Status of AKK Fragmentation Functions

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Global Analyses of Unpolarized FFs

light charged (l.c.h.) $\pi^{\pm}$, $K^{\pm}$, $p/\bar{p}$,
strange neutral (s.n.h.) K_S^0, $\Lambda/\overline{\Lambda}$

Albino-Kramer-Kniehl (2005/6)

- e^+e^-: OPAL etc., $\sqrt{s} = M_Z$
- OPAL u, d, s tagged
- h^+, h^- not distinguished
- $x > 0.1$

de Florian-Sassot-Stratmann (2007)

- e^+e^-: OPAL, TASSO, etc.
- OPAL u, d, s tagged
- pp: BRAHMS, STAR, PHENIX
- ep: HERMES ($\pi^{\pm}$, $K^{\pm}$)
- h^+, h^- separately
- $x > .05$

Hirai-Kumano-Nagai-Sudoh (2007)

- e^+e^-: OPAL, TASSO, etc.
- OPAL u, d, s tagged
- FF exp. errors
- h^+, h^- separately
- $x > .05$

Summary of new AKK fit

Experiment

- l.c.h. ($\pi^\pm$, $K^\pm$, $p/\bar{p}$) + s.n.h. (K^0_S, $\Lambda/\bar{\Lambda}$)
- e^+e^-, $x > .05$:
  - $\sqrt{s} = M_Z$ (OPAL, ALEPH...)
  - l, c, b tag (OPAL: u, d, s, c, b)
  - + $12\mathrm{GeV} < \sqrt{s} < M_Z$ (TASSO...)
- pp: BRAHMS, STAR, PHENIX ($p_T > 2$ GeV)
- $p\bar{p}$: CDF, 2005 (K^0_S, $\Lambda/\bar{\Lambda}$)
- norm. errors $\in$ covariance matrix
- exclude ep: HERMES ($\pi^\pm$, $K^\pm$):
  - $Q \lesssim 2$ GeV — fixed order?
  - $ep(Q) \sim e^+e^-(\sqrt{s})$
  - compare low Q with low $\sqrt{s}$
  - different *systematic* theory error

Theory

- FF basis:
  - $\pm$ sum ($h^\pm$) (e^+e^- $\therefore$ accurate)
  - $\pm$ diff ($\Delta_c h^\pm$) (pp $\therefore$ less so)
- non pert. constraints:
  - Strong:FF$(\sqrt{2}\mathrm{GeV}) = N x^\alpha (1-x)^\beta$, $\times(1+\gamma(1-x)^\delta)$ for $h^\pm$
  - Weak: SU(2) isospin $u \leftrightarrow d \in \pi^\pm$
  - Weaker:$D_{\bar{q}}^{(\Delta_c)h^\pm} = D_q^{(-\Delta_c)h^\pm}$ (QCD)
- hadron mass:
  - fitted (e^+e^-) $\rightarrow$ subtract low x, $\sqrt{s}$ effects
  - baryons accurate (+1%)
  - mesons OK
- large x res. (e^+e^-) — improves fit

General calculation

Factorization theorem I

IS $\to \sum_j$ parton $j + X \to h + X$

$$\frac{d\sigma^h}{dx}(x,E) =$$
$$\sum_j \int_x^1 \frac{dz}{z} \frac{d\sigma^j}{d(x/z)} \left(\frac{x}{z}, a_s(E), \frac{E}{M_f}\right) D_j^h(z, M_f)$$

$(E = \sqrt{s}, p_T)$

Neglect higher twist $O(1/E)$

Scale: $\mu^2, M_f^2 = kE^2$

$(k = 1, 1/4, 4)$

Scheme: $\overline{\rm MS}$

Factorization theorem II

DGLAP evolution

$$\frac{d}{d\ln M_f} D_i^h(x, M_f) =$$
$$\sum_j \int_x^1 \frac{dz}{z} P_{ij}\left(\frac{x}{z}, a_s(M_f)\right) D_j^h(z, M_f)$$

$D_i^h(x, M_f)$ universal

$D_q^h(x, M_f < m_q^t \sim m_q)$:

Extrinsic $= 0$,

Intrinsic not in evolution

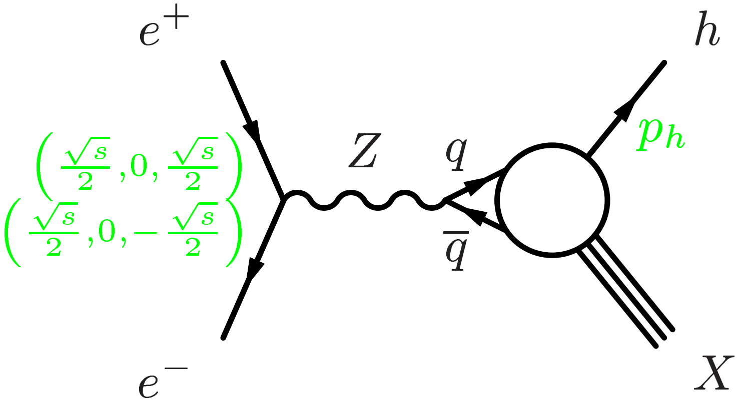
Calculations to NLO

CTEQ6.5S0 PDFs $\to \Lambda_5^{\overline{\rm MS}} = 226$ MeV

Processes

$$e^+e^- \to h + X$$

$$\frac{d\sigma^h}{dx}\left(x=\frac{2|\mathbf{p}_h|}{\sqrt{s}}, \sqrt{s}\right) =$$

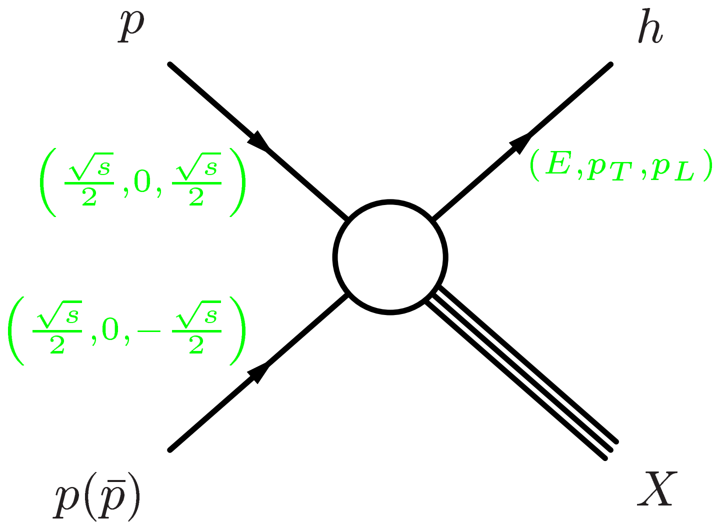


ALEPH, DELPHI, TASSO, OPAL, ...

$\longrightarrow$ precise $q \to h^{\pm}$ FFs (no $s - d$)

$$pp(\overline{p}) \to h + X$$

$$\frac{d^2\sigma^h}{dp_T\,dy}\left(x=\frac{2p_T}{\sqrt{s}}\cosh y, y=\frac{1}{2}\ln\frac{E+p_L}{E-p_L}, \sqrt{s}\right) =$$



- CDF (2005) ($p\bar{p} \to K^0_S, \Lambda/\overline{\Lambda}$, $\sqrt{s} = 630$ GeV)
  $\to$ FF shape + relative (not overall) norms
- BRAHMS, PHENIX, STAR (2006,2007) @ RHIC (pp, $\sqrt{s} = 200$ GeV)

$\to$ remaining FFs (gluon, $q \to \Delta_c h^{\pm}$)

$pp(\overline{p})$ in Mellin Space

N- (Mellin) space $\left(f(N) = \int_0^1 dx x^{N-1} f(x)\right)$:
x-convolutions $\longrightarrow$ products in N-space
N-space calculations fast and accurate

$pp(\overline{p})$ XS from convolution with FFs:

$$F\left(x = \frac{2p_T}{\sqrt{s}} \cosh y\right) = \int_x^1 \frac{dz}{z} \widehat{F}\left(\frac{x}{z}\right) z D(z)$$

$$\therefore F(N) = \widehat{F}(N) D(N+1)$$

$\widehat{F}(N)$ analytic calculation impossible

solution: $\widehat{F}$ as Chebyshev expansion

$$\widehat{F}(z)/\ln(1 - x/z) = \sum_k c_k T_k(X)$$

$-1 < X < 1 \leftarrow$ linear map $\rightarrow x < z < 1$
$\longrightarrow$ semi-analytic $\widehat{F}(N)$
$\longrightarrow F(x)$ calc. is quick, to few parts per mil

Systematic Errors

$$P(\{f_i^e\},\{f_i^t\}) \propto \exp\left[-\frac{1}{2}\chi^2 = \sum_i \left(\frac{f_i^t - f_i^e}{\sigma_i}\right)^2\right]$$

$pp(\overline{p})$, low $\sqrt{s}$ e^+e^-: norm. uncertainty(s)

= systematic error(s)

systematic error, Kth source:

- $\Delta_K f_i^e = \lambda_K \sigma_i^K$
- $P(\lambda_K) \propto \exp\left[-\frac{1}{2}\lambda_K^2\right]$

  ($\longrightarrow$ expect $|\lambda_K| \lesssim 1$)

$$\chi^2 = \sum_i \left(\frac{f_i^t - f_i^e - \sum_K \lambda_K \sigma_i^K}{\sigma_i}\right)^2 + \sum_K \lambda_K^2$$

Minimize χ^2 w.r.t. λ_K (Max prob):

- $\lambda_K = \sum_i \frac{f_i^t - f_i^e}{\sigma_i^2}\left(\sigma_i^K - \sum_{jkL} \sigma_i^L \sigma_j^L \left(C^{-1}\right)_{jk} \sigma_k^K\right)$
- $\chi^2 = \sum_{ij} (f_i^t - f_i^e) \left(C^{-1}\right)_{ij} (f_j^t - f_j^e)$

  (Covariance $C_{ij} = \sigma_i^2 \delta_{ij} + \sum_K \sigma_i^K \sigma_j^K$)

Unknown systematic effects:

Cancel for data sets from large no. of different exps.(?)

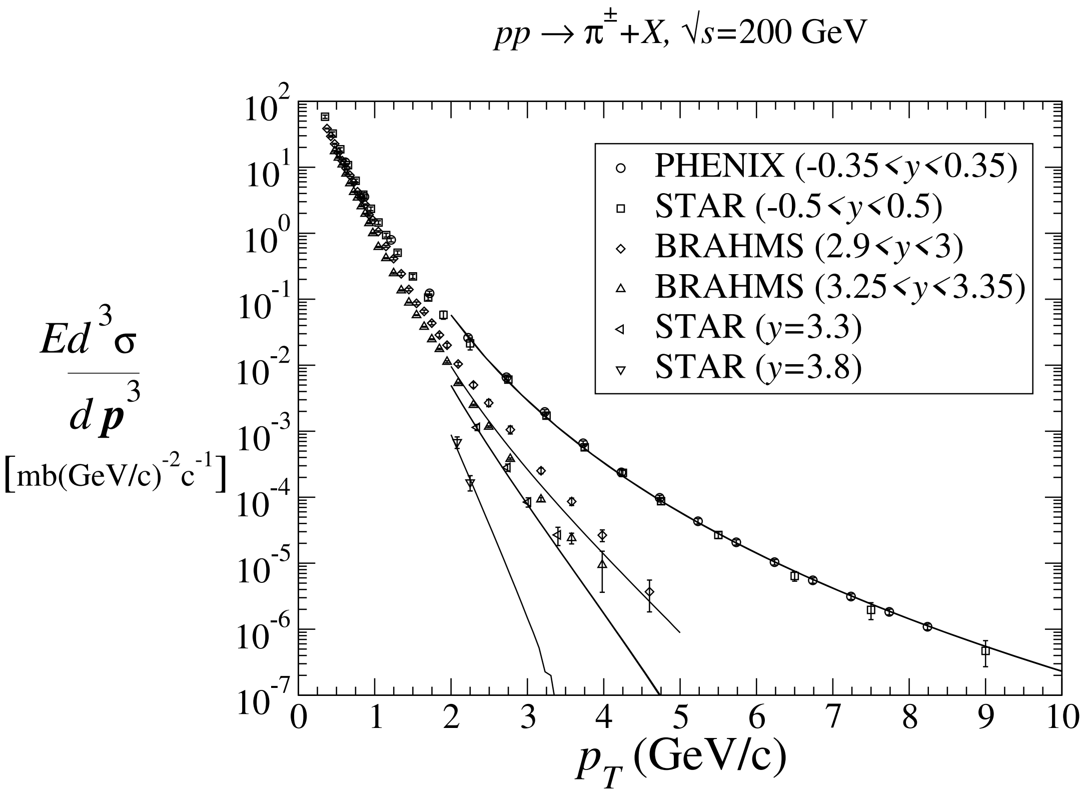
Systematic Errors — $\pi^{\pm}$

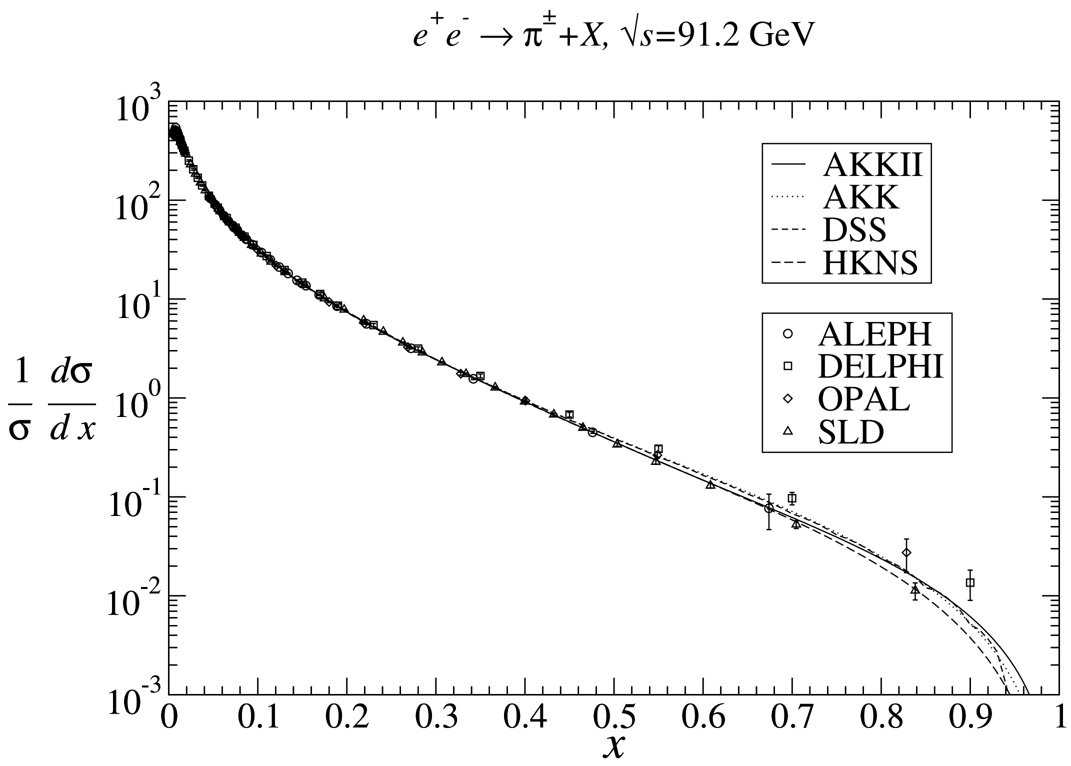
Collaboration	$\sqrt{s}$ (GeV)	# data	Norm. (%)	$\chi^2_{\rm DF}$	λ_K	Shift (%)
TASSO	12	5	20	0.50	0.21	4.3
TASSO	14	10	8.5	0.92	-1.26	-10.7
TASSO	22	1	6.3	0.01	-0.08	-0.5
TASSO	30	4	20	0.57	0.69	13.8
TASSO	34	10	6	1.07	0.62	3.7
TASSO	44	7	6	1.99	0.66	3.9
ALEPH	91.2	22	3	0.61	-0.55	-1.6
BRAHMS, $y \in [2.9, 3]$	200	8	11,7,8(13),	0.96	-1.76, -1.12, -1.22, -0.32, -0.13	-19.4, -7.9, -9.7, -0.6, -0.1
$y \in [3.25, 3.35]$		7	2,1(3)	2.68	-2.01, -1.28, -1.80, -0.37, -0.32	-22.2, -9.0, -14.4, -0.7, -0.3
PHENIX (π^0), $\|\eta\| < 0.35$	200	13	9.7	0.54	-0.48	-4.6
STAR (π^0), $\eta = 3.3$	200	4	16	0.70	-0.70	-11.3
STAR (π^0), $\eta = 3.8$	200	2	16	0.57	-0.31	-5.0
STAR, $\|y\| < 0.5$	200	10	11.7	0.49	-0.34	-3.9

Systematic Errors — $p/\overline{p}$

Collaboration	$\sqrt{s}$ (GeV)	# data	Norm. (%)	$\chi^2_{\rm DF}$	λ_K	Shift (%)
TASSO	12	3	20	0.49	-0.31	-6.2
TASSO	14	9	8.5	2.33	-0.58	-4.9
TASSO	22	9	6.3	1.36	-0.98	-6.2
TASSO	30	3	20	0.52	-0.82	-16.5
JADE	34	2	14	6.04	-0.82	-16.5
TASSO	34	7	6	1.12	-1.63	-9.8
ALEPH	91.2	18	3	0.62	-1.77	-5.3
BRAHMS, $y \in [2.9, 3]$	200	7	11,7,8(13),	2.38	-2.66, -1.69, -1.65, -0.48, -0.13	-29.3, -11.8, -13.2, -1.0, -0.1
$y \in [3.25, 3.35]$		5	2,1(3)	5.21	-3.64, -2.32, -2.47, -0.66, -0.26	-40.1, -16.2, -19.7, -1.3, -0.3
STAR, $\|y\| < 0.5$	200	8	11.7	3.08	-1.89	-22.1

Hadron Mass Effects





$\therefore$ “naive” FO calculation is sufficient

Main caveat: mass effects for heavier particles

(Second caveat: Complex decay patterns)

Hadron Mass Effects

Particle	Fitted mass (MeV)	True mass (MeV)
$\pi^\pm$	156.9	139.6
$K^\pm$	340.7	493.7
$p/\bar{p}$	949.2	938.3
K^0_S	363.2	497.6
$\Lambda/\overline{\Lambda}$	1127.0	1115.7

$\pi^\pm$: large excess

$\longrightarrow$ decays from much heavier $\rho(770) \to \pi^+ + \pi^-$

Ks: large undershoot

$\longrightarrow$ complicated decay channels, e.g. K resonance $\to \pi + K$

baryons ($p/\bar{p}$, $\Lambda/\overline{\Lambda}$): $\simeq$ 1% excess

$\longrightarrow$ decays from slightly heavier resonances

good environment to study partonic fragmentation

Hadron Mass Effects

factorization theorem: x = available c.m. light cone momenta $(p^0 + |\mathbf{p}|)$ fraction

$$e^+e^- \to h(p_h) + X$$

$$p_h = \left(+ = \frac{x\sqrt{s}}{\sqrt{2}}, - = \frac{m_h^2}{\sqrt{2}x\sqrt{s}}, (1,2) = \mathbf{0}\right)$$

$\longrightarrow$ x from x_E, x_p

e.g. $x_p = x\left(1 - \frac{m_h^2}{sx^2}\right)$

$\longrightarrow$ low x, $\sqrt{s}$ effect

$$pp(\overline{p}) \to h(p) + X$$

Partonic c.m. frame: parton $\mathbf{l}$ $\parallel$ hadron $\mathbf{p}$

$$\longrightarrow x = \frac{p^+}{l^+} = \frac{\sqrt{|\mathbf{p}|^2 + m_h^2} + |\mathbf{p}|}{2|\mathbf{l}|}$$

$$\longrightarrow \frac{d\mathbf{p}^3}{E} = \frac{|\mathbf{p}|^2}{\sqrt{|\mathbf{p}|^2 + m_h^2}} d|\mathbf{p}| d\Omega$$

$$\longrightarrow \frac{d\mathbf{l}^3}{|\mathbf{l}|} = |\mathbf{l}| d|\mathbf{l}| d\Omega$$

mass correction factor

$$E\frac{d^3}{d\mathbf{p}^3}\sigma^{h_1 h_2 \to h(p)+X}(p, \sqrt{s}) =$$

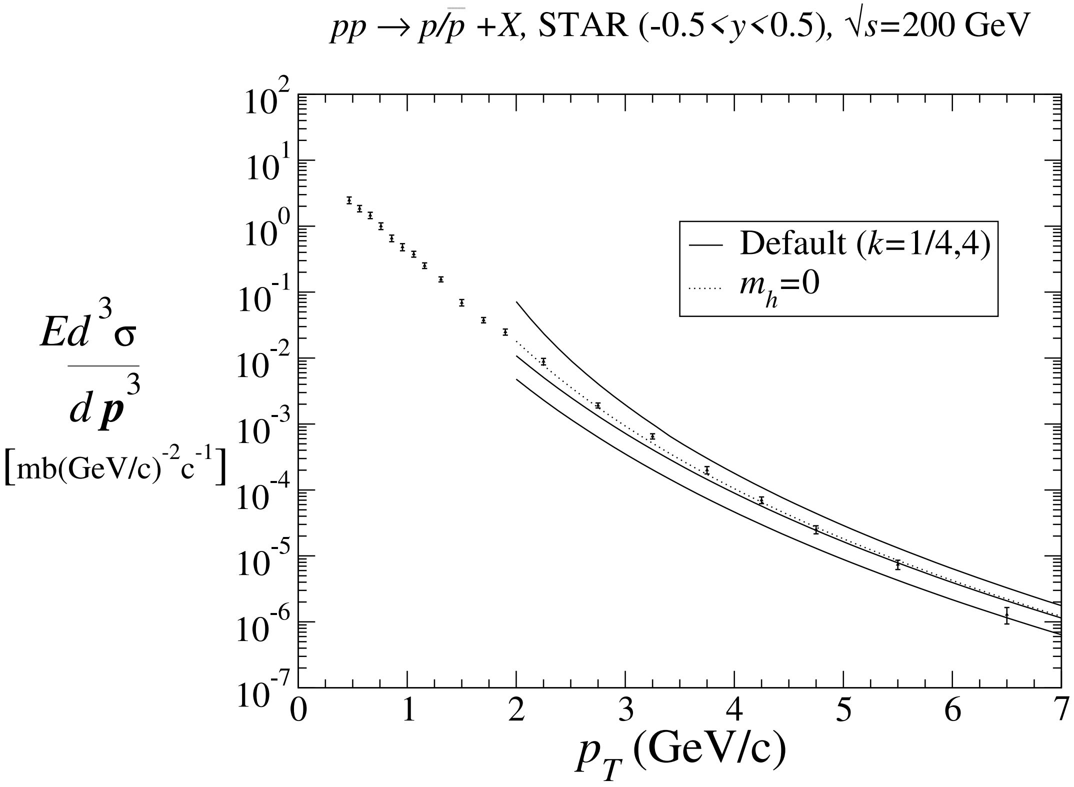
$$\sum_{i_1 i_2} \int dx_1 dx_2 f_{i_1}^{h_1}(x_1, M_f^2) f_{i_2}^{h_2}(x_2, M_f^2)$$

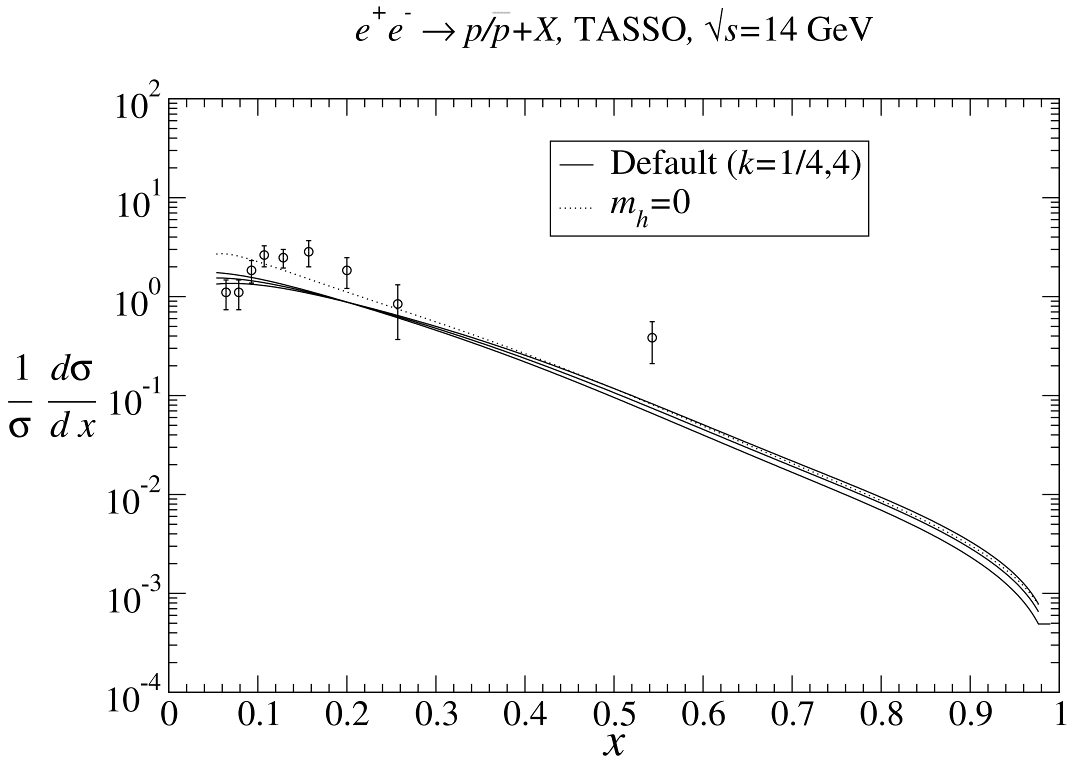
$$\times \sum_i \int dx D_i^h(x, M_f^2)$$

$$\times \frac{1}{R^2(x_1, x_2, y, m_h^2/p_T^2)} \frac{1}{x^2} |\mathbf{l}| \frac{d^3}{d\mathbf{l}^3} \sigma^{i_1 i_2 \to i(l)+X}(l, x_1 x_2 \sqrt{s})$$

Hadron Mass Effects

(use true mass) (fit mass)

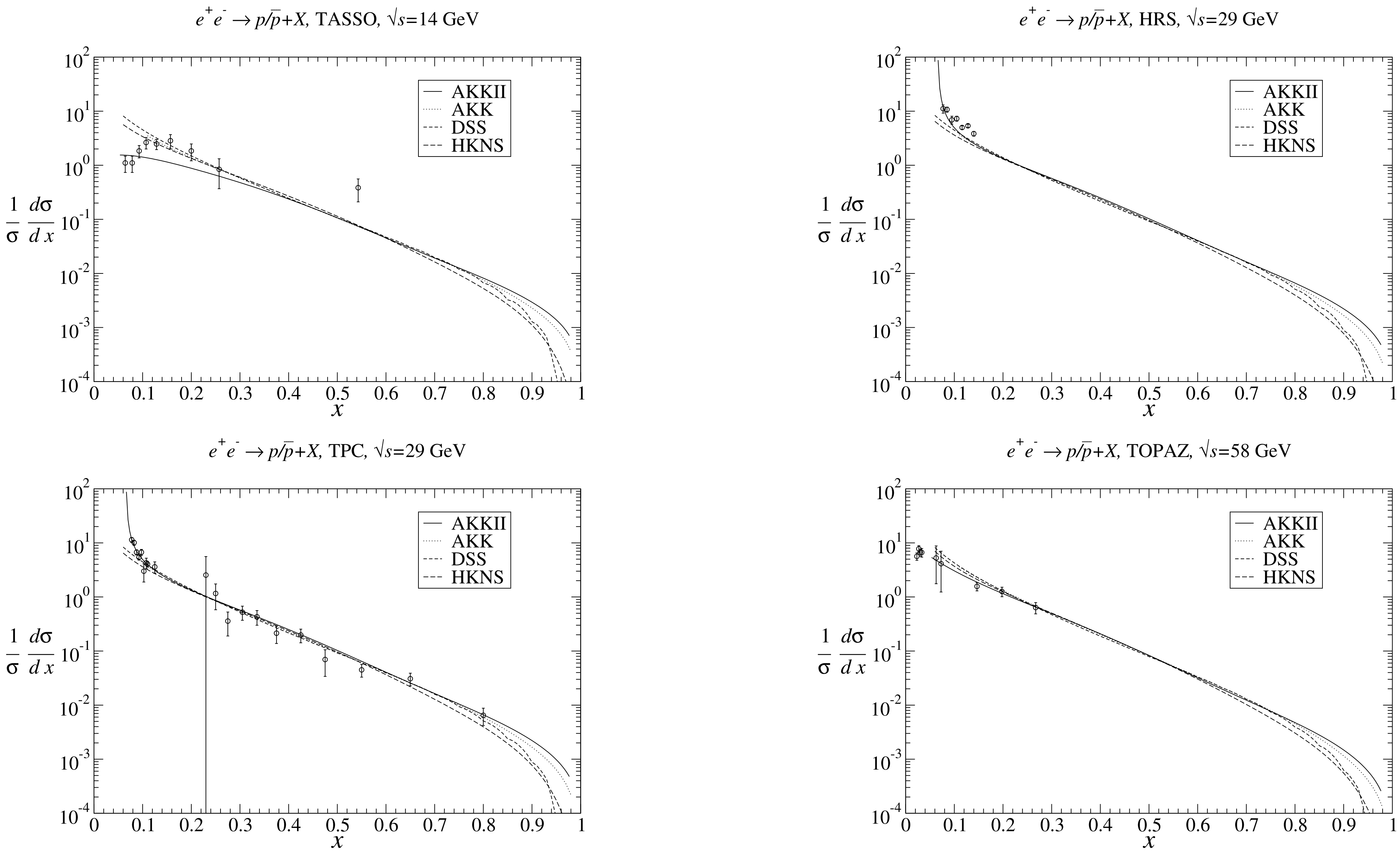




similar results at high rapidity

$\pi^{\pm}$: mass effects negligible

Hadron Mass Effects



Large x Resummation — e^+e^-

e^+e^- quark coefficient function C_q

fixed order approach fails at large N:

$$C_q(a_s,N) \simeq 1 + A a_s \ln N + B a_s^2 \ln^2 N + \ldots$$

general solution: resum $(a_s \ln N)^n$ into single function $f(a_s \ln N)$

(also $a_s(a_s \ln N)^n$ into $a_s g(a_s \ln N)$ etc.):

$$C_q(a_s,N) = C_q^r(a_s,N) \times \left(\sum_n a_s^n C_q^{\mathrm{FO}(n)}(N)\right)$$

NLO solution: resum $(a_s \ln N)^n$ ($r=0$), $a_s(a_s \ln N)^n$ ($r=1$) via Cacciari et al. result

$$C_q(a_s,N) = C_q^{r=0,1}(a_s,N) \times \left(1 + a_s(C_q^{(1)} - C_q^{r=0,1\ (1)})\right)$$

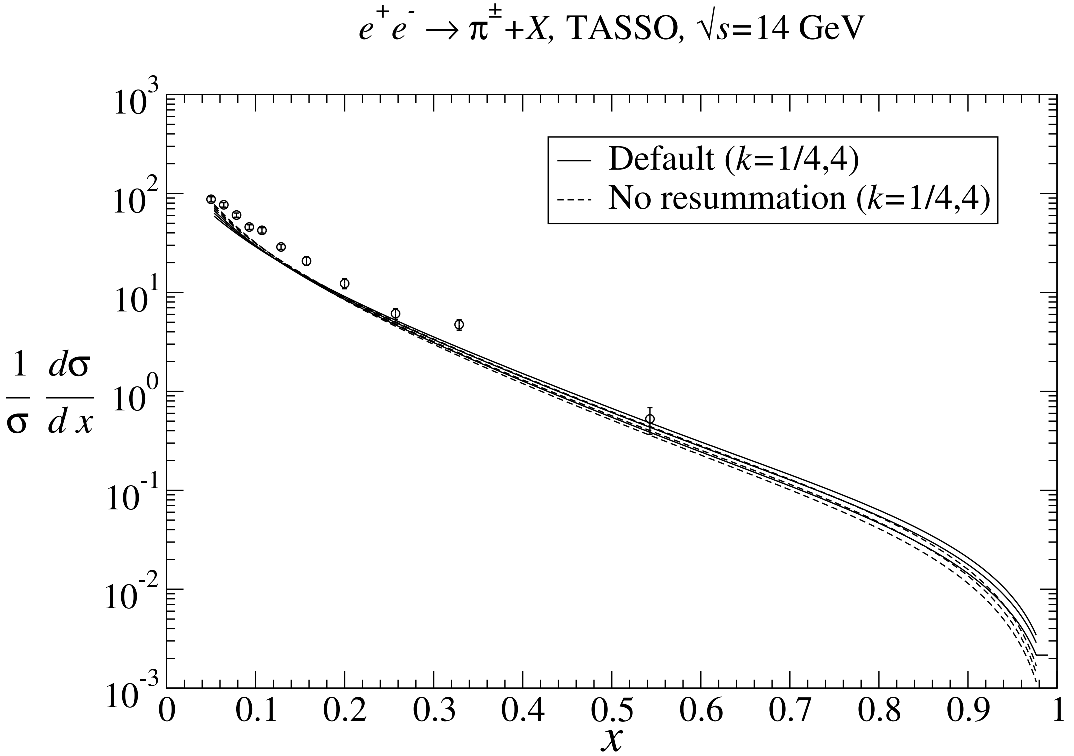
also resum DGLAP evolution in e^+e^- and $pp(\overline{p})$

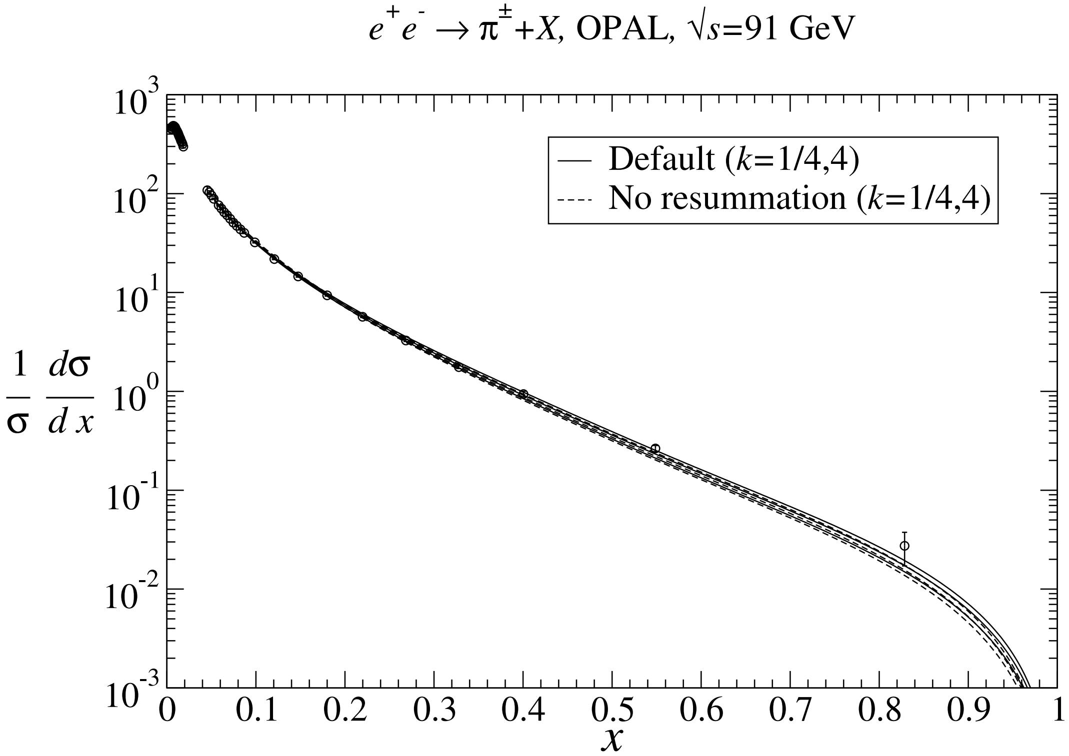
$pp(\overline{p})$, finite δ(rapidity): no resummation (de Florian+Vogelsang,PRD71:114004,2005)

effects:

- enhances XS at large x (more so at small $\sqrt{s}$)
- reduces theoretical error(?)

Large x Resummation — e^+e^-





H	χ^2	
	Main fit	Unres. fit
$\pi^\pm$	519.7	520.8
$K^\pm$	417.3	488.2
$p/\bar{p}$	525.1	538.0
K_S^0	318.6	318.8
$\Lambda/\overline{\Lambda}$	272.3	326.0

Partonic contributions — pp

$$F^h(pp \to h + X) = \sum_i \widehat{F}^i(pp \to i + X) \otimes D_i^h(i \to h + X)$$

($\widehat{F}$ contains PDFs f_j, f_k, and $jk \to i + X$ subprocesses)

3 "types" of fragmenting parton i: $u - \bar{u}$, $d - \bar{d}$ (valence) and sea (g, $2\bar{u}$, $2\bar{d}$, $2s = 2\bar{s}$, ...)

$\longrightarrow$ partonic decomposition of F^h:

$$F^{h^\pm} = \underbrace{\widehat{F}^{u_v}}_{\widehat{F}^{u_v} = \widehat{F}^u - \widehat{F}^{\bar{u}} > 0} D_u^{h^\pm} + \underbrace{\widehat{F}^{d_v}}_{\widehat{F}^{d_v} = \widehat{F}^d - \widehat{F}^{\bar{d}} > 0} D_d^{h^\pm} + \sum_{i = g, q_{\rm sea}} \widehat{F}^i D_i^{h^\pm}$$

$$\widehat{F}^{u_v} > \widehat{F}^{d_v}:$$

- $D_u^{\pi^+} = D_d^{\pi^-}$ ($\gg D_u^{\pi^-} = D_d^{\pi^+}$) $\to$ $(pp \to u_v \to \pi^\pm) > (pp \to d_v \to \pi^\pm)$ $\to$ $pp \to \pi^+ > pp \to \pi^-$
- $D_u^{p/\overline{p}} > D_d^{p/\overline{p}} \to (pp \to u_v \to p/\overline{p}) >> (pp \to d_v \to p/\overline{p})$, $D_{u,d}^p \gg D_{u,d}^{\overline{p}} \to pp \to p \gg pp \to \overline{p}$
- $D_{s,\bar{s}}^{K^\pm} > D_u^{K^\pm} \gg D_d^{K^\pm} \to (pp \to \text{sea} \to K^\pm) \gg (pp \to u_v \to K^\pm) \gg (pp \to d_v \to K^\pm)$

$D_{u,d}^{\pi^\pm, p/\overline{p}} \gg D_{\rm sea}^{\pi^\pm, p/\overline{p}}$ but $\widehat{F}^{u_v, d_v} \ll \widehat{F}^{\rm sea}$ — sea or valence?

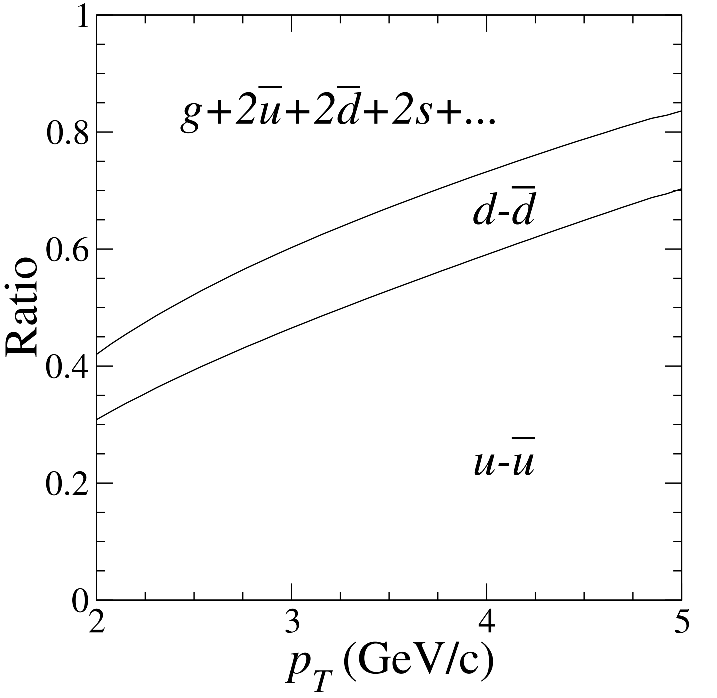
more physical, gives same qualitative results, bad for graphics:

$$F_{pp} = \underbrace{(F_{p\overline{p}} - F_{\overline{p}\overline{p}})\Big|_{u_v}}_{\propto f_{u_v}^2} + \underbrace{(F_{p\overline{p}} - F_{\overline{p}\overline{p}})\Big|_{d_v}}_{\propto f_{d_v}^2} + (F_{pp} + F_{\overline{p}\overline{p}} - F_{p\overline{p}})$$

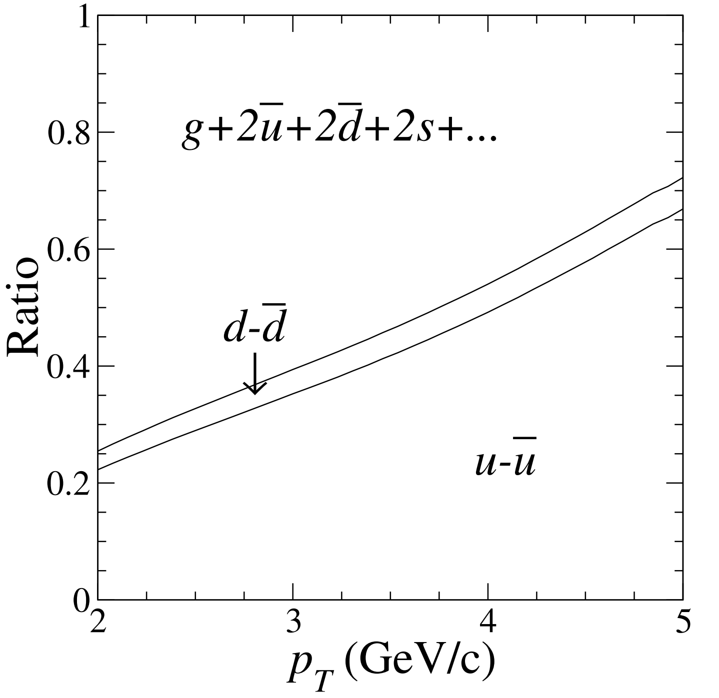
Partonic contributions — pp

charge-sign unidentified

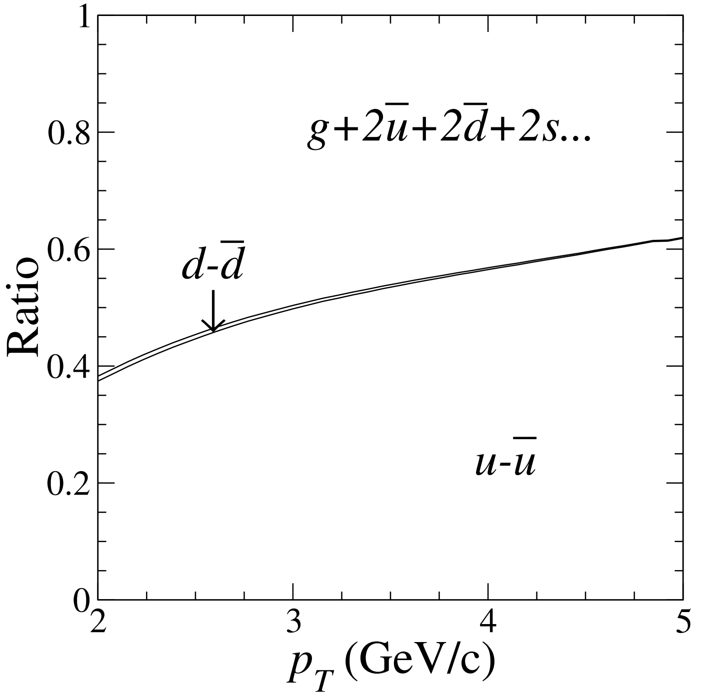
$pp \to \pi^{\pm} + X$ $(2.9<y<3)$, $M_f = p_T$, $\sqrt{s}=200$ GeV



$pp \to p/\bar{p} + X$ $(2.9<y<3)$, $M_f = p_T$, $\sqrt{s}=200$ GeV



$pp \to K^{\pm} + X$ $(2.9<y<3)$, $M_f = p_T$, $\sqrt{s}=200$ GeV



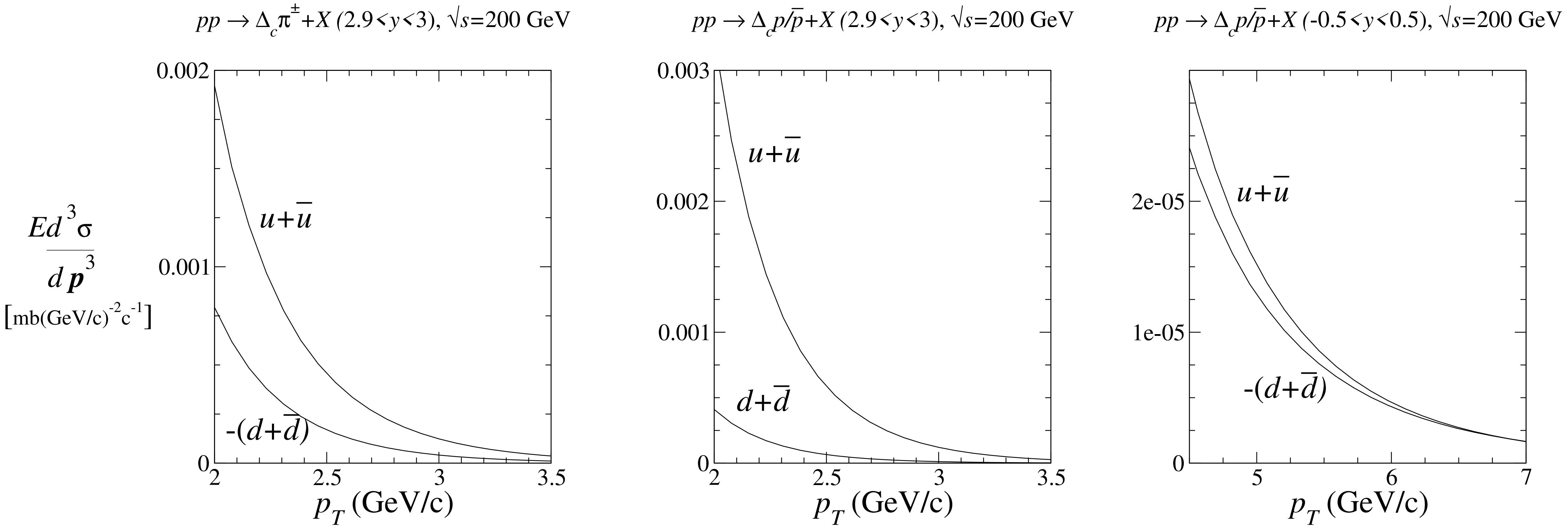
less valence at lower rapidity

(physically, pp sea dominates)

Partonic contributions — pp

charge-sign assymetry

$$F^{\Delta_c h^\pm} = \widehat{F}^{u_v} D_u^{\Delta_c h^\pm} + \widehat{F}^{d_v} D_d^{\Delta_c h^\pm}$$

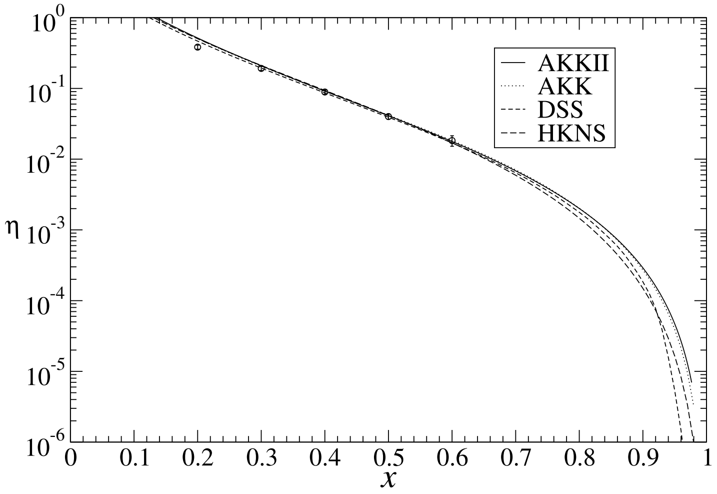


negative $d, \bar{d} \to \Delta_c p/\overline{p}$ unexpected

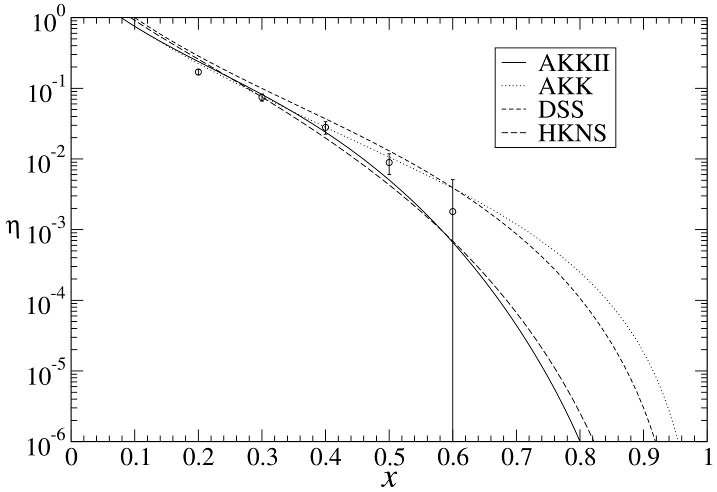
Partonic contributions — e^+e^-

OPAL tagging probabilities are obvious physical candidates

$e^+e^- \to u\bar{u} \to \pi^{\pm}+X$, OPAL, $\sqrt{s}$=91 GeV



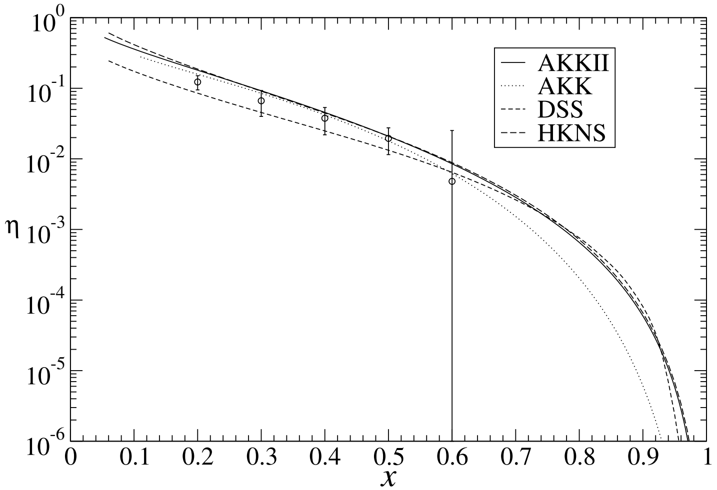
$e^+e^- \to s\bar{s} \to \pi^{\pm}+X$, OPAL, $\sqrt{s}$=91 GeV



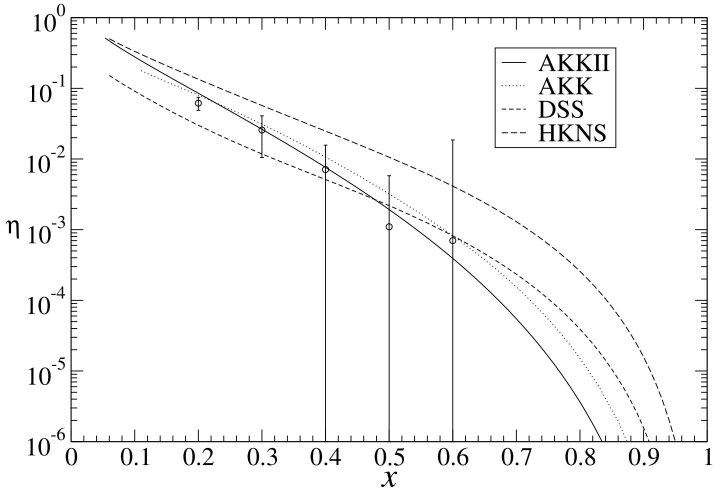
unfavoured quarks:

large x uncertain

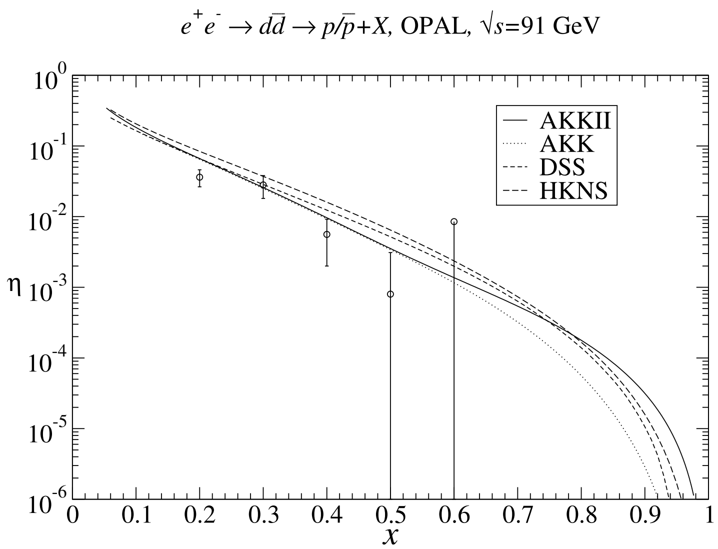
$e^+e^- \to u\bar{u} \to K^{\pm}+X$, OPAL, $\sqrt{s}$=91 GeV

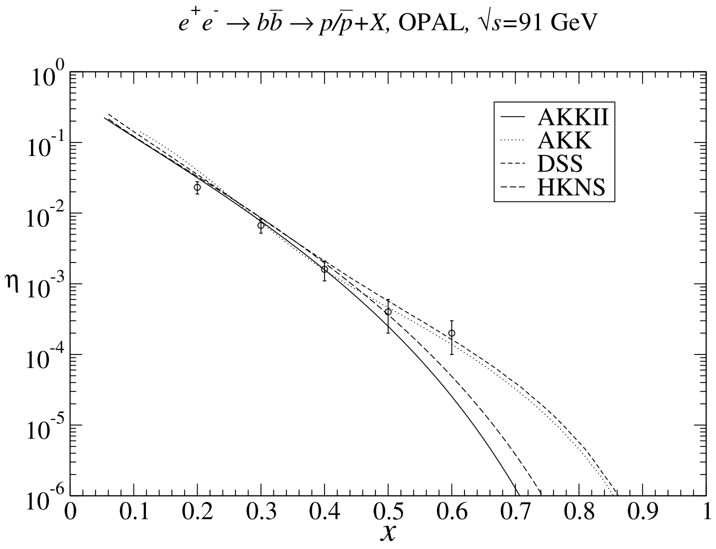


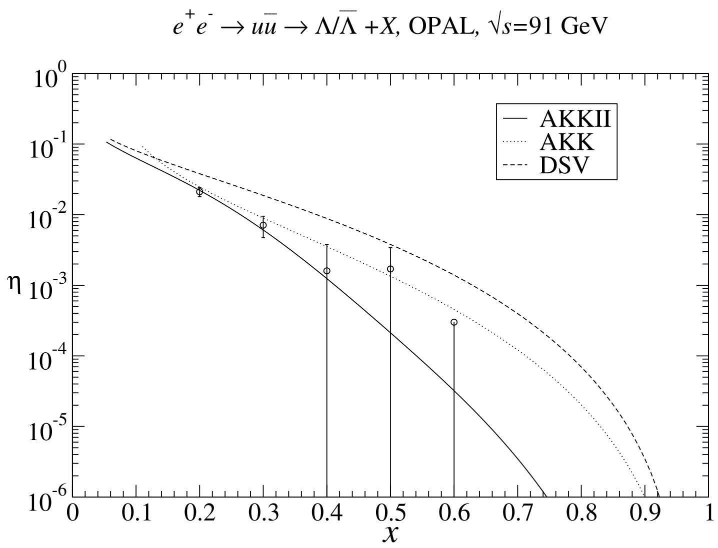
$e^+e^- \to d\bar{d} \to K^{\pm}+X$, OPAL, $\sqrt{s}$=91 GeV

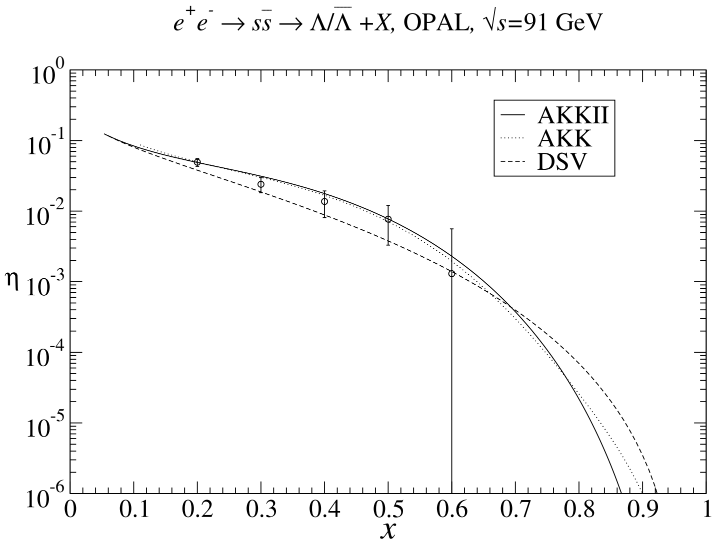


Partonic contributions — e^+e^-





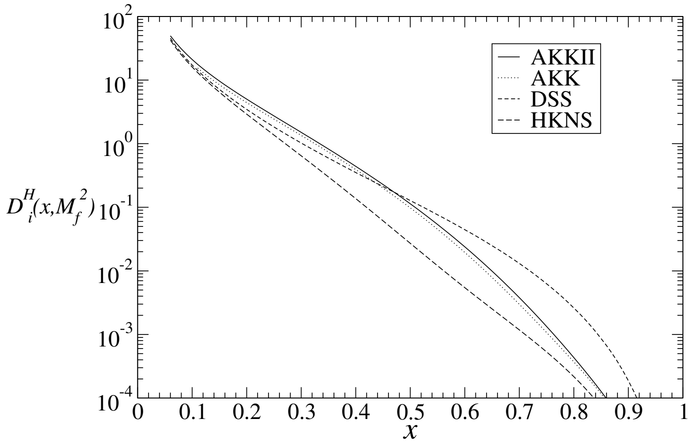




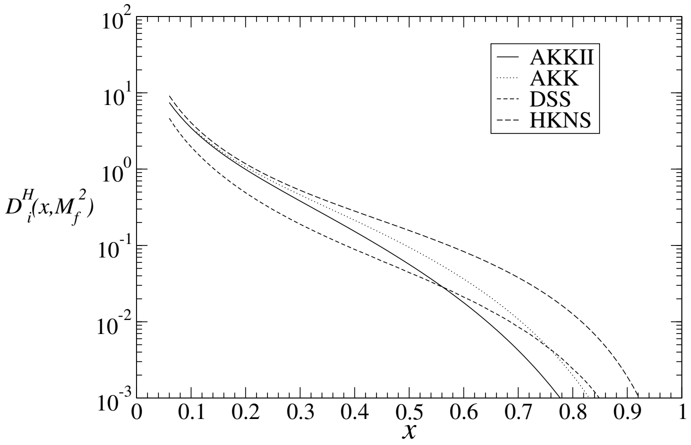
$\Lambda/\bar{\Lambda}$: s more favoured than u, d

Gluon FFs

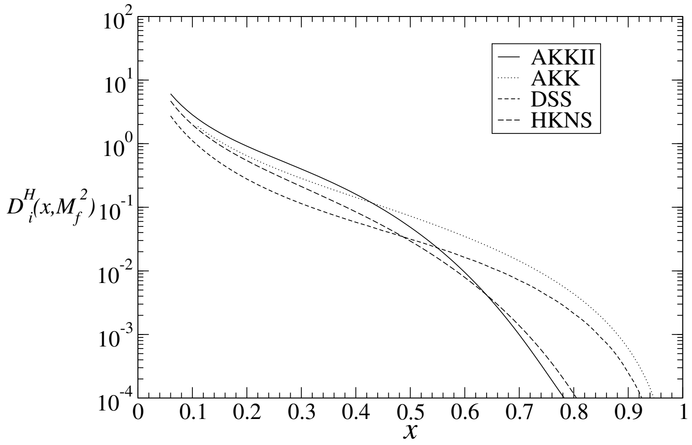
$H=\pi^{\pm}$, $i=g$, M_f=91.2 GeV



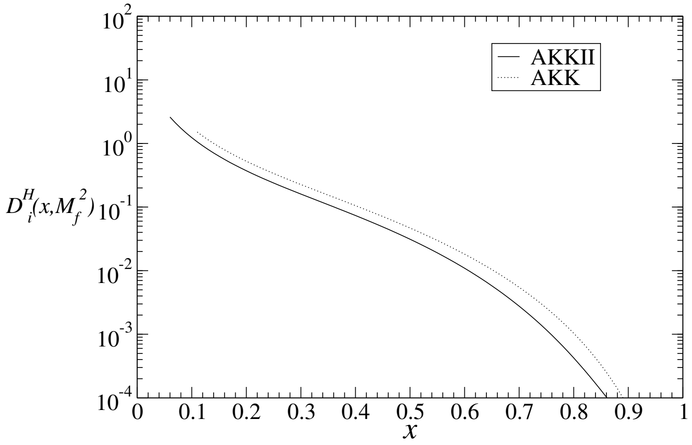
$H=K^{\pm}$, $i=g$, M_f=91.2 GeV



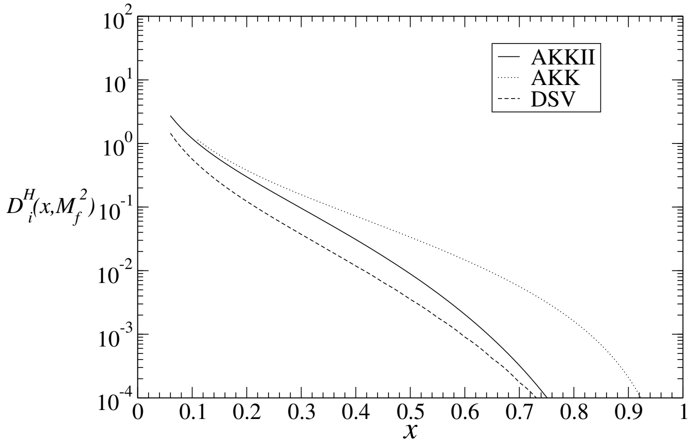
$H=p/\bar{p}$, $i=g$, M_f=91.2 GeV



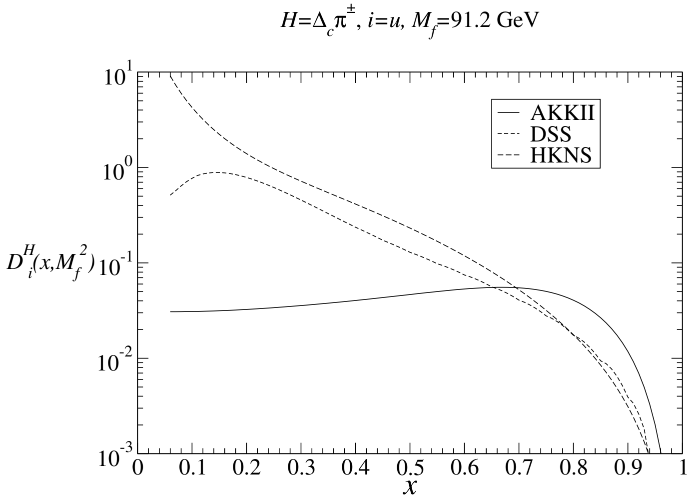
$H=K_S^0$, $i=g$, M_f=91.2 GeV

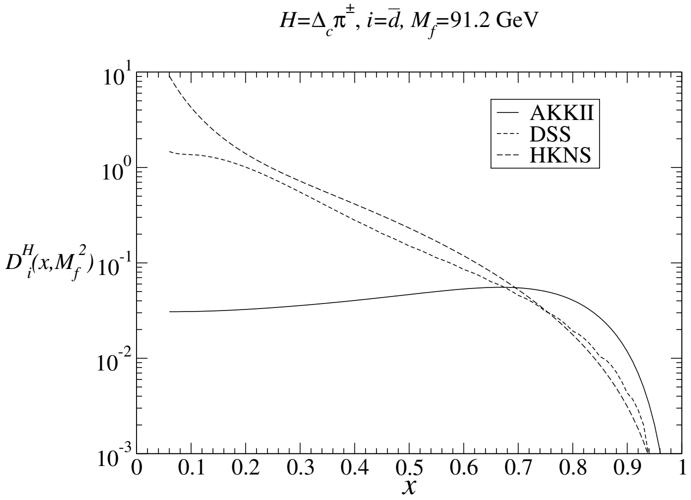


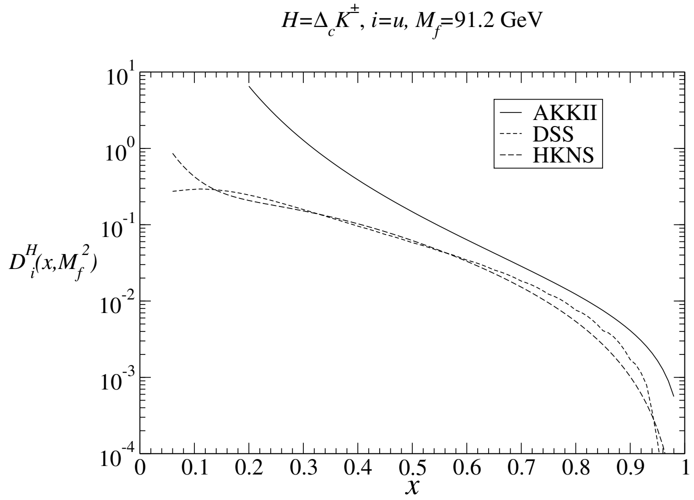
$H=\Lambda/\bar{\Lambda}$, $i=g$, M_f=91.2 GeV

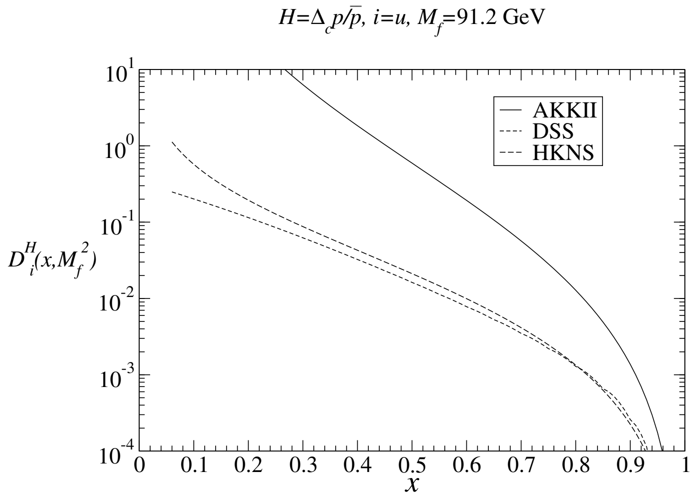


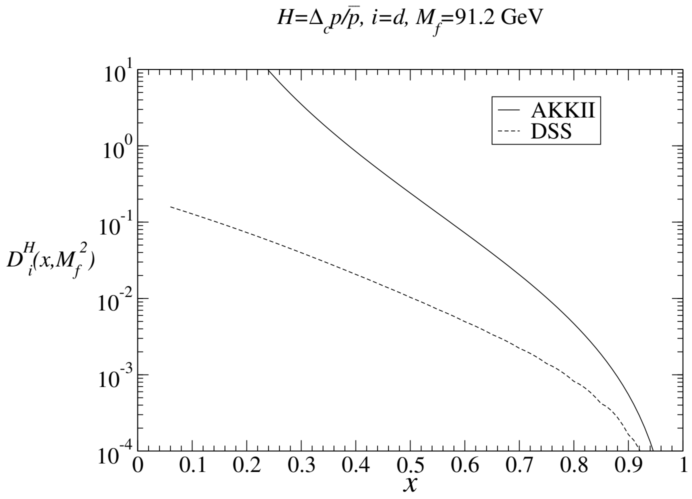
Charge-sign assymetry FFs











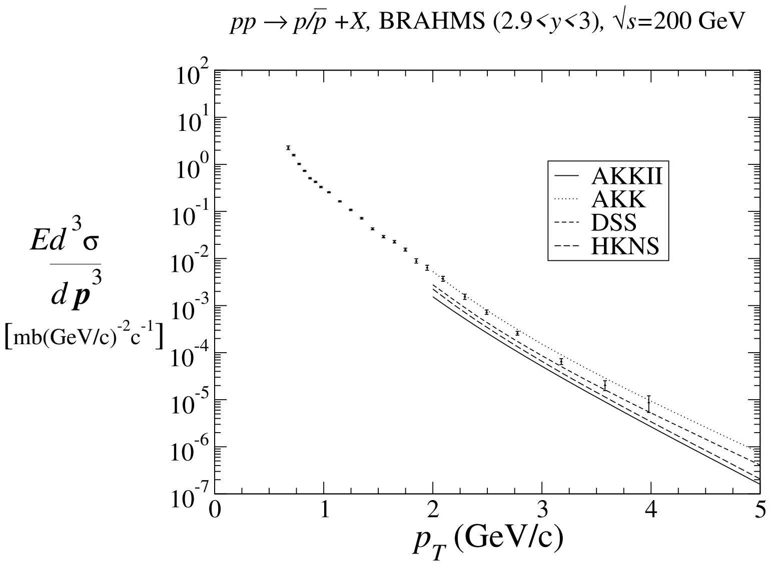
BRAHMS $\longrightarrow$ higher FF

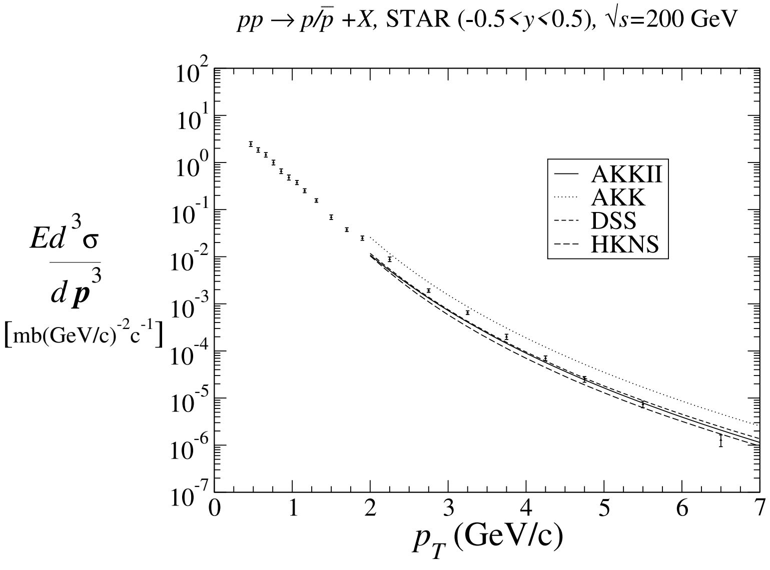
(HKNS: negative)

HKNS: no $\Delta_c h^{\pm}$ data

HKNS $\sim$ DSS $\longrightarrow$ $\sim$ non perturbative assumptions

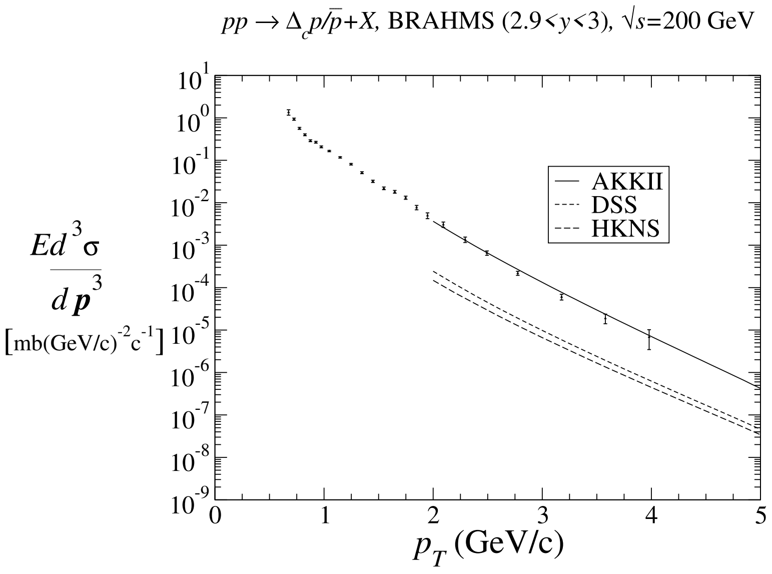
$pp \to$ baryon issues

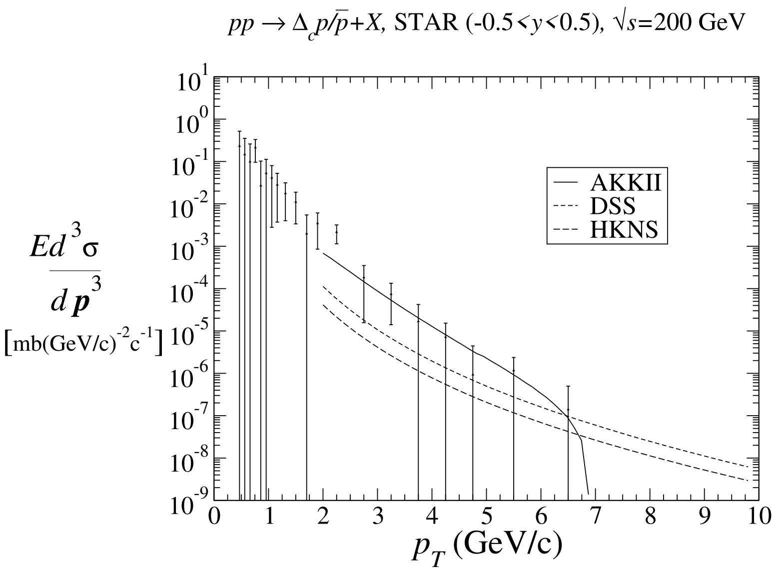




new fit:

slightly lower $p/\overline{p}$



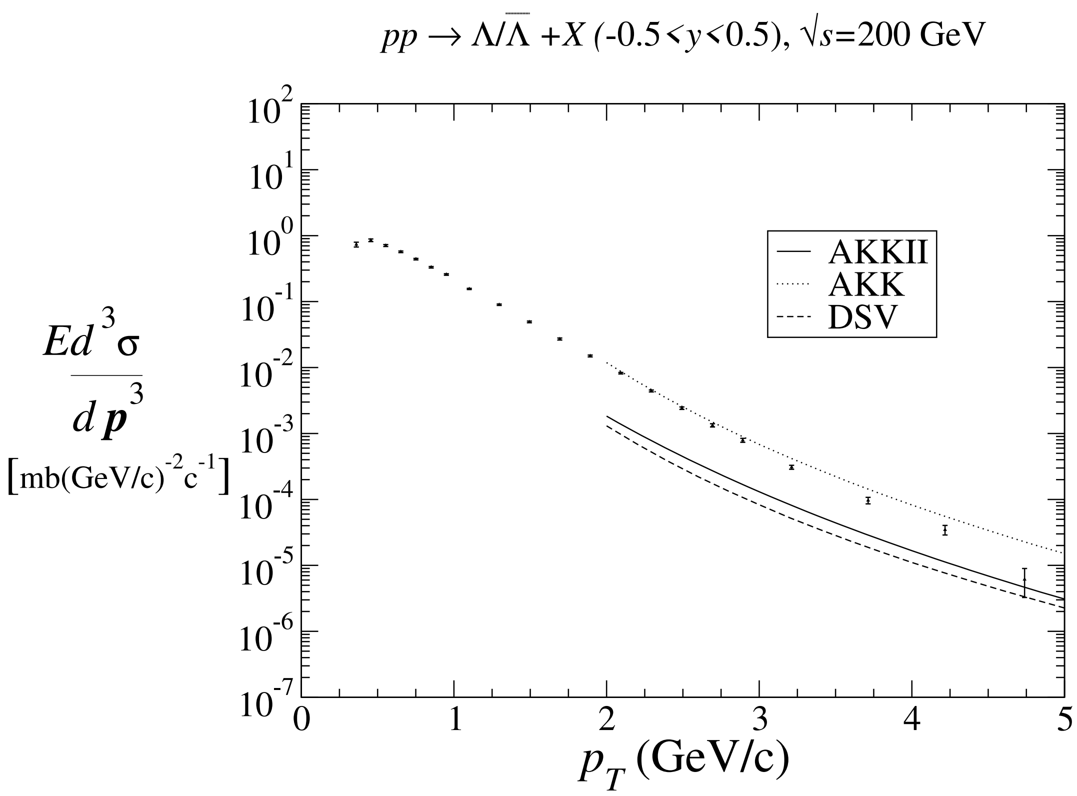


much higher

$\Delta_c p/\overline{p}$

$pp \to$ baryon issues

previous AKK gluon FF: $\Lambda/\overline{\Lambda}$ fixed to 1/3 $p/\overline{p}$
$\longrightarrow$ good agreement with STAR



full error analysis required to determine (in)consistency between e^+e^- and pp

Summary

- FFs for $\pi^{\pm}$, $K^{\pm}$, $p/\overline{p}$, K_S^0 and $\Lambda/\overline{\Lambda}$,
- FFs for $\Delta_c \pi^{\pm}$, $\Delta_c K^{\pm}$, $\Delta_c p/\overline{p}$, *separate* fits
- data from e^+e^-, $pp(\overline{p})$
- hadron mass effects, fitted in e^+e^-
- slight overshoot for pions: decays from heavier particles?
- mass deficiency for kaons: complicated decay channels?
- mass good for baryons,
  better stage for partonic fragmentation?
- investigate baryon problems further
  namely $\Lambda/\overline{\Lambda}$ undershoot, –ve valence $d \to \Delta_c p/\overline{p}$ @ STAR
- e^+e^- large x resummation generally improves fit
- norm. error as systematic errors in covariance matrix, λ_K fitted

Future work

- full error analysis for FFs
  $\longrightarrow$ (in)consistencies with other FFs + data
- additional theory input
- future high Q^2 ep from HERA — quark tagging?