Colour Coherence & Gluon and Quark Fragmentation

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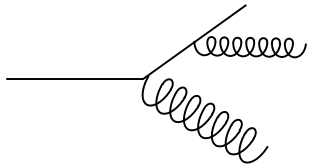
Outline

- Colour Coherence Warmup
- Interjet Coherence in 3 Jet Events (PLB605:37)
- Strategy & Limits of Gluon Measurements (cmp EPJC13:573)
- Multiplicity in 3 Jet Events and 2 Gluon Systems (EPJC44:311)
- Leading Protons in Gluon Jets (EPJC17:207)
- Summary

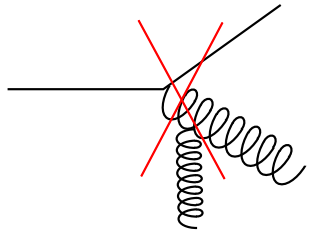
Colour Coherence Warmup

QCD wave nature reflects in coherent parton emissons.

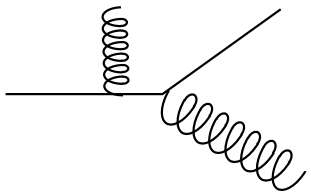


Observed in two classes of phenomena:

- Intrajet coherence:
  governs dependence of soft radiation on parton prehistory within jets (angular ordering)
- Interjet coherence:
  soft particle angular flow determined by geometry and colour topology of complete jet ensemble



"Coherent" predictions
$\longrightarrow$ determine evolution scales of the radiation.



Evolution scales directly connecting to Lorentz–invariance.

Momentum Spectra – Small x

Colour coherence limits gluon emission
at small x / large ξ

$$\xi = -\ln x = -\ln \frac{2E_h}{E_{cm}}$$

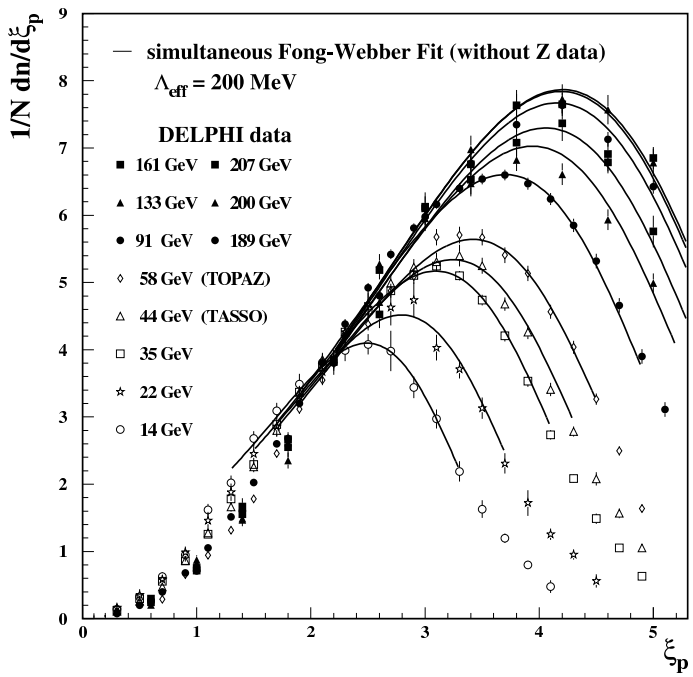
Predicted by MLLA "limiting spectrum"

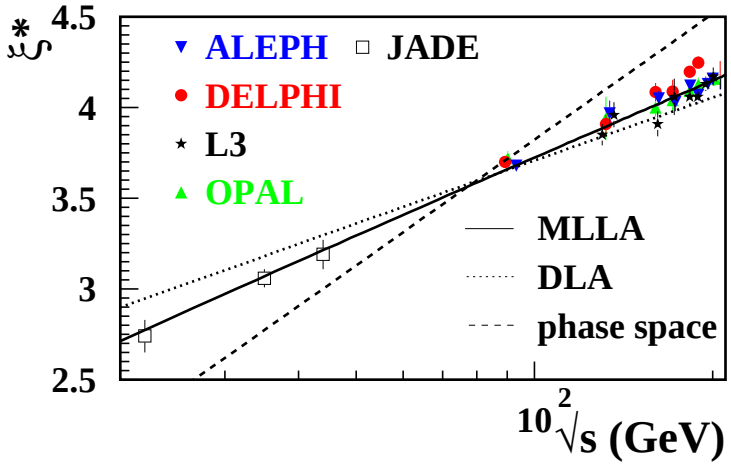
Coherence $\longrightarrow$ change of peak position ξ^*

$$\xi^*(E) \sim \sqrt{\log E}$$

slower than expected from phase space

$$\xi^*(E) \sim \log E$$





Momentum Spectra – Small p

Alternative view:

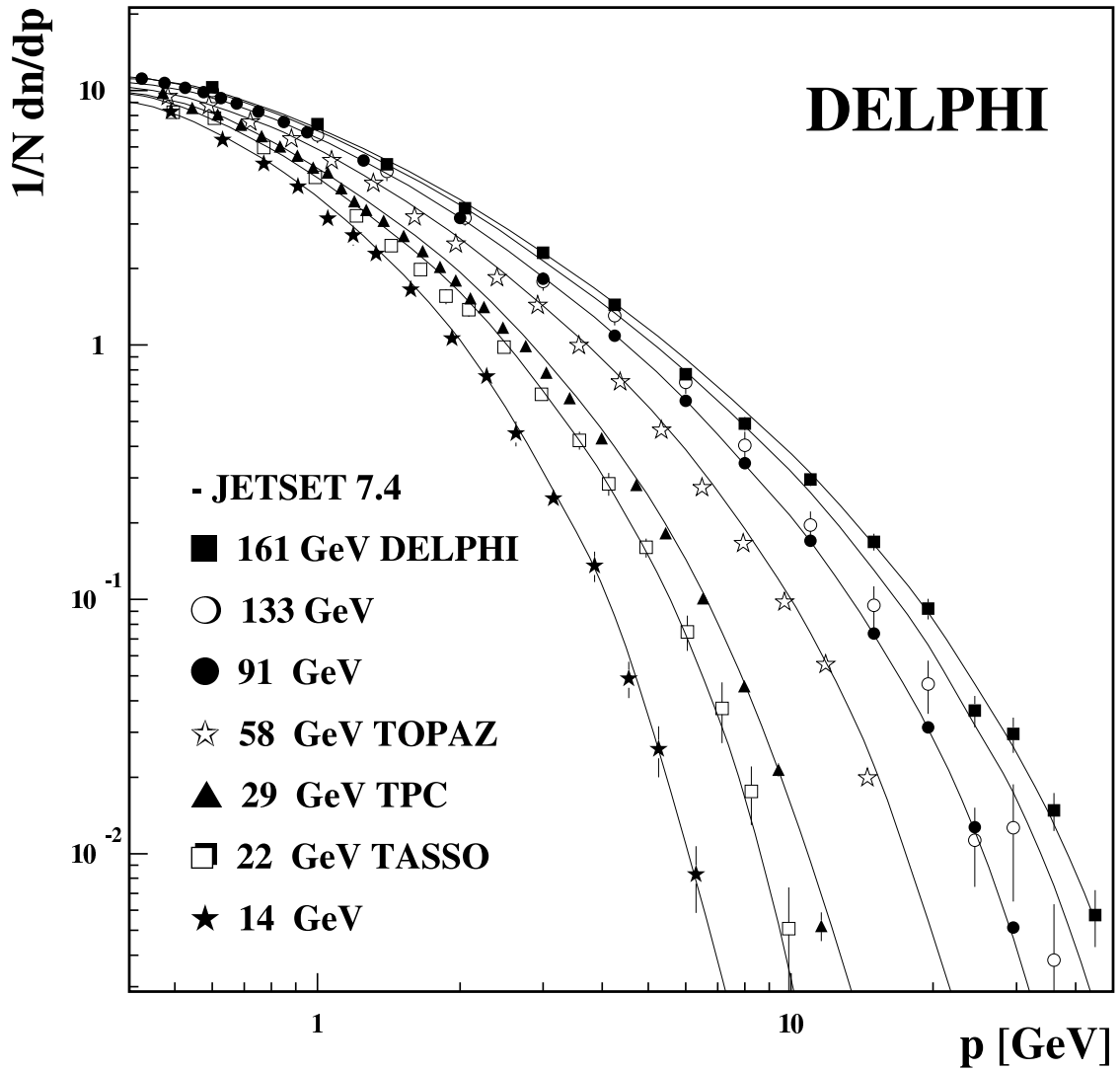
small p,
high wavelength hadrons

only resolve

colour charge of initial $q\bar{q}$ pair.

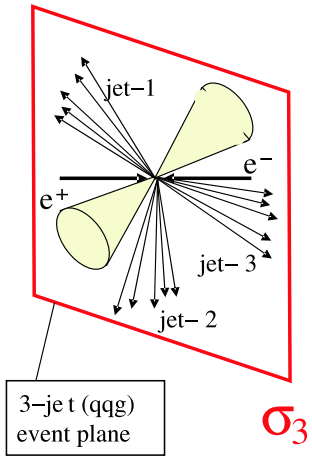
At small p:

particle production is least
affected by finite energy
or leading particle effects!



Coherent Particle Production at Large Angles

- Soft gluons at large angles:
  Large wave-length $\longrightarrow$ small resolution $\longrightarrow$ coherent emission
  Effective colour-charge depends on event topology



- Compare gluon radiation $\perp$ to $q\bar{q}g$ plane $\quad\leftrightarrow\quad$ $\perp$ to $q\bar{q}$ axis.

$$\frac{N_\perp^{q\bar{q}g}}{N_\perp^{q\bar{q}}} = \frac{C_A}{C_F}\cdot r_t = \underbrace{\frac{C_A}{C_F}}\cdot\frac{1}{4}\left[\widehat{q\,g}+\widehat{\bar{q}\,g}-\underbrace{\frac{1}{N_C^2}\widehat{q\,\bar{q}}}\right]$$

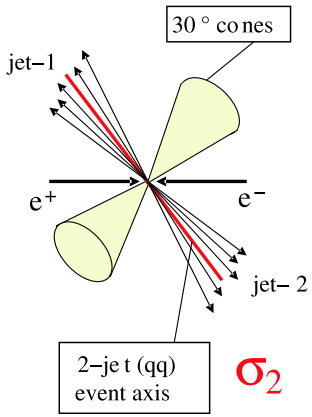
$$\widehat{i\,j} = 2\sin^2\frac{\vartheta_{ij}}{2}$$

Khoze, Ochs, Lupia

Ratio directly proportional to C_A/C_F in LO. No corrections !
Destructive gluon-interference term $\propto 1/N_C^2$

- Experimentally identify partons with k_t-jets (at fixed $y_{\rm cut}$):
  defines 2 and 3-jet events, excludes $\geq$ 4-jet events $\leftrightarrow$ LO

  Compare multiplicity $\perp$ to 3-jet plane to the one $\perp$ 2-jet axis

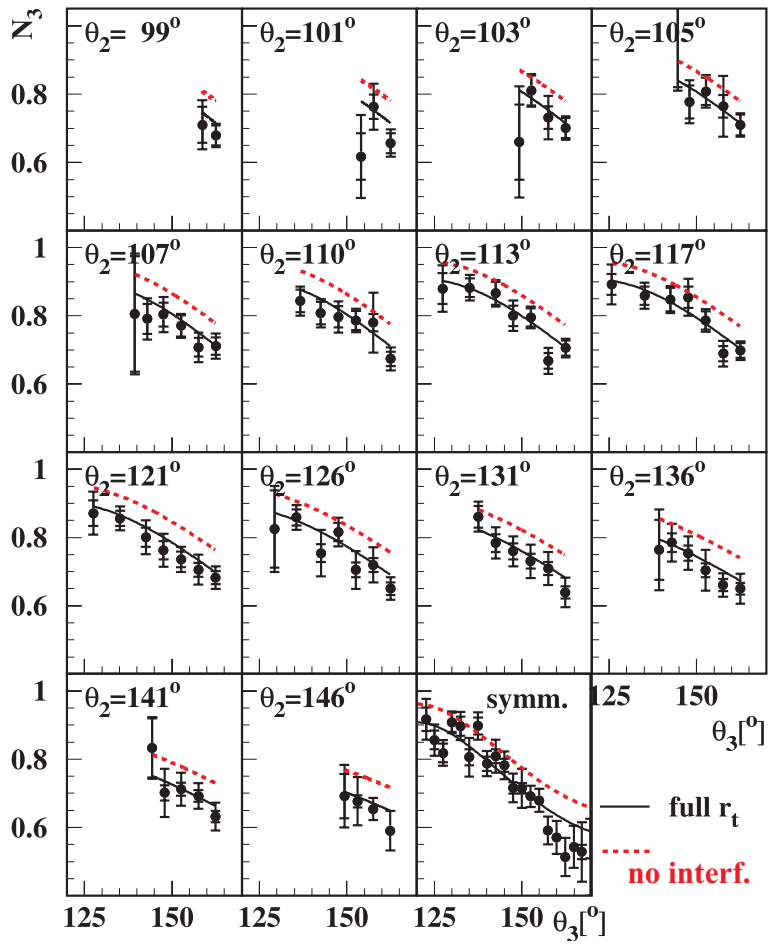


The Destructive Interference Term $\propto 1/N_C^2$

- Compare to absolute prediction,
  solid complete r_t,
  dashed line: without $1/N_C^2$-term

- Data only described with interference
  Fit amplitude k of $1/N_C^2$-term:
  general 3 jet topologies:
  $$k = 1.39 \pm 0.05_{(\mathrm{stat.})} \pm 0.28_{(\mathrm{sys.})}$$
  symmetric 3 jet topologies:
  $$k = 1.30 \pm 0.06_{(\mathrm{stat.})} \pm 0.33_{(\mathrm{sys.})}$$
  Syst. error from variation of
  y_{cut}, θ_{cone} and cluster algorithms

- Compatible with $k = 1$
  $k = 0$ excluded with $> 95\% CL$.



Clean observation of colour coherence!

The Connection to C_A/C_F

- Expect homogenous linear relation

$$\frac{N_3}{N_2} = r_t \cdot \frac{C_A}{C_F} = r_t \cdot \text{slope}$$

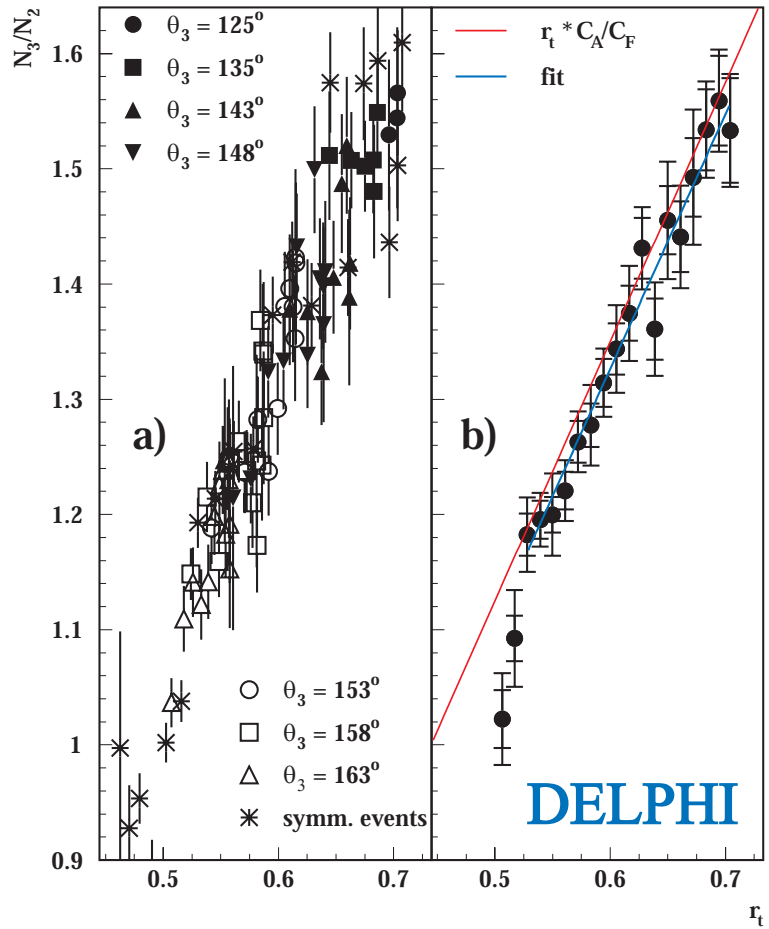
- Data incl. stat. and syst. errors.
- Measured slope from fit:

general 3 jet topologies:

$$2.182 \pm 0.009_{(\text{stat.})} \pm 0.055_{(\text{sys.})}$$

symm. 3 jet topologies:

$$2.205 \pm 0.006_{(\text{stat.})} \pm 0.073_{(\text{sys.})}$$



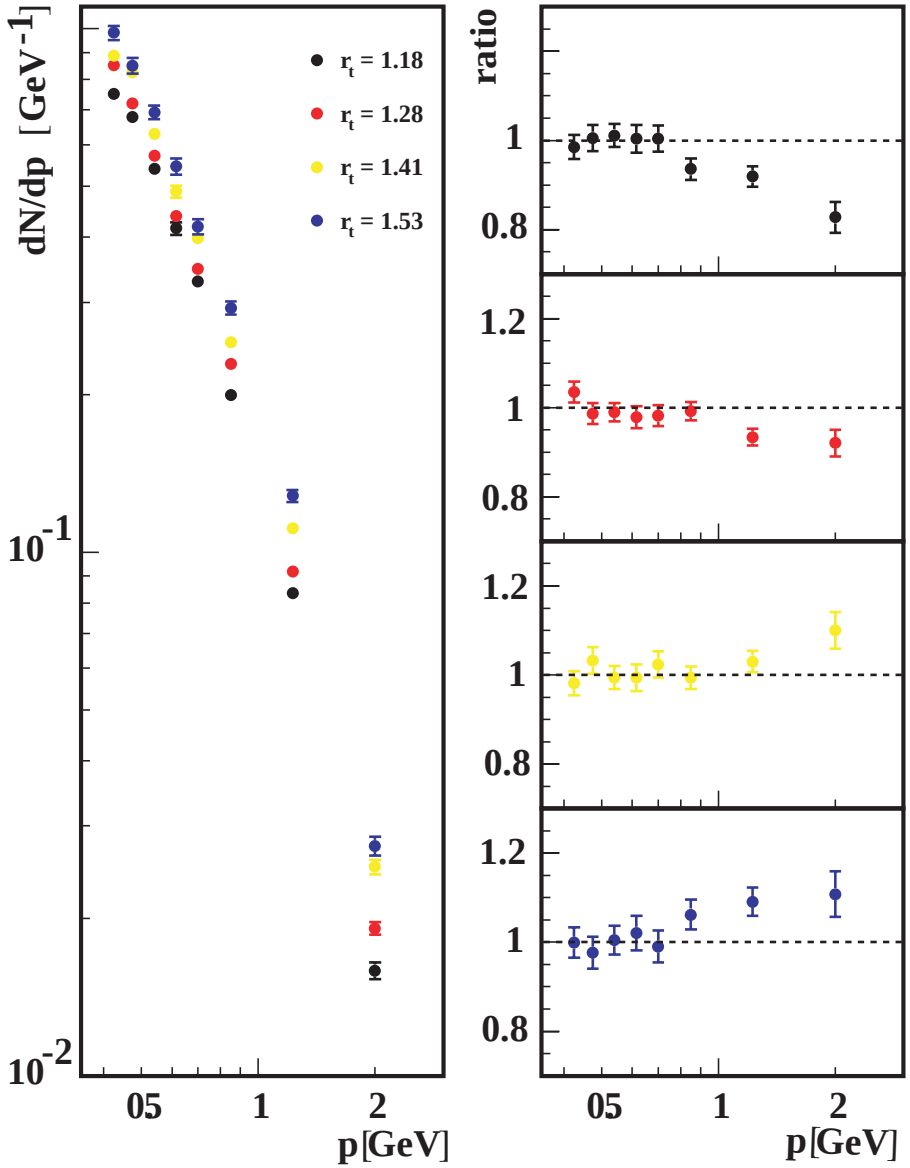
Amazingly good agreement with expectation slope $= C_A/C_F = 2.25$

Momentum-Distribution as Function of r_t

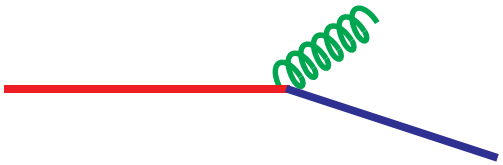
- Use r_t as single scale
- Momentum distributions scale for $p \lesssim 1$GeV

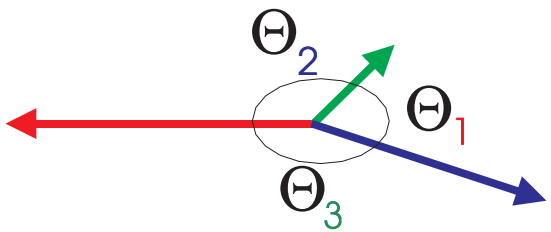
  Right: renormalised to average/theo. expectation
- Pert. expectation fulfilled for very low energy hadrons $\rightarrow$

  Local Parton Hadron Duality



Comparing Gluons and Quarks in e^+e^- 3 Jet Events





- assign partons $\leftrightarrow$ jets at tree level
- identify quarks using E-ordering and displaced vertices (heavy q's)
- unfold light-, b-quark, g-contribution by inverting purity matrix
- parton kinematics from event topology involves hadronisation $\longrightarrow$ resolution
- dynamical studies require evolution scales for jets $\longrightarrow$ transverse momentum like scales:

$$\kappa = Q = E_{jet} \cdot \sin\frac{\theta_{ij}}{2} \quad (\widehat{=}\frac{\sqrt{s}}{2})$$

- scales from QM calculations obey Lorentz invariance
- assignment of particles to jets requires jet algorithms $\longrightarrow$ ambiguities

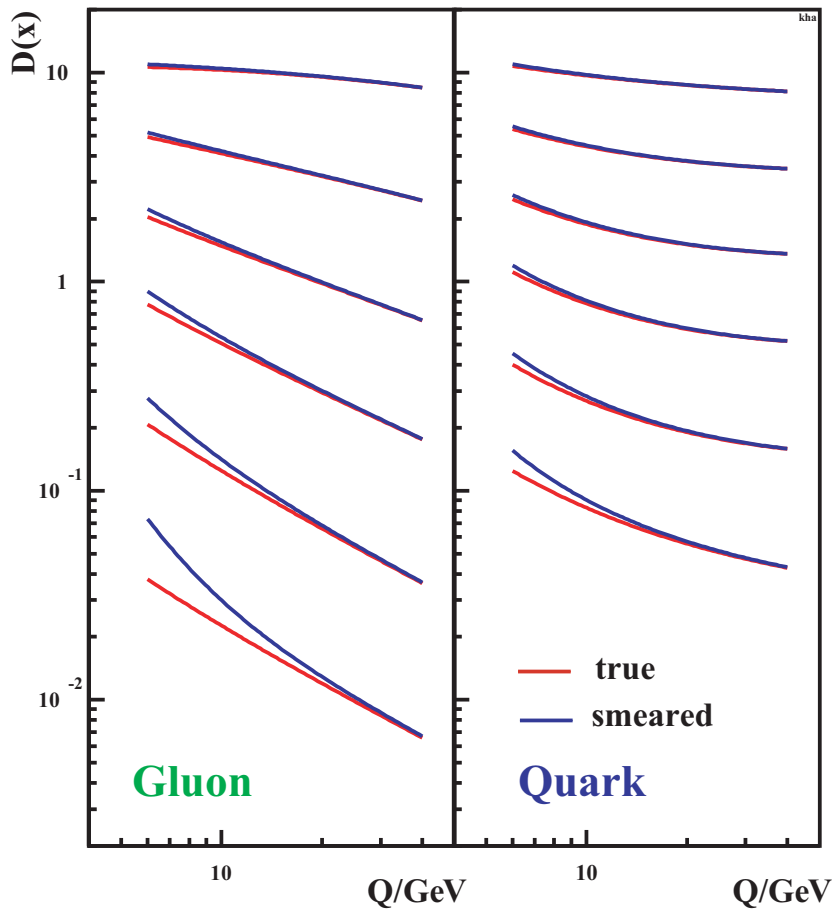
Problems can be minimised by

- analysing only 3-jet event multiplicity
- use fully symmetric situation (may require boosting)
- analysing fast hadrons only (high x fragmentation)

Influence of Hadronisation on Gluon Measurements

Jets measured from hadrons, parton resolution limited by hadronisation!

- Assume $E_{jet} = E_{parton}$
- Hadronisation smearing
  $\longrightarrow$ jet-parton $\sphericalangle$-resolution $\sim 3^\circ$ at Z
  $\longrightarrow$ parton energy smearing $\propto\sim 1/E_{parton}$
- Effect on FF's $\propto$ FF fall-off (error propagation)
  $\Rightarrow$ strong overestimate of gluon FF at small E and high x



This principle problem is there for any jet analysis!

- Less of a problem for multiplicities (small "slow" variations)

Only proper way out for FF's: include hadronisation smearing in FF fits!

Gluon and Quark Fragm. Funct. in 3 Jet Events

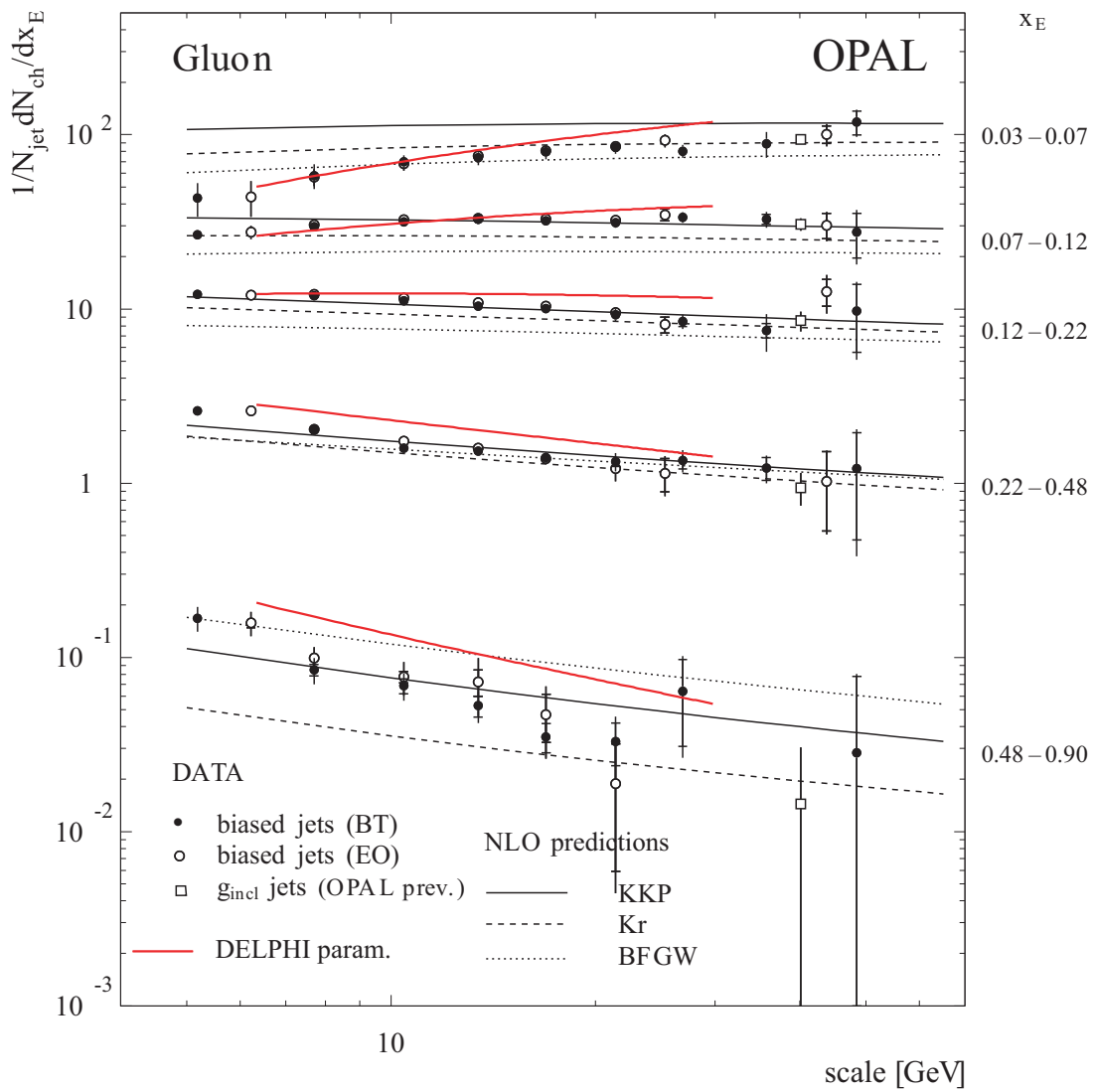
For illustration compare

- OPAL's "inclusive" and "boosted" analysis
- DELPHI old 3 jet analysis (parameterised –, different x bins)

to theoretical parameterisations (KKP –, Kr - -, BFGW . . .)

Energy slope at high x stronger than expected from parameterisation (for both OPAL and DELPHI)

Discrepancy partly due to smearing effect described.



Topology Dependence of 3 Jet Event Multiplicity

Measure overall multiplicity of 3 jet events
Prediction accounts for coherence effects by choice of scales

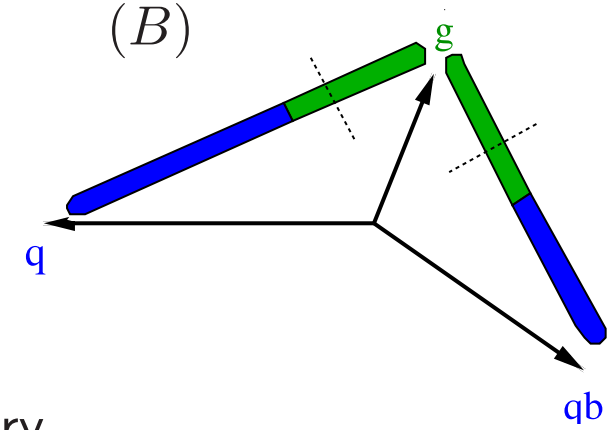
Eden
Gustafson
Khoze

$$N_{q\bar{q}g}(L_{q\bar{q}}, \kappa_{Lu}, \kappa_{Le}) = N_{q\bar{q}}(L_{q\bar{q}}, \kappa_{Lu}) + \frac{1}{2} N_{gg}(\kappa_{Le}) \qquad (A)$$

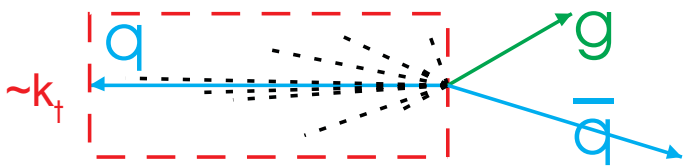
$$N_{q\bar{q}g}(L_{q\bar{q}}, \kappa_{Lu}, \kappa_{Lu}) = N_{q\bar{q}}(L, \kappa_{Lu}) + \frac{1}{2} N_{gg}(\kappa_{Lu}) \qquad (B)$$

$$\text{with} \quad L = \ln\left(\frac{s}{\Lambda^2}\right), \qquad L_{q\bar{q}} = \ln\left(\frac{s_{q\bar{q}}}{\Lambda^2}\right),$$

$$\kappa_{Lu} = \ln\left(\frac{s_{qg} s_{\bar{q}g}}{s\Lambda^2}\right), \qquad \kappa_{Le} = \ln\left(\frac{s_{qg} s_{\bar{q}g}}{s_{q\bar{q}}\Lambda^2}\right)$$



Division of multiplicity in $q\bar{q}$ and gluon part is arbitrary
$\longrightarrow$ differing definitions of gluon multiplicity $\leftrightarrow$ differing scales.



The phase space of the $q\bar{q}$-pair is restricted by the gluon jet $\longrightarrow$ requires correction

Topology Dependence of 3 Jet Event Multiplicity

- In the Dipole Model energy slopes of gg and $q\bar{q}$ systems are related by:

$$\left.\frac{dN_{gg}(L')}{dL'}\right|_{L'=L+c_g-c_q} = \frac{C_A}{C_F}\left(1-\frac{\alpha_0 c_r}{L}\right)\frac{d}{dL}N_{q\bar{q}}(L)$$

- $N_{q\bar{q}}(E_{\rm cm})$ measured by various e^+e^--experiments
- Solution leaves constant of integration free

$\longrightarrow$ To be determined from a single measurement of N_{gg} for the prediction.

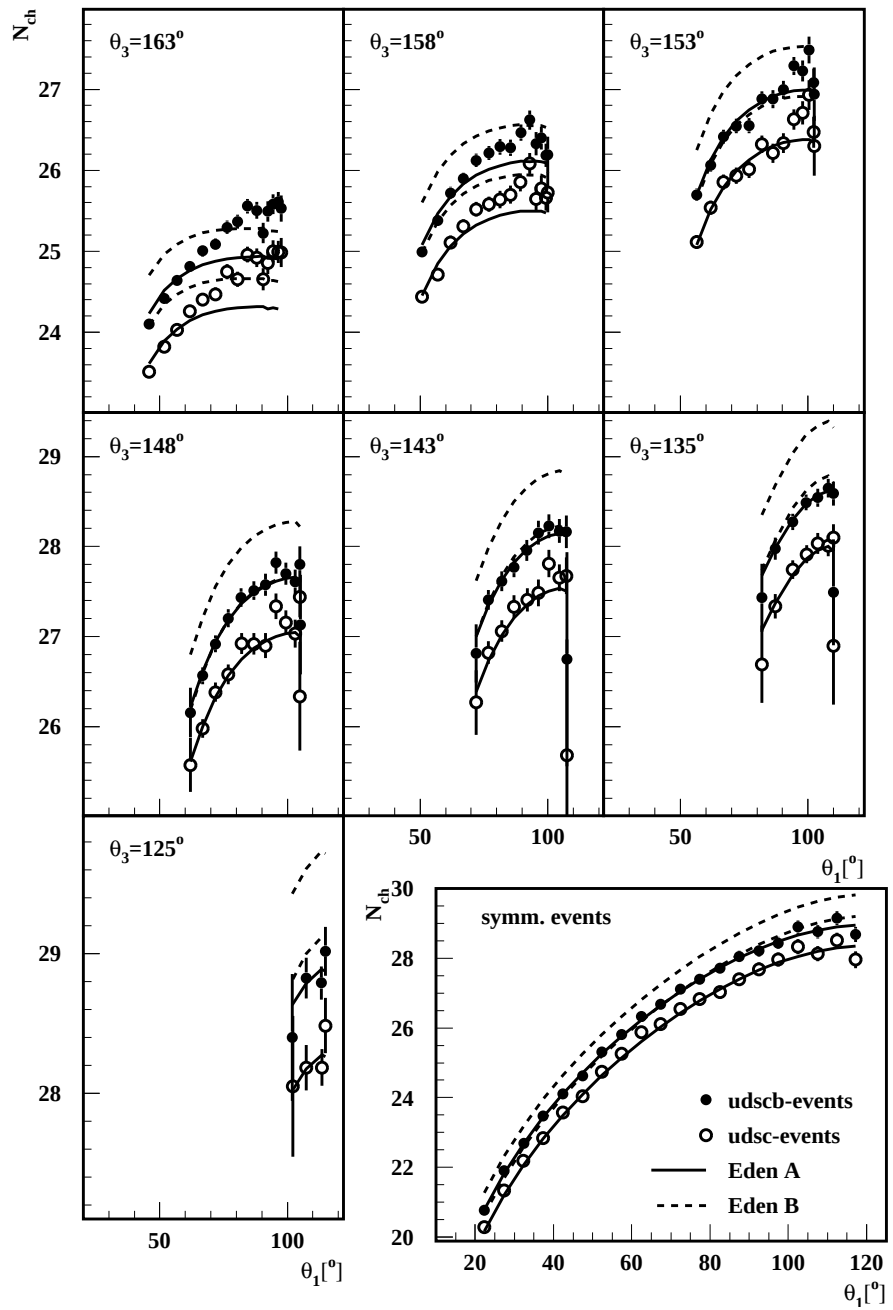
$\longrightarrow$ Take CLEO-data from $\chi_b'(J=2) \to gg$ decay at $E_{\rm cm} = 9.9132$GeV

Analysis:

- Select 3 jet events without cut on y_{cut} (AoD, Cambridge, Durham, PHYJET)
- Apply (tiny) hadronisation correction
- Compare general and symmetric topologies
- Compare $udscb$ and $udsc$ events $\longrightarrow$ constant offset $N_0 \sim 0.6$ due to b-events
- Compare solutions Eden (A) and (B)
- Leave N_0 free $\longrightarrow$ only slope determines measurement of C_A/C_F !!!

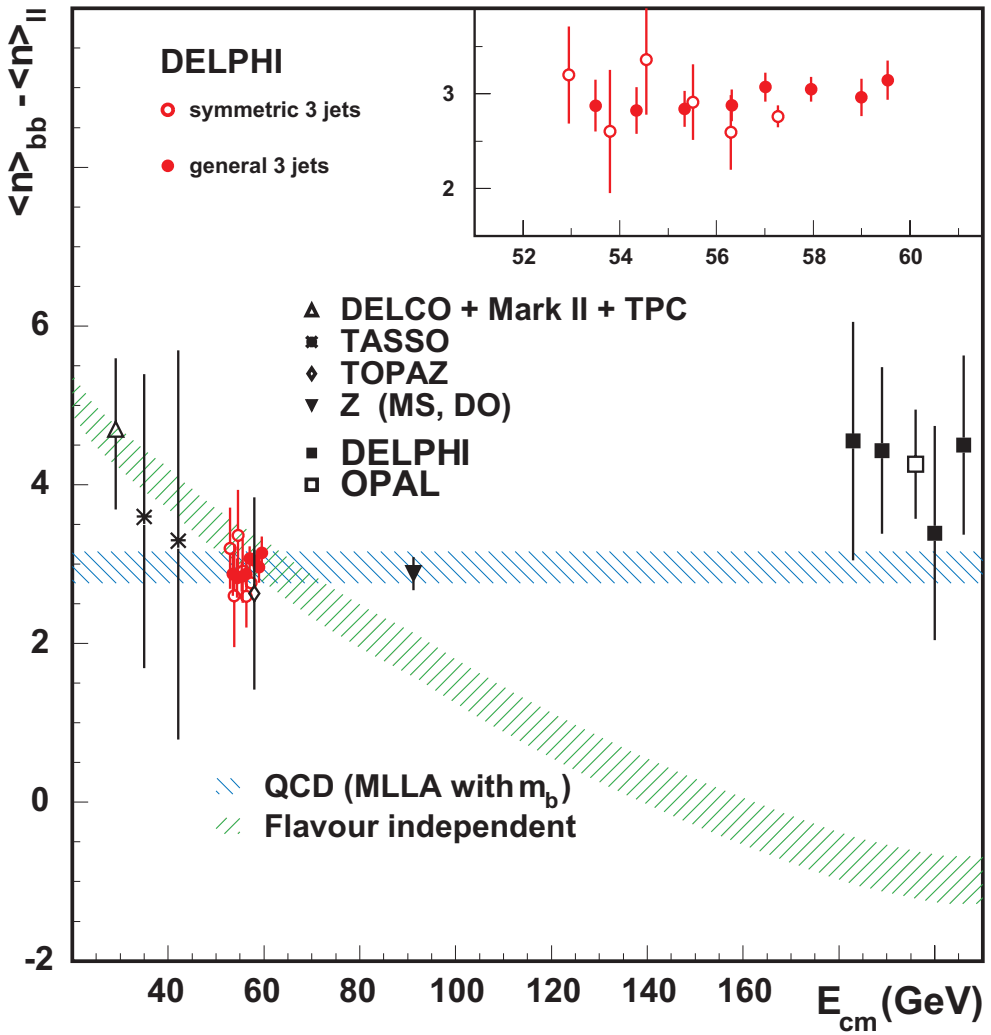
The 3 Jet Multiplicity

- Compare $udsc$ (○) and $udscb$ (●) data
- Eden A ——
  - Very good agreement for symmetric and general topologies
- Eden B - - -
  - Multiplicity overestimated by ∼0.6
  - Slope too high
  - χ^2 inacceptable in global fit
- Solution Eden B is incompatible with DELPHI data!
- Further DELPHI analysis bases only on Eden A
- OPAL uses Eden B (sym. events only)



Is N_0 Constant? Connecting to $\delta_{bl} = N_{b\bar{b}} - N_{l\bar{l}}$

- No topology dependence of the multiplicity difference δ_{bl} of $udscb$ and $uds(c)$ events is observed.
- Consistent with:
  – previous measurements of δ_{bl}
  – QCD expectation.
- Probably a new precise measurement of δ_{bl} at reduced C.M.S. energy.



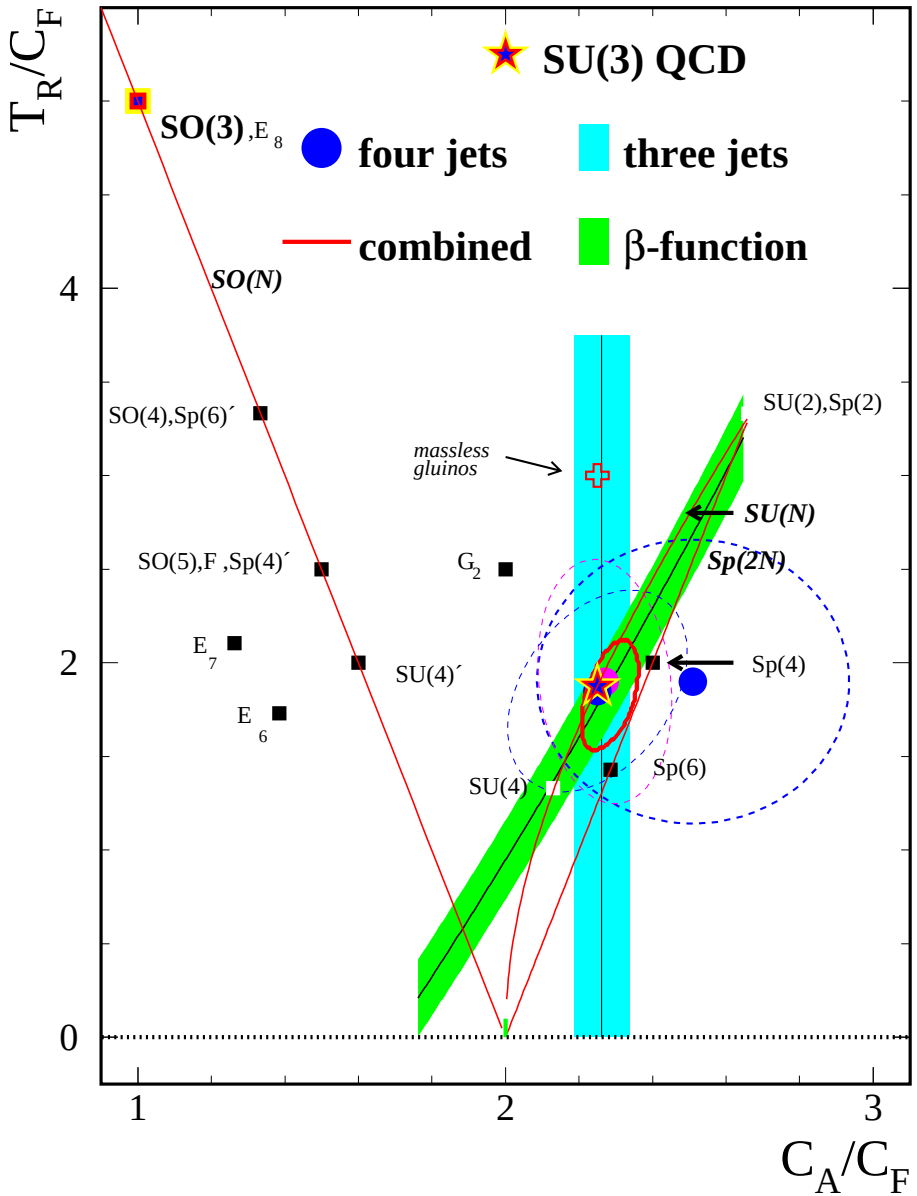
Result for C_A/C_F

- We want C_A/C_F from slope w.r.t. angles: $\rightarrow N_0$ varied freely!
- Experimentally advantageous:
  $\rightarrow$ insensitive to normalisation
  $\rightarrow$ no b-tagging systematics

$$\frac{C_A}{C_F} = 2.261 \quad \pm 0.014_{\text{stat.}} \pm 0.036_{\text{exp.}} \pm 0.052_{\text{theo.}} \pm 0.041_{\text{clus.}}$$

- Precise ($\sim 3\%$) measurement.
- Together with β-function and 4 jet $\sphericalangle$-distributions fixes QCD group structure to SU(3).

- Most important measurements based on energy variation!



Energy dependence of the Gluon Multiplicity N_{gg}

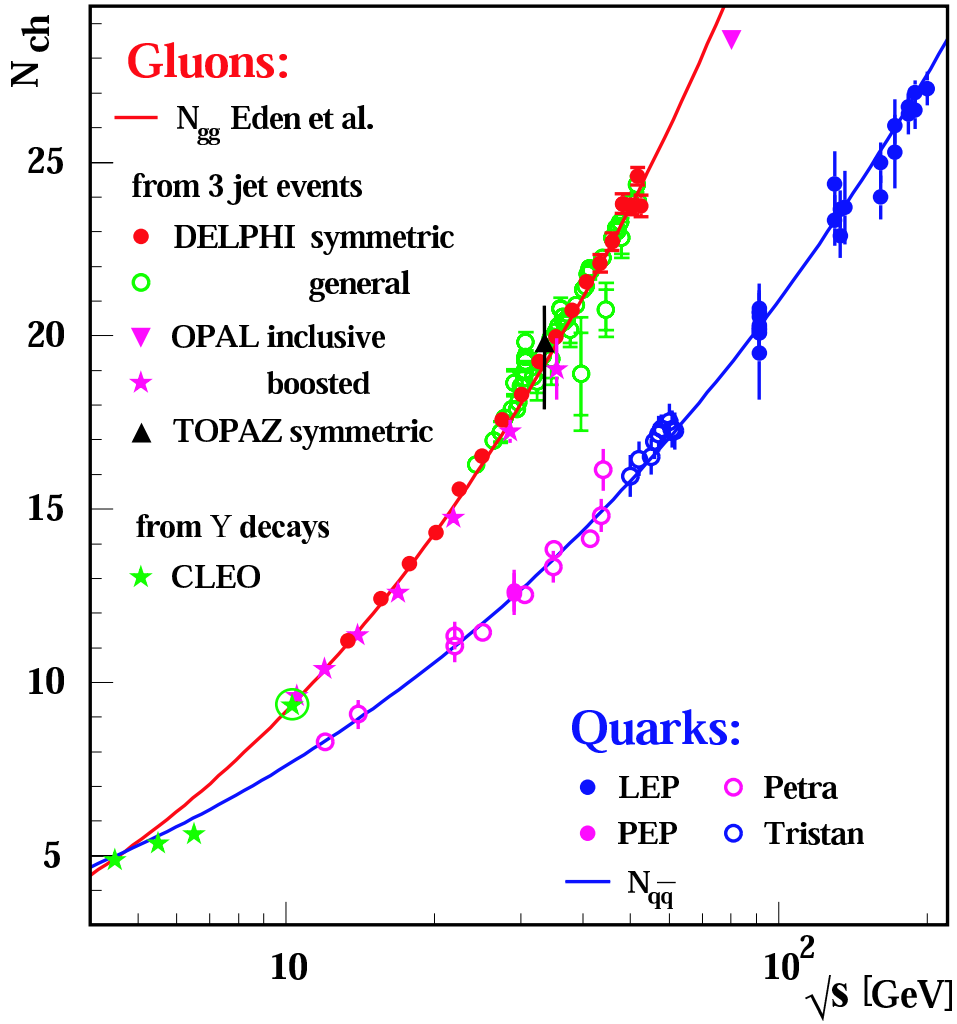
- Determine Gluon contribution:

  $N_{gg}(\kappa_{Le}) = 2 \cdot$
  $(N_{q\bar{q}g}(\theta_1) - N_{q\bar{q}}(L_{q\bar{q}}, \kappa_{Lu}) - N_0)$

- Agreement between:
  - general & symmetric topologies
  - experiments
    (except similar OPAL anal.)
  - data and Eden prediction

- Gluon measurements start to compete with quark data

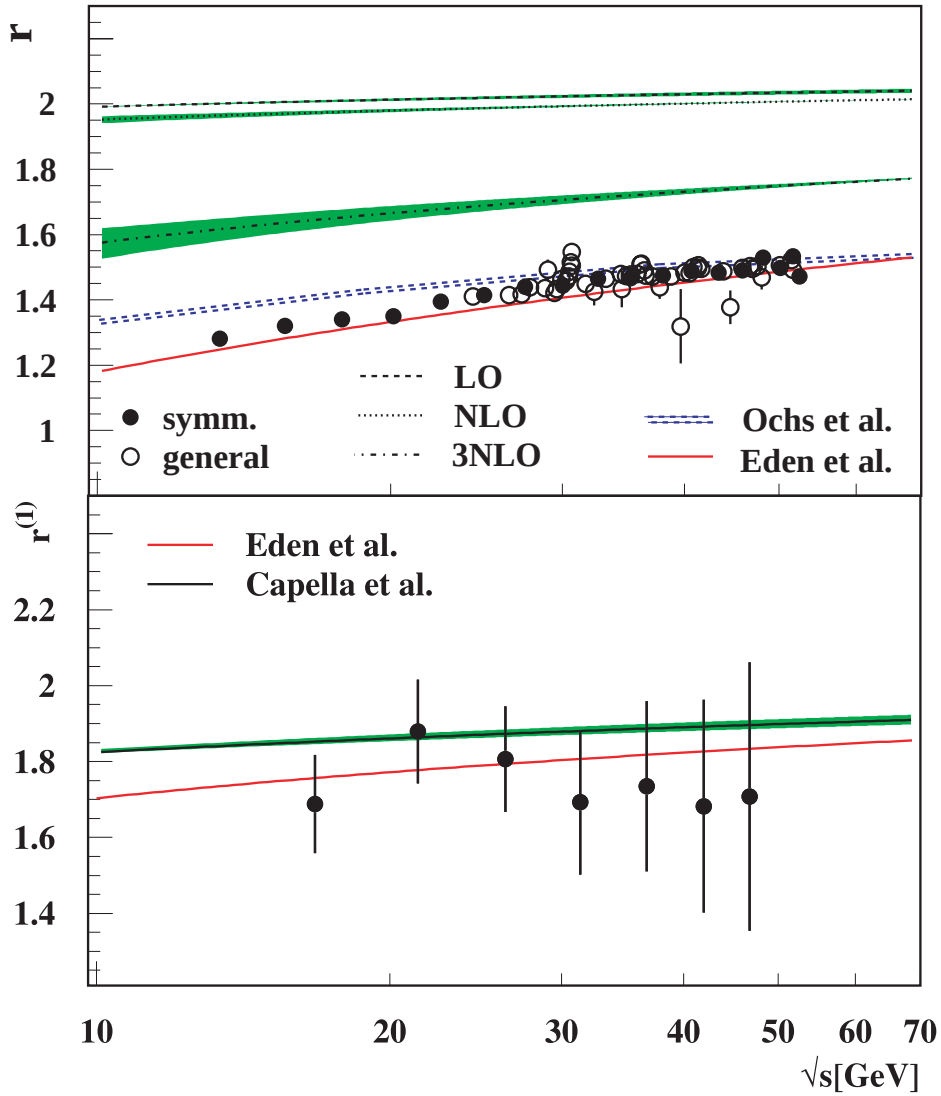
- N_{gg}: E-slope $\sim$ twice that of $N_{q\bar{q}}$
  $\rightarrow$ illustrates colour factor ratio



Ratio of the Multiplicities r and the Slopes $r^{(1)}$

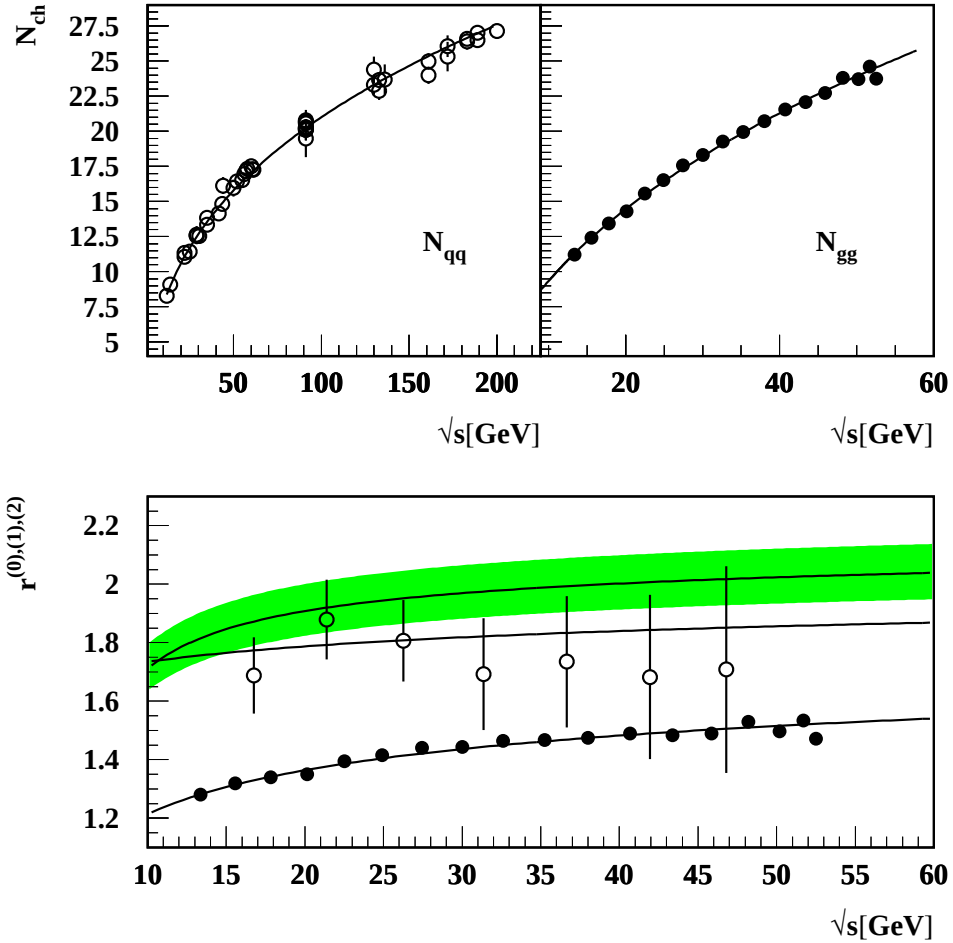
- r:
  - Large differences between LO, NLO, 3NLO, numeric predictions
  - Data only described by Eden (using experimental input)
  - Non-pert. effects are important!

- $r^{(1)}$:
  - Direct measurement of the linear slopes (N_0 free)
  - Difference to C_A/C_F smaller
  - 3NLO & Eden pred. similar in agreement with data
  - Non-pert. effects less important
  - Precision of data does not allow to measure 2'nd derivative $r^{(2)}$



Comparing Ratios r, $r^{(1)}$, $\ldots r^{(2)}$?

- Fit of 3NLO-prediction of $N_{q\bar{q}}$ and N_{gg} to data
- Good description of data with $\Lambda_q \neq \Lambda_g$
- r, $r^{(1)}$ and $r^{(2)}$ can be calculated
- But:
  Determination of $r^{(2)}$
  based on parametrisation used
  $\longrightarrow$ strong correlation with the multiplicity measurement



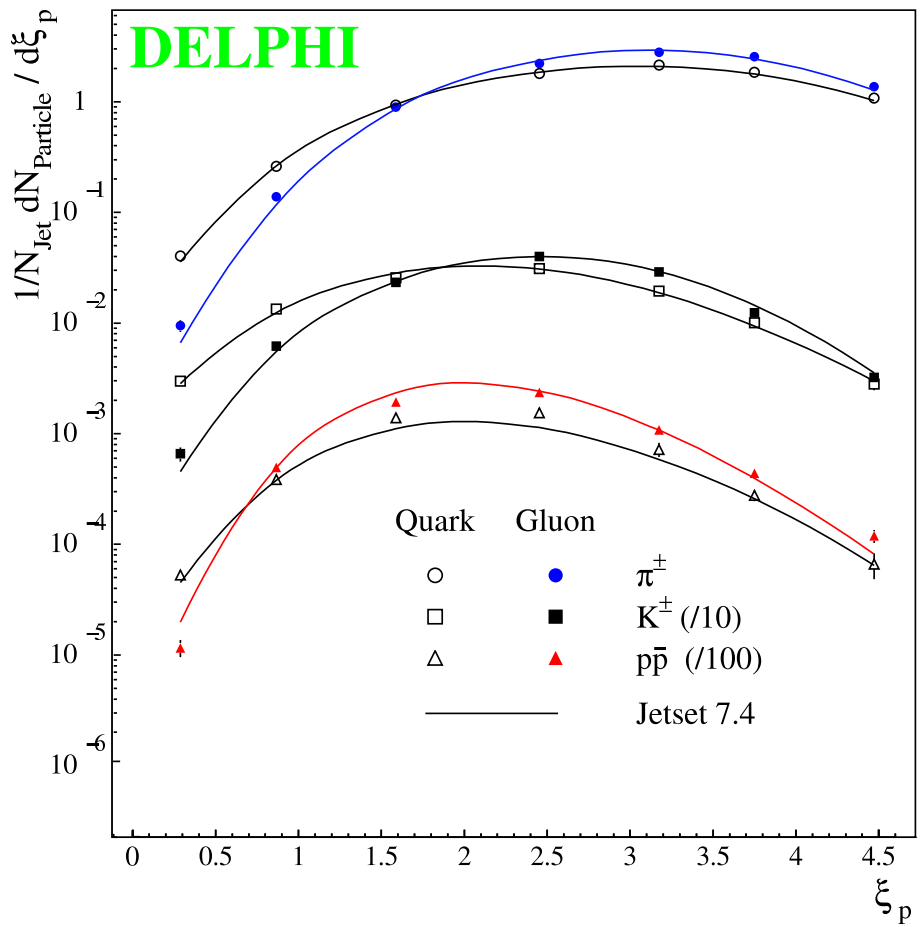
Gluon to Quark Ratio for Identified Particles

Find charged particle production
in gluon and quark jets
in amazing agreement with
QCD expectation C_A/C_F.

$\longrightarrow$ Only tiny differences
for identified particles possible!

But high baryon yield at
$\Upsilon(4S) \longrightarrow ggg$ (ARGUS/CLEO).

More protons also at LEP ($\sim$10–20%)



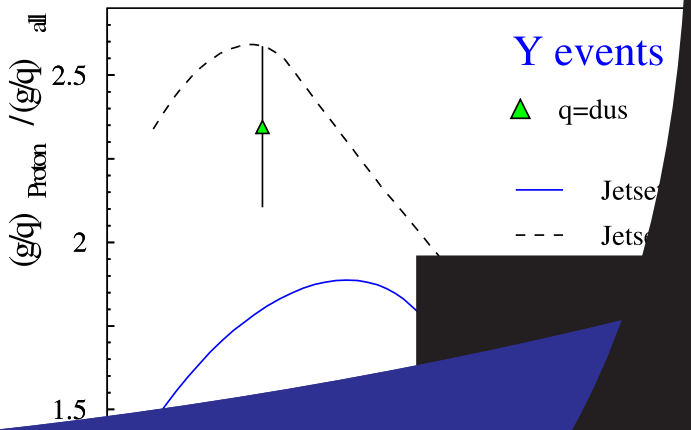
Main differences only for fast (leading) particles

rk Ratio for Identified Protons

$rotons/(g/q)_{all} \Rightarrow$

small ξ, high x only.

le



Summary

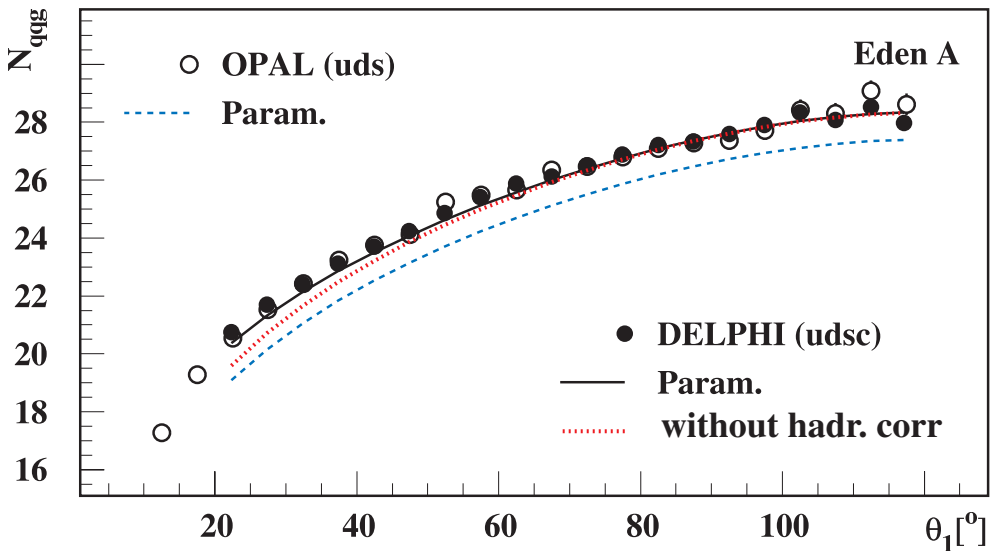
- Hadronisation causes unavoidable resolution problems when measuring fragmentation in multijet events

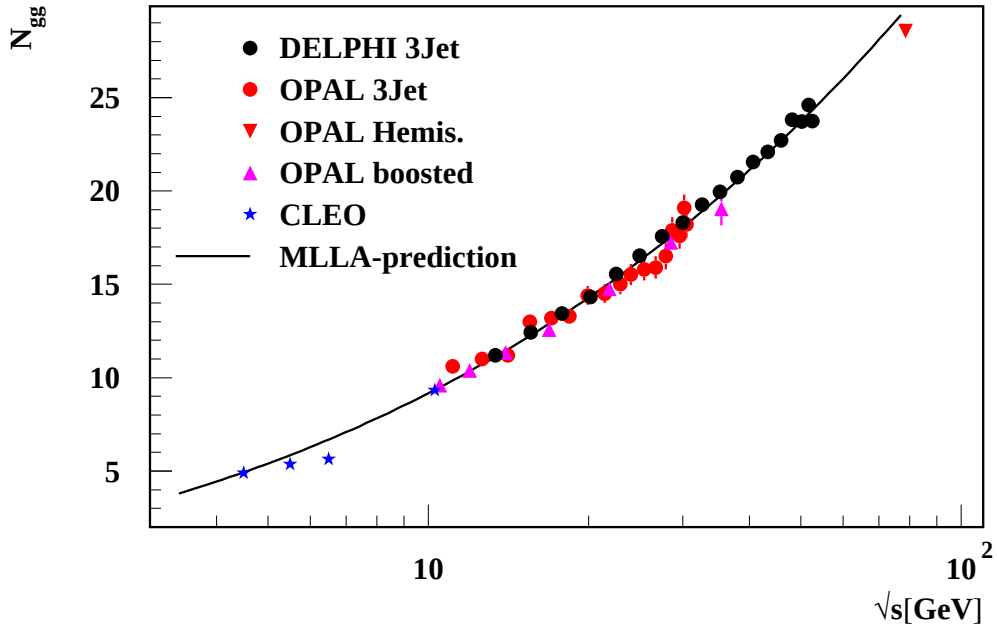
- Convincing measurement of colour coherence from soft particles in 3 jet events
  – destructive interference term observed
  – amazing agreement with LO prediction $\leftrightarrow$ LPHD

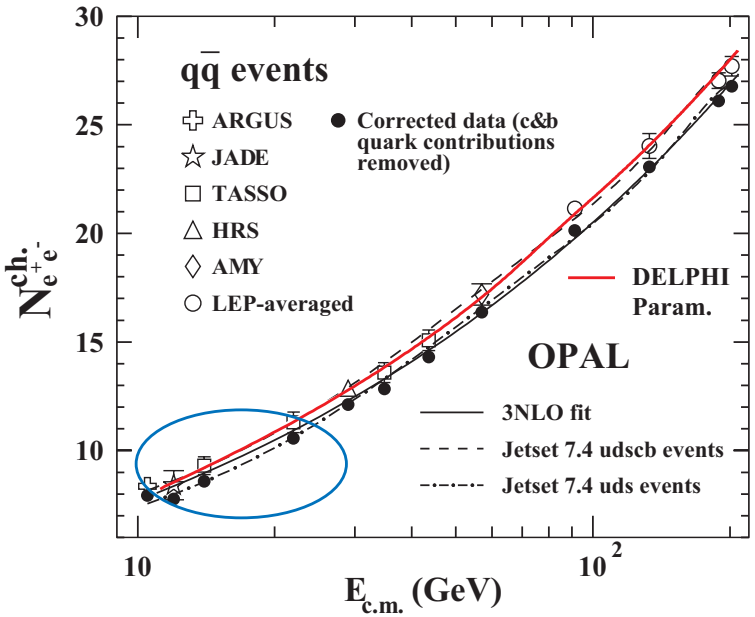
- Consistent measurements of the energy dependence of the gluon multiplicity
  – gluon slope $\sim$ twice quark slope
  – Precise measurement of C_A/C_F
  – QCD gauge group is fixed by measurement of hadronic properties: gluon/quark multiplicity, β-function

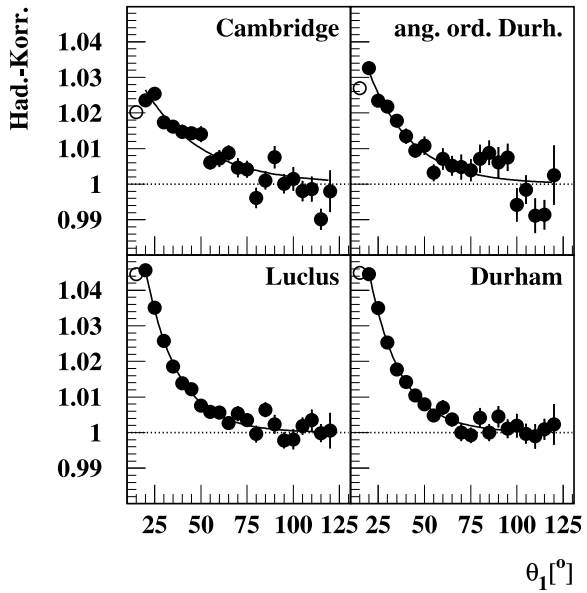
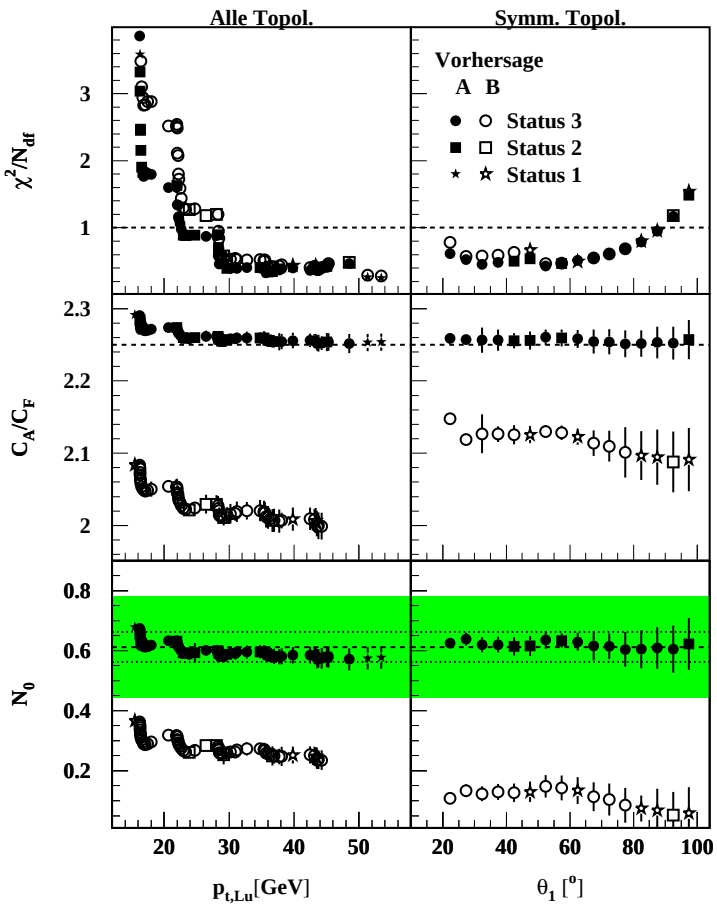
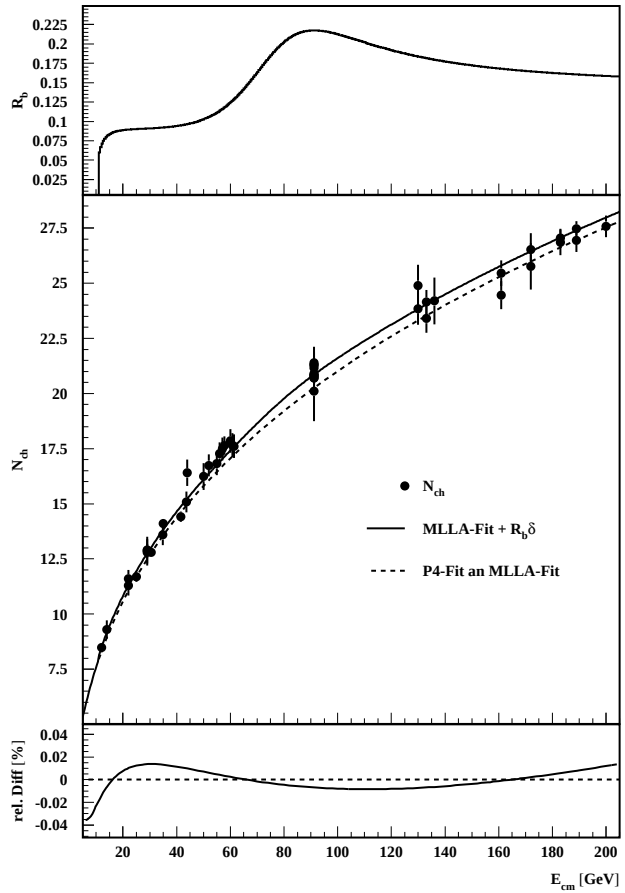
- Differences between gluon and quark fragmentation observed for fast, leading particles (e.g. protons)

Detailed Comparison to Opal









The C_A/C_F Fit to the 3 Jet Multiplicity