

A tool for extended scalar sectors

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Acknowledgments





Outline



Motivation: Which scalar sector at the LHC?

ScannerS: A tool to constrain the parameter space

- The Strategy
- Structure & features
- First tests & Outlook



Standard Model $\begin{aligned} &\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \overline{\psi} \mathcal{B} \psi + hc. \\ &+ \psi_i \mathcal{Y}_i \mathcal{Y}_j \psi_j \phi + hc. + |D_{\mu} \phi|^2 - |D_{\mu}$

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Non-abelian field theory coupled to fermions which explains:

Standard Model

$$= \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \overline{\psi} \mathcal{B} \psi + i$$

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 Electromagnetic force & weak and strong nuclear forces;

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$$\Psi_{i} \Psi_{i} \Psi_{j} \Phi + h.c.$$



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Scalar sector prone to coupling to hidden sectors!

Only SM singlets with dimension <4 are: $\textit{H}^{\dagger}\textit{H}\,, \textit{B}_{\mu\nu}\,, \textit{H}^{\dagger}\textit{L}$ James Wells lectures arXiv:0909.4541

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Extended scalar sectors can address these problems.

LHC designed to find new TeV scale physics \rightarrow Higgs boson?

Crucial interaction between theorists & experimentalists!





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Efficient tools are essential:

- Feynman rules: Feynrules, LanHEP,...
- Matrix Elements+MC generators: CalcHEP,CompHEP,FeynArts/FormCalc, Madgraph/MadEvent, Herwig,Pythia, MC@NLO, POWHEG,...
- Detector Simulation: AcerDet, Delphes, PGS,...+ Experiments





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We propose a new tool dedicated to: Scanning the parameter space of extended scalar sectors

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Vacuum linear conditions: VEVs

Scalar potential for given model:

• Linear in couplings λ_a

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model:

$$\frac{m^{2}}{2}H^{\dagger}H + \frac{\lambda}{4}(H^{\dagger}H)^{2}, SM$$

$$+ \frac{\delta_{2}}{2}H^{\dagger}H |S|^{2} + \frac{b_{2}}{2}|S|^{2} + \dots, BSM$$

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• Expansion around minimum

$$\phi_i = \mathbf{v}_i + M_{ij}H_j \Rightarrow \langle \partial_i V \rangle_a \lambda_a = 0$$

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• Numerically inexpensive to eliminate $\lambda_{a_1} = \Lambda_{a_1 a_2} \lambda_{a_2}$

$$\frac{\partial^2 V}{\partial H_i \partial H_j} \bigg|_{H_i=0} \to \mathsf{M}^T \Big\langle \widehat{\partial}^2 V \Big\rangle_{a_2} \mathsf{M} \lambda_{a_2} = \mathsf{Diag}(m_i^2)$$
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Delay non-linear/expensive tasks to increase efficiency!



Tree level unitarity & vacuum stability

Tree level unitarity in 2 \rightarrow 2 high energy scattering:

 $a^{(0)}_{jj}\sim\sum_{a_4}\ldots\lambda_{a_4}$ Lee, Quigg, Thacker; PRD16, Vol.5 (1977)

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Example: Variation of Electroweak Precision Observables $\Delta S, \Delta T, \Delta U$ for n-Singlets (implemented):



ScannerS @ Work





Note: Includes routines to identify Goldstones and other "a priori" eigen-directions at a certain point v_i .

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Tests: Some singlet extensions

Complex Scalar field with broken U(1):

$$V = \frac{m^2}{2}H^{\dagger}H + \frac{\lambda}{4}(H^{\dagger}H)^2 + \frac{\delta_2}{2}H^{\dagger}H|\mathbb{S}|^2 + \frac{b_2}{2}|\mathbb{S}|^2 + \frac{d_2}{4}|\mathbb{S}|^4 + \left(\frac{b_1}{4}\mathbb{S}^2 + a_1\mathbb{S} + c.c.\right)$$

- Model 0: Standard Model Higgs only!
- Model 1, (S) = 0: When a₁ = 0 and b₁ ∈ ℝ⁺. <u>Two stable</u> dark matter candidates. ⇔ Barger et al. PRD77,035005 (2008)
- Model 2, $\langle \Im(\mathbb{S}) \rangle = 0$: When $a_1 \in \mathbb{R}^+$ and $b_1 \in \mathbb{R}$. A stable dark matter candidate. Higgs mixes with other scalar. Barger et al. PRD 79,015018 (2009)
- *Model 3*, $\langle \Re(\mathbb{S}) \rangle \neq 0$, $\langle \Im(\mathbb{S}) \rangle \neq 0$: Can be parametrised by $a_1 \in \mathbb{R}^+$ and $b_1 \in \mathbb{C}$. Three scalars mixing. \rightarrow In progress!

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- Here ξ_i^2 is the reduction factor of the corresponding Higgs signal compared to SM.



In progress

The main points we are addressing:

- General algorithm to determine boundedness from below
- General algorithm to check minimum is Global: Already have a hybrid Monte Carlo/bracketing strategy applied to Model 3.

 \rightarrow Maybe hybrid MC possible in general

Interface with LanHep and Micromegas for models with dark matter candidates

Conclusions

- Scalar extensions of SM, common when addressing BSM problems: Hierarchy problem, Dark matter, Neutrino masses, Flavour structure,...
- Tool development is crucial in the interaction with experimentalists to identify candidate models
- We are developing a scanning tool for the Higgs sector. We have included:
 - Minimum generation, with an inverted scanning strategy, retaining physical states information
 - Tree level unitarity checking routines
 - Vacuum stability & Global (test models only for now)
 - Flexible structure for user defined analysis

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Look out for the release of **ScannerS**oon!

THANK YOU!

BACKUP

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The Standard Model – Interactions









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If $\Lambda_{cutoff} \sim M_4 \sim 10^{16} \text{ TeV} \Rightarrow$ fine tuning of $\sim 10^{-16}$

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