QCD Corrections and Non-linear Mappings

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CHIPP Plenary Meeting 2012

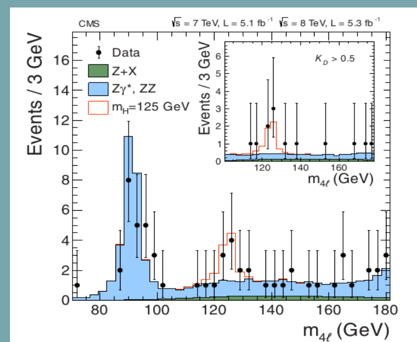
The LHC is a QCD machine

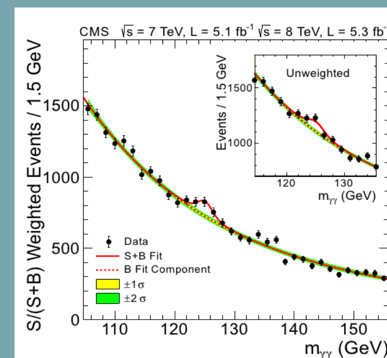
The dynamics of collisions is therefore predominantly governed by QCD

QCD Corrections ?

- The LHC is build primarily (not just to study QCD) to answer a number of fundamental questions:
  - What is the mechanism of electroweak symmetry breaking?
  - What is the origin of mass?
  - Are Supersymmetry, extra dimensions are realized at the TeV Scale?
  - Dark matter?
- QCD corrections will be crucial to answer these questions

Is the bump in the LHC data the Standard Model Higgs boson?





Observation of a new boson at a mass of 125 GeV

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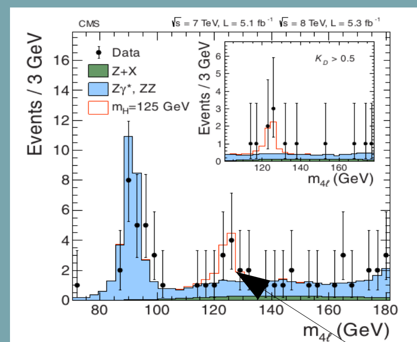
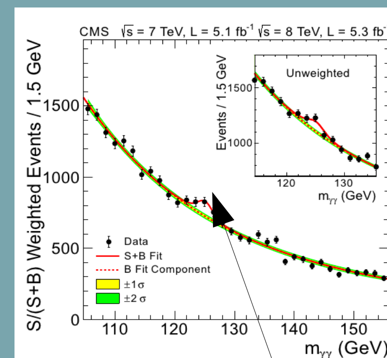
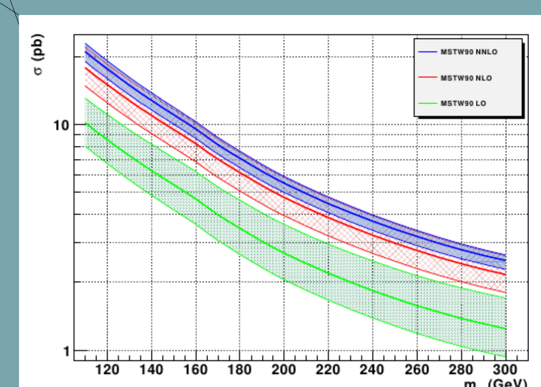




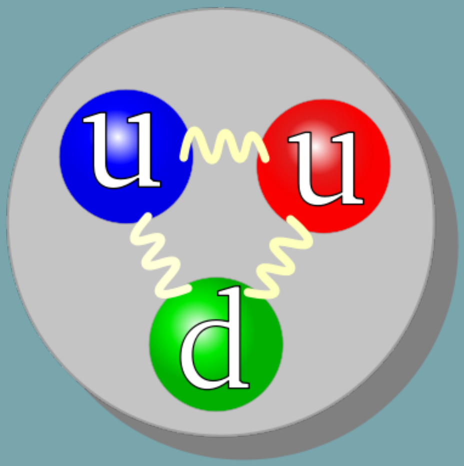
Observation of a new boson at a mass of 125 GeV

- To determine how *Standard* this (Higgs) boson is requires not only precise measurement, but also precise knowledge of the Higgs production cross-section.

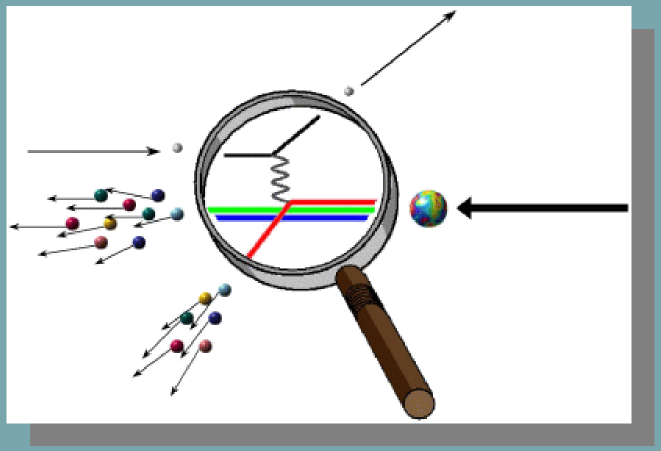


QCD

Confinement



Asymptotic freedom

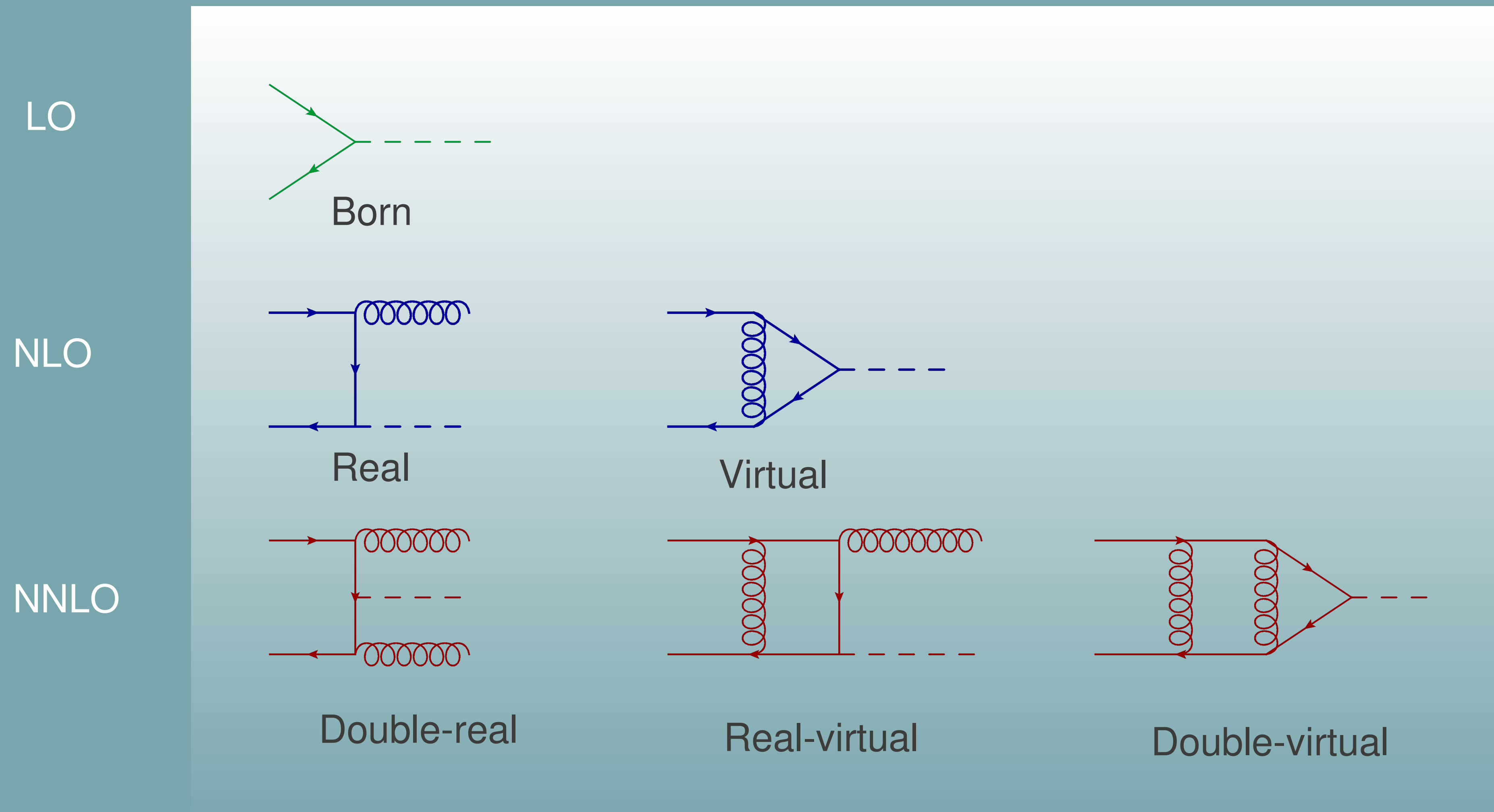


- At high energies QCD becomes weakly coupled, allowing for a perturbative expansion

$$\sigma = \sigma_{\mathrm{LO}} + \alpha_s \sigma_{\mathrm{NLO}} + \alpha_s^2 \sigma_{\mathrm{NNLO}} + \mathcal{O}(\alpha_s^3)$$

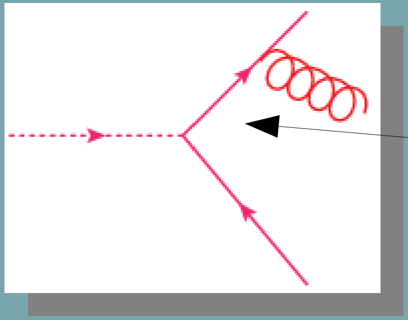
Example: bb to Higgs at NNLO

The coefficients of the expansion can be computed via Feynman Diagrams:



Infra-red singularities at NLO

- Consider the effect of QCD radiation



$$\frac{1}{(p_g + p_q)^2} = \frac{1}{2E_g E_q (1 - \cos\theta_{qg})} \to \infty$$

Soft

Collinear

- At **NLO**, infra-red (IR) singularities naturally factorise in energies and angles
- Factorised singularities can be dealt with using the plus-distribution expansion

Laurent expanding factorised singularities

$$\text{Dimension} = 4 - 2\epsilon$$

$$\int_0^1 dx\, x^{\epsilon} \frac{1}{x} = \frac{1}{\epsilon}$$

Allows to extract the pole for an arbitrary numerator function:

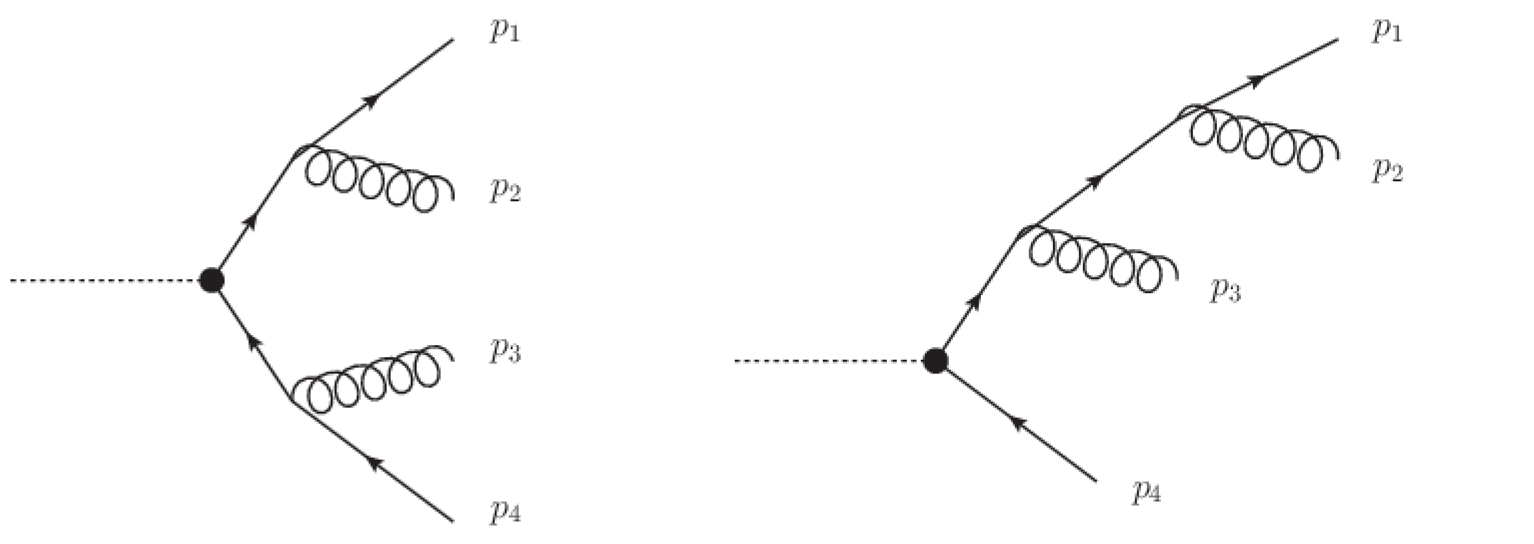
$$\int_0^1 dx\, x^{\epsilon} \frac{f(x)}{x} = \frac{f(0)}{\epsilon} + \int_0^1 dx\, x^{\epsilon} \frac{f(x) - f(0)}{x}$$

Infinite contribution

Finite contribution

Infra-red Singularities at NNLO

- Different ”topologies” correspond to different singularity structures



$$\sim \frac{1}{s_{12}s_{34}} = \frac{1}{(p_1+p_2)^2(p_3+p_4)^2}$$

$$\sim \frac{1}{s_{123}s_{12}} = \frac{1}{(p_1+p_2)^2(p_1+p_2+p_3)^2}$$

Infra-red Singularities at NNLO

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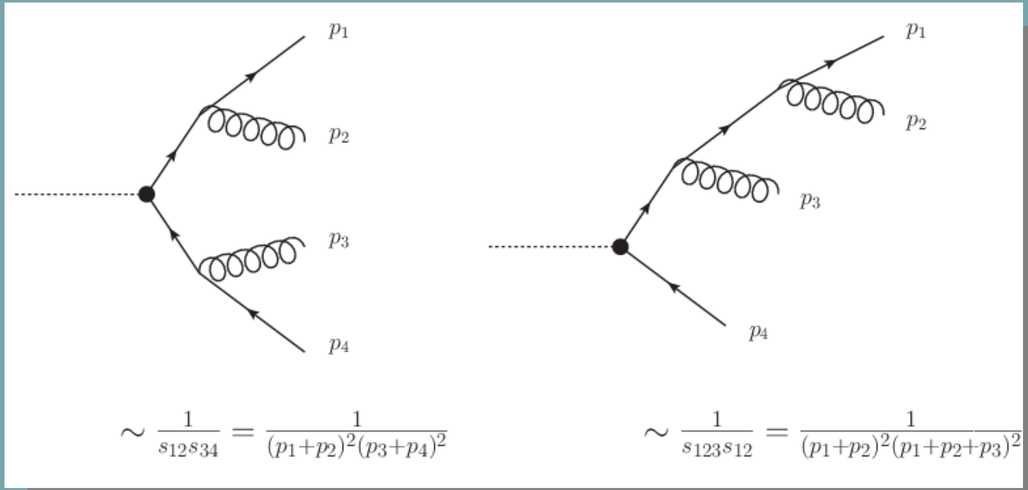


- Singularities do not always factorise in energies and angles e.g.

$$\frac{1}{s_{12}s_{123}} = \frac{1}{s_{12}(s_{12}+s_{23}+s_{31})}$$

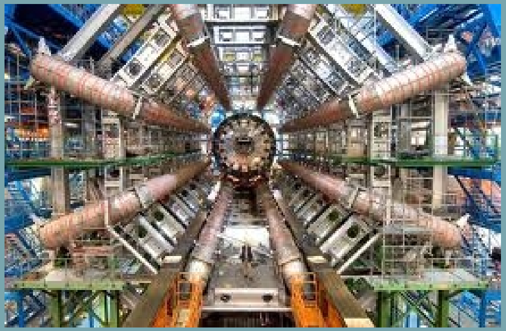
Overlapping singularities!

$$= \frac{1}{4E_1E_2(1-\cos\theta_{12})\Big(E_1E_2(1-\cos\theta_{12})+E_1E_3(1-\cos\theta_{13})+E_2E_3(1-\cos\theta_{23})\Big)}$$

Fully differential?

- Finite observables are sums of infrared divergent quantities:

$$\mathrm{FiniteObservable} = \sum_j \int_{\mathrm{boundaries}} \prod_i dp_i \mathrm{DivergentAmplitude}_j(\{p_k\})$$



- Integration boundaries must be kept arbitrary in order to simulate the experimental set-up realistically.

A historical perspective on IR subtraction

- Subtraction schemes for NLO: (fully differential works for all initial and final states)
  - 1991: phase space slicing [Glover, Giele,Kosower]
  - 1995: FKS (residue subtraction) [Firxione, Kunszt, Signer]
  - 1996: Dipole subtraction [Catani, Seymour]

A historical perspective on IR subtraction

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  - 1996: Dipole subtraction [Catani, Seymour]
- Subtraction for NNLO: (only e+e- and hadronic collisions)
  - 2003: *Sector Decomposition* [Anastasiou, Melnikov,Petriello]
    2003: e+e- $\to$ 2jets; 2005: pp(gg) $\to$ H; 2006: pp$\to$ W,Z;
  - 2005: Antennas [Glover, Gehrmann, Gehrmann-De Ridder] (fairly general)
    2007: e+e- $\to$ 3jets
  - 2006: subtraction scheme for jets [Somogyi, Trocsanyi, Del Duca] (a general proposal!)
  - 2007: Kt-subtraction:[Catani, Grazzini] (color singlets)
    2007: pp(gg) $\to$ H; 2011 pp $\to$WH; 2012 pp $\to$ diphoton;
  - 2010: General subtraction with sector decomposition [Czakon] (proposal, With proof of concept for top anti-top production)
  - 2010: *Non-linear Mappings* [Anastasiou, FH, Lazopoulos]: (massive colored final states and color-singlets) 2012: pp(bb) $\to$ H

Example with Sector Decomposition:

- Consider an overlapping divergence at x=0=y:

$$I = \int_0^1 dx dy \frac{x^\epsilon}{x(ax+y)}$$

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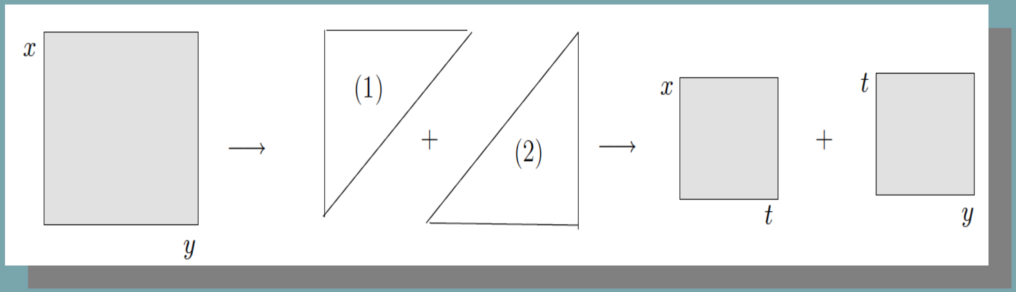

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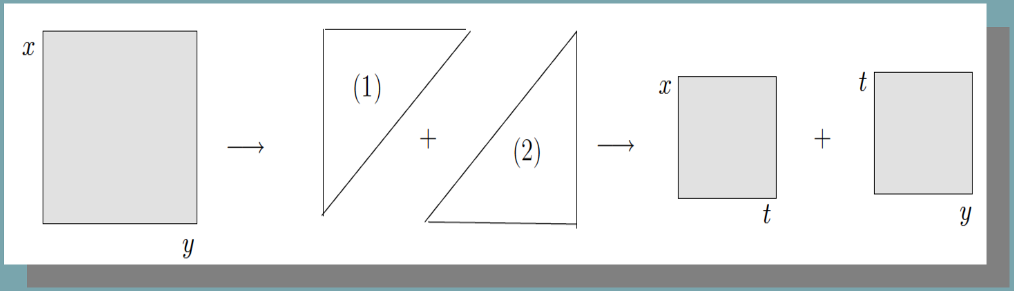

- Rescale

$$I = \int_0^1 dx dt \frac{(x)^\epsilon}{x(a+t)} + \int_0^1 dt dy \frac{(yt)^\epsilon}{yt(at+1)}$$

$$y = tx \qquad\qquad x = ty$$

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- We reconsider

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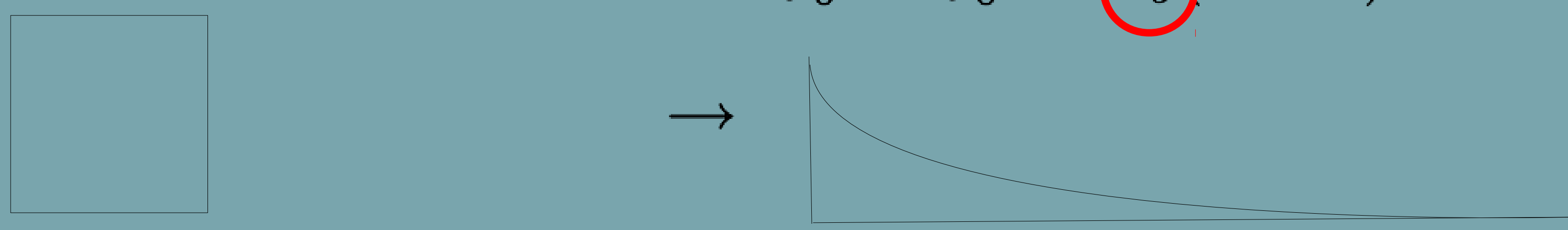
$$x \longrightarrow xy \qquad \mapsto \int_0^1 dy \int_0^{\frac{1}{y}} dx \frac{(xy)^\epsilon}{xy(ax+1)}$$



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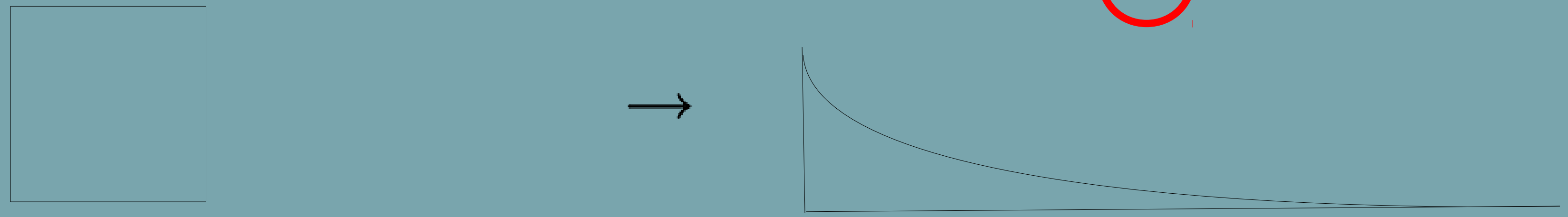

- to remap boundaries, use a nonlinear transformation

$$x \mapsto \frac{x(y/a)}{1-x+(y/a)} \qquad \mapsto \int_0^1 dx dy \frac{(xy)^\epsilon}{xy} \left(a(1-x)+y\right)^{-\epsilon}$$

Properties of the non-linear mapping

- The mapping can be expressed as:

$$x \mapsto \alpha(x, A, B) \equiv \frac{xA}{xA + \bar{x}B}$$

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$$\alpha(\alpha(x, A_1, B_1), A_2, B_2) = \alpha(x, A_1A_2, B_1B_2)$$

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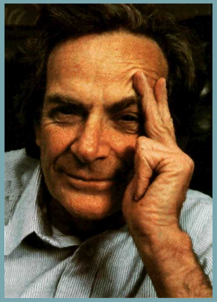
- Abelian composition law: $$\alpha(\alpha(x, A_1, B_1), A_2, B_2) = \alpha(x, A_1 A_2, B_1 B_2)$$
- Inverse: $$\alpha^{-1}(x, A, B) = \alpha(x, B, A)$$
- The mapping represents an abelian group!

Properties of the non-linear mapping

- The mapping $x \mapsto \alpha(x, A_1, A_2)$ can be used to derive Feynman's identity:



$$\int_0^1 dx \frac{1}{[xA_2 + \bar{x}A_1]^2} \mapsto \frac{1}{A_1 A_2} \int_0^1 dx = \frac{1}{A_1 A_2}$$

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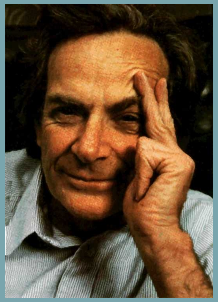

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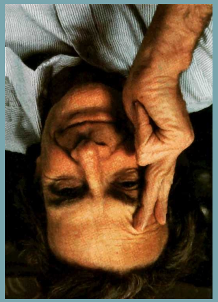
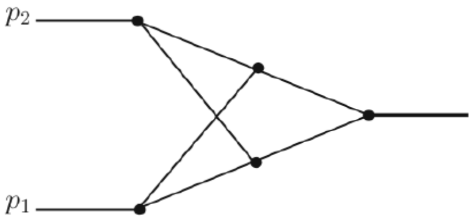
- This result can be generalised to what I shall term *the Fenyman Inversion trick:*

$$\int_0^1 dx \frac{f(x)}{[xA_1 + \bar{x}A_2]^n} \mapsto \int_0^1 dx \frac{[xA_2 + \bar{x}A_1]^{n-2}}{A_1^{n-1} A_2^{n-1}} f(\alpha(x, A_2, A_1))$$



A more complicated example

- A Feynman parameterisation of the non-planar massless two-loop triangle:



$$p_{12} = \Gamma(2+2\epsilon)\int_0^1 dx_1 dx_2 dx dy dz \frac{z\bar{z}^{-1-\epsilon}y^{1+\epsilon}\bar{y}^{-1-\epsilon}}{\left[x\bar{x}(1-yz)+yz(x\bar{x}_1\bar{x}_2+\bar{x}x_1x_2)\right]^{2+2\epsilon}}$$

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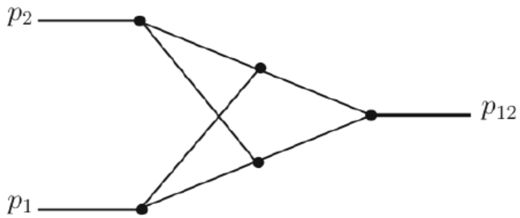

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- Shrink the product yz into a single variable via

$$y \mapsto \alpha(y, 1, \bar{z})$$
$$z \mapsto \alpha(z, 1, y)$$

Obtain:

$$= \Gamma(2+2\epsilon)\int_0^1 dx_1 dx_2 dx dy dz \, \frac{y^{1+\epsilon}(z\bar{z})^{-1-\epsilon}\bar{y}^{-1-2\epsilon}}{\left[x\bar{x}\bar{y} + y(x\bar{x}_1\bar{x}_2 + \bar{x}x_1x_2)\right]^{2+2\epsilon}}$$

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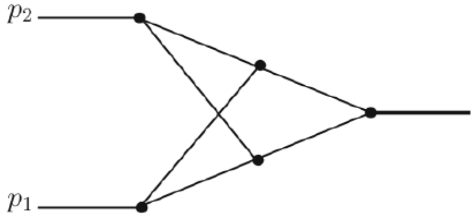

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$$\begin{aligned} y &\mapsto \alpha(y, 1, \bar{z}) \\ z &\mapsto \alpha(z, 1, y) \end{aligned}$$

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- Notice the form of the denominator is $yA_1 + \bar{y}A_2$

A more complicated example

- The *Feynman inversion trick* thus leads us to apply

$$y \mapsto \alpha\big(y, x\bar{x}, x\bar{x}_1\bar{x}_2 + \bar{x}x_1x_2\big)$$

- We obtain

$$\begin{aligned} = \quad & \Gamma(2+2\epsilon)\int_0^1 dx_1 dx_2 dx_3 dy dz\, y^{1+\epsilon}\bar{y}^{-1-2\epsilon}(z\bar{z})^{-1-\epsilon}(x\bar{x})^{-\epsilon} \qquad (4. \\ & \times \quad \left\{x\bar{x}y + \bar{y}(x\bar{x}_1\bar{x}_2 + \bar{x}x_1x_2)\right\}^{3\epsilon}\left[x\bar{x}_1\bar{x}_2 + \bar{x}x_1x_2\right]^{-2-4\epsilon} \end{aligned}$$

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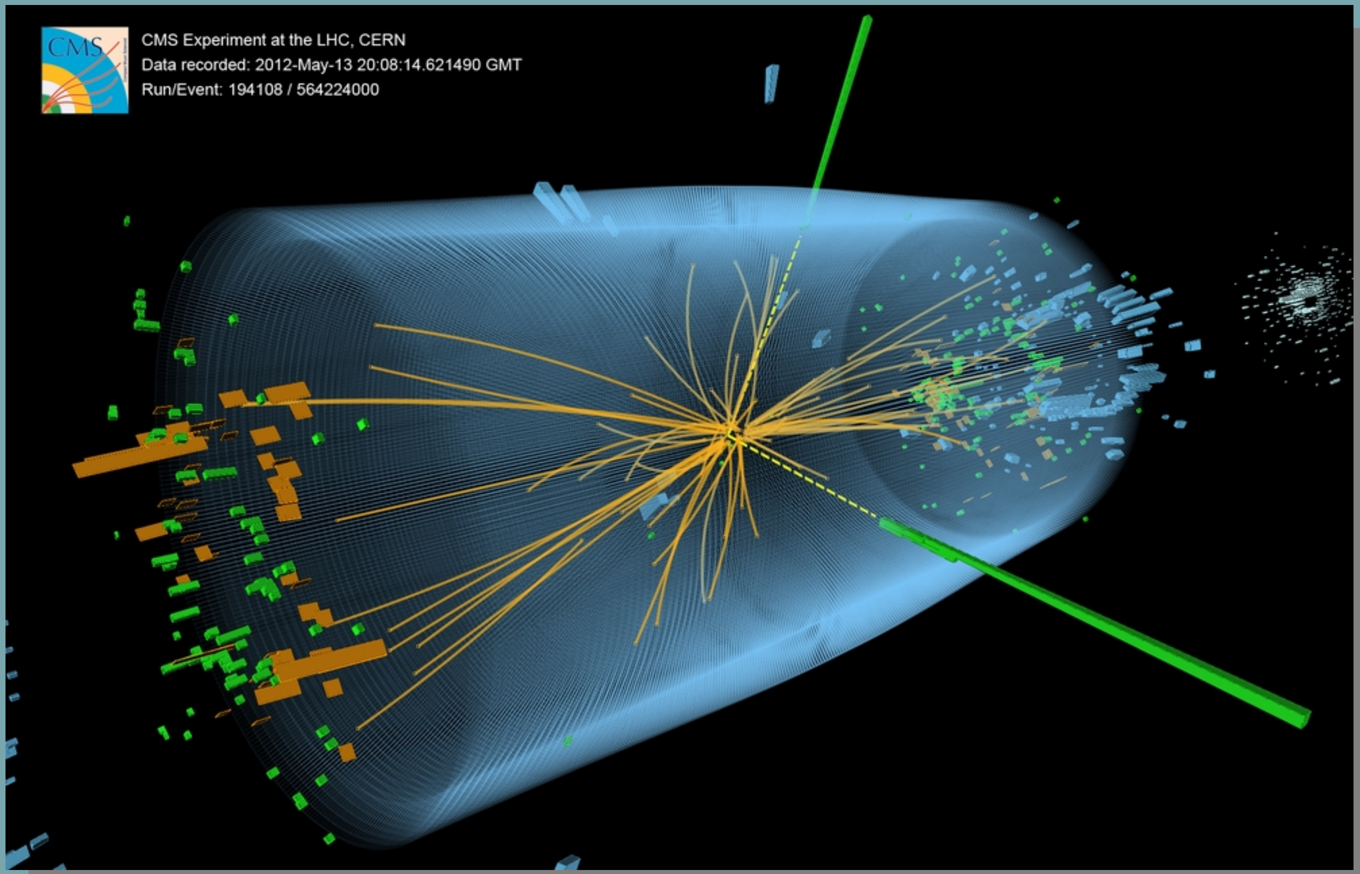
- The singular denominator is (again!) of the form $x A + \bar{x} B$ and can be fully factorised with $x \mapsto \alpha(x, x_1x_2, \bar{x}_1\bar{x}_2)$. We obtain

$$\begin{aligned} I &= \Gamma(2+2\epsilon)\int_0^1 dx_1dx_2dxdydz\, y^{1+\epsilon}\bar{y}^{-1-2\epsilon}(z\bar{z})^{-1-\epsilon}(x\bar{x})^{-\epsilon} \qquad (4.43) \\ &\times \quad (x_1\bar{x}_1)^{-1-2\epsilon}(x_2\bar{x}_2)^{-1-2\epsilon}\left\{x\bar{x}y + \bar{y}(xx_1x_2 + \bar{x}\bar{x}_1\bar{x}_2)\right\}^{3\epsilon}, \end{aligned}$$

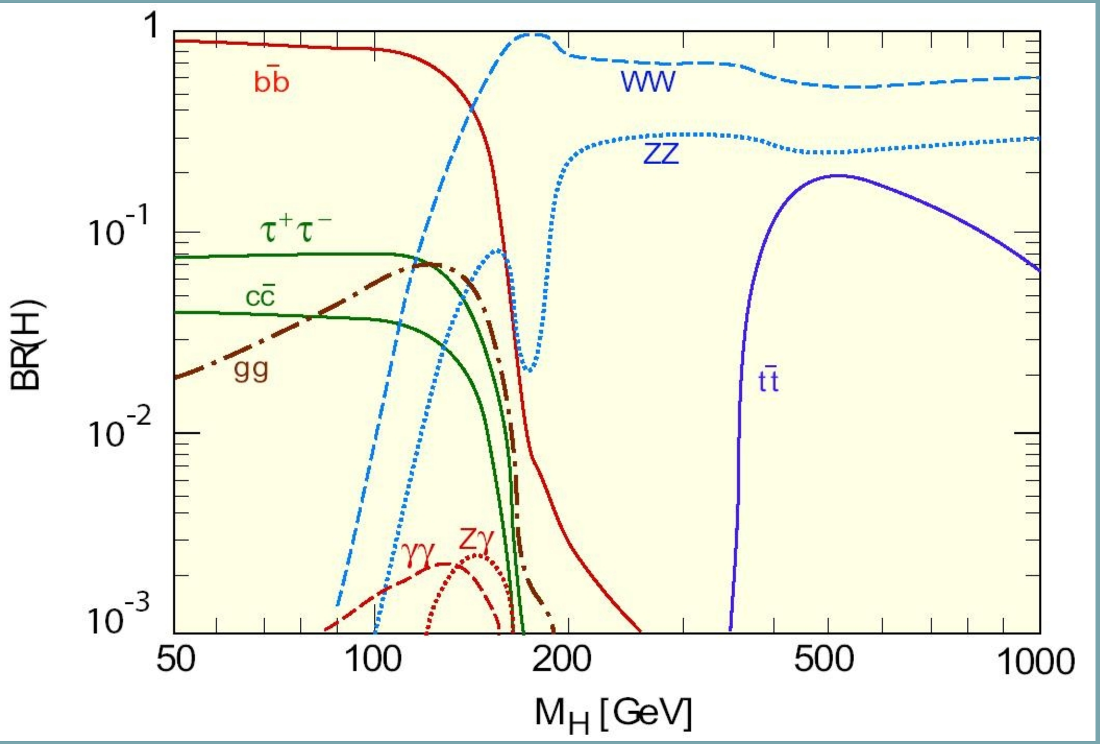
64 sectors with Sector Decomposition!

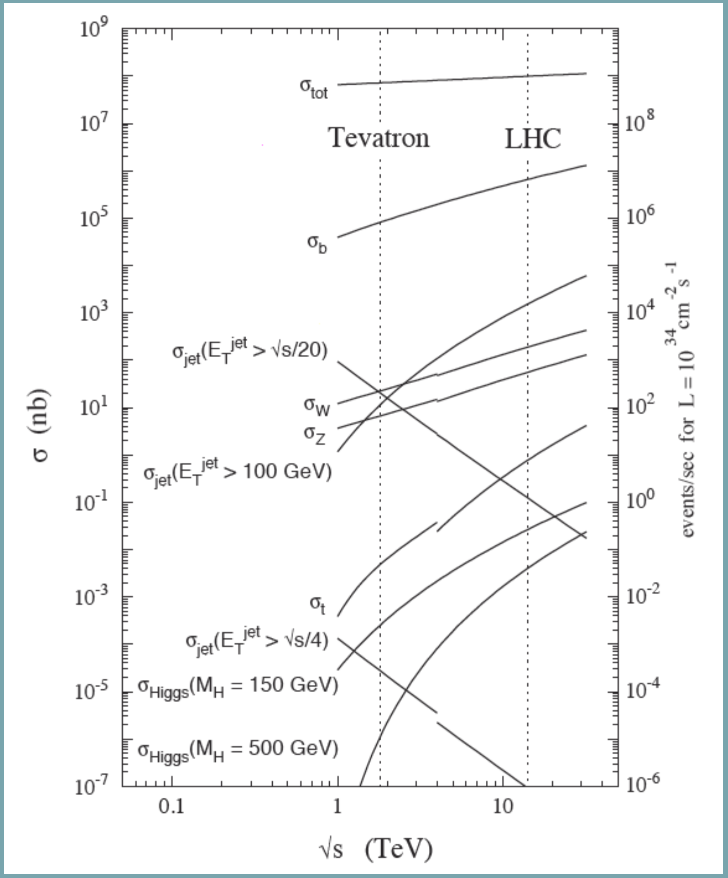


Applications of the Non-linear Mapping approach to LHC observables

Higgs decay into bottom quarks

- DOMINANT Decay for the light Higgs.

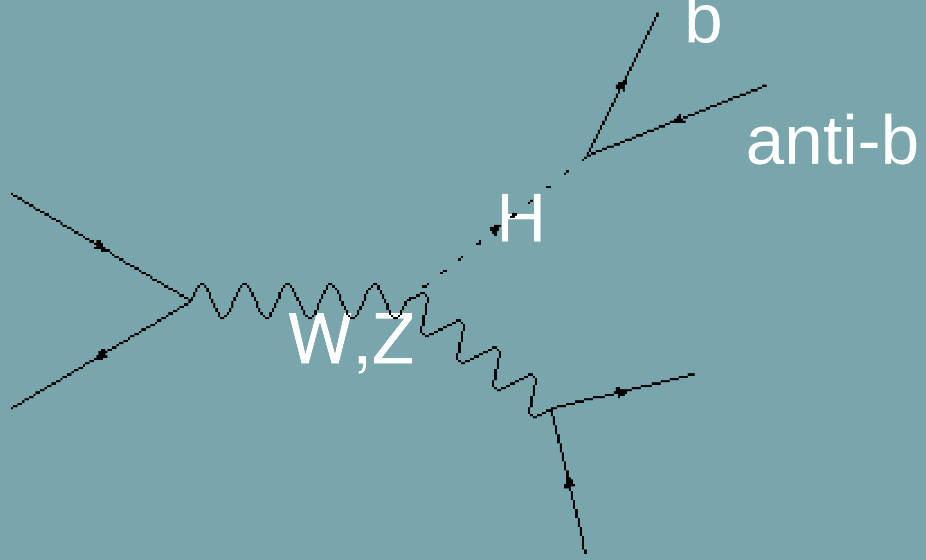




But still impossible to see in the b anti-b backgound!

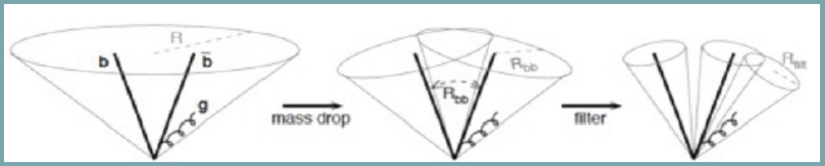
Only chance is associated Higgs

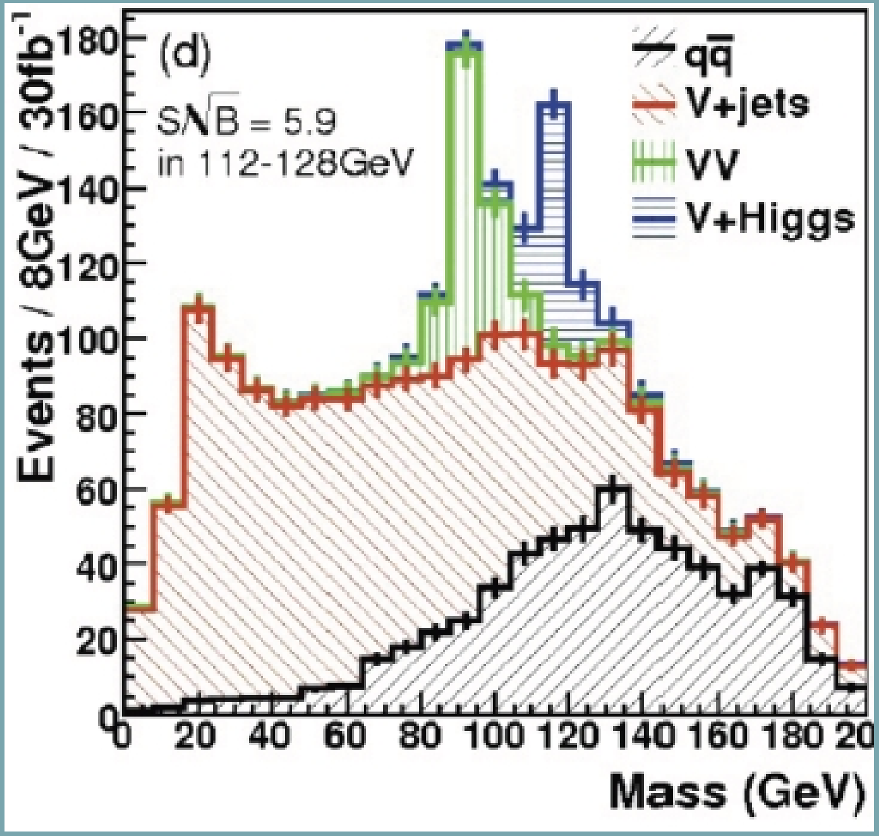
- A good chance is to search in associated production: HW, HZ, Httbar



- This is still not enough cross section to overcome dijet, single top and VV production backgrounds, even at the 14 TeV LHV with 30 inverse fb.

FAT Jet Analysis

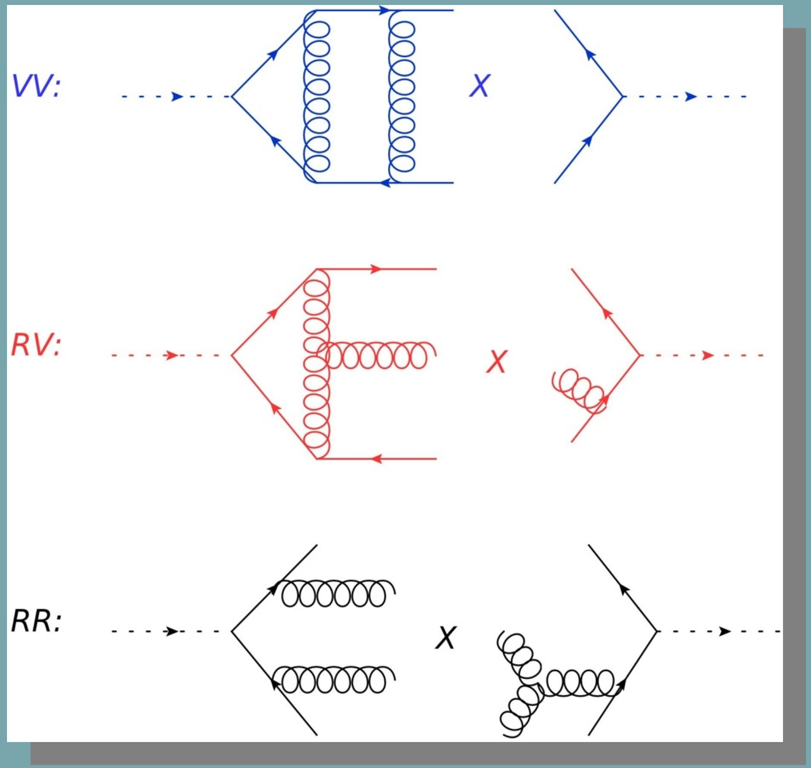




- Uses highly boosted events to cut out major background
- Gluon radiation off bottom quarks is important for fat jet analysis [Butterworth, Davison, Rubin, Salam]
- Search strategy relies on **higher order effects**!

1st fully differential computation of H to bottom quarks @ NNLO

- DOUBLE VIRTUAL:
  Easily done analytically (use AIR, Master Integrals known since 1987)
- REAL-VIRTUAL:
  Nontrivial overlapping singularities between virtual and real
- DOUBLE REAL:
  Difficult overlapping structures, perfect testing ground for non-linear mapping



Differential Results for H to bb at NNLO

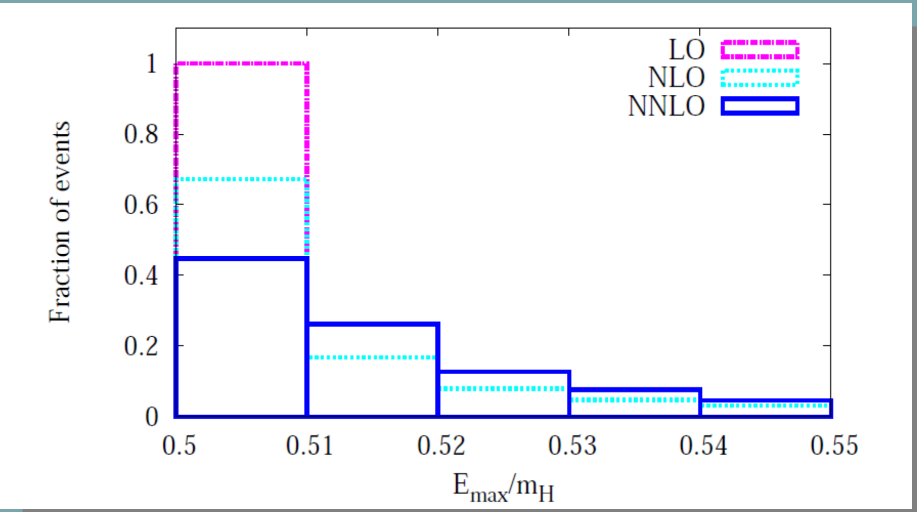
- Jet rates with the JADE jet algorithm:

$$\Gamma^{NNLO}_{H\to b\bar{b}}(\text{2JetRate}) = \Gamma^{LO}_{H\to b\bar{b}}\left[1 - \left(\frac{\alpha_s}{\pi}\right)13.591(6) - \left(\frac{\alpha_s}{\pi}\right)^2 307(2) + \mathcal{O}(\alpha_s^3)\right]$$

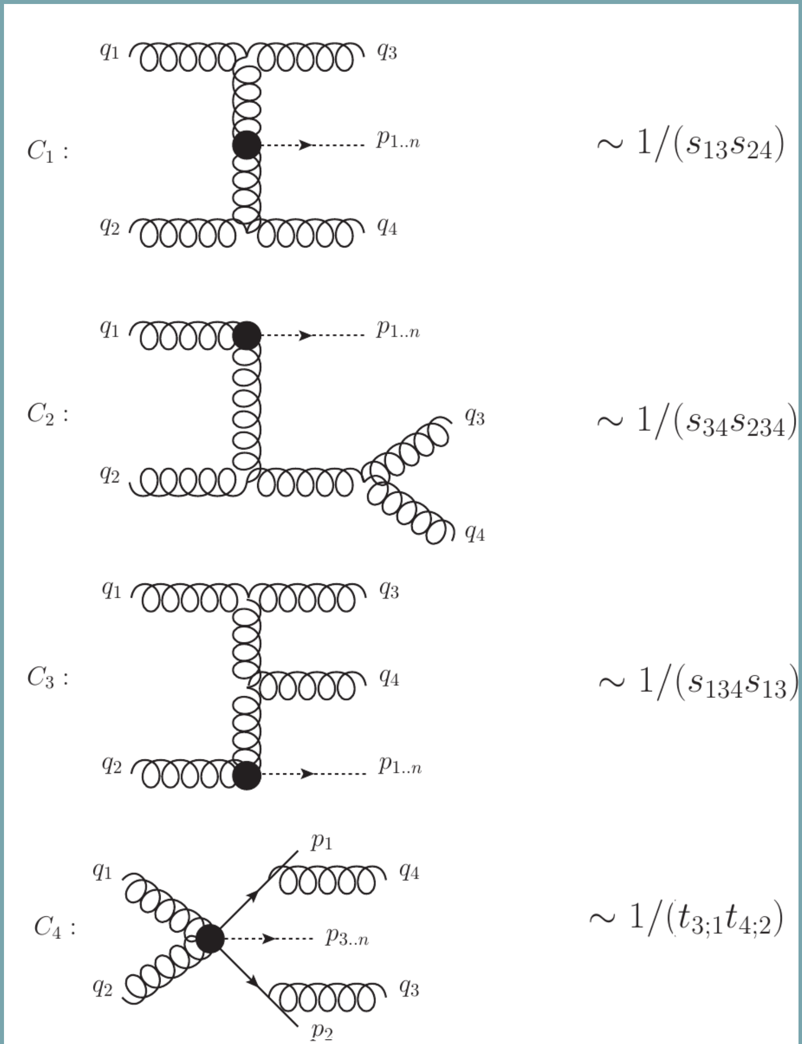
$$\Gamma^{NLO}_{H\to b\bar{b}}(\text{3JetRate}) = \Gamma^{LO}_{H\to b\bar{b}}\left[+\left(\frac{\alpha_s}{\pi}\right)19.258(4) + \left(\frac{\alpha_s}{\pi}\right)^2 241(2) + \mathcal{O}(\alpha_s^3)\right]$$

$$\Gamma^{LO}_{H\to b\bar{b}}(\text{4JetRate}) = \Gamma^{LO}_{H\to b\bar{b}}\left[+\left(\frac{\alpha_s}{\pi}\right)^2 94.1(1) + \mathcal{O}(\alpha_s^3)\right]$$

- The maximum energy of the leading jet in the 2 jet rate:



Double real corrections to massive final states in hadronic collisions



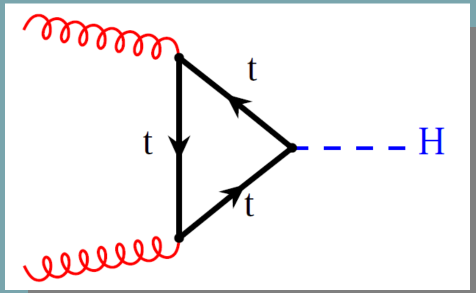
- Developed a strategy to factorise all interferences of singular denominators.
- The method is applicable to a large number of final states, including:

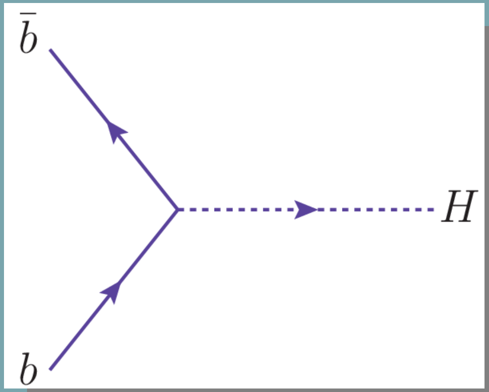
  pp $\rightarrow$ H, W, Z, WH, ZH, WW, ZZ, ttbar, HH,...

- The method could be extended to include also jets in the final states

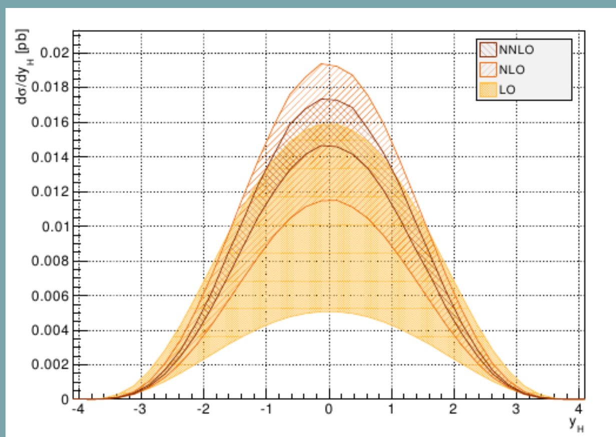
Higgs production in bottom quark fusion

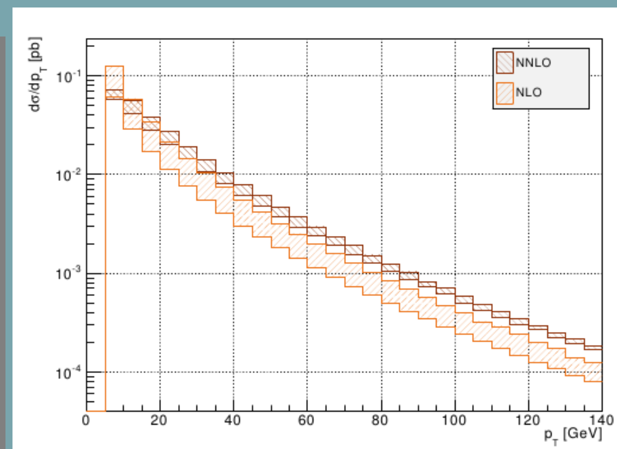
- In the Standard Model gluon fusion is dominant
- BSM models (e.g. 2HDM, MSSM, dynamical yukawas) can enhance the coupling between bottom quarks and the Higgs boson
- The bottom fusion channel can (still) become dominant
- Motivates a fully differential NNLO computation

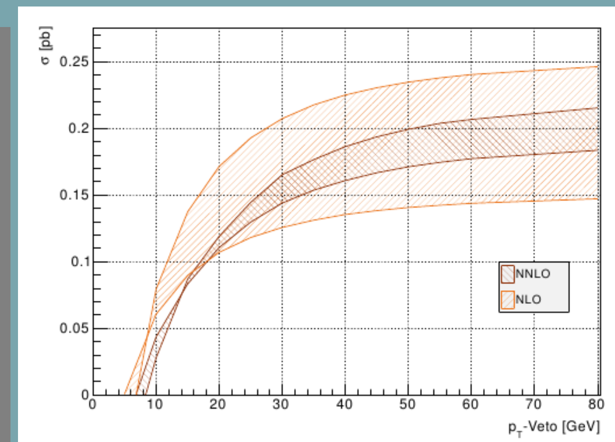


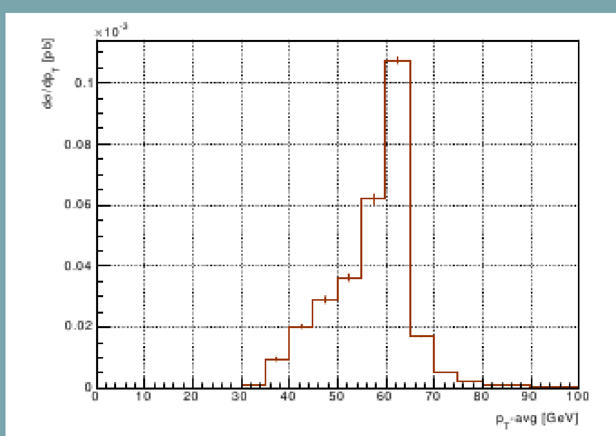


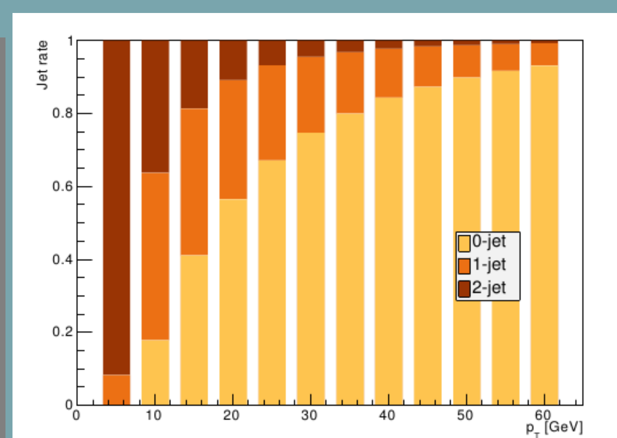
1st fully differential Results for bb to H at NNLO

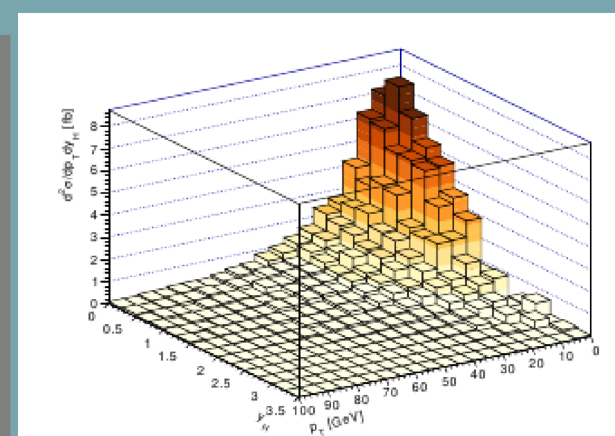












Summary

- Established a method for the factorisation of overlapping singularities via a non-linear mapping
- Showed that the method works for maximally singular loop integrals
- Derived a method for double real corrections for the hadronic production of massive states and color singlets
- Computed the fully differential H to bb decay rate at NNLO
  - Presented jet rates and the distribution Energy of the leading jet
- Computed the fully differential bb to H cross section at NNLO
  - Presented a number of distributions

Conclusions & Outlook

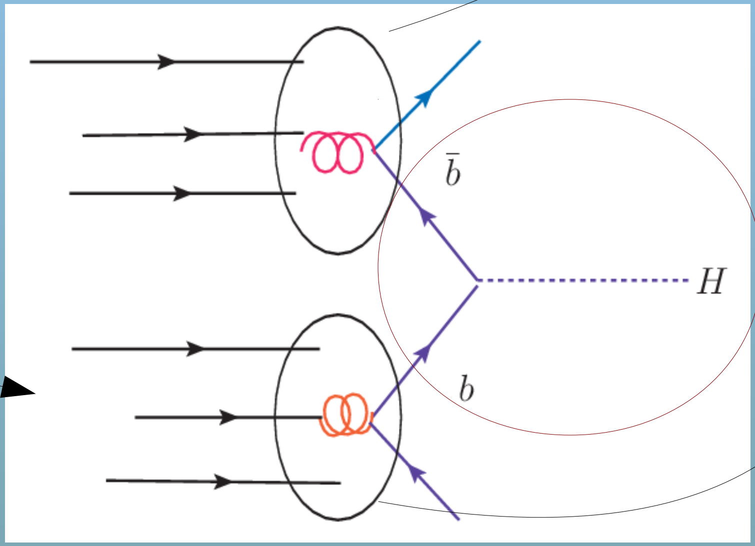
- Our non-linear mapping is well suited for factorising singularities at NNLO and beyond
- We found that the numerical convergence of observables was surprisingly fast and believe that the method is more efficient than the sector decomposition approach
- We are looking forward to combine our code for H to bb with a code for the production of a Higgs boson in the associated production
- For the bb to H the future could be bright if the Higgs sector turns out not to be minimal, i.e. enhanced Yukawa couplings etc

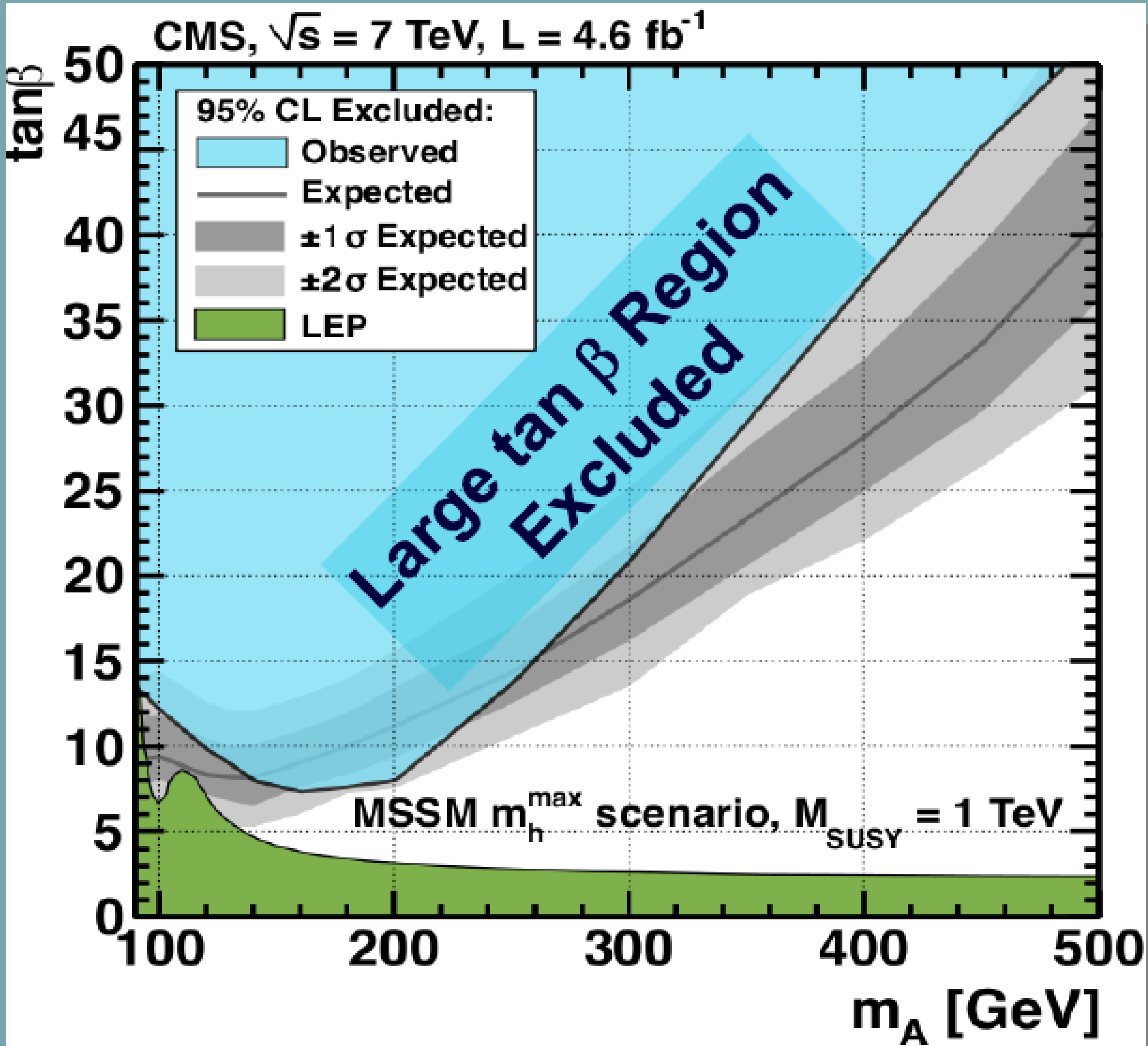
Hadronic cross section for bb to H

- The hadronic cross section is a convolution of parton distribution functions and the partonic cross-section:

$$\sigma_{P_1 P_2 \to H+X}\,[\mathcal{J}] \;=\; \sum_{i_1,i_2} \int_0^1 dx_1 dx_2\, \theta(x_1 x_2 - \tau) f_{i_1}(x_1) f_{i_2}(x_2) \sigma_{i_1 i_2 \to HX}[\mathcal{J}]$$

Colliding Protons





Limit obtained by scanning tan(β) for each Mass hypothesis M_A: