $f_0(980)$

Fragmentation functions for f_0(980)



$$h = f_0(980)$$

$$z \equiv \frac{E_h}{\sqrt{s}/2} = \frac{2E_h}{Q} = \frac{E_h}{E_q}, \qquad s = Q^2$$

$$F^h(z,Q^2) = \frac{1}{\sigma_{tot}} \frac{d\sigma(e^+e^- \to hX)}{dz}$$

σ_{tot} = total hadronic cross section

$$F^h(z,Q^2) = \sum_i \int_z^1 \frac{dy}{y} C_i\left(\frac{z}{y}, Q^2\right) D_i^h(y,Q^2)$$

Initial functions

$$D_u^{f_0}(z,Q_0^2) = D_d^{f_0}(z,Q_0^2) = N_u^{f_0} z^{\alpha_u^{f_0}} (1-z)^{\beta_u^{f_0}}$$

$$D_s^{f_0}(z,Q_0^2) = N_s^{f_0} z^{\alpha_s^{f_0}} (1-z)^{\beta_s^{f_0}}$$

$$D_g^{f_0}(z,Q_0^2) = N_g^{f_0} z^{\alpha_g^{f_0}} (1-z)^{\beta_g^{f_0}}$$

$$D_c^{f_0}(z,m_c^2) = N_c^{f_0} z^{\alpha_c^{f_0}} (1-z)^{\beta_c^{f_0}}$$

$$D_b^{f_0}(z,m_b^2) = N_b^{f_0} z^{\alpha_b^{f_0}} (1-z)^{\beta_b^{f_0}}$$

- $D_q^{f_0}(z,Q_0^2) = D_{\bar{q}}^{f_0}(z,Q_0^2)$
- $Q_0 = 1$ GeV
  $m_c = 1.43$ GeV
  $m_b = 4.3$ GeV

$$N = M \frac{\Gamma(\alpha+\beta+3)}{\Gamma(\alpha+2)\Gamma(\beta+1)}, \qquad M \equiv \int_0^1 z D(z)\, dz$$

Experimental data for f_0

Total number of data: only 23

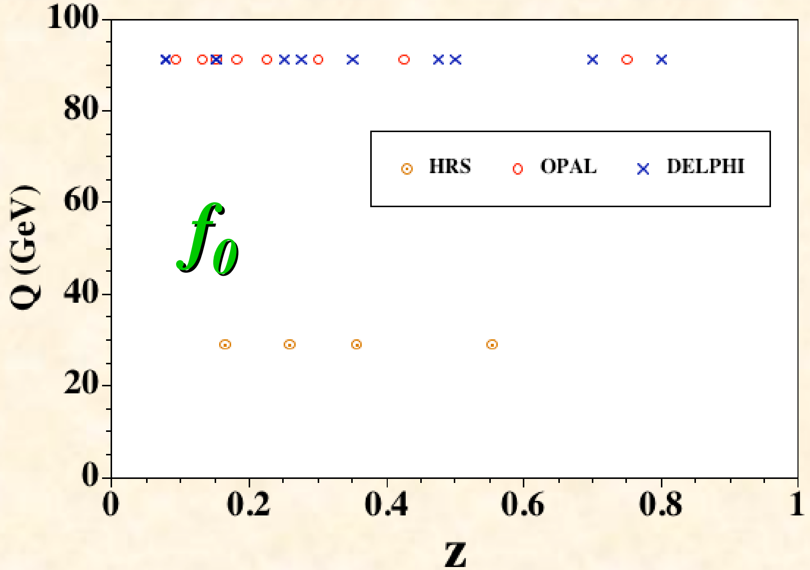
Exp. collaboration	$\sqrt{s}$ (GeV)	# of data
HRS	29	4
OPAL	91.2	8
DELPHI	91.2	11

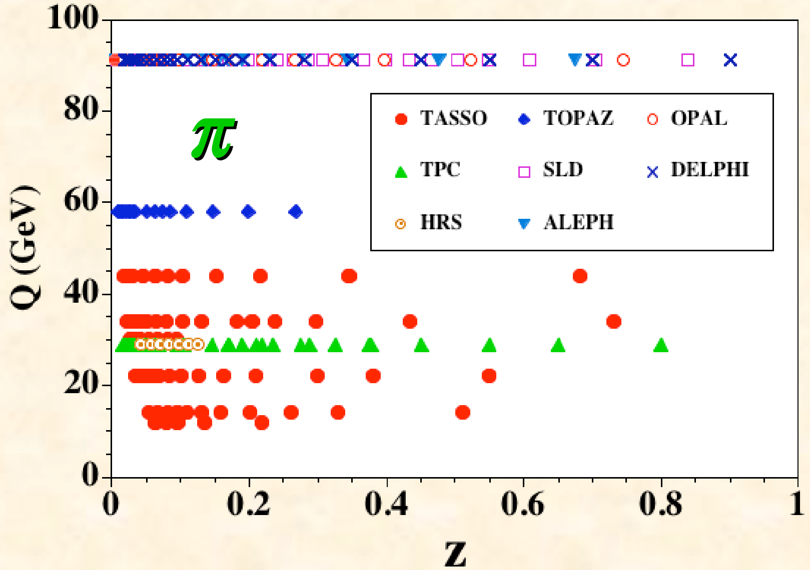
pion

Total number of data: 264

Exp. collaboration	$\sqrt{s}$ (GeV)	# of data
TASSO	12,14,22,30,34,44	29
TCP	29	18
HRS	29	2
TOPAZ	58	4
SLD	91.2	29
SLD [light quark]		29
SLD [c quark]		29
SLD [b quark]		29
ALEPH	91.2	22
OPAL	91.2	22
DELPHI	91.2	17
DELPHI [light quark]		17
DELPHI [b quark]		17

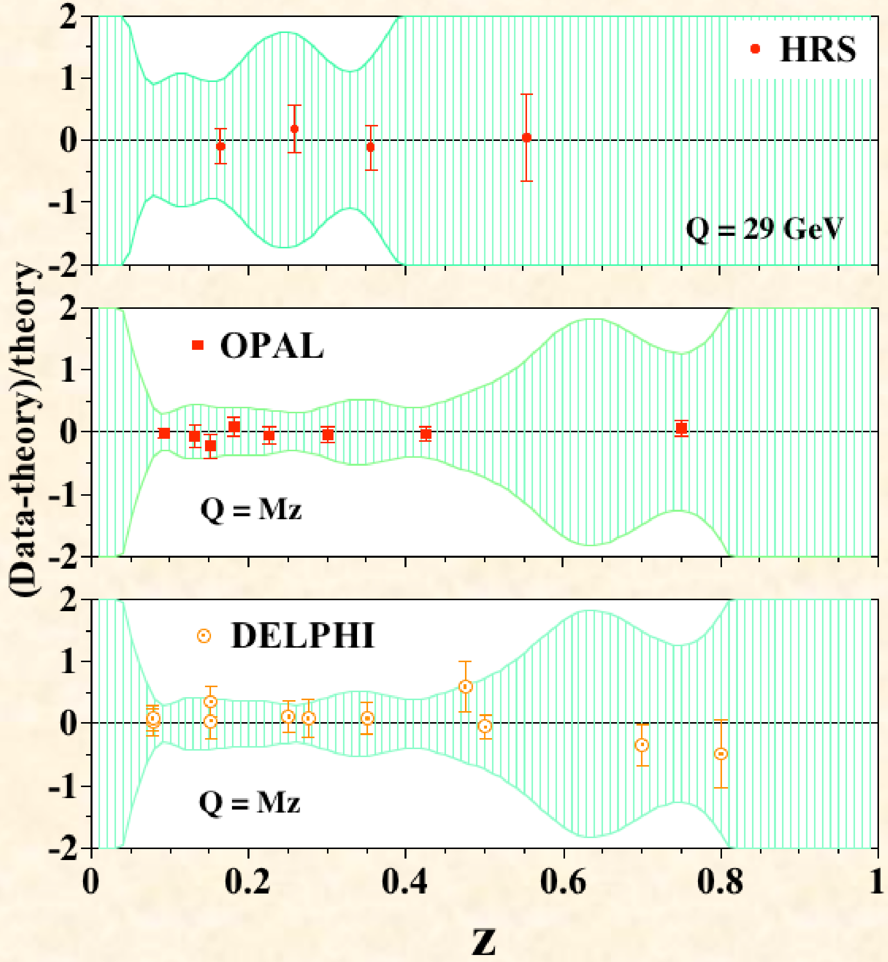
One could foresee the difficulty in getting reliable FFs for f_0 *at this stage.*





Analysis results: Comparison with data

$$\chi^2 / \text{d.o.f.} = 0.907$$



HRS, PRL 57, 1990 (1986)
OPAL, EPJ C4,19 (1998)
DELPHI, PL 449B, 364 (1999), ZP C65, 587 (1995)

Rational difference between data and theory

$$\frac{F^{f_0}(z,Q^2)_{\text{data}} - F^{f_0}(z,Q^2)_{\text{theory}}}{F^{f_0}(z,Q^2)_{\text{theory}}}$$

- **Uncertainties of determined FFs are very large!**
- **Only a few data at small Q^2**

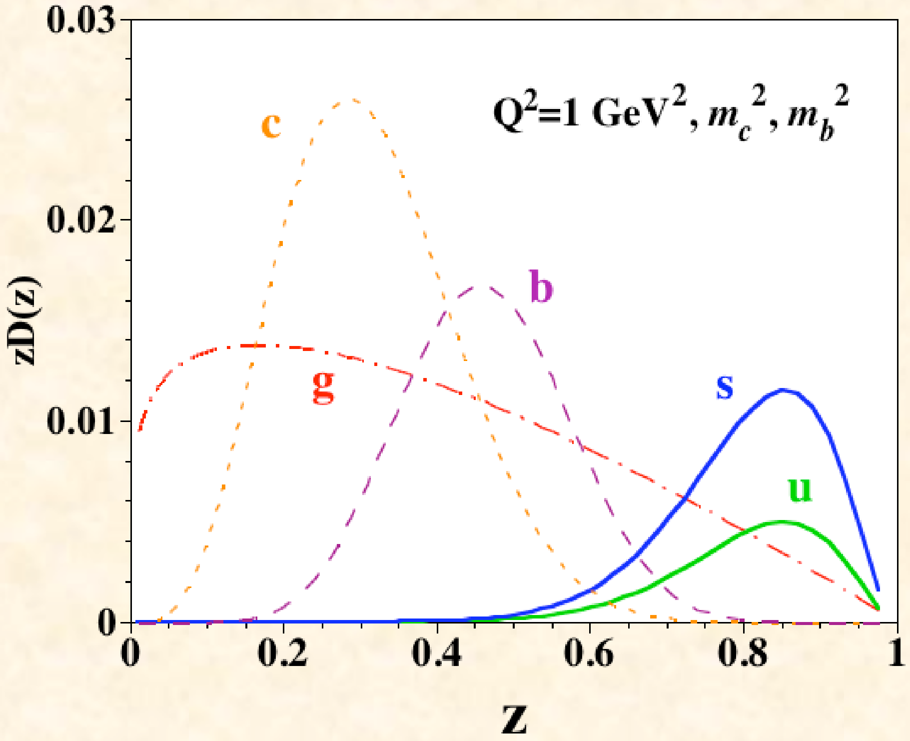
  $\rightarrow$ difficult to determine the FFs (f_0) especially at small Q^2

  $\rightarrow$ difficult to find scaling violation
  = gluon FF cannot be fixed
  = quark FFs should have extra uncertainties due to error correlation with the gluon

Results on the fragmentation functions

- **Functional forms**

  (1) $D_u^{f_0}(z)$, $D_s^{f_0}(z)$ have peaks at large z
  (2) $z_u^{\max} \sim z_s^{\max}$



(1) and (2) indicate tetraquark structure

$$f_0 \sim \frac{1}{\sqrt{2}}(u\bar{u}s\bar{s} + d\bar{d}s\bar{s})$$

- **2nd moments:** $\dfrac{M_u}{M_s} = 0.43$

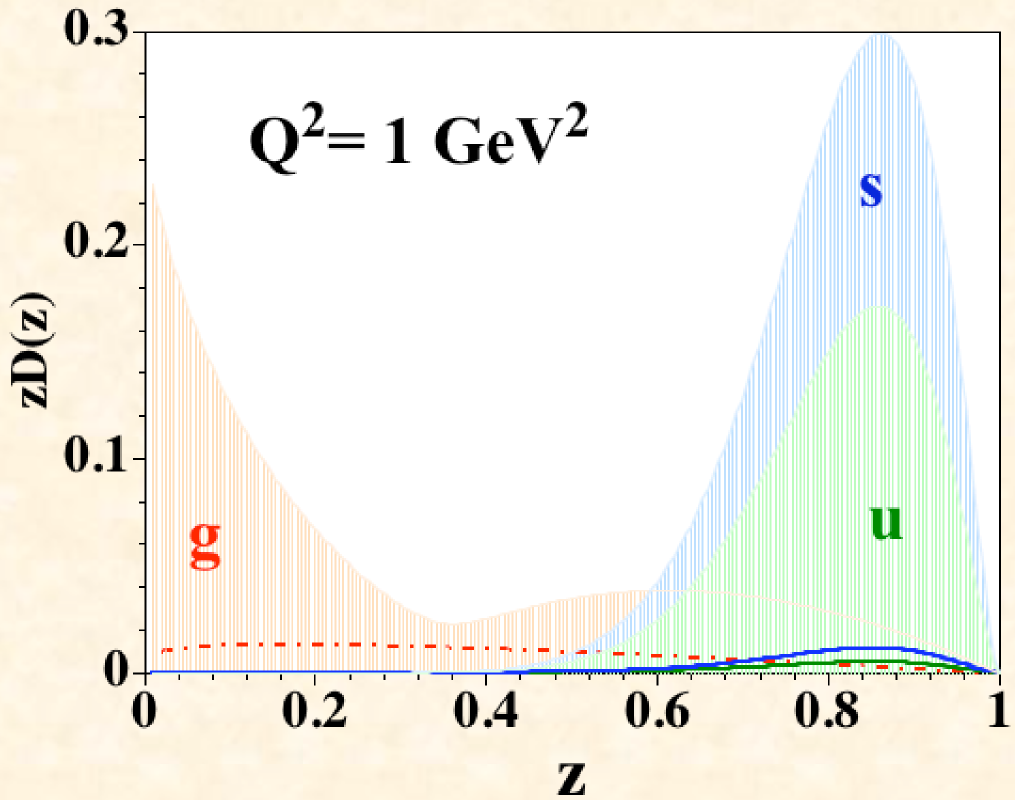
This relation indicates $s\bar{s}$-like structure (or admixture)

$$f_0 \sim s\bar{s}$$

$\Rightarrow$ Why do we get the conflicting results?

$\rightarrow$ Uncertainties of the FFs should be taken into account (next page).

Large uncertainties



2nd moments

$$M_u = 0.0012 \pm 0.0107$$
$$M_s = 0.0027 \pm 0.0183$$
$$M_g = 0.0090 \pm 0.0046$$

$$\rightarrow M_u / M_s = 0.43 \pm 6.73$$

The uncertainties are order-of-magnitude larger than the distributions and their moments themselves.

At this stage, the determined FFs are not accurate enough to discuss internal structure of f_0(980).

→ Accurate data are awaited not only for f_0(980) but also for other exotic and "ordinary" hadrons.

Requests for experimentalist *(hopefully, not so demanding)*

- **Accurate data on f_0(980) and other exotic hadrons, as well as ordinary ones**

- **Accurate data especially at small Q^2**

  e.g. Belle, c.m. energy = 10.58 GeV

  → Determination of scaling violation
  (mainly, gluon fragmentation function)

- **Charm- and bottom-quark tagging**

  Charm and bottom functions can be determined.

  → Remaining functions, which are important for judging whether or not the hadron is exotic, should be determined much accurately.

Summary on Part II

Exotic hadrons could be found by studying fragmentation functions. As an example, the $f_0(980)$ meson was investigated.

(1) We proposed to use 2nd moments and functional forms as criteria for finding quark configuration.

(2) Global analysis of $e^+ + e^- \rightarrow f_0 + X$ data

The results *may* indicate $s\bar{s}$ or $qq\bar{q}\bar{q}$ structure. However, …

- **Large uncertainties in the determined FFs**

→ The obtained FFs are not accurate enough to discuss the quark configuration of $f_0(980)$.

(3) Accurate experimental data are important

→ Small-Q^2 data as well as large-Q^2 (M_z^2) ones

→ *c*- and *b*-quark tagging

The End

The End