Inclusive $k_\perp$ distribution and two-particle correlations inside one jet at Modified Leading Logarithmic Approximation (MLLA)

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References :

- *R. Perez Ramos, F. Arléo & B. Machet* : "Next-to-MLLA corrections to $k_\perp$ distributions and 2-particle correlation in a jet", hep-ph/0512236, arXiv :0712.2212 [hep-ph] ;

- *F.Arléo, R. Perez Ramos & B. Machet* : Hadronic single inclusive transverse momentum ($k_\perp$) distributions inside jets beyond MLLA, Phys.Rev.Lett.100 :052002,2008 ;

- *R. Perez Ramos & B. Machet* : "MLLA inclusive hadronic distributions inside one jet at high energy colliders", hep-ph/0512236, JHEP 04 (2006) 043, p.1-35 ;

- *R. Perez Ramos* : "Two-particle correlations inside one jet at "Modified Leading Logarithmic Approximation" of Quantum Chromodynamics ; I Exact solution of the evolution equations at small x", hep-ph/0605083, JHEP 06 (2006) 019, p.1-50 ;

- *R. Perez Ramos* : "Single inclusive distribution and two-particle correlations inside one jet at "Modified Leading Logarithmic Approximation" of Quantum Chromodynamics ; II Steepest descent evaluation at small x, JHEP 09 (2006) 014, p.1-21.

- Perturbative Quantum Chromodynamics :
  - $*$ Non-Abelian theory $\boldsymbol{SU(3)_c}$;
  - $*$ Problems ;
  - $*$ Feynman diagrams, resummation approaches, probabilistic interpretation of cascading processes ;
  - $*$ Infrared and collinear singularities ;
  - $*$ Resummation schemes in QCD ($\rightarrow$ MLLA and Next-to-MLLA) ;
  - $*$ Coherence effects in QED and QCD ;
- Some tools in QCD.

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- Single inclusive differential one-particle distribution as a function of $k_\perp$ $\left(\dfrac{d\sigma}{\sigma d \ln k_\perp}\right)$ in MLLA and NMLLA ; comparison with CDF preliminary data ;
- Two-particles energy correlations inside jets in MLLA and NMLLA ;
- Conclusions.

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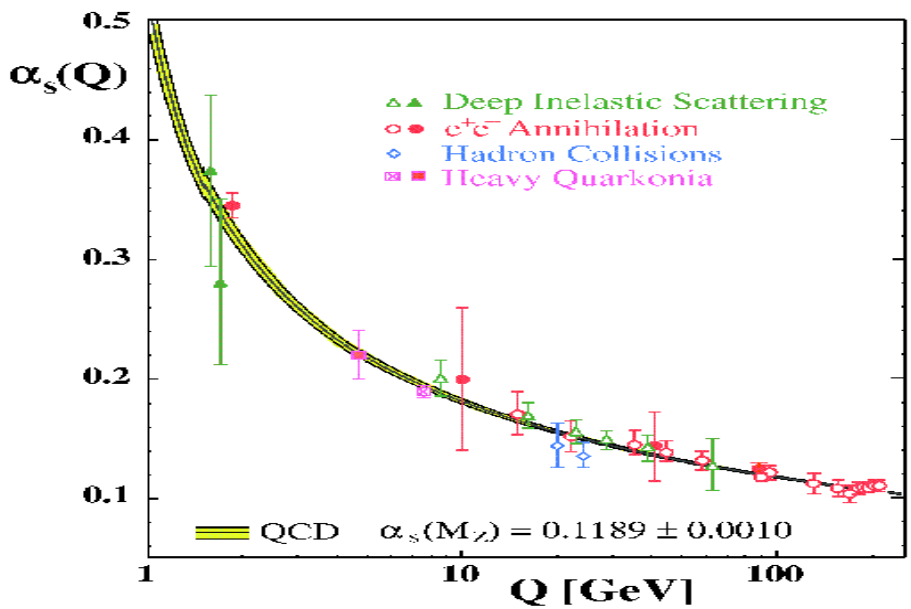
Perturbative Quantum Chromodynamics : Jet Calculus

QCD : $SU(3)_c$ non-Abelian gauge theory of quarks and gluons

$$\alpha_s(Q^2) = \frac{4\pi}{\beta \ln \frac{Q^2}{\Lambda^2_{QCD}}}, \quad \beta = \frac{1}{4N_c}\left(\frac{11}{3}N_c - \frac{2}{3}n_f\right); \quad N_c = 3$$

non-perturbative domain

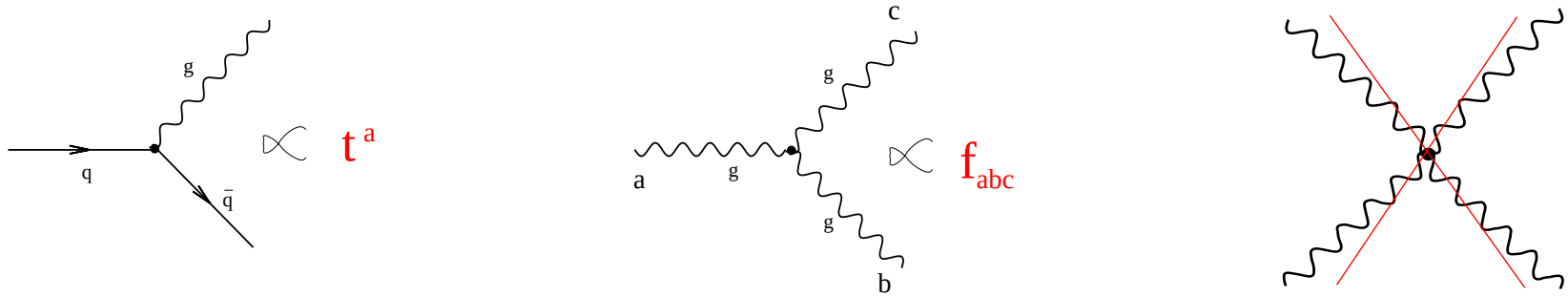
$\alpha_s \geq 1$



perturbative domain (asymptotic freedom)

$Q \gg \Lambda_{QCD} \Rightarrow \alpha_s \ll 1$

Vertices $q\bar{q}g$, ggg & $gggg$



Problems

• $\alpha_s(Q^2 \to \Lambda^2_{QCD}) \to \infty$;

• Infrared and collinear singularities $\Rightarrow$ problems on the convergence of the resumed series through their large logarithmic contributions ;

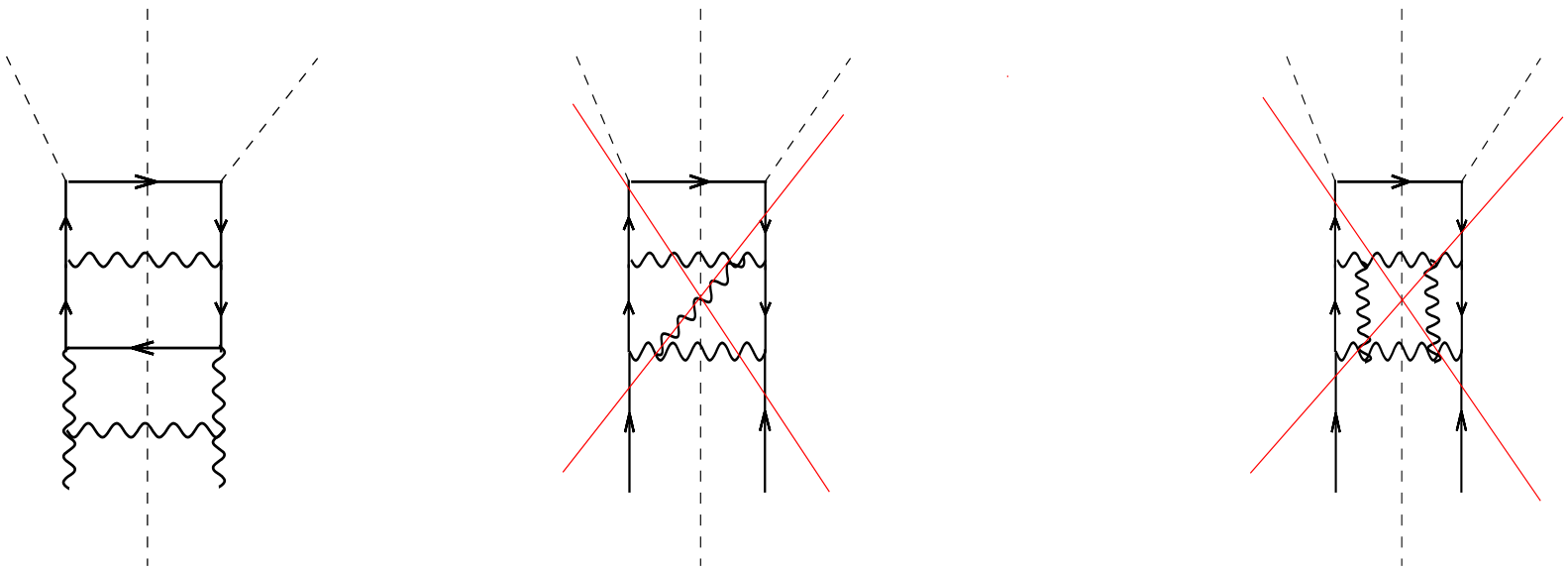
• Hadronization (confinement).

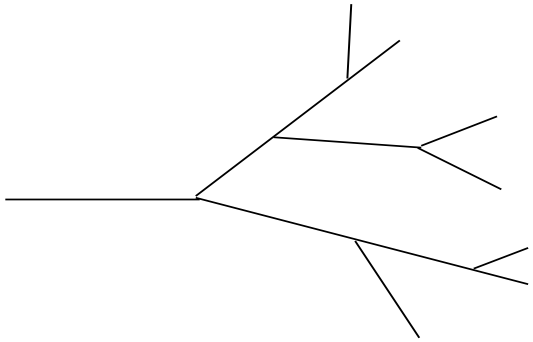
Different approaches

• Exclusive Processes : resummation of all Feynman digrams at a given order in α_s ;

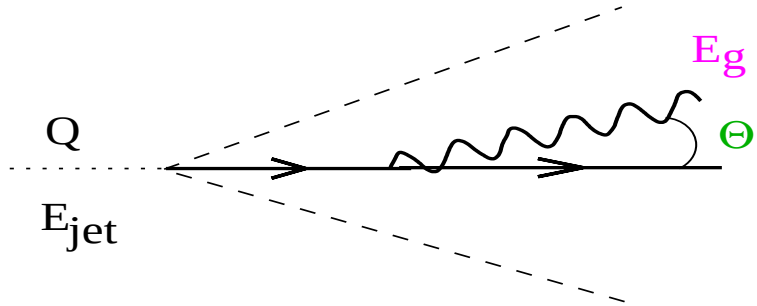
• Inclusive processes (jet calculus) : resummation of all leading, sub-leading logarithmic contributions (...) coming from the emission of soft and/or collinear gluons in jets.

Planar Gauge $\Rightarrow$ only ladder diagrams count in $|$ amplitude $|^2 \Rightarrow$ branching processes and probabilistic interpretation : vertices can be replaced by splitting functions.



amplitude  "probabilistic branching processes"

Infrared and collinear singularities



• collinear singularity : $\Theta \to \Theta_{min} \equiv Q_0/E_g$ $(k_\perp \approx E_g\Theta \geq Q_0) \Rightarrow$ divergence in $\log\Theta \Rightarrow \alpha_s \log\Theta$

$\Rightarrow$ resume over all log. contributions coming from the emission of all collinear gluons inside a cone of opening angle $\Theta > \Theta_{min}$;

• Infrared singularities : when $E_g \ll E_{jet}$ $(x = E_g/E_{jet} \ll 1) \Rightarrow$ divergence in $\log(1/x) \Rightarrow \alpha_s \log(1/x)$

• in general : Infrared + collinear $\Rightarrow \alpha_s \log(1/x) \log\Theta$.

Some resummation schemes

• LLA : $\left(\alpha_s \log \frac{k_\perp^2}{\mu^2}\right)^n$, resummation of single logs (Deep Inelastic Scattering) ;

• DLA : $(\alpha_s \log(1/x) \log \Theta)^n$; $\alpha_s \log^2 \sim 1$ ($\log \sim \alpha_s^{-1/2}$) $\Rightarrow$ main ingredient to the estimation of inclusive observables in jets ; neglects the recoil of the parent parton $\Rightarrow$ overestimates particle production ;

• Single Logs (SL) :

$*$ $(\alpha_s \log \Theta)^n \sim \alpha_s^{n/2}$: emission of hard collinear partons ($z \sim 1$) ;

$*$ other possible origins like the running of α_s : corrections $\propto \beta_0$;

• MLLA : $\left(\underbrace{\alpha_s \log \log}_{DLA(1)} + \underbrace{\alpha_s \log}_{SL(\sqrt{\alpha_s})}\right)^n$; SL corrections to DLA "restore" energy conservation and take into account the running of α_s (evolution of the jet) ;

• NMLLA : $\left(\underbrace{\alpha_s \log \log}_{DLA(1)} + \underbrace{\alpha_s \log}_{SL(\sqrt{\alpha_s})} + \underbrace{\alpha_s \log \log^{-1}}_{NSL(\alpha_s)}\right)^n$; NSL corrections take better account of energy conservation and allow the increase the range of validity in "x".

$$** \; D\left(\log z + \log(1/x)\right) \overset{z\sim 1}{\approx} D\left(\log(1/x)\right) + \frac{\partial D}{\partial \log(1/x)} \log z + \mathcal{O}(\log^2 z)$$

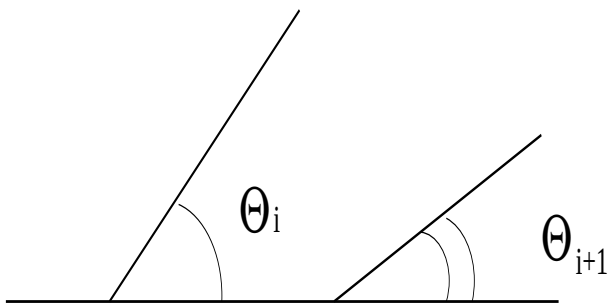
Coherence effects in QED and QCD

• QED : accelerated electrical charges coherently radiate bremsstrahlung photons ($\Theta_\gamma <$ scattering Θ) ;

• QCD : accelerated color charges coherently radiate bremsstrahlung gluons (Angular Ordering : AO) :

∗ in DLA $\Theta_{i+1} \ll \Theta_i$ ("strict" AO) ;

∗ in MLLA $\Theta_{i+1} \leq \Theta_i$ ("exact" AO).



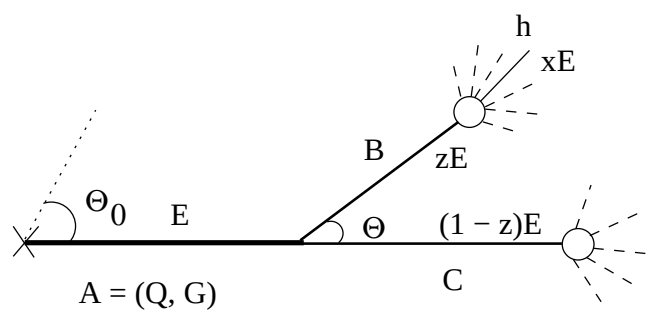
⇒ the parton tree as a cypress rather than an oak. . .

Some tools in QCD

• Exact solution of approached integro-differential equations : MLLA evolution equations at $x \ll 1$

- $*$ for the one-particle inclusive distributions ;
- $*$ for two-particle correlations (iterative solutions).

They follow from taking variational differentiation over a generating functional Z which satisfies the MLLA Master Equation :



$$\frac{d}{d\ln\Theta} Z_A\left(p,\Theta;\{u\}\right) = \frac{1}{2}\sum_{B,C}\int_0^1 dz\, \Phi_A^{B[C]}(z)\, \frac{\alpha_s\left(k_\perp^2\right)}{\pi} \Big(Z_B\big(zp,\Theta;\{u\}\big)\, Z_C\big((1-z)p,\Theta;\{u\}\big) - Z_A\big(p,\Theta;\{u\}\big)\Big)$$

- $*$ inclusive one-particle spectrum : $D = \frac{\delta}{\delta u} Z(u)$;
- $*$ two-particle 4-momentum correlation $D^{(2)} = \frac{\delta^2}{\delta u_1 \delta u_2} Z(u)$;
- $*$ etc. . . (just like in quantum field theories).

• Solution of Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) partonic evolution equations in Mellin's space (valid for $x \sim 1$) (we use it in certain integrals) ;

$$\frac{d}{d\ln Q^2} D(x, Q^2) = \int_x^1 \frac{dz}{z} \frac{\alpha_s(Q^2)}{2\pi} \Phi(z)\, D\left(\frac{x}{z}, Q^2\right)$$

$$\underbrace{D(j, Q^2)}_{\text{Known !}} = \int_x^1 dx\, x^{j-1} D(x, Q^2)$$

• Derivatives $\frac{d^n}{dj^n} D(j, Q^2) = \int_x^1 dx\, x^{j-1} \ln^n x\, D(x, Q^2)$, $n = 1, 2, \ldots$

• Numerical calculation (Fortran-90).

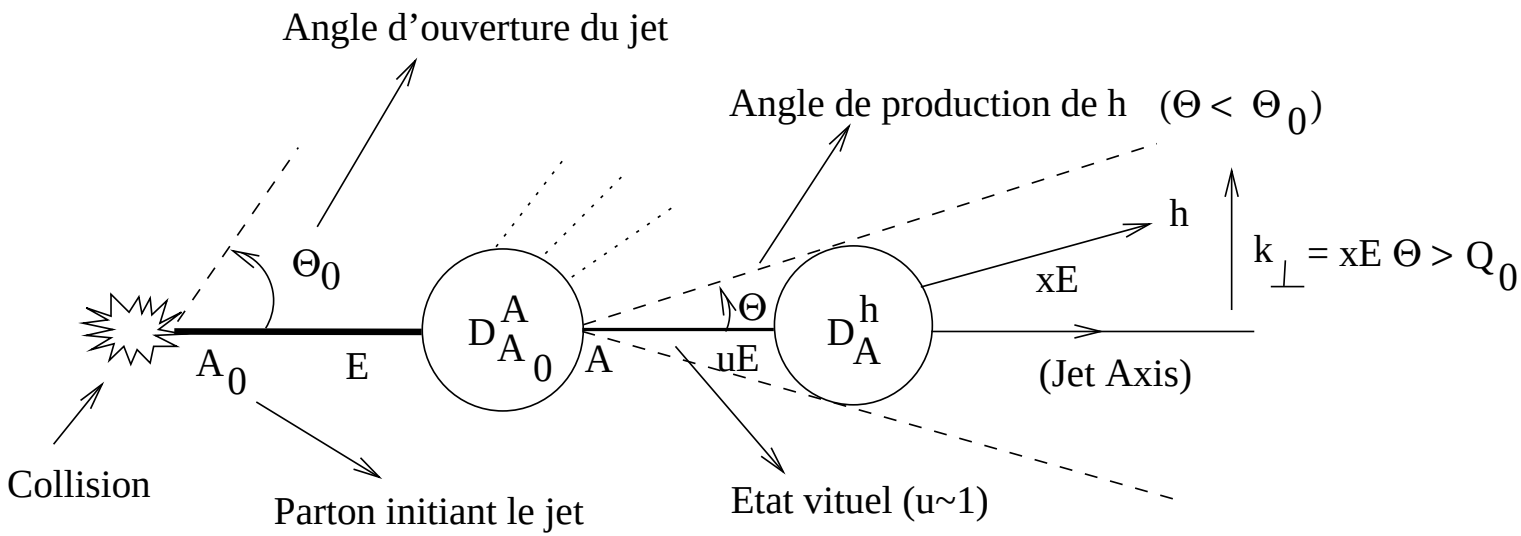
Single inclusive one-particle differential cross section $\frac{1}{\sigma}\frac{d\sigma}{d\ln k_\perp}$ as a function of $k_\perp$

R. Perez Ramos, F. Arléo & B. Machet, arXiv :0712.2212 [hep-ph]

F.Arléo, R. Perez Ramos & B. Machet, Phys.Rev.Lett.100 :052002,2008

R. Perez Ramos & B. Machet, JHEP 04 (2006) 043

Setting up the problem



$$\underbrace{\left(\frac{d\sigma}{d\ln k_\perp}\right)_{g\ or\ q}}_{\text{Measurable !}} = \int dx \left(\frac{d^2\sigma}{dx\, d\ln k_\perp}\right)_{g\ or\ q}$$

where

$$\left(\frac{d^2\sigma}{dx\, d\ln k_\perp}\right)_{A_0} = \frac{d}{d\ln k_\perp}\left[\sum_A \int_x^1 du D^A_{A_0}\left(u, E\Theta_0, uE\Theta\right) D^h_A\left(\frac{x}{u}, uE\Theta, Q_0\right)\right]$$

$\Rightarrow$ valid $\forall\, x < u$ and dominated by $u \sim 1$! !

Small x ($x \ll 1$) approximation $\Rightarrow$ analytical results

$$\boxed{D^A_{A_0}(u = \mathcal{O}(1)\ldots) : \text{DGLAP}\,; \quad D^h_A(\tfrac{x}{u} \ll 1\ldots) : \text{(N)MLLA}}$$

- Perturbative Parameter of the expansion, kinematics and variables :

$$\gamma_0^2 = \alpha_s\left(k_\perp^2\right)\frac{1}{2\pi} = \frac{1}{\beta \ln \dfrac{Q}{\Lambda_{QCD}}} = \frac{1}{\beta Y_{\Theta_0}}, \quad Y_{\Theta_0} = \ln\frac{Q}{\Lambda_{QCD}}, \quad \beta = \frac{1}{4N_c}\left(\frac{11N_c}{3} - \frac{2}{3}n_f\right);$$

$$\ell = \ln\left(1/x\right), \quad y = \ln\left(k_\perp/\Lambda\right), \qquad k_\perp \approx xE\Theta, \quad \Lambda_{QCD} \approx 0.253\ \text{GeV}$$

$\Rightarrow$ Double differential inclusive cross section :

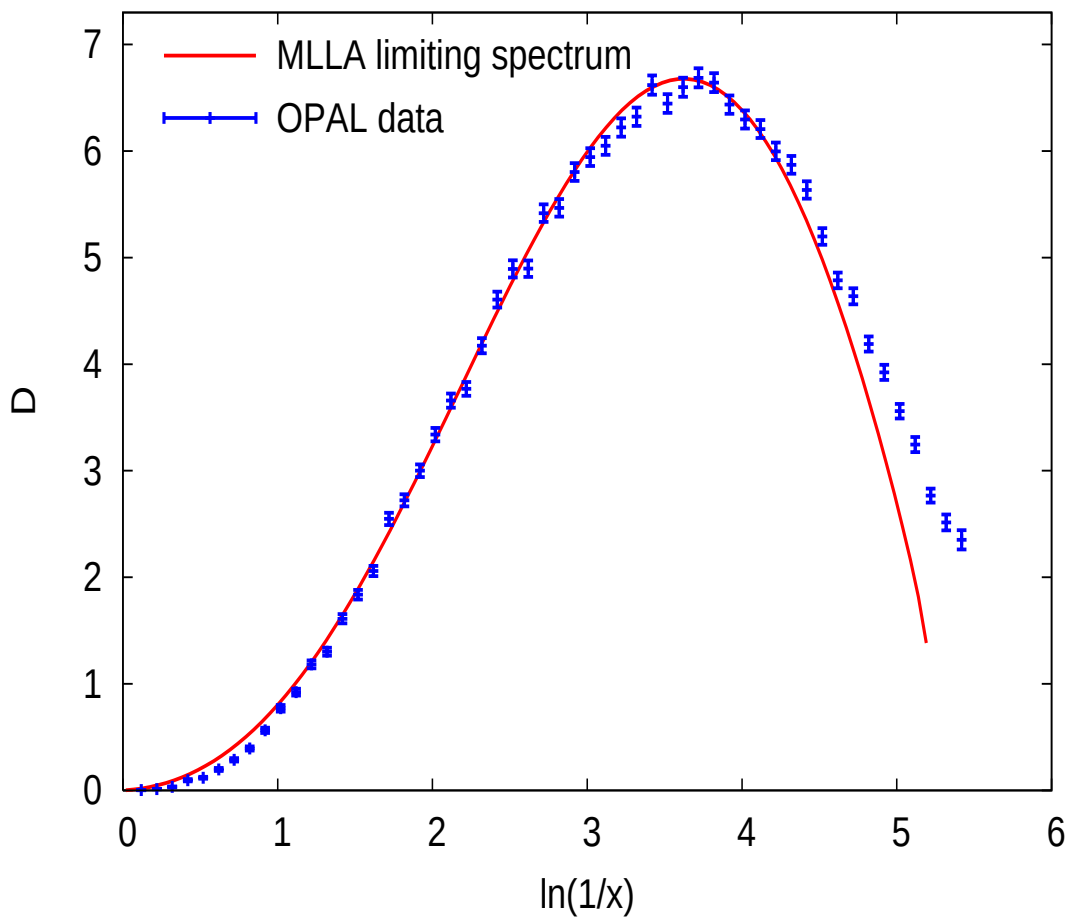
$$\left(\frac{d^2\sigma}{d\ell dy}\right)_{A_0} \overset{x\ll 1}{\approx} \frac{d}{dy}\left[< C >_{A_0}(\ell, y)\tilde{D}^h_g(\ell, y)\right]$$

- $< C >_{A_0}(\ell, y) \simeq c_0(\ell, y) \times 1 + c_1(\ell, y) \times \sqrt{\alpha_s} + c_2(\ell, y) \times \alpha_s + \mathcal{O}(\alpha_s^{3/2})$:

$\boxed{\text{color current}}$ (varying color factor) ; describes the evolution of the jet between Θ_0 and Θ.

- $\tilde{D}^h_g(\ell, y)$: ($\boxed{\text{(N)MLLA evolution equations}}$) describes the hump-backed plateau in the limit $Q \gg Q_0 \sim \Lambda_{QCD}$ (limiting spectrum). $D_g^{\text{NMLLA}} \approx D_g^{\text{MLLA}}$.

$*$ Reminder : MLLA hump-backed plateau $\mathcal{K}^{ch} \times \frac{1}{\sigma}\frac{d\sigma}{d\ln(1/x)}$



$* \; Q \gg Q_0 \sim \Lambda_{QCD}, \gamma_0 \approx 0.5$

"Next-to-MLLA" evolution equations for the inclusive spectrum D_A^h

"Next-to-MLLA" coupled system of integral equations for $D_g^h \equiv G$ and $D_q^h \equiv Q$:

$$\underbrace{G(\ell, y)}_{gluon\ jet} = \delta(\ell) + \int_0^\ell d\ell' \int_0^y dy' \gamma_0^2(\ell' + y') \Big[\underbrace{1}_{DLA} - (a_1 + a_2\, \psi_\ell(\ell', y'))\, \delta(\ell' - \ell) \Big] G(\ell', y'),$$

$$\underbrace{Q(\ell, y)}_{quark\ jet} = \delta(\ell) + \frac{C_F}{N_c} \ldots G(\ell', y') \ldots$$

$$\ell = \ln(1/x) \equiv \ln \frac{E}{k_0}, \quad y = \ln \frac{k_\perp}{Q_0}, \quad \lambda = \ln \frac{Q_0}{\Lambda_{QCD}}$$

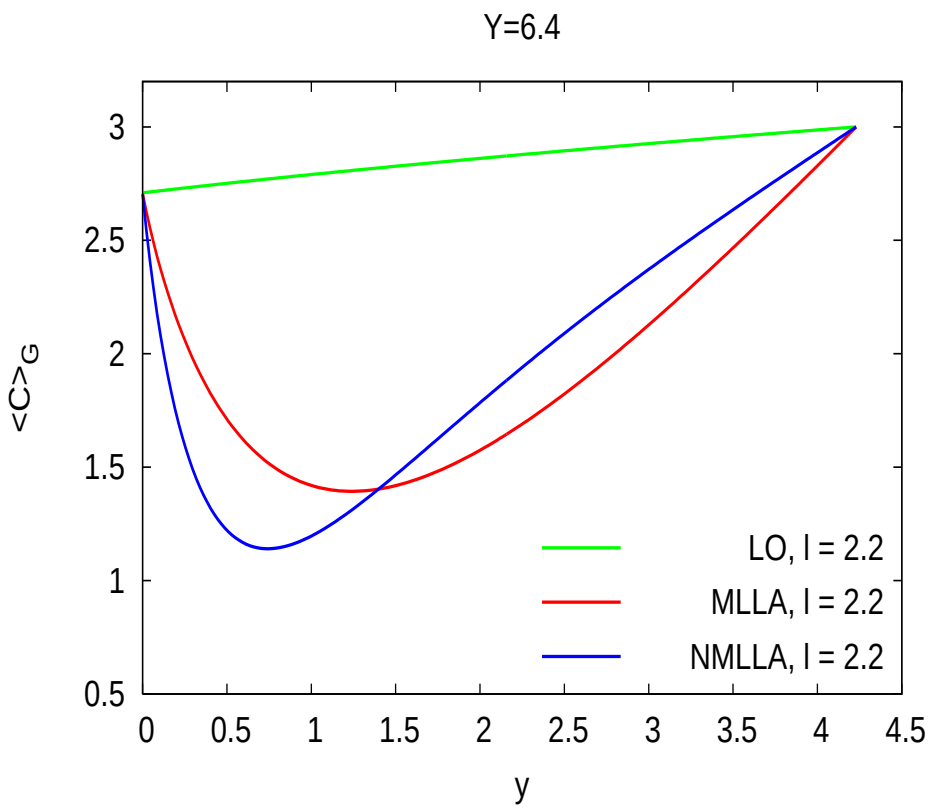
$$\gamma_0^2(\ell + y) = \frac{1}{\beta(\ell + y + \lambda)}, \quad Y = \ell + y = \ln \frac{E\Theta}{Q_0}$$

$$\boxed{G(\ell, y) = \frac{1}{\sigma} \frac{d\sigma}{d\ell}}$$

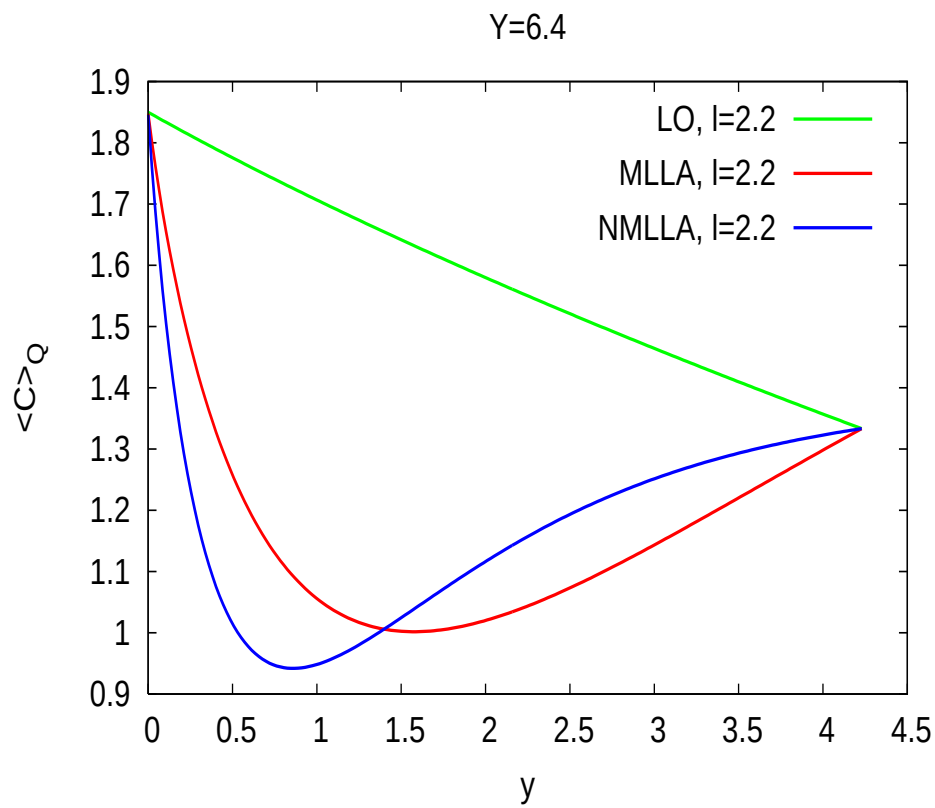
- DLA term : $\propto 1 \Rightarrow \mathcal{O}(1)$
- hard corrections : $\propto a_1 \approx 0.935,\ \mathcal{O}(\sqrt{\alpha_s})$(MLLA); $\propto a_2 \approx 0.06,\ \mathcal{O}(\alpha_s)$(NMLLA) since $\psi_\ell \equiv G_\ell / G = \mathcal{O}(\sqrt{\alpha_s})$
- $G(\ell, y) = (\ell + y + \lambda) \int \frac{d\omega d\nu}{(2\pi i)^2} e^{\omega\ell + \nu y} \int_0^\infty \frac{ds}{\nu + s} \left(\frac{\omega(\nu + s)}{\nu(\omega + s)} \right)^{1/\beta_0(\omega - \nu)} \left(\frac{\nu}{\nu + s} \right)^{a_1/\beta_0} e^{-\lambda s}$
- $Q \simeq \frac{C_F}{N_c} \left(1 + r_1 \sqrt{\alpha_s} + r_2 \alpha_s \right) G$

The color current $< C >_{A_0}$ at $Q = 155$ GeV (Tevatron)

Gluon



Quark



- NMLLA sources : $< C >_{A_0} = \mathcal{O}(\alpha_s)$ and $\frac{d}{dy} < C >_{A_0} = \mathcal{O}(\alpha_s)$

$$\Rightarrow \frac{d^2\sigma}{d\ell dy}(\ell, y) \to \frac{d\sigma}{dy}(y) = \int d\ell \left(\frac{d^2\sigma}{d\ell dy} \right).$$

Limitations of the method $\Rightarrow$ ranges of validity in MLLA and NMLLA :

2 contradictory conditions should be simultaneously satisfied : $x \ll 1$ and $\alpha_s(k_\perp) \ll 1$;

- At too large $k_\perp$, the small x approximation fails ($k_\perp \approx xE\Theta$) because of positivity problems $\Rightarrow \exists \quad k_{\perp max} = f(Q)$ (NMLLA>MLLA).

- When $k_\perp \to \Lambda_{QCD}$, the perturbative expansion fails ($\alpha_s(k_\perp) \to \infty$).

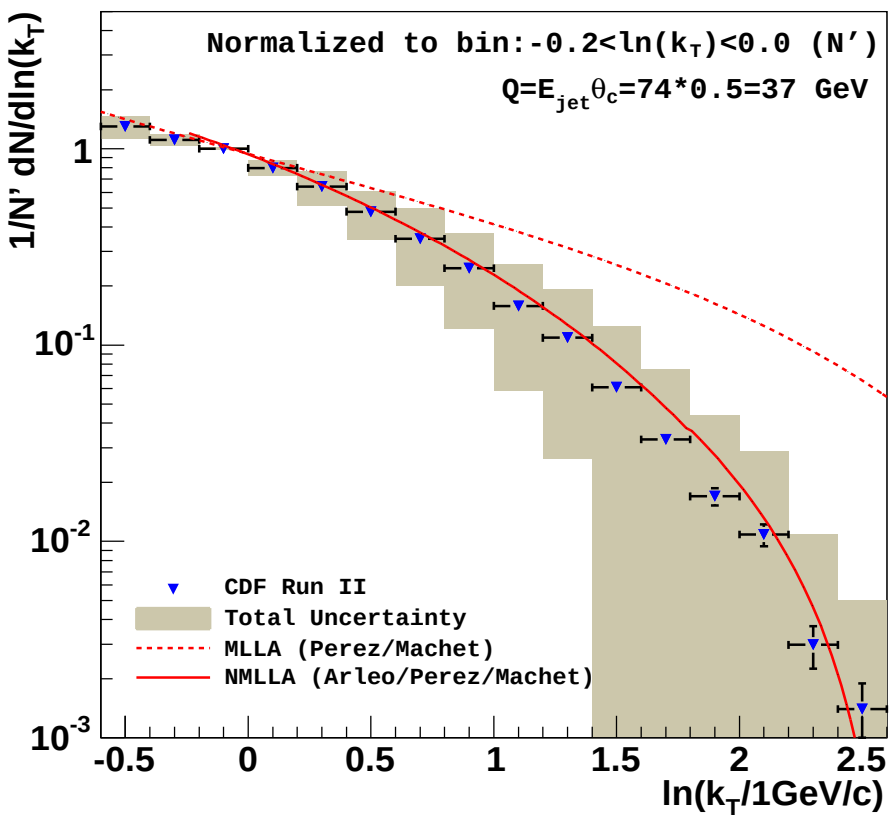
$$k_\perp > k_{\perp\, min} \Rightarrow y > 1.4 \Leftrightarrow k_\perp > 1 \text{ GeV}$$

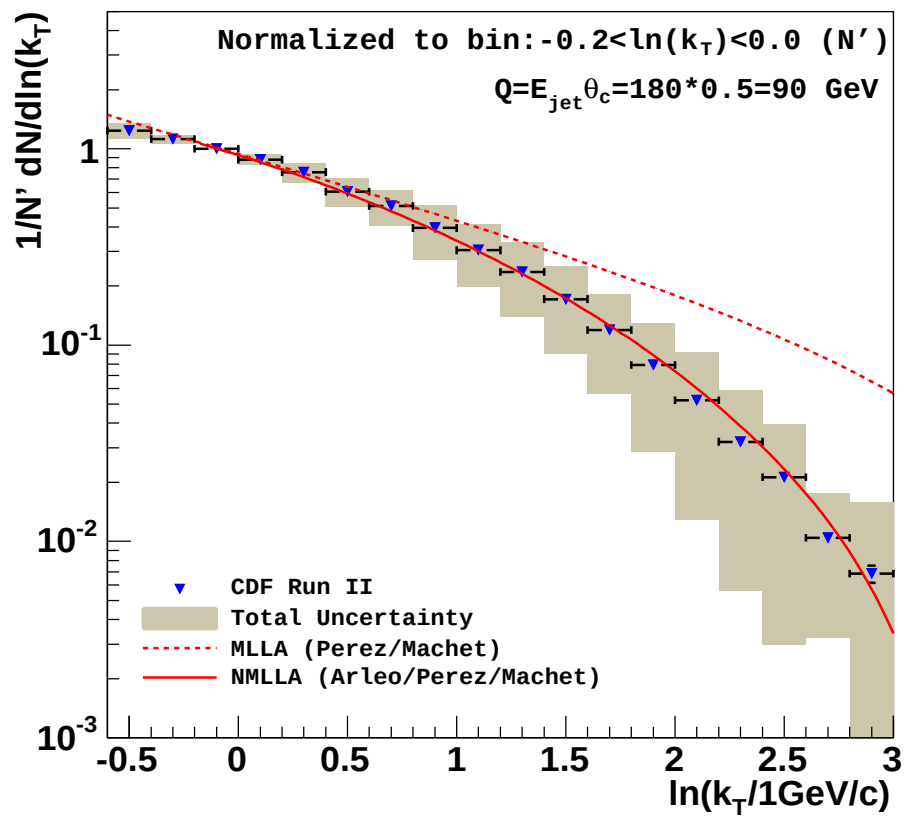
$\Rightarrow$ $\exists$ range of validity : $\boxed{k_{\perp\, min} \leq k_\perp \leq k_{\perp\, max}}$ (NMLLA>MLLA) which increases when energy scale increases.

$\to$ LHC will supply tests of pQCD in a much larger domain of $k_\perp$ than the Tevatron.

$\Rightarrow$ Hadronic single inclusive one-particle cross section as a function of $y = \ln(k_\perp/1GeV)$ for $Q_0 \to \Lambda_{QCD} = 250$ **MeV** :

$$\left(\frac{d\sigma}{d\ln k_\perp}\right)^{ch} = \omega \left(\frac{d\sigma}{d\ln k_\perp}\right)_g + (1-\omega)\left(\frac{d\sigma}{d\ln k_\perp}\right)_q$$





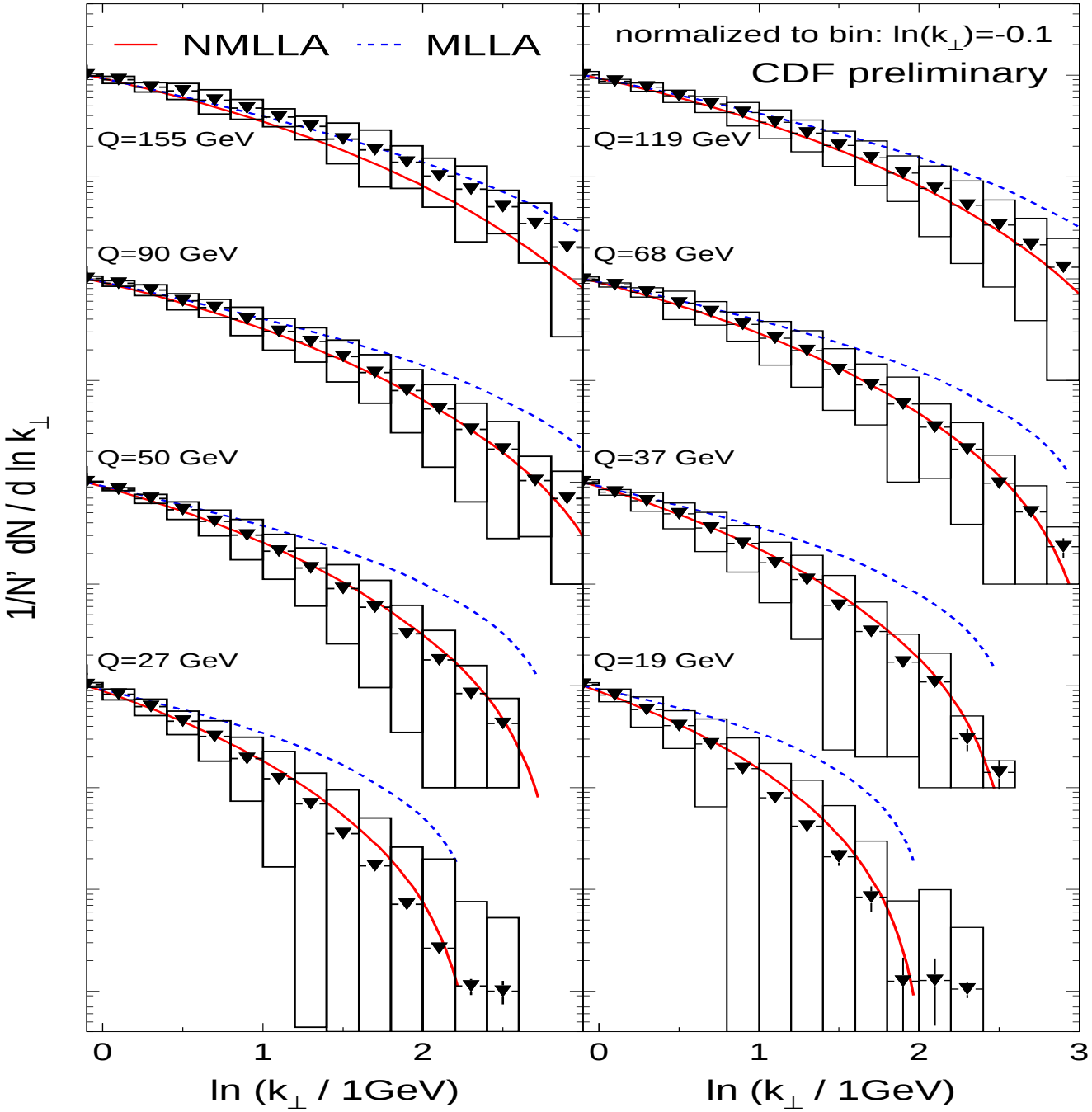
$MLLA : 0 \leq \ln(k_\perp/1\,GeV) \leq 1.0$

$MLLA : 0 \leq \ln(k_\perp/1\,GeV) \leq 1.6$

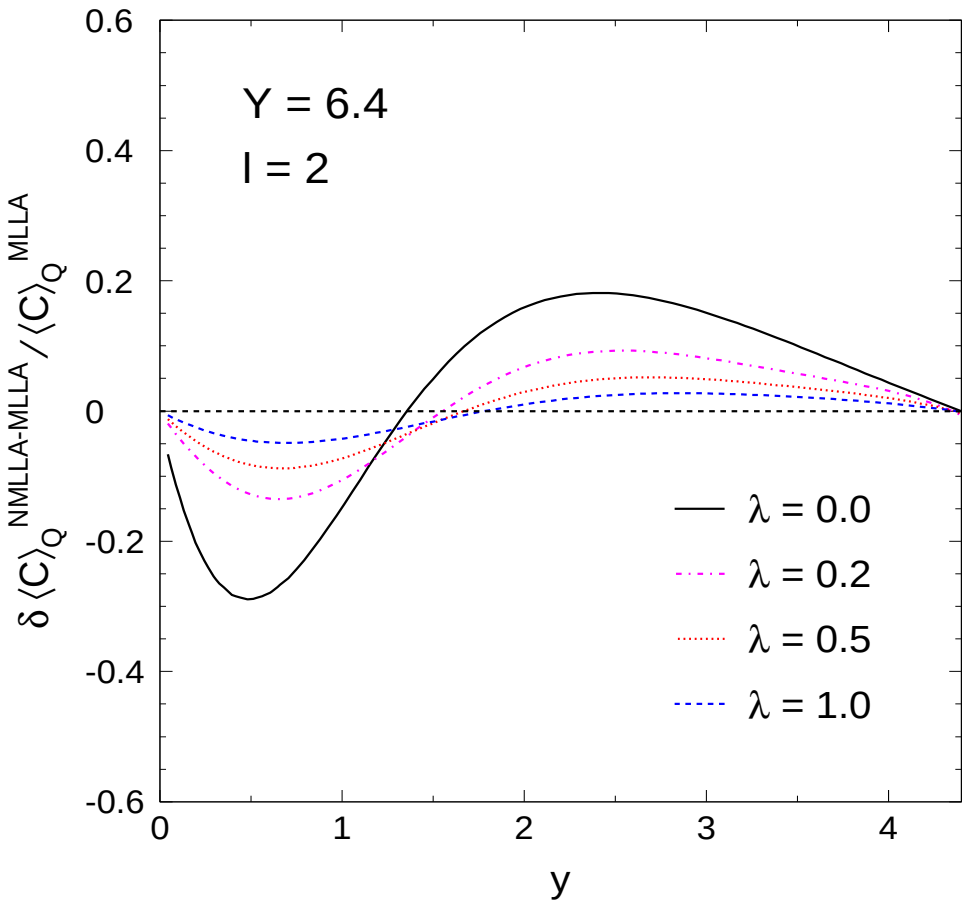
$NMLLA : 0 \leq \ln(k_\perp/1\,GeV) \leq 2.4$

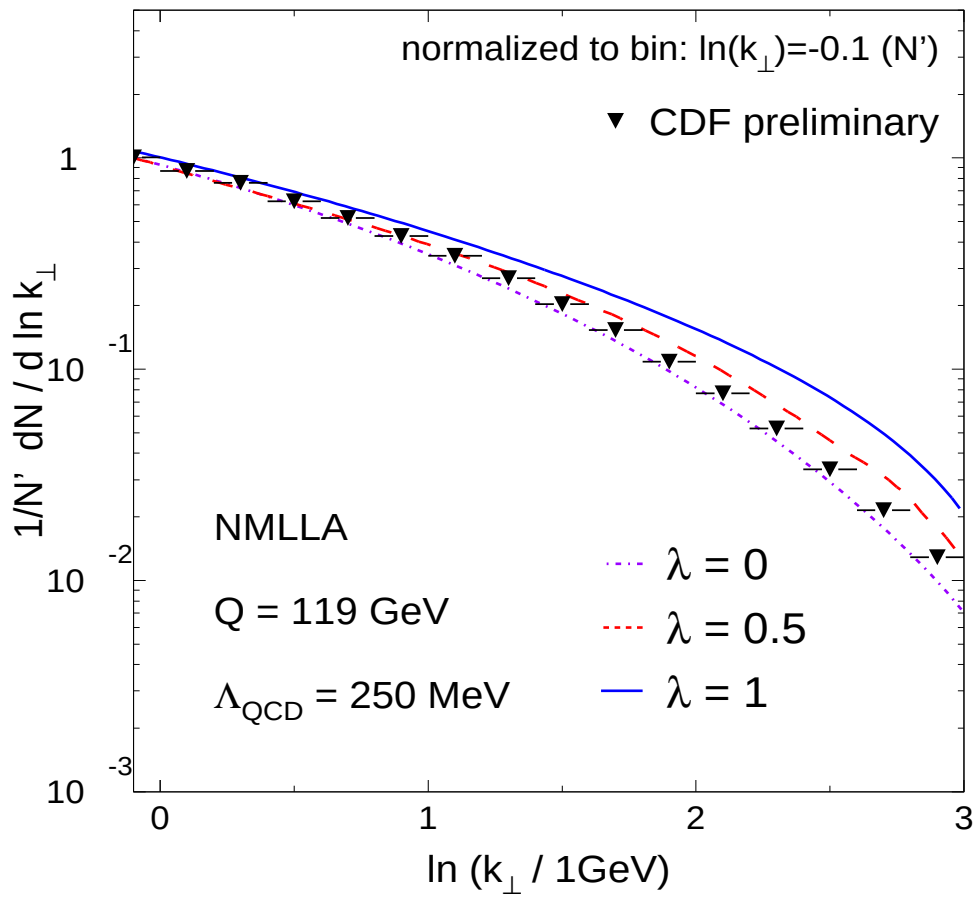
$NMLLA : 0 \leq \ln(k_\perp/1\,GeV) \leq 2.8$

Phys.Rev.Lett.100 :052002,2008



Going beyond the limiting spectrum approximation $\lambda = \ln(Q_0/\Lambda_{QCD}) \neq 0$





$\Rightarrow$ Limiting spectrum ($\lambda \to 0$) is the most successful candidate in the description of the data !

Conclusions for $\dfrac{d\sigma}{\sigma\, d\ln k_\perp}$

• Very good agreement between NMLLA predictions and the CDF data in a broader $k_\perp$ range than in MLLA ; NMLLA$\rightarrow$MLLA asymptotically ;

• Further test of LPHD hypothesis (partons roughly behave as hadrons) ; pQCD successfully predicts the shape of $\frac{1}{\sigma}\frac{d\sigma}{d\ln k_\perp}$ and dominates over non-perturbative effects : this has also been confirmed for multiplicities, multiplicity correlators (KNO problems where an analogous set of NMLLA corrections was included !), hump-backed plateau ;

• Limiting spectrum proves once again to be the most successful to describing the data ;

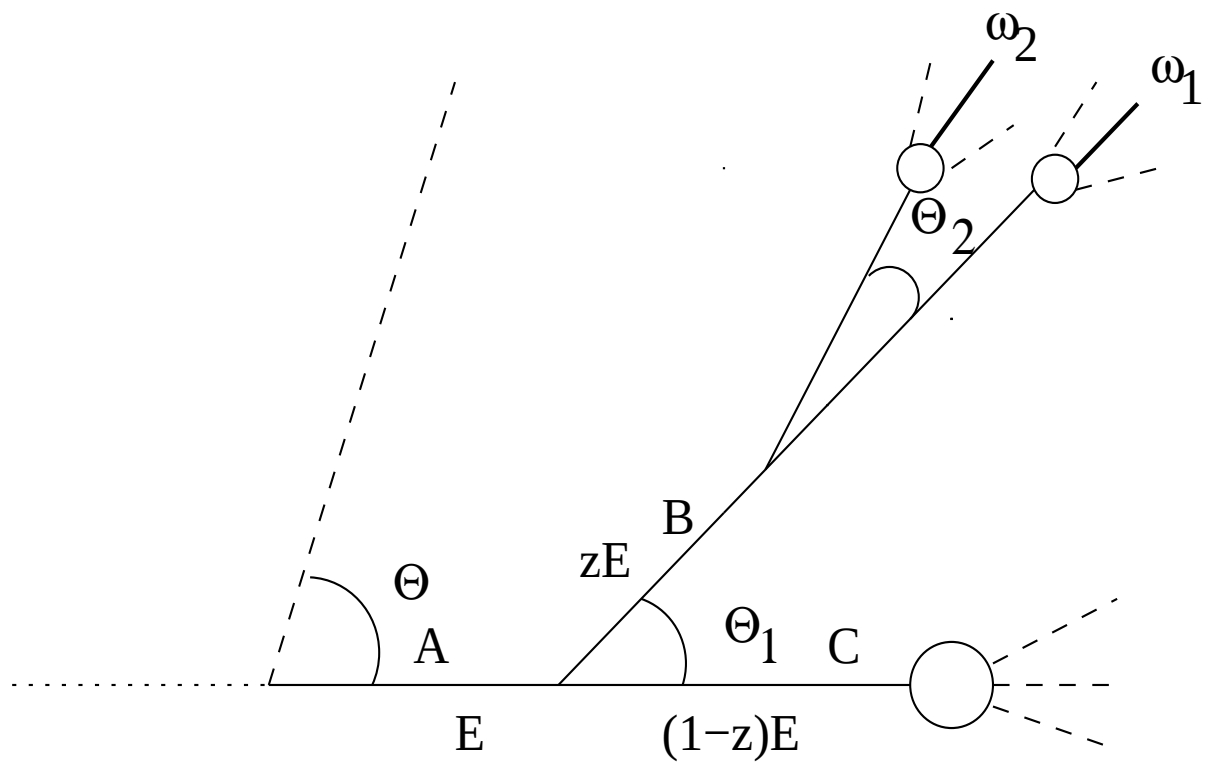
• Efficiency of jet calculus in the estimation of inclusive observables ;

• Correlations : less inclusive (2 particles) ; role of non-perturbative effects ?

Two-particle correlations inside jets

R. Perez-Ramos, JHEP 06 (2006) 019

R. Perez-Ramos, JHEP 09 (2006) 014



Exact Angular Ordering : $\Theta \geq \Theta_1 \geq \Theta_2$

$$G^{(2)}, Q^{(2)}(\omega_1, \omega_2) \equiv \omega_1 \omega_2 \frac{d^2\sigma}{d\omega_1 d\omega_2} \; ? \; ?$$

MLLA evolution equations for two-particle correlations inside quark and gluon jets

System of two coupled evolution integral equations :

$$G^{(2)}(\ell_1, y_2, \eta) - G_1(\ell_1, y_1) G_2(\ell_2, y_2) = \int_0^{\ell_1} d\ell \int_0^{y_2} dy\, \gamma_0^2(\ell + y) \Big[\underbrace{1}_{DLA} - a\delta(\ell - \ell_1) \Big] G^{(2)}(\ell, y, \eta)$$

$$+ (a - b) \int_0^{y_2} dy\, \gamma_0^2(\ell_1 + y) G(\ell_1, y + \eta) G(\ell_1 + \eta, y),$$

$$Q^{(2)}(\ell_1, y_2, \eta) - Q_1(\ell_1, y_1) Q_2(\ell_2, y_2) = \ldots$$

DLA term : $\propto 1 \Rightarrow \mathcal{O}(1)$; MLLA corrections : $\propto a, (a - b) \Rightarrow \mathcal{O}(\sqrt{\alpha_s})$

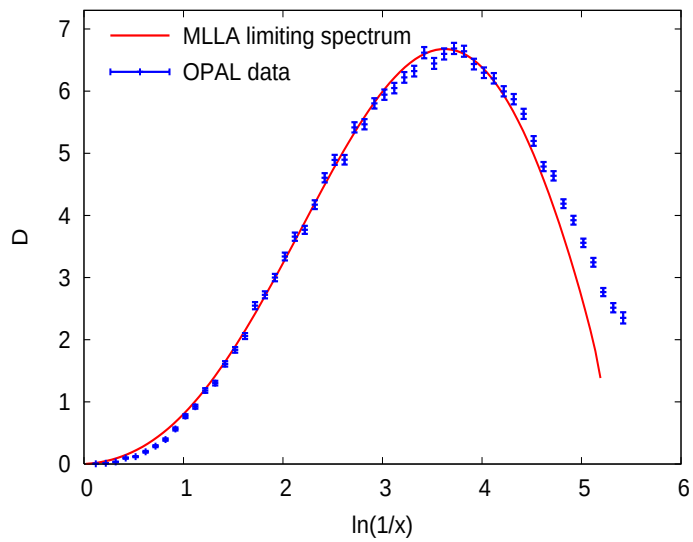
$$\eta = \ell_2 - \ell_1 = y_1 - y_2 = \ln(x_1/x_2) > 0$$

Correlator : $\boxed{\mathcal{C}_g = \dfrac{G^{(2)}(\ell_1, \ell_2, Y)}{G(\ell_1, Y)\, G(\ell_2, Y)}, \qquad \mathcal{C}_q = \dfrac{Q^{(2)}(\ell_1, \ell_2, Y)}{Q(\ell_1, Y)\, Q(\ell_2, Y)}}$

Independent of the normalization $\rightarrow$ refined test of partonic dynamics !

G, Q : Inclusive spectrum $\propto x \dfrac{d\sigma}{dx}$; $G^{(2)}, Q^{(2)}$: $\propto x_1 x_2 \dfrac{d^2\sigma}{dx_1 dx_2}$.

Fong and Webber's approximation (1990) : first and only MLLA calculation



$$G^{ch}\left(\ell = \ln(1/x), \ln\frac{Q}{Q_0}\right) = \mathcal{K}^{ch} \times \frac{1}{\sigma}\frac{d\sigma}{d\ln(1/x)}$$

$Q \gg Q_0 \sim \Lambda_{QCD}$, $\gamma_0 \approx 0.5$; $\ell_1 \approx \ell_2 \approx \frac{1}{2}\ln\frac{Q}{Q_0}$

Perturbative expansion in $1+\sqrt{\alpha_s}$ at small x near the hump region : $\ln(1/x)_{max} = \frac{1}{2}\ln(Q/Q_0)$:

$$R(\ell_1, \ell_2, Y) \equiv \frac{1}{2}(1 + C_q) = 1.375 - 1.125\left(\frac{l_1 - l_2}{\ln\frac{Q}{Q_0}}\right)^2 - \left[1.262 - 0.877\frac{(l_1 + l_2)}{\ln\frac{Q}{Q_0}}\right]\frac{1}{\sqrt{\ln\frac{Q}{Q_0}}}.$$

Analytical solution of the evolution equations at small x

The normalized correlator $\mathcal{C}_g(\ell_1, y_2, \eta) = \dfrac{G^{(2)}}{G_1\, G_2} \longrightarrow 1$ if independent emission.

The analytical solution at small x reads

$$\mathcal{C}_g - 1 = \frac{1 - \delta_1 - b\left(\psi_{1,\ell} + \psi_{2,\ell} - [\beta\gamma_0^2]\right) - [a\chi_\ell + \delta_2]}{1 + \Delta + \delta_1 + \left[a\left(\chi_\ell + [\beta\gamma_0^2]\right) + \delta_2\right]}$$

- Terms in black expressed through the logarithmic derivatives of the inclusive spectrum ;
- Δ (DLA) decreases the correlation when $x_2 \neq x_1$;
- the solution is obtained iteratively over $\mathcal{C}_g$: all the terms in orange in its r.h.s. depend on $\mathcal{C}_g$ and are small corrections ($\chi = \ln \mathcal{C}_g \ldots$) ;
- it mixes $\mathcal{O}(\sqrt{\alpha_s})$ (MLLA) and $\mathcal{O}(\alpha_s)$ (NMLLA) corrections.

$$\chi_\ell = \frac{\partial \chi}{\partial \ell} = \mathcal{O}(\gamma_0^2), \qquad \chi_y = \frac{\partial \chi}{\partial y} = \mathcal{O}(\gamma_0^2);$$

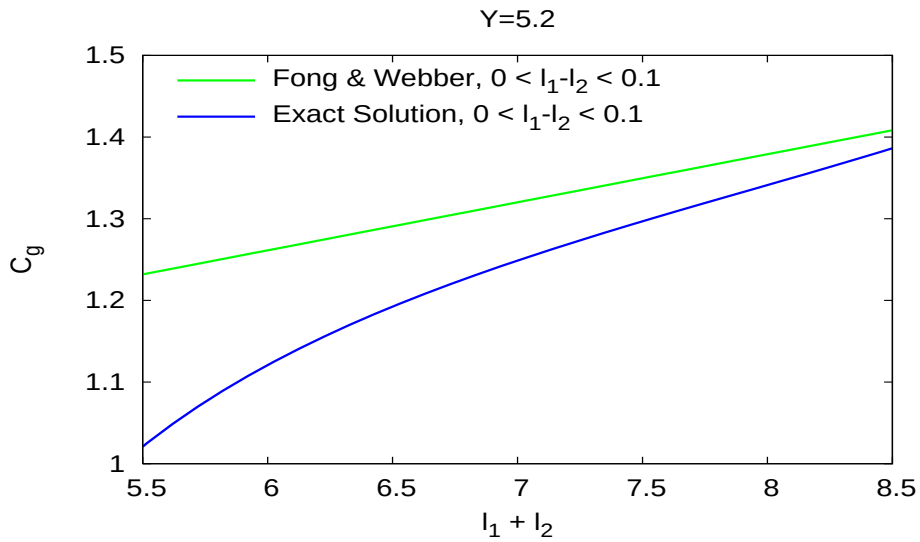
$$\psi_i = \ln G_i, \qquad \psi_{i,\ell} = \frac{1}{G_i}\frac{\partial G_i}{\partial \ell} = \mathcal{O}(\gamma_0), \qquad \psi_{i,y} = \frac{1}{G_i}\frac{\partial G_i}{\partial y} = \mathcal{O}(\gamma_0);$$

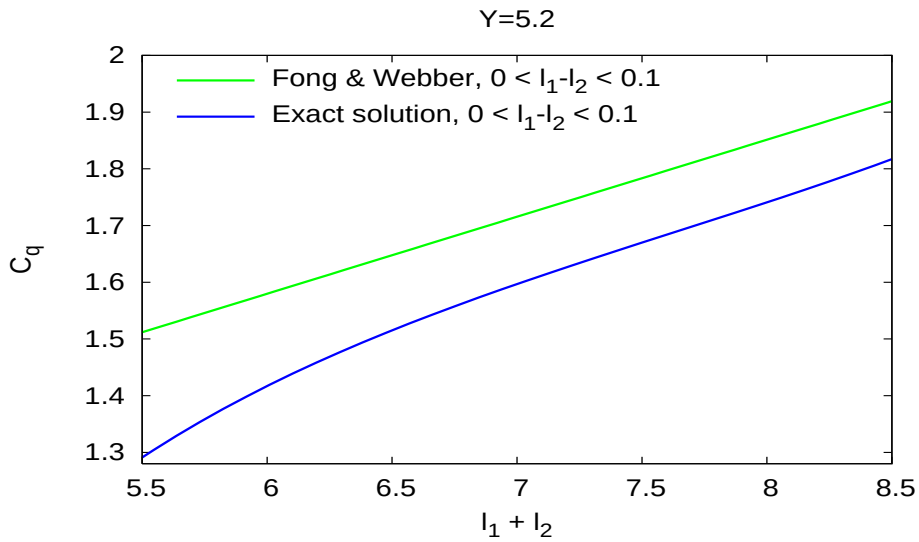
$$\Delta = \gamma_0^{-2}\Big(\psi_{1,\ell}\psi_{2,y} + \psi_{1,y}\psi_{2,\ell}\Big) = \mathcal{O}(1);$$

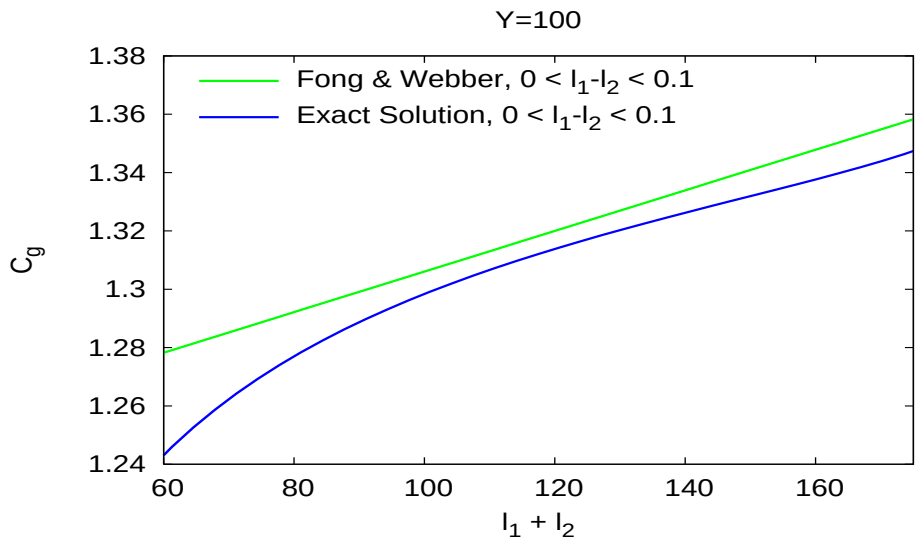
$$\delta_1 = \gamma_0^{-2}\Big[\chi_\ell(\psi_{1,y} + \psi_{2,y}) + \chi_y(\psi_{1,\ell} + \psi_{2,\ell})\Big] = \mathcal{O}(\gamma_0);$$

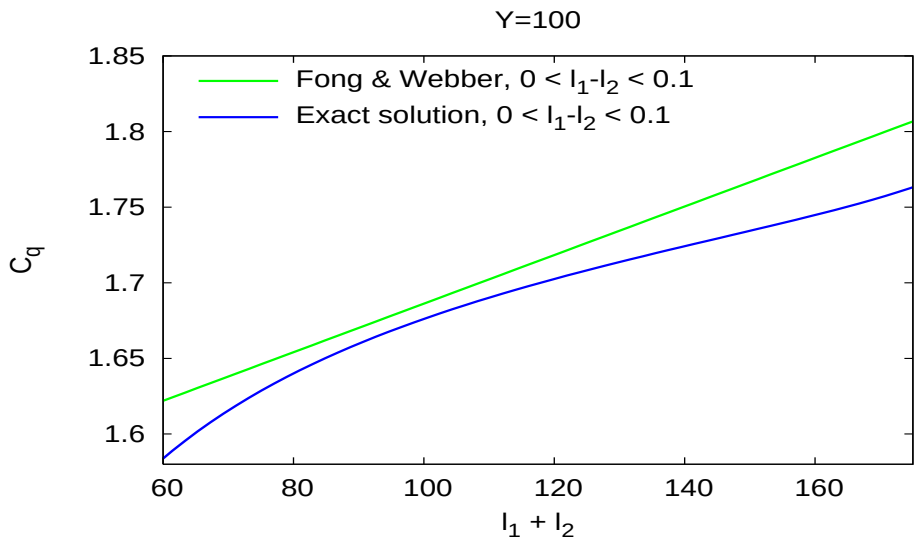
$$\delta_2 = \gamma_0^{-2}\Big(\chi_\ell\chi_y + \chi_{\ell\, y}\Big) = \mathcal{O}(\gamma_0^2), \qquad \delta_c = \delta_1 + \delta_2 + a\chi_\ell \simeq 10^{-2}.$$

Comparison with Fong & Webber's predictions









GLUON JET

QUARK JET

$*$ gaussian approximation of the ”hump-backed plateau” :

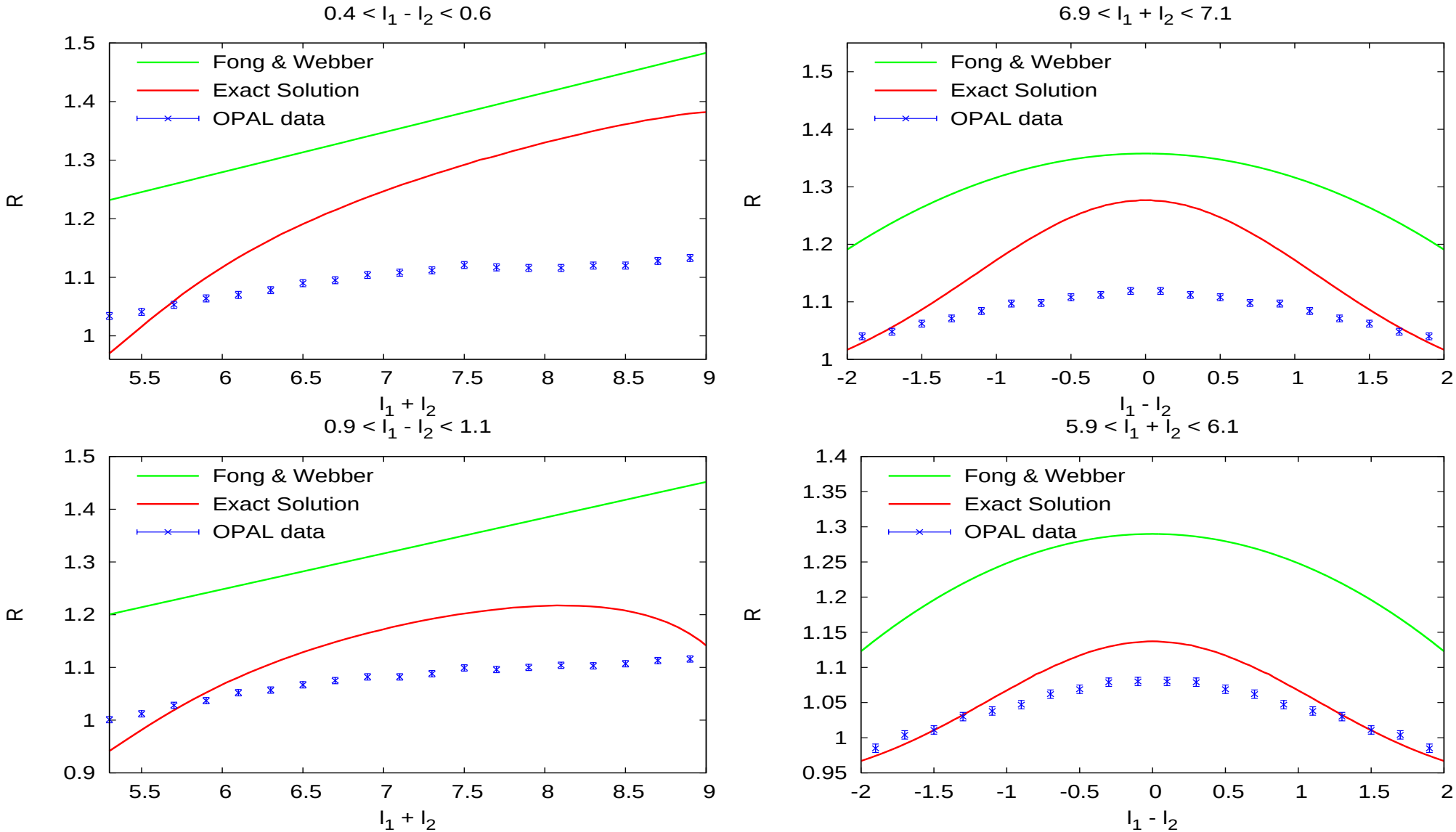
$$G(\ell_i, Y) \simeq \exp\left[-\frac{(\ell_i - Y/2)^2}{Y^{3/2}}\right]$$

$$\mathcal{C}_g(\ell_1, \ell_2, Y) \approx 1 + \frac{1 - \left(5b - 3b\frac{\ell_1 + \ell_2}{Y}\right)\gamma_0 + \mathcal{O}(\gamma_0^2)}{3 + 9\left(\frac{\ell_1 - \ell_2}{Y}\right)^2 - \left(2\beta + 5a - 3a\frac{\ell_1 + \ell_2}{Y}\right)\gamma_0 + \mathcal{O}(\gamma_0^2)}$$

$*$ expanded in $1 + \gamma_0 (\sim \sqrt{\alpha_s})$:

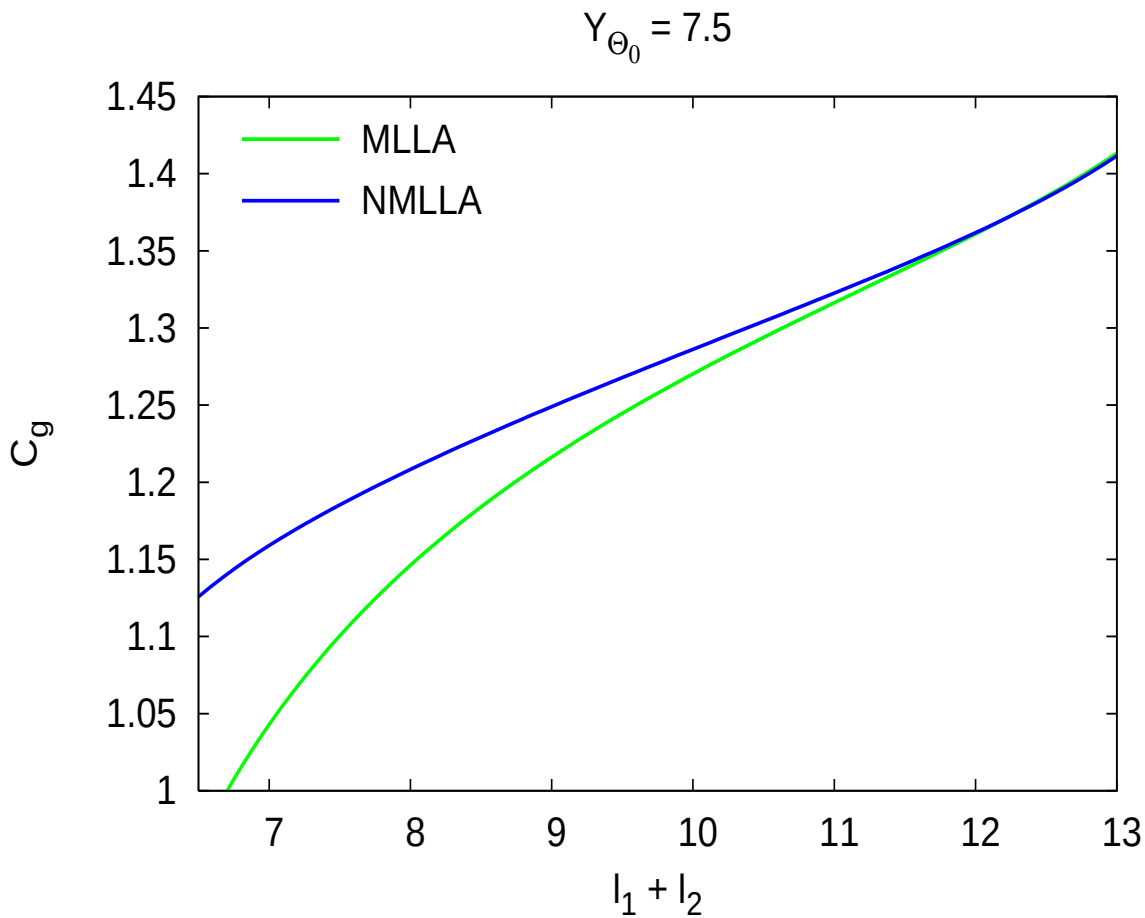
$$\mathcal{C}_g^{\text{FW}} \approx \frac{4}{3} - \left(\frac{\ell_1 - \ell_2}{Y}\right)^2 + \left[-\frac{5}{3}\left(b - \frac{1}{3}a\right) + \frac{2}{9}\beta + \left(b - \frac{1}{3}a\right)\left(\frac{\ell_1 + \ell_2}{Y}\right)\right]\gamma_0 + \mathcal{O}(\gamma_0^2)$$

Comparison with existing data : OPAL, $Q = 91.2$ GeV ; JHEP 04 (2006) 043



$\Rightarrow$ Forthcoming data from CDF will be compared with our predictions.

Comparison between MLLA and NMLLA predictions



⇒ NMLLA terms do not bring better agreement with OPAL data

Conclusions

• In both approaches (exact and Fong-Webber's) : correlations are the strongest when $x_1 = x_2$;

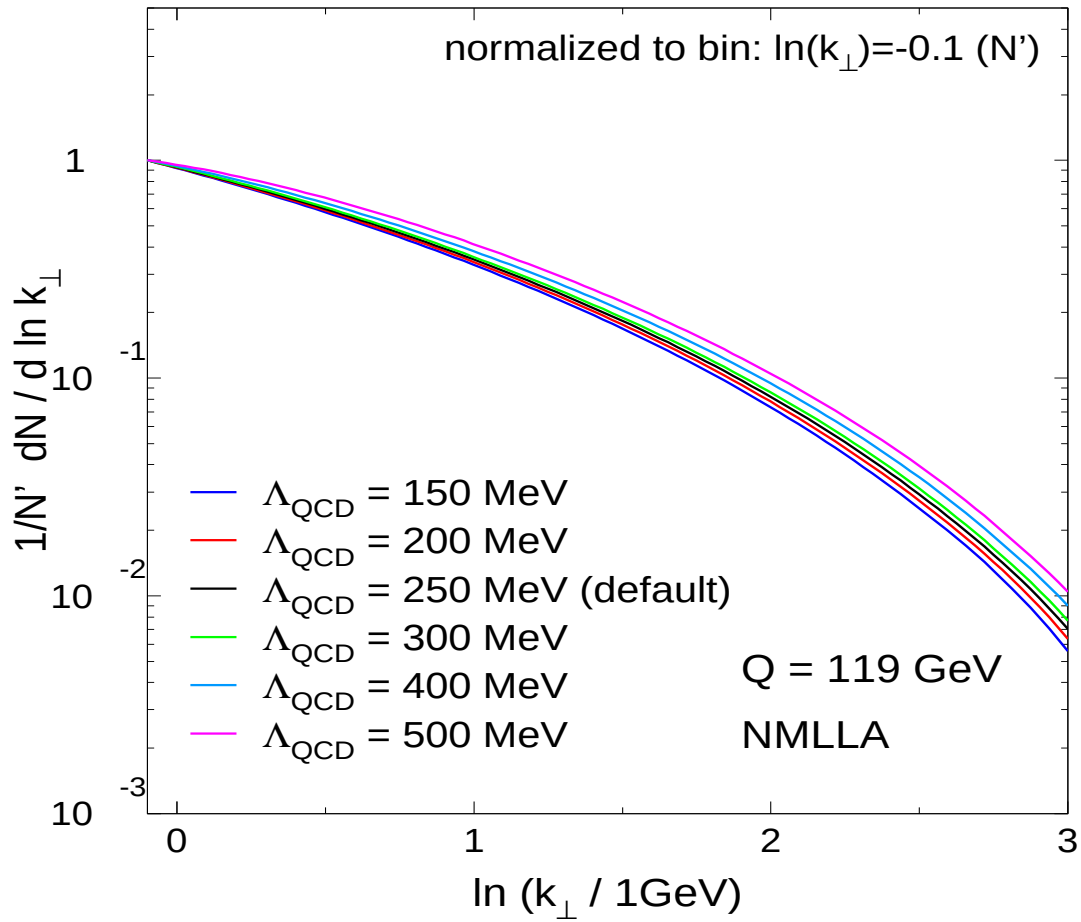
• The behavior of the correlation as predicted by the exact solution of the MLLA evolution equations is different :

$*$ it flattens when (x_1, x_2) decrease ;

$*$ and it starts decreasing as (x_1, x_2) gets smaller : decorrelation following from the emission of gluons at larger angles (destructive interferences occuring in phase space) ;

• The exact solution comes closer to the data and roughly follows the shape but a disagreement subsists ; non-trivial hadronization effects might play an important role (specially at relative small transverse momenta : large resonances from bound states !).

Varying $\Lambda_{QCD} \Rightarrow \mathcal{O}(\alpha_s)$



$\Rightarrow$ The shape and normalization of our prediction stay pretty close to the one evaluated with $\Lambda_{QCD} = 250$ MeV !

Scaling violation in DGLAP $\Rightarrow \mathcal{O}(\alpha_s)$

One writes

$$D^A_{A_0}(u, E\Theta_0, uE\Theta) = e^{\ln u \frac{d}{d\ln(E\Theta)}} D^A_{A_0}(u, E\Theta_0, E\Theta)$$

where

$$\frac{d}{d\ln(E\Theta)} = \frac{d}{d\xi}\frac{d\xi}{d\ln(E\Theta)} = -\frac{1}{b}\frac{1}{\ln(E\Theta)}\frac{d}{d\xi}$$

so that

$$e^{\ln u \frac{d}{d\ln(E\Theta)}} = 1 - \frac{1}{b}\frac{\ln u}{\ln(E\Theta)}\frac{d}{d\xi} + \mathcal{O}(\alpha_s^2).$$

Finally, the integral in the convolution can be approached by

$$\int_x^1 du\, u\, D^A_{A_0}(u, E\Theta_0, uE\Theta) \approx \int_x^1 du u D^A_{A_0}(u, E\Theta_0, E\Theta) - \frac{1}{b}\frac{1}{\ln\frac{E\Theta}{\Lambda}}\int_x^1 du\, u\, \ln u \frac{\partial D^A_{A_0}}{\partial \xi} + \mathcal{O}(\alpha_s^2).$$

We can now estimate the order of magnitude of this correction. Taking, for exemple, the analytic form of $D^q_q(u)$ (non-singlet combination of quark distributions) in the $u \to 1$ limit

we are aimed to compare

$$I = \int_x^1 du\, u(1-u)^{-1+4C_F\xi}$$

and

$$\delta I = \frac{4C_F}{b(\ell + y + \lambda)} \int_x^1 du\, u\, \ln u\, \ln(1-u)\, (1-u)^{-1+4C_F\xi}.$$

where $\xi = \frac{1}{b} \ln\left(\frac{Y_{\Theta_0}}{Y_\Theta}\right)$. For instance, $\xi(Y_{\Theta_0} = 6, Y_\Theta = 3) = 0.08$ is a typical value at LEP or Tevatron energy scales. In that case $\delta I/I \times 100 \approx 4\%$.