The LHC transverse coupled-bunch instability

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EPFL PhD thesis

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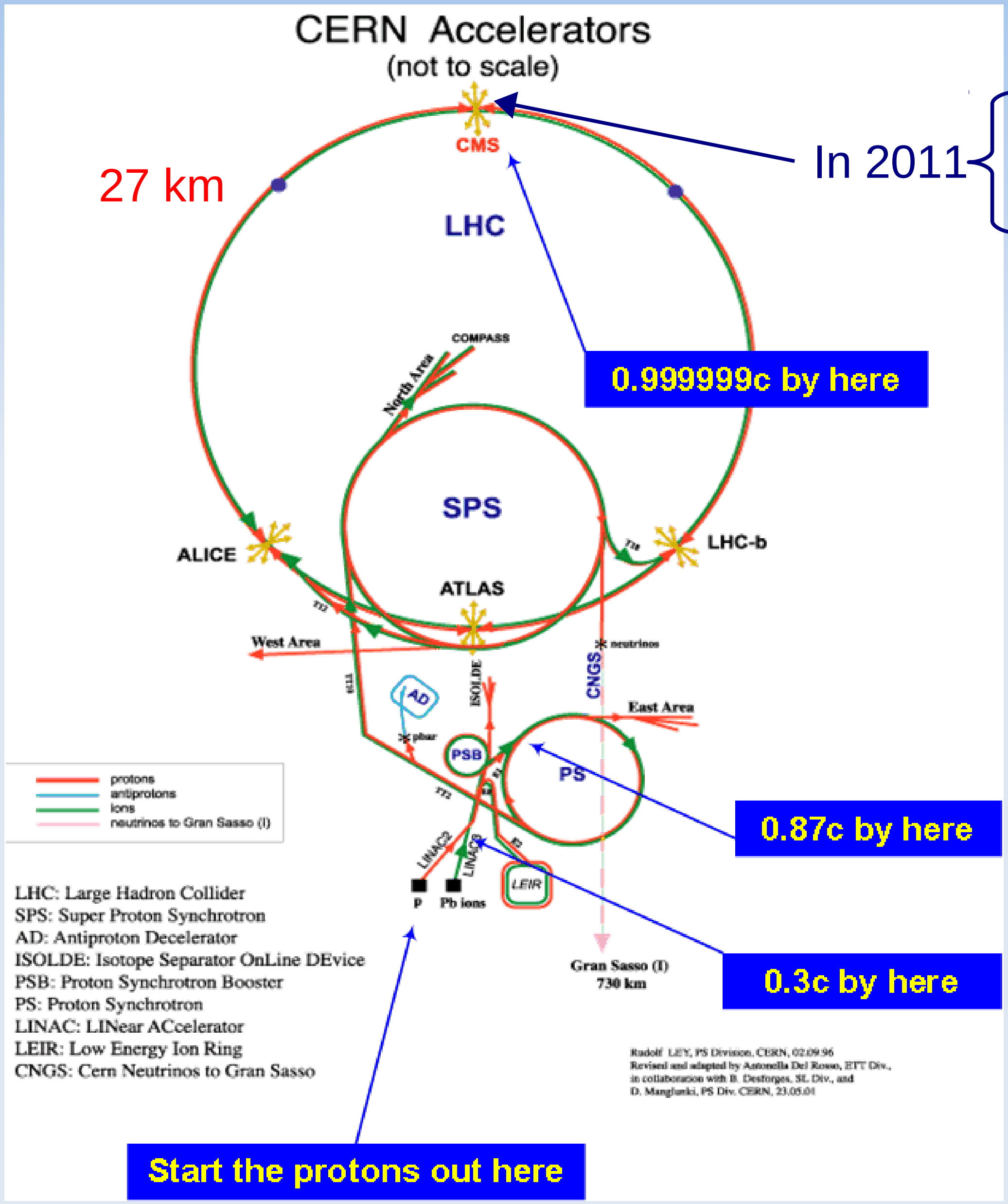
The LHC transverse coupled-bunch instability

- Context
- About impedances and wake fields
- The LHC impedance model
- A new multibunch simulation tool
- Instabilities in the LHC and comparison with experiments

The Large Hadron Collider (LHC)



In 2011
- 3.5 TeV/proton
- 2.10^{14} protons per beam
- 25 µm size

Aim: study very rare phenomena

→ requires high-density and high-intensity beams.

Beam dynamics in the LHC

- Beams guided along an orbit thanks to dipole magnets, and focused transversally thanks to quadrupole magnets.

  → in transverse, at first order, charge q of momentum p_0 governed by Hill's equation:

$$\frac{d^2 x}{ds^2} + K(s)\, x = 0 \Rightarrow x(s) = \sqrt{\underbrace{\epsilon_x}_{\text{Emittance}} \underbrace{\beta_x(s)}_{\text{Beta function}}} \cos\left(\overbrace{\mu_x(s)}^{\text{Phase function}} - \mu_{x_0}\right)$$

s → longitudinal coordinate along the orbit
x → transverse coordinate
v → velocity along the orbit

Tune Q_{x0} = number of oscillations per turn = $\mu_x(\mathit{circumference})/(2\pi)$.

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- Beams also accelerated with a sinusoidal electric field (in RF cavity)
  → creates longitudinal motion & momentum deviation $\delta = (p-p_0)/p_0$
  → for each particle, different tune: $Q_x = Q_{x0} + Q'_x\delta$

Beam dynamics in the LHC

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  Phase function

  $$\frac{d^2x}{ds^2} + K(s)x = 0 \;\Rightarrow\; x(s) = \sqrt{\epsilon_x \beta_x(s)} \cos\left(\mu_x(s) - \mu_{x_0}\right)$$

  Emittance Beta function

  s → longitudinal coordinate along the orbit
  x → transverse coordinate
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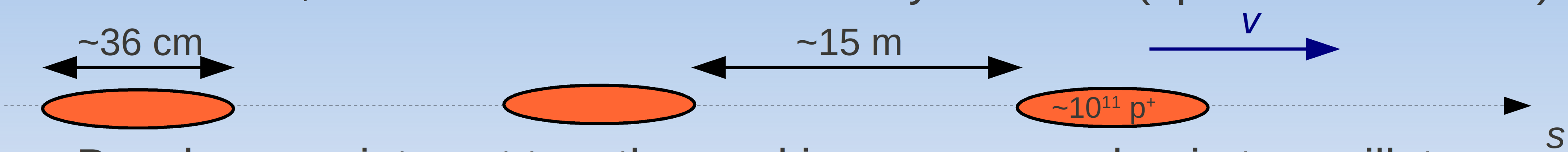
- This picture is valid for single particle in the ring. In reality, we have to take into account collective effects → particles are interacting

  $$\frac{d^2x}{ds^2} + K(s)x = \frac{F_x}{p_0 v}$$

  Force due to other particles, increasing with number of particles

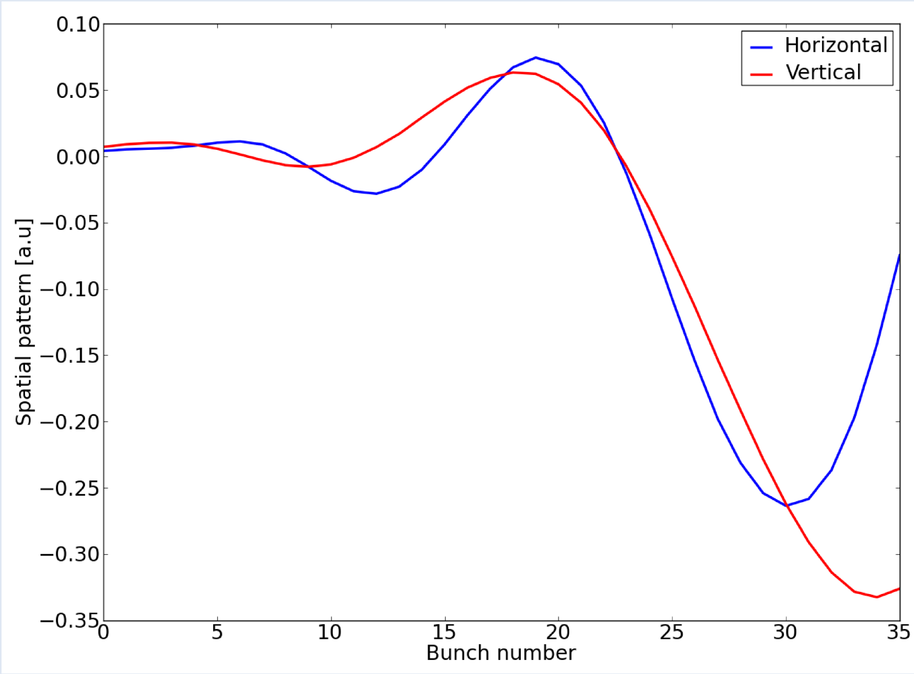
Coupled-bunch instabilities

- In the LHC, the beams are made of many bunches (up to 1380 in 2011)



Bunches can interact together and in some cases begin to oscillate.

Example with 36 bunches in the LHC: oscillation pattern along the bunch train (simulation result):

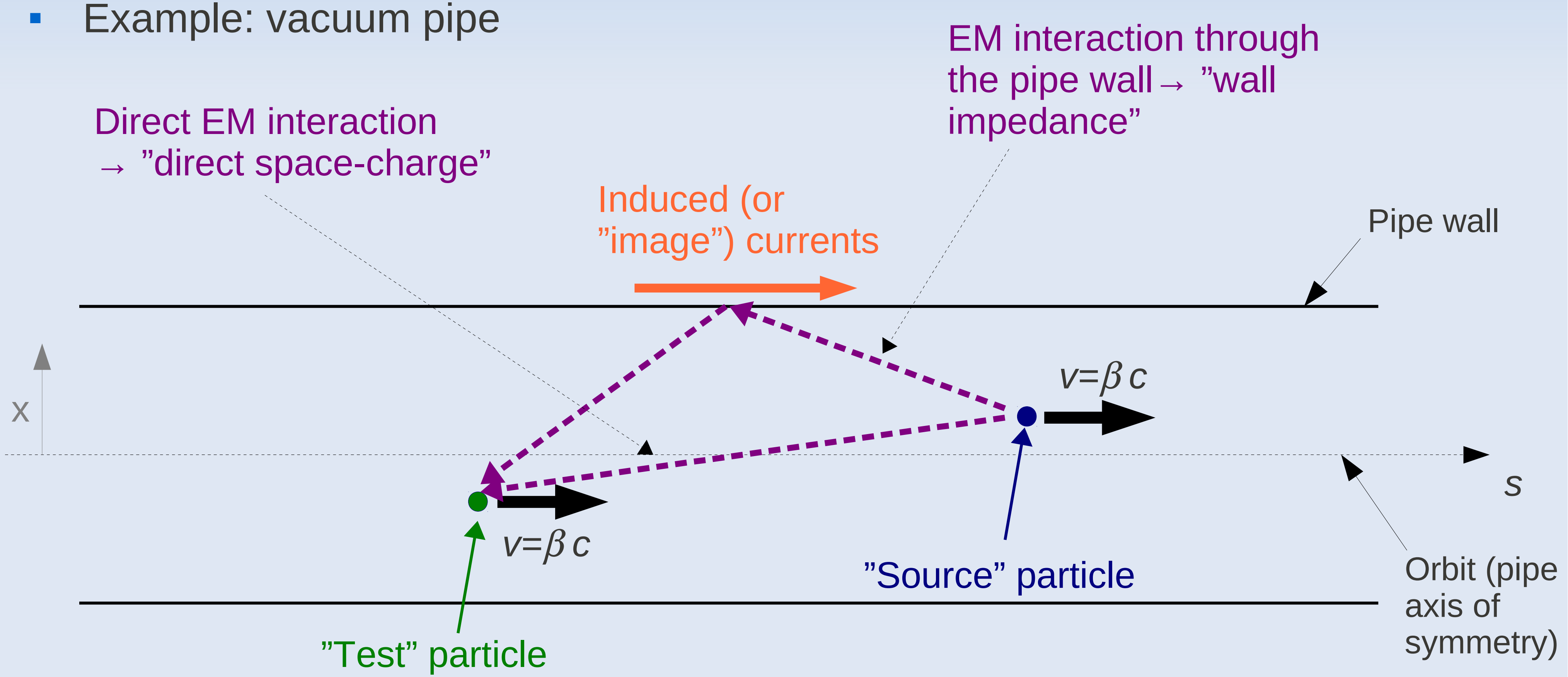


→ **Coupled-bunch instabilities**

- Must be damped by feedback system and/or Landau damping (otherwise beams are lost).
- Important to study them to know if damping mechanisms are sufficient.

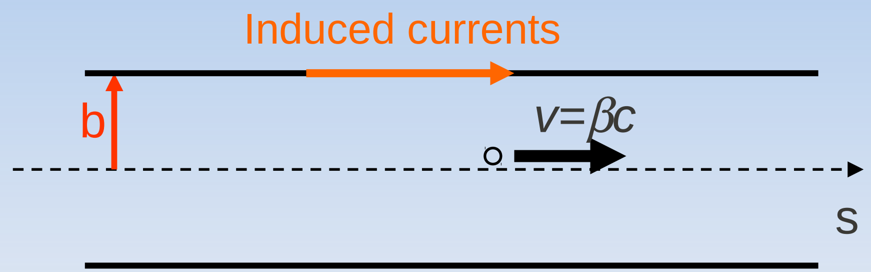
Wake fields and impedances

- One source of instabilities are self-generated fields: electromagnetic (EM) fields created by a beam particle inside a structure (vacuum pipe, cavity, collimator, etc.), and felt by another particle.
  → results in an EM force, called wake field in time domain, beam-coupling impedance in frequency domain.
- Example: vacuum pipe



What is so particular about the impedance of the LHC collimators ?

- Classic approach for a ”cylindrical” collimator: classic thick wall formula



If wall thickness and radius *b* >> skin depth of the conductor $\delta = \sqrt{\frac{2}{\sigma\omega\mu_0\mu_r}}$

then the image currents are almost at the boundary and one obtains the transverse wall impedance as (see e.g. Chao)

$$Z_x^{\mathrm{wall}}(\omega) = \frac{jLZ_0}{2\pi b^2\beta\gamma^2} + (1+j)\beta\frac{LZ_0\delta}{2\pi b^3}$$

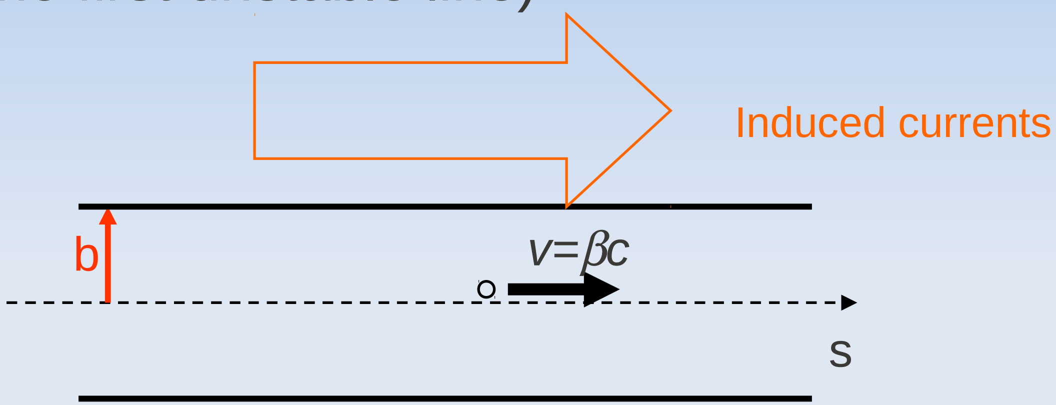
L= resistive length
μ_r= permeability
$Z_0 = \mu_0 . c$

Indirect space-charge, small in the LHC

Resistive-wall impedance → goes as $1/b^3$ → can be very large in narrow collimators (b~2mm)

Wall impedance in the LHC collimators

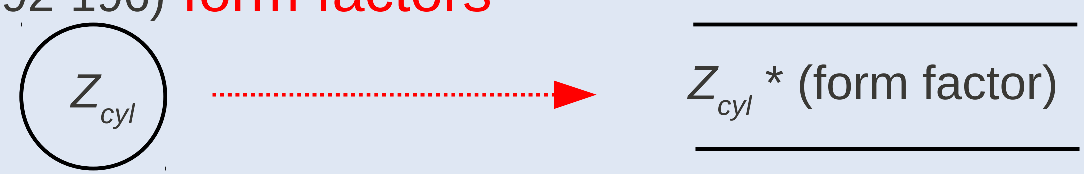
- BUT in the case of graphite collimators, $b<\delta$ at low frequency (LHC: 8 kHz is the first unstable line)



  - Classical "skin-depth" approximation breaks down, needs a more general theory.

- From cylindrical to flat (collimator-like) geometries, use usually "Yokoya" (KEK 92-196) form factors

Cross sections:

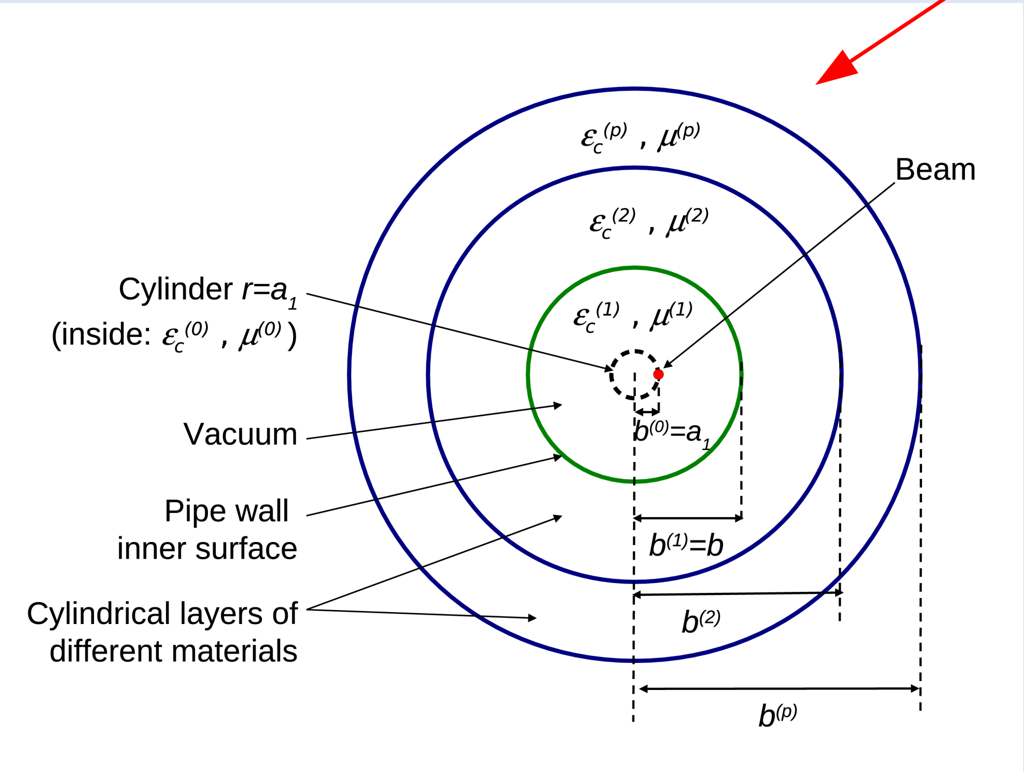


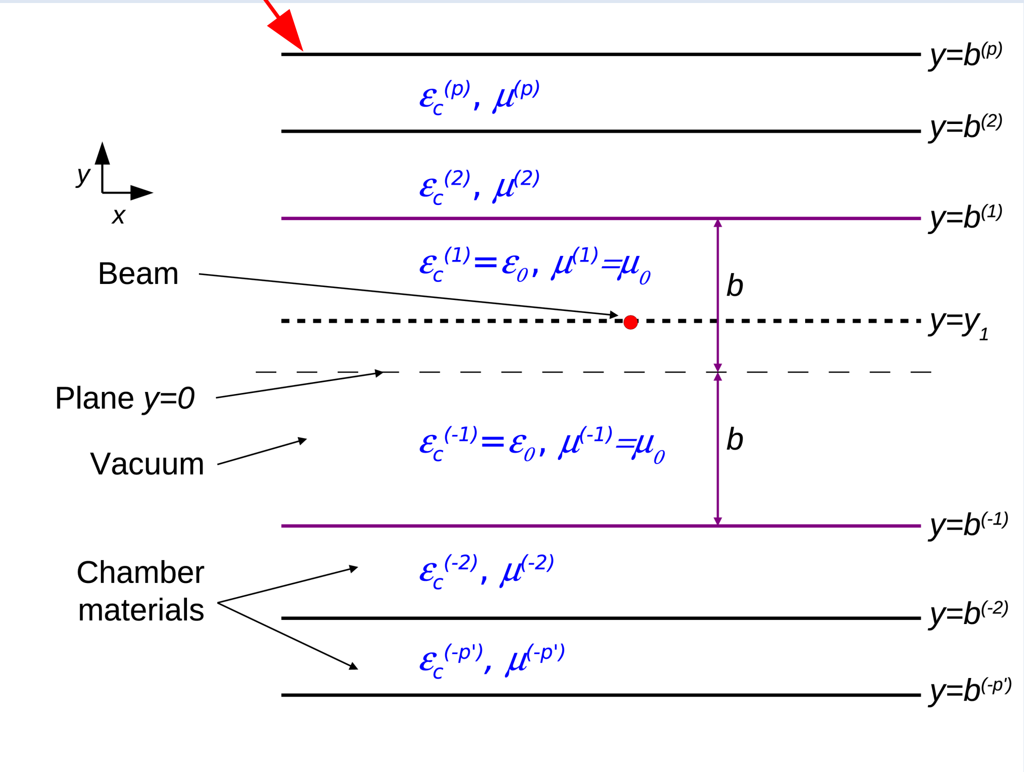
→ also relies on skin-depth approximation, again a more general theory is required.

Wall impedance: theory

- 2D models: consider a longitudinally smooth element in the ring, of infinite length, and integrate the EM force from the source particle to the test particle, over a finite length.

  ⇒ Neglect thus all edge effects.
- Main advantage: for simple geometries, EM fields obtained (semi-) analytically without any other assumptions (except linearity, isotropy and homogeneity).
- Cross sections studied: multilayer axisymmetric and flat chambers





Wall impedance: theory

- Essence of the formalism initially from B. Zotter (1969), in axisymmetric only.
- Start from Maxwell equations in frequency domain:

$$\begin{aligned} \mathrm{div}\vec{D} &= \rho, \\ \vec{\mathrm{curl}}\,\vec{H} - j\omega\vec{D} &= \vec{J}, \\ \vec{\mathrm{curl}}\,\vec{E} + j\omega\vec{B} &= 0, \\ \mathrm{div}\vec{B} &= 0, \end{aligned}$$

with

$$\begin{aligned} \vec{D} &= \varepsilon_c(\omega)\vec{E} = \varepsilon_0\varepsilon_1(\omega)\vec{E}, \\ \vec{B} &= \mu(\omega)\vec{H} = \mu_0\mu_1(\omega)\vec{H}, \end{aligned}$$

Complex permittivity

Complex permeability

and

$$\rho(r,\theta,s;\omega) = \frac{Q}{a_1}\delta(r-a_1)\delta_p(\theta-\theta_1)e^{-jks}$$

(point-like source)

- Playing with vector operations, get wave equations: e.g. for *E*

$$\nabla^2\vec{E} + \omega^2\varepsilon_c\mu\vec{E} = \frac{1}{\varepsilon_c}\vec{\mathrm{grad}}\rho + j\omega\mu\rho v\vec{e}_s$$

- Idea: decompose fields and source charge density thanks to Fourier transforms

e.g. axisymmetric

$$\rho(r,\theta,s;\omega) = \int_{-\infty}^{\infty} \mathrm{d}k' e^{-jk's}\delta(k'-k)\sum_{m=0}^{\infty}\frac{Q\cos(m\theta)}{\pi v a_1(1+\delta_{m0})}\delta(r-a_1)$$

Continuous Fourier transform

Fourier series decomposition

Wall impedance: theory

- Write wave equations for the longitudinal components E_s and H_s,then identify the terms (drop integrals and sums), obtaining second order differential equations. Solutions:
  - combination of modified Bessel functions of the radial coordinate *r*, in axisymmetric,
  - combination of exponentials of the vertical coordinate in the flat case.
- Transverse components obtained from the longitudinal ones, thanks to Maxwell eqs.
- Integration constants determined from field matching (continuity of tangential field components) between adjacent layers. Instead of solving the full system by ”brute force”, use analytical trick: relate constants between adjacent layers by 4 x 4 matrices:

  Constants (layer $p+1$) = M_p^{p+1} . constants (layer p)

  in the end:

  Constants (last layer) = M . constants (first layer).

  ⇒ Only need to multiply simple 4x4 matrices and do a final inversion, to get all the constants.
- Finally, put back the Fourier transforms and/or series. In flat case, additional algebra to get a simple form.

Wall impedance: theory

- Electric field longitudinal component in the vacuum:

axisymmetric

$$E_s^{vac} = Ce^{-jks}\left[K_0\left(\frac{k}{\gamma}\sqrt{a_1^2+r^2-2a_1 r\cos\theta}\right)-2\sum_{m=0}^{\infty}\frac{\alpha_{\mathrm{TM}}(m)\cos(m\theta)}{1+\delta_{m0}}I_m\left(\frac{ka_1}{\gamma}\right)I_m\left(\frac{kr}{\gamma}\right)\right]$$

Direct space-charge term Constants Wall term

flat

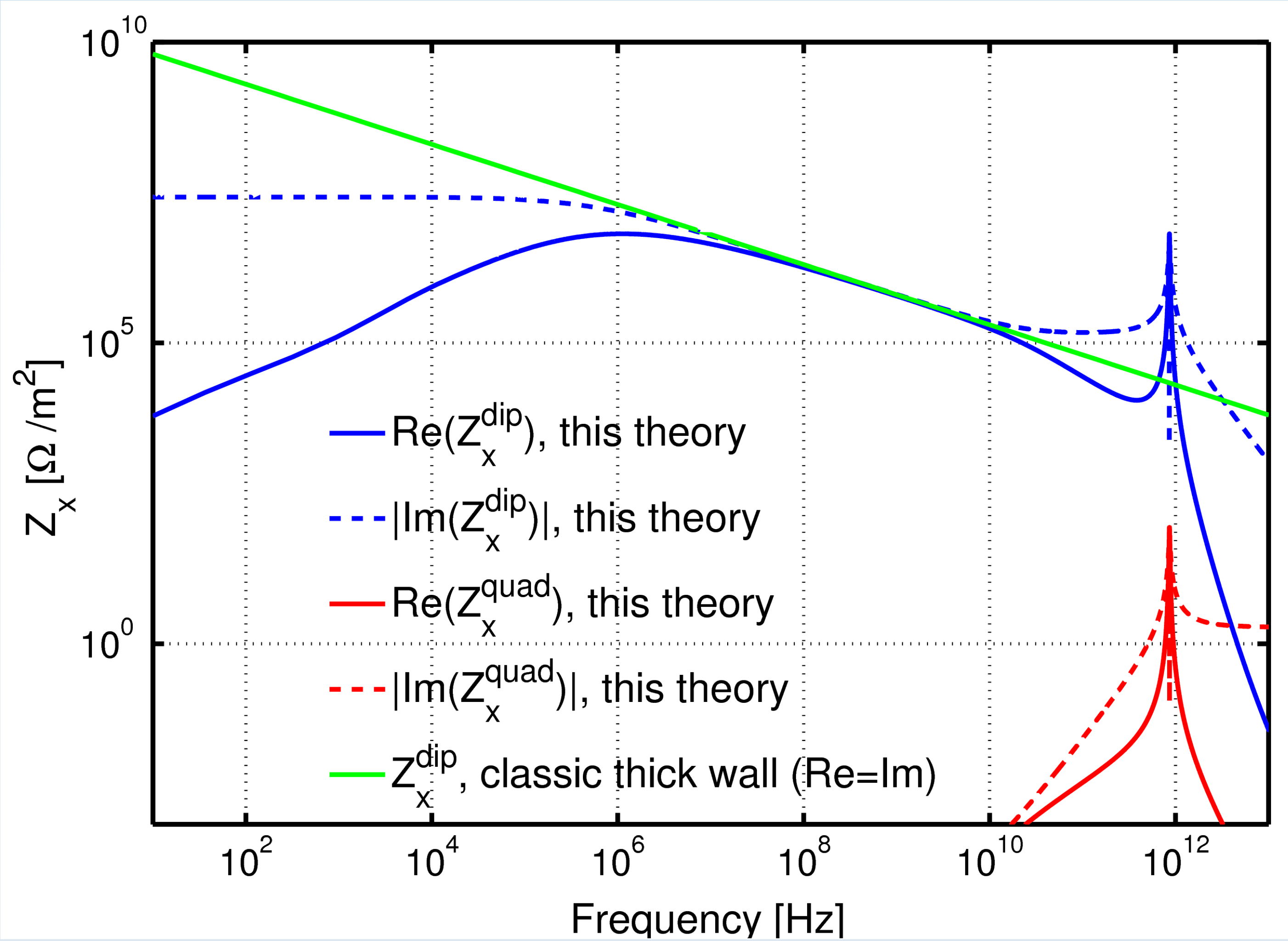
$$E_s^{vac} = Ce^{-jks}\left[K_0\left(\frac{k}{\gamma}\sqrt{x^2+(y-y_1)^2}\right)-4\sum_{m,n=0}^{+\infty}\frac{\alpha_{mn}\cos\left(n\theta-\frac{n\pi}{2}\right)}{(1+\delta_{m0})(1+\delta_{n0})}I_m\left(\frac{ky_1}{\gamma}\right)I_n\left(\frac{kr}{\gamma}\right)\right]$$

In the "wall term": only first terms of the sums are relevant when sufficiently close to the orbit → linear terms ($m\leq 1$, $n\leq 2$).

- From E_s we can get the EM force in vacuum, then upon integration over a finite length L and normalization (by the test and sources charges) we obtain the **beam-coupling impedances** as simple functions of the first few $\alpha_{TM}(m)$ (axisymmetric) or α_{mn} (flat). Again, keep only linear terms: dipolar terms proportional to source coordinates, quadrupolar ones to test coordinates.

Wall impedance: results in axisymmetric

- 25mm-thick graphite of radius 1.5 mm (surrounded by stainless steel), compared to classic formula:



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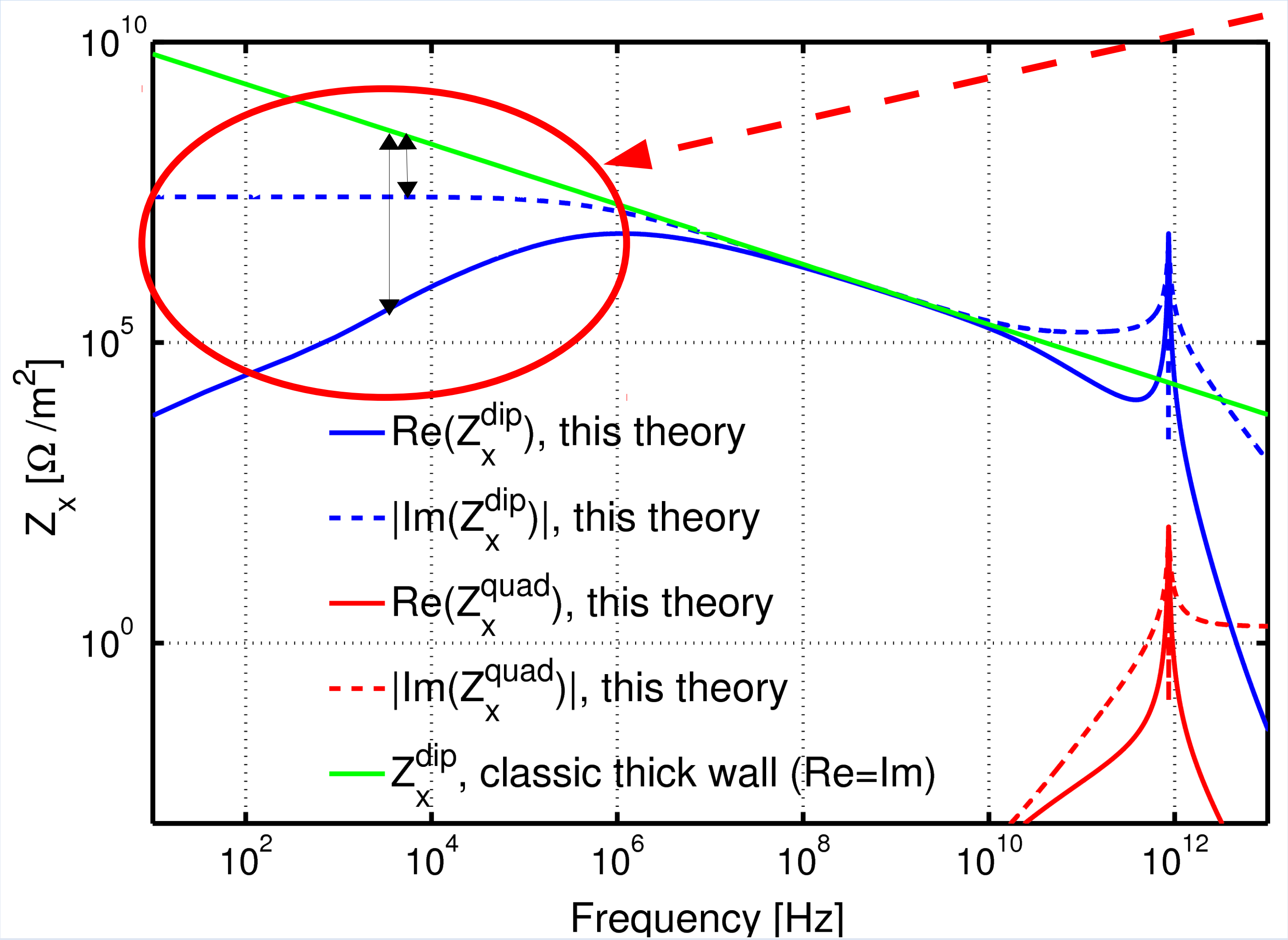

Low frequencies: importance of general theory w.r.t classic formula (factor ~10 for imag. part, >100 for real part)

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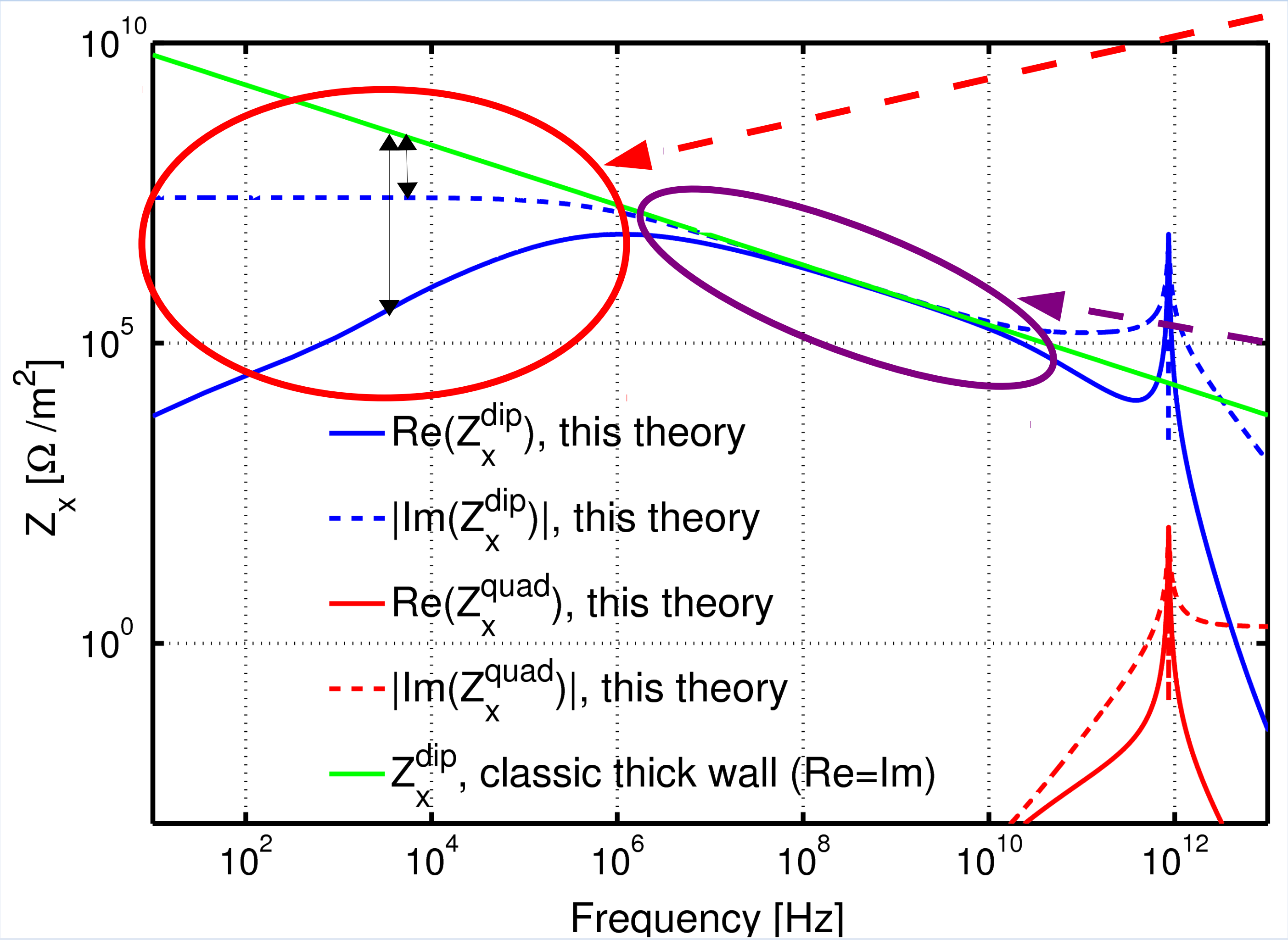

Low frequencies: importance of general theory w.r.t classic formula (factor ~10 for imag. part, >100 for real part)

Intermediate frequencies: classic formula valid (skin depth approximation).

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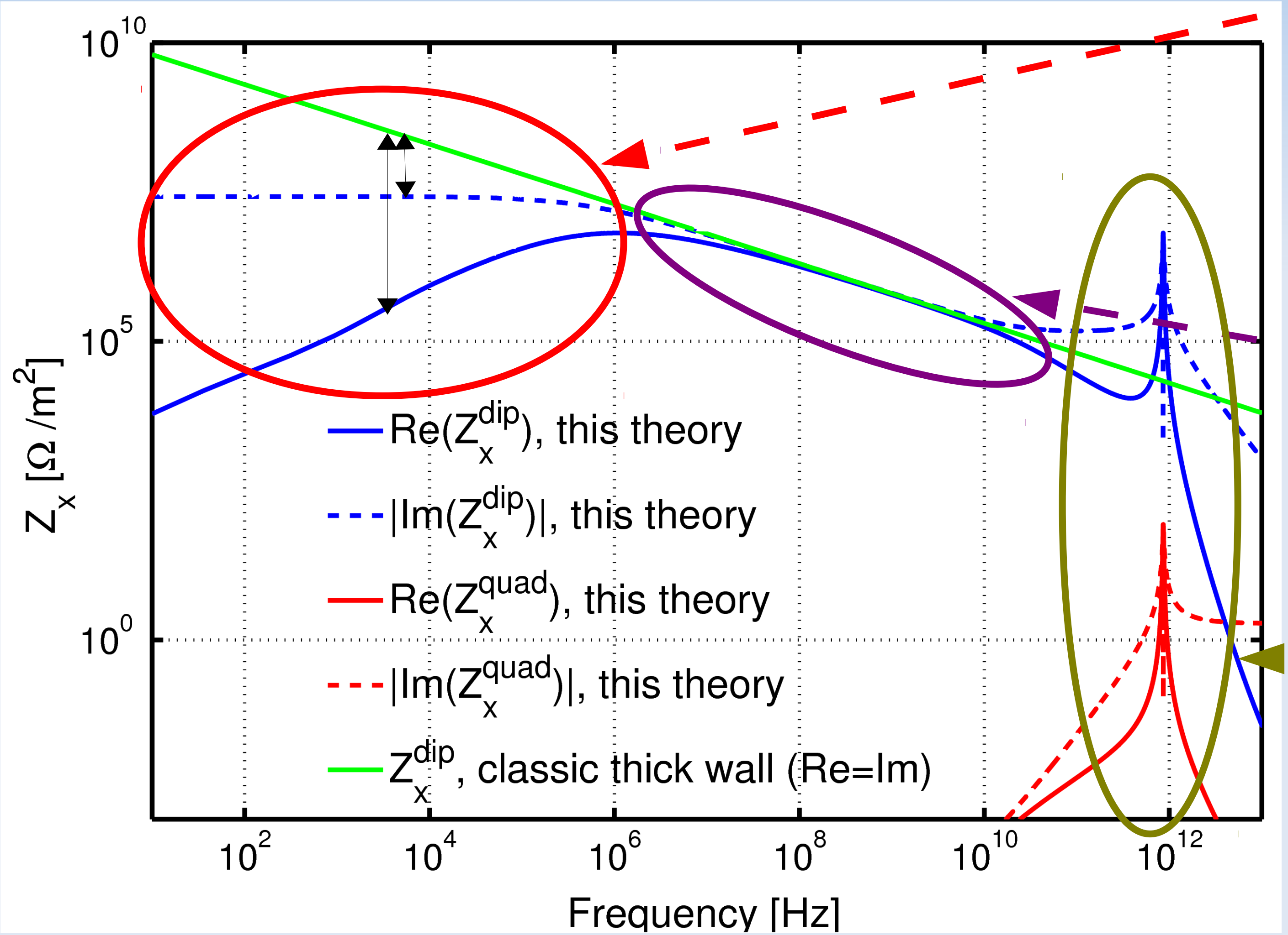


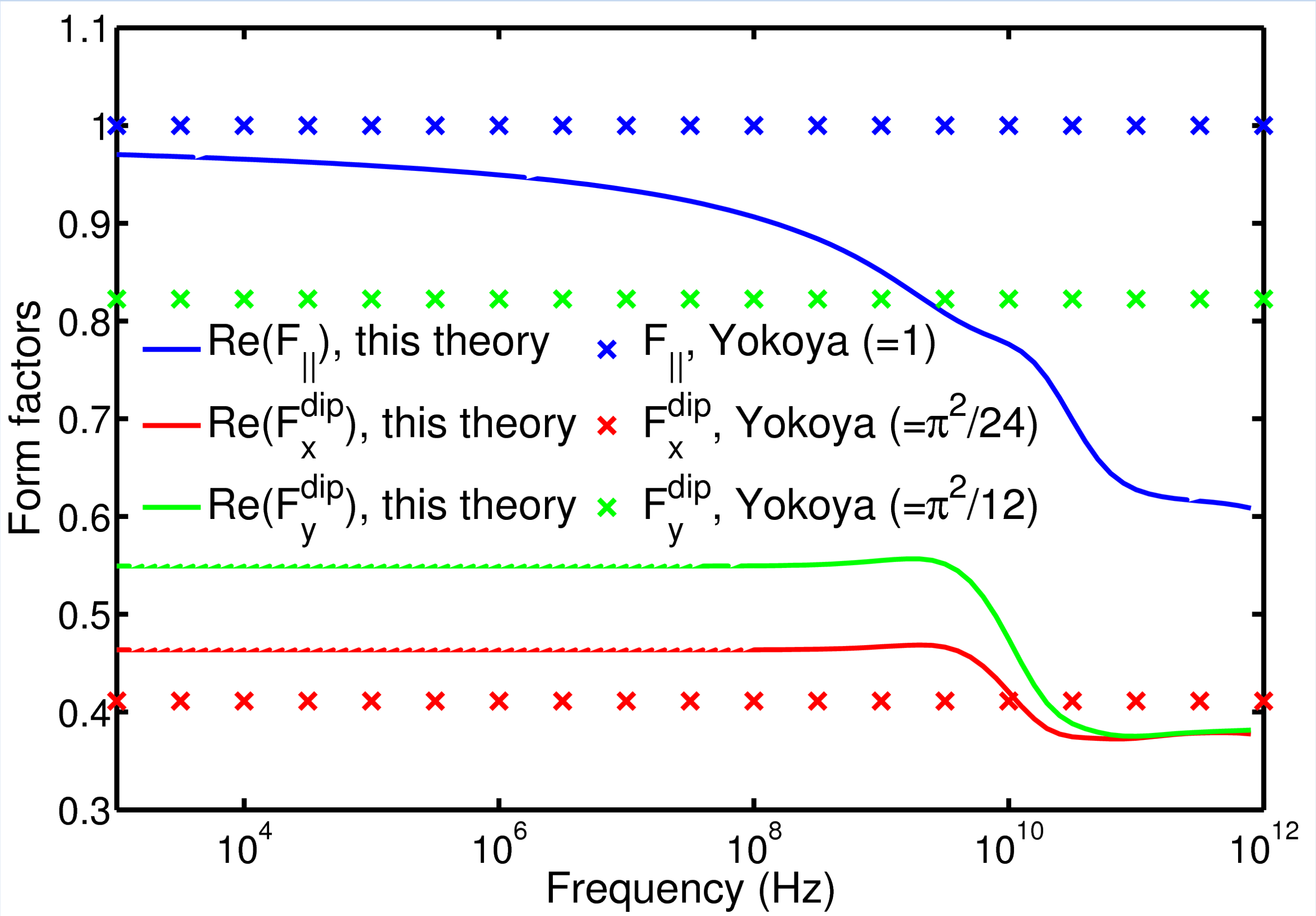
Low frequencies: importance of general theory w.r.t classic formula (factor ~10 for imag. part, >100 for real part)

Intermediate frequencies: classic formula valid (skin depth approximation).

High frequencies: resonance + new quadrupolar term (in this theory only).

Wall impedance: form factors

- Ratio of flat chamber impedances w.r.t longitudinal and transverse dipolar axisymmetric ones → generalize Yokoya factors (Part. Acc., 1993, p. 511). In the case of a single-layer ceramic (hBN) at 450 GeV:



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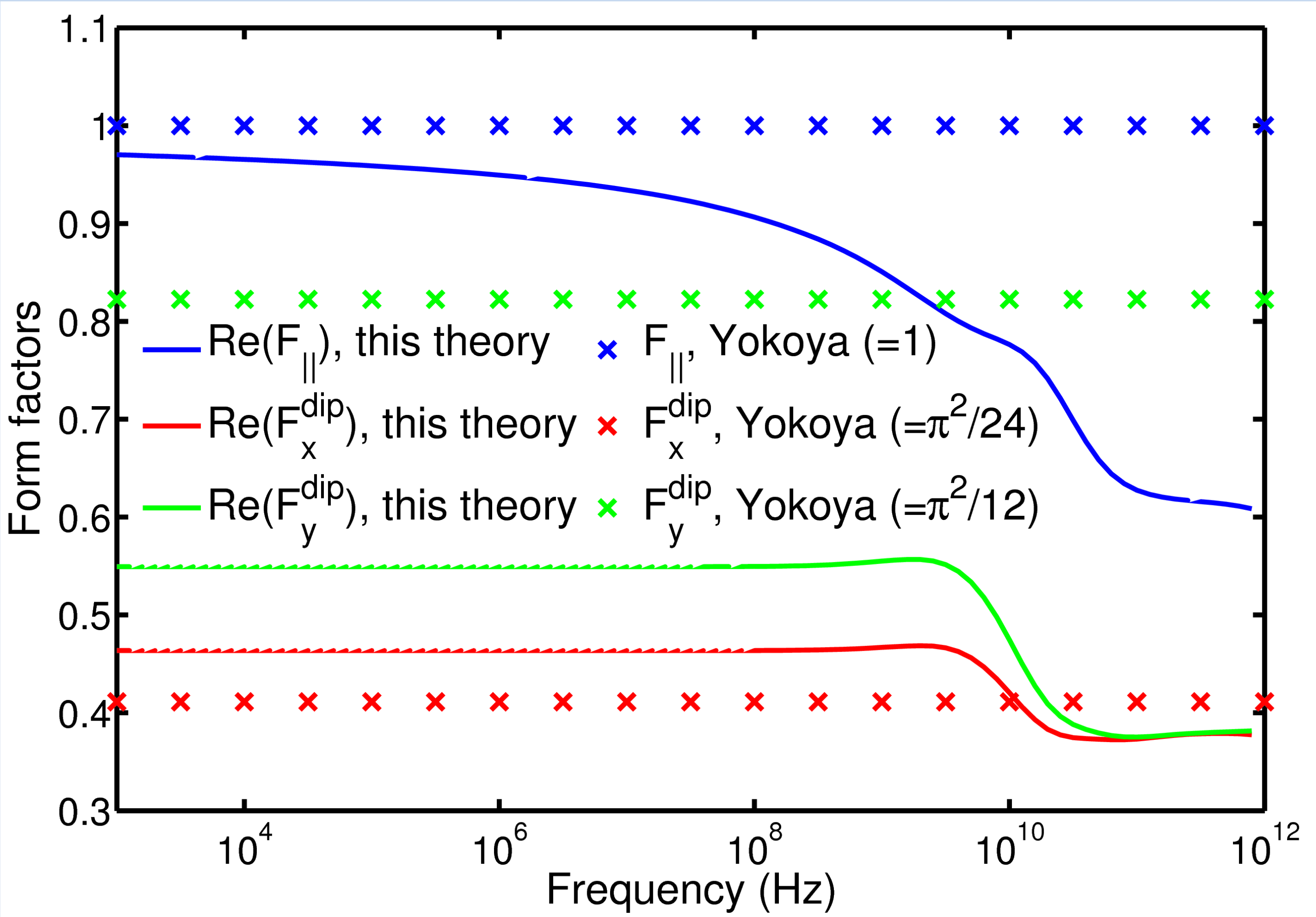


⇒In this particular case, frequency dependent form factors quite ≠ from the Yokoya factors.

(Note: this is a virtual example, not an actual LHC case)

⇒We can get such form factors for any material or material combination (i.e. several layers).

Wake functions

- Wake functions are the Fourier transforms of the impedances, e.g.

$$W_x(\tau) = -\frac{j}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}\omega e^{j\omega\tau} Z_x(\omega)$$ for a test particle at τ seconds behind the source

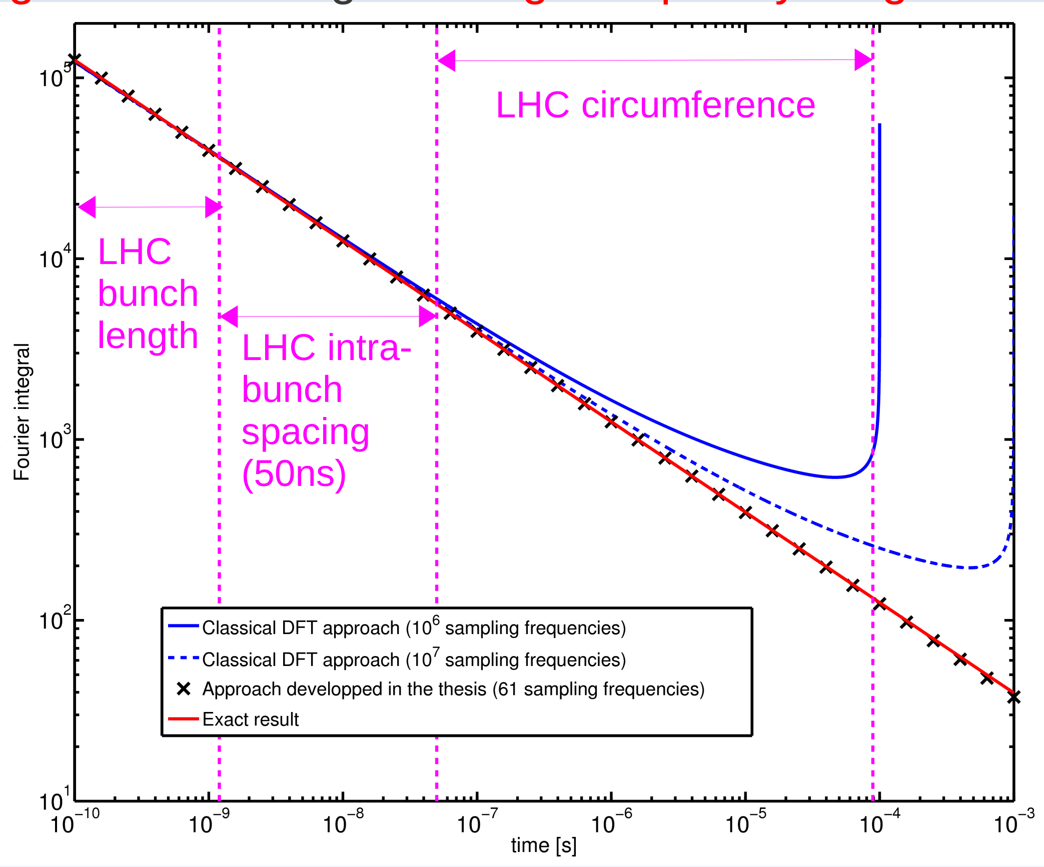
In principle, straightforward to obtain from the impedances: ”do a FFT”.

In practice, usual method with discrete Fourier transform (DFT) with evenly spaced frequency mesh not accurate enough when dealing with large frequency range.

⇒ developped a ”new” method (based on idea from 1928): given any frequency sampling, on each subinterval replace the impedance by its cubic interpolation, and integrate it analytically.

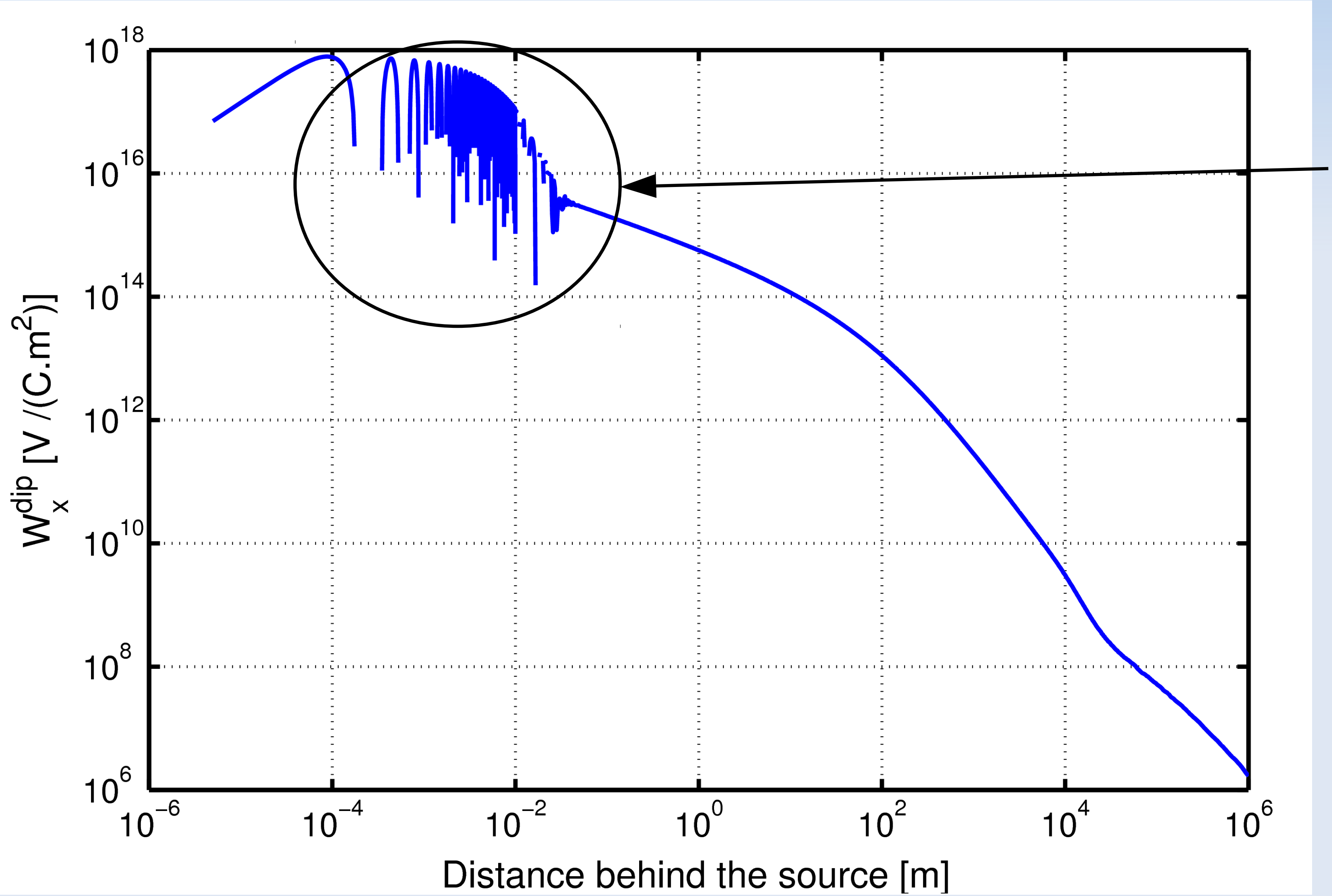
Example with $Z_x = 1/\sqrt{|\omega|}$

→ clearly DFT fails (and also slower + heavy memory load)



Wake function: results (axisymmetric)

- 25mm-thick graphite of radius 1.5 mm (surrounded by stainless steel):



High frequency oscillations (due to the THz resonance).

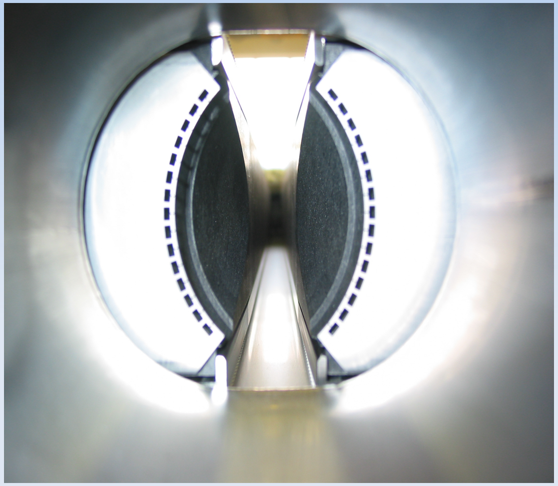
The LHC impedance model

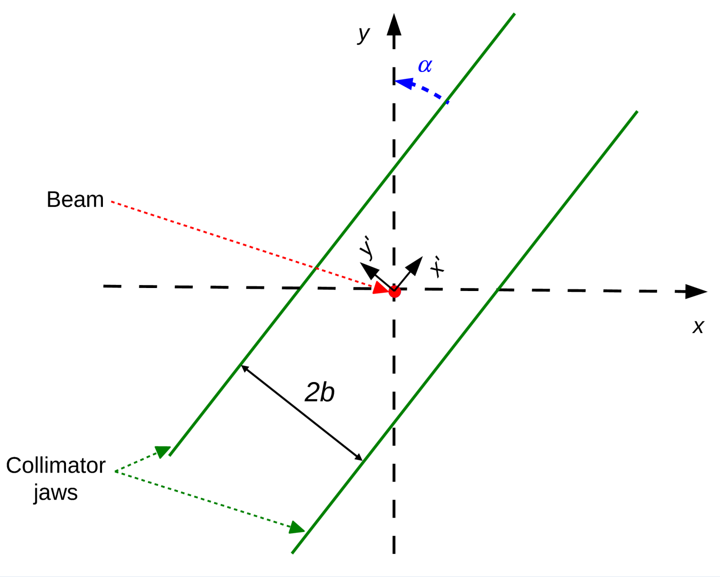
- Outline:
  - Identify highest impedance contributors (physical elements around the beam) with simple criteria: length of the element – aperture – conductivity → we chose the 44 collimators, beam screens and vacuum pipe.
  - For each of them, evaluate wall impedances and wake functions from the theories and tools presented. Note: this assumes longitudinal smoothness.
  - Compute a broad-band model to take into account some non-smooth features (simple estimates from the LHC design report).
  - Sum all these contributions into a model applied at a single-location around the ring (as a ”thin lens”)

    → can do that by weighting each contribution with the beta function at its real position.

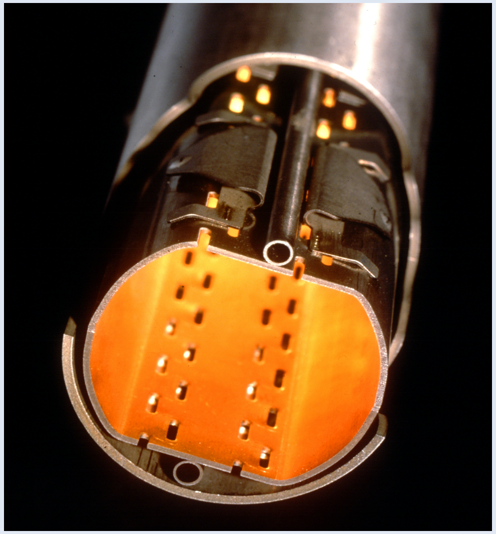
The LHC impedance model

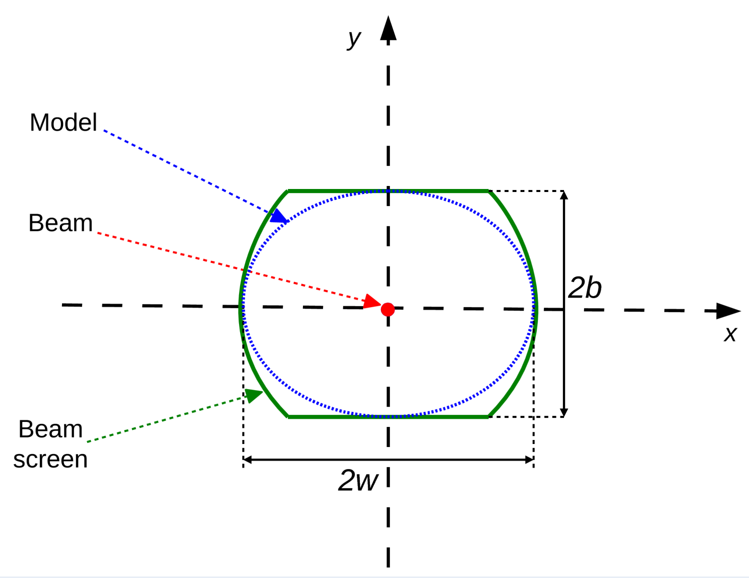
- Examples of simplifications made to compute the impedances:
  - Collimators:





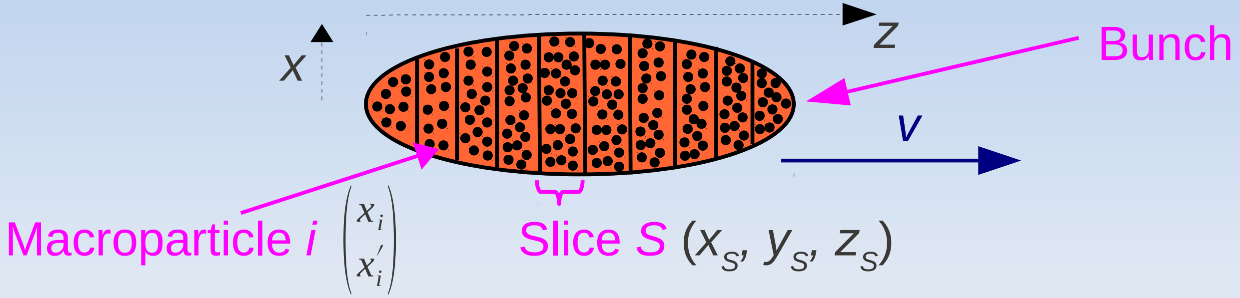
  - Beam screens:





Multibunch simulation code

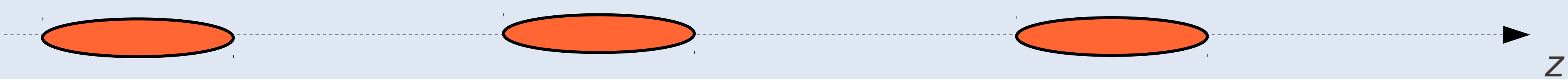
- HEADTAIL: beam dynamics simulation code, using macroparticles
  - Pre-existing single-bunch version (*G. Rumolo et al*, PRST-AB, 2002):



Each turn:
- macropart. *i* receives kick from the wake of all preceding slices: $\begin{pmatrix} x_i \\ x'_i \end{pmatrix} \rightarrow \begin{pmatrix} x_i \\ x'_i + \Delta x'_i(x_S, x_{S_i}, z_S - z_{S_i}) \end{pmatrix}$
- then it is transported through the machine lattice: $\begin{pmatrix} x_i \\ x'_i \end{pmatrix} \rightarrow M \cdot \begin{pmatrix} x_i \\ x'_i \end{pmatrix}$

(similar treatment for the other components of the macroparticle y_i, z_i).

  - Extension of the code: allow several bunches + parallelization over the bunches (extensive use of EPFL clusters).

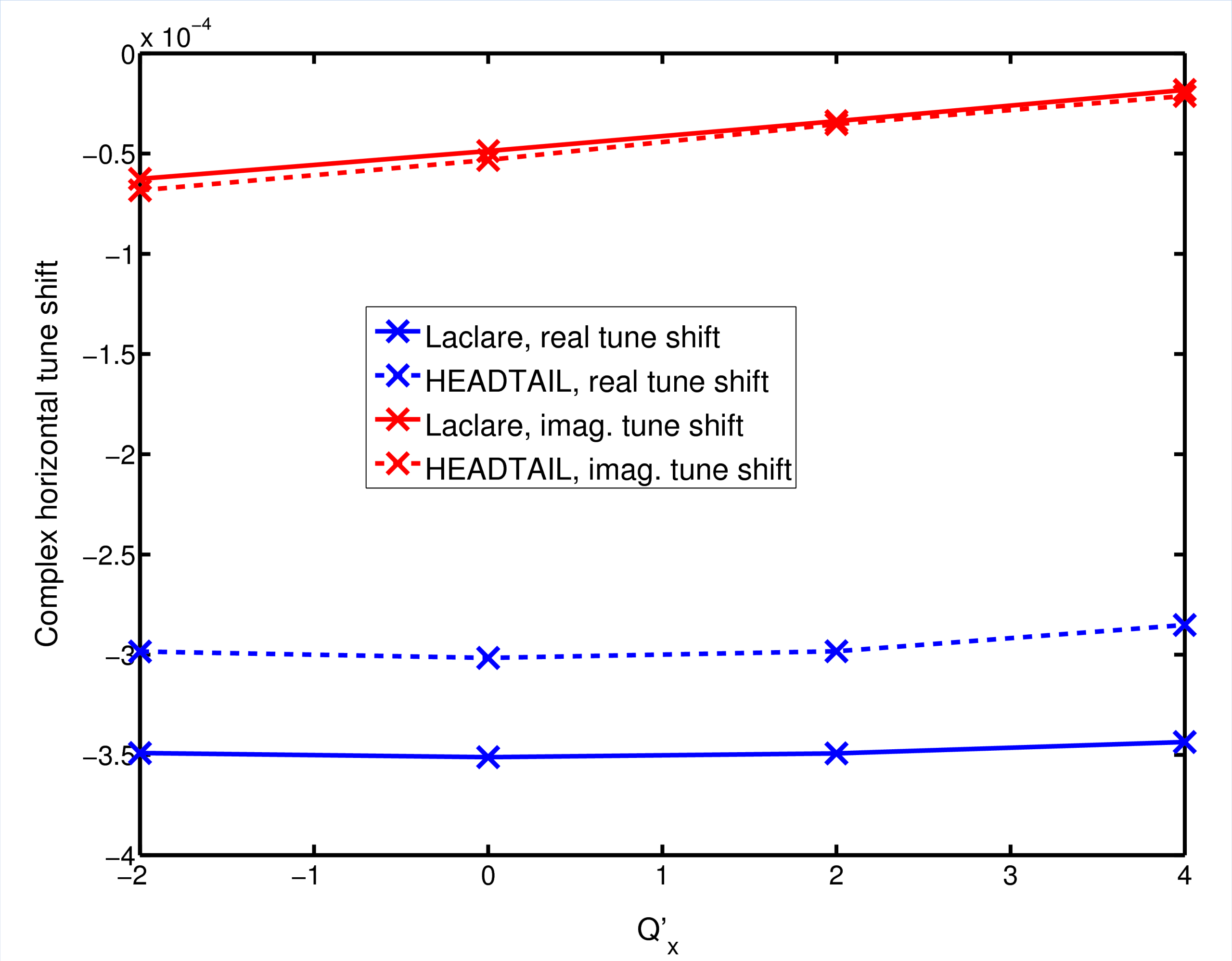


Parallelization quite efficient because each bunch can be treated independently → communication between processors only once per turn.

Multibunch simulation code

- New HEADTAIL multibunch code benchmarked with respect to Laclare's theory, in simplified cases (dipolar impedance & equidistant bunches):

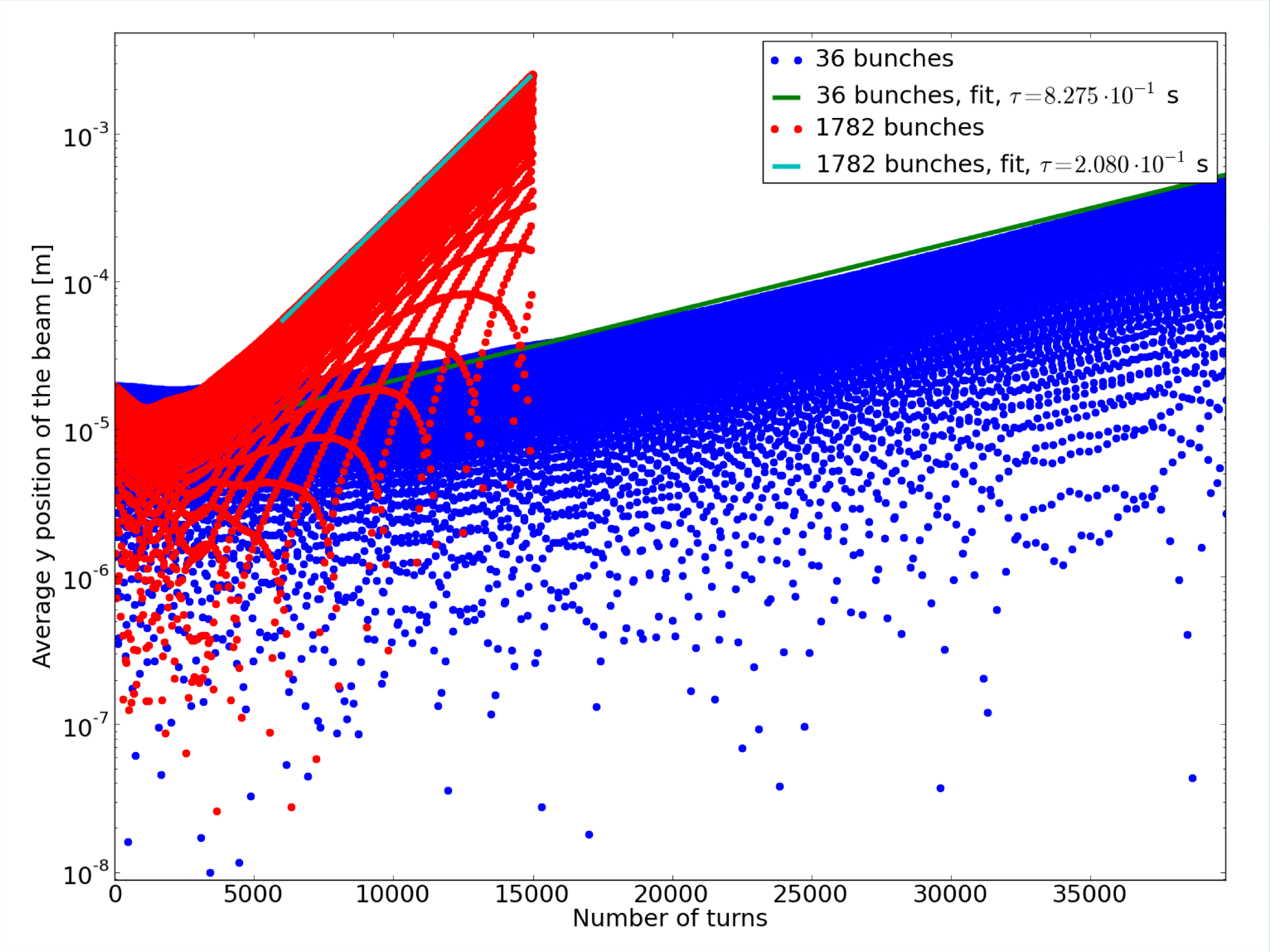
Complex tune shift = modification to the tune due to the most unstable mode.



→ New HEADTAIL reliable, and also more general than available theories or codes.

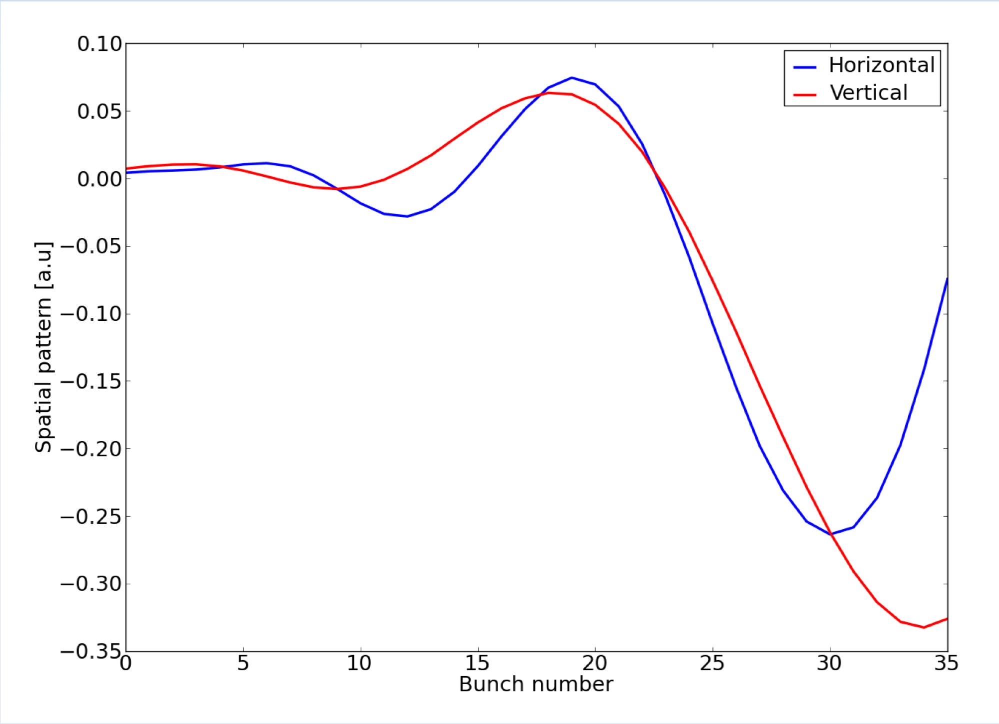
Simulations of LHC coupled-bunch instabilities: effect of the number of bunches

- A completely filled machine (1782 bunches) is at worst only 4 times more critical than a single bunch train of 36 bunches (with the same spacing – 50 ns):

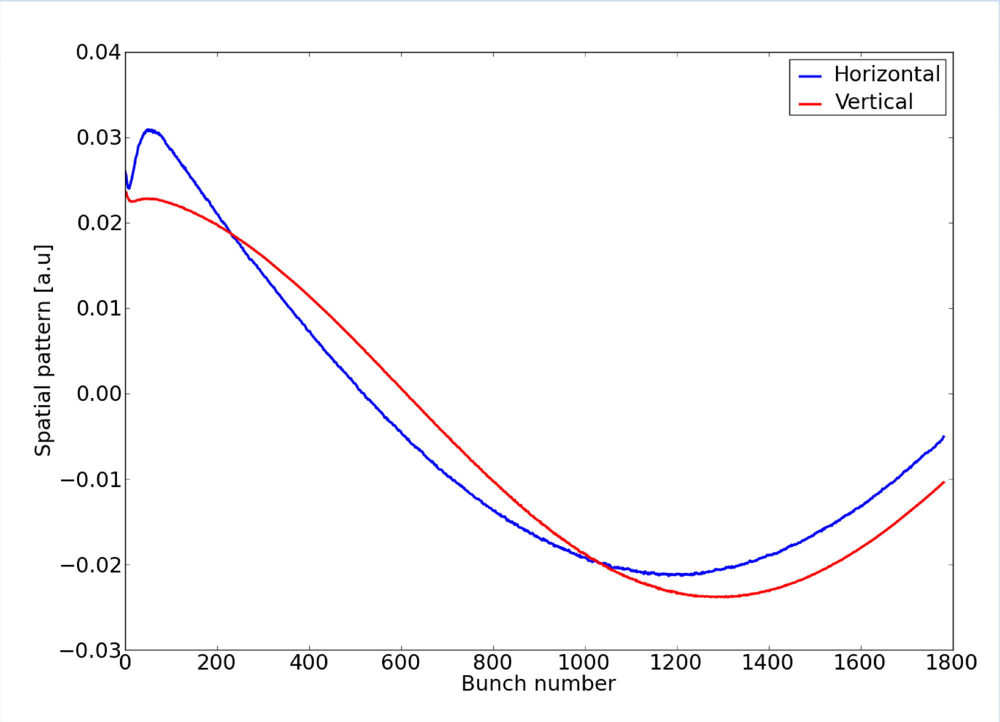


Simulations of LHC coupled-bunch instabilities: effect of the number of bunches

- Oscillation pattern along the bunch train exhibits a smaller wavelength with 36 bunches than with 1782 (50ns spacing):



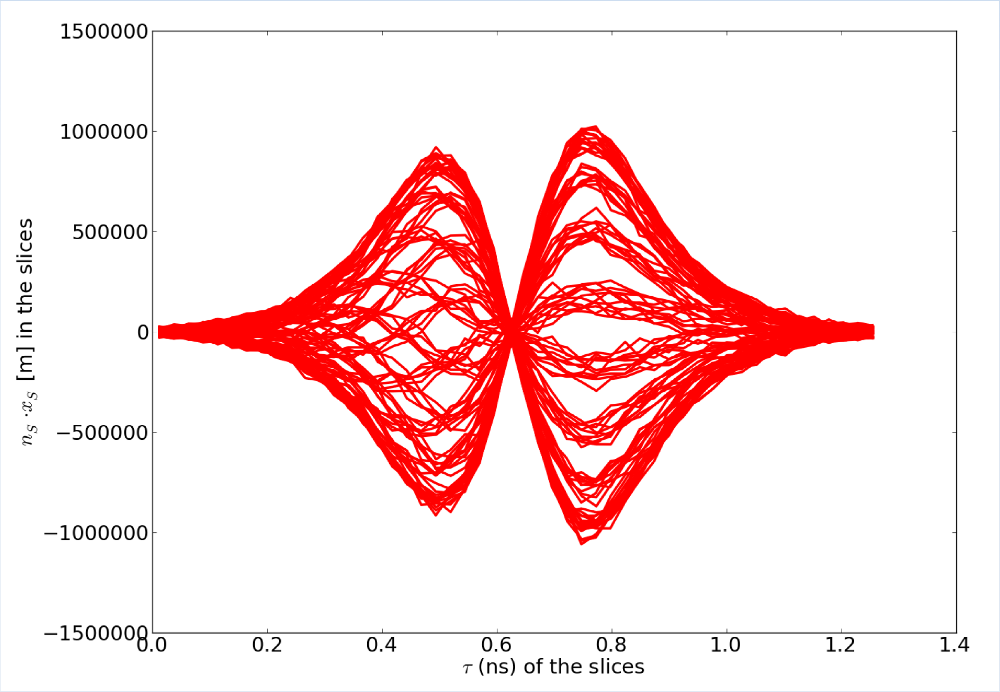
36 bunches (50 ns)



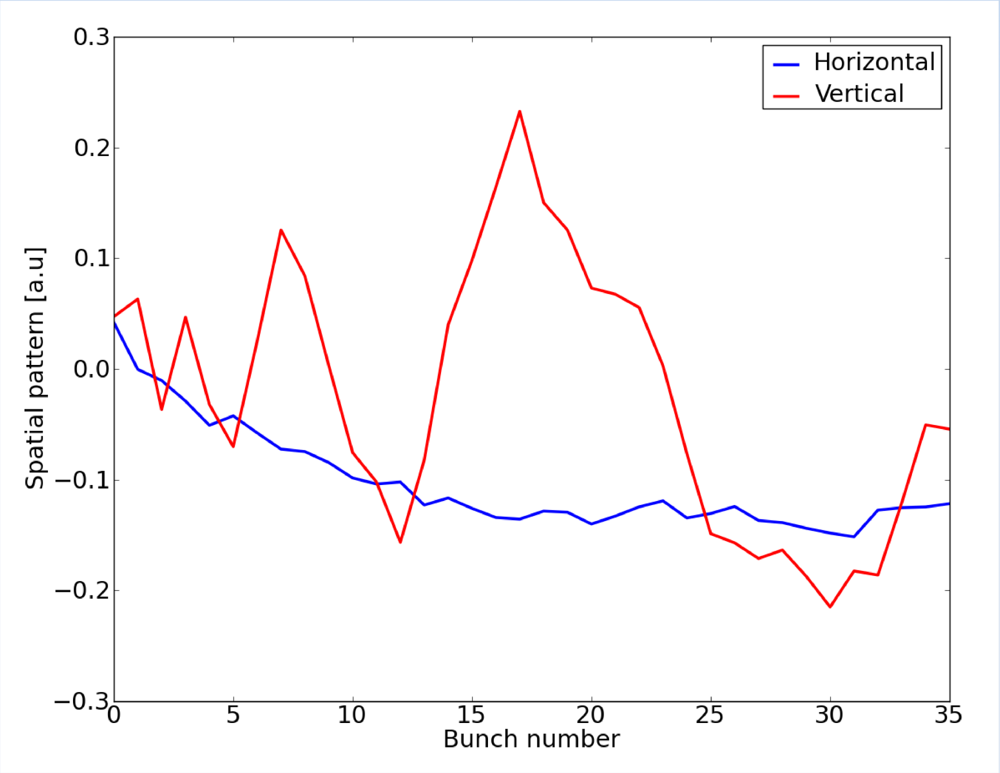
1782 bunches (50 ns)

LHC coupled-bunch instabilities with intrabunch motion

- The code also allows studying coupled-bunch instabilities with non-rigid bunches: for 36 bunches (50ns spacing), with high intensity and high chromaticity:



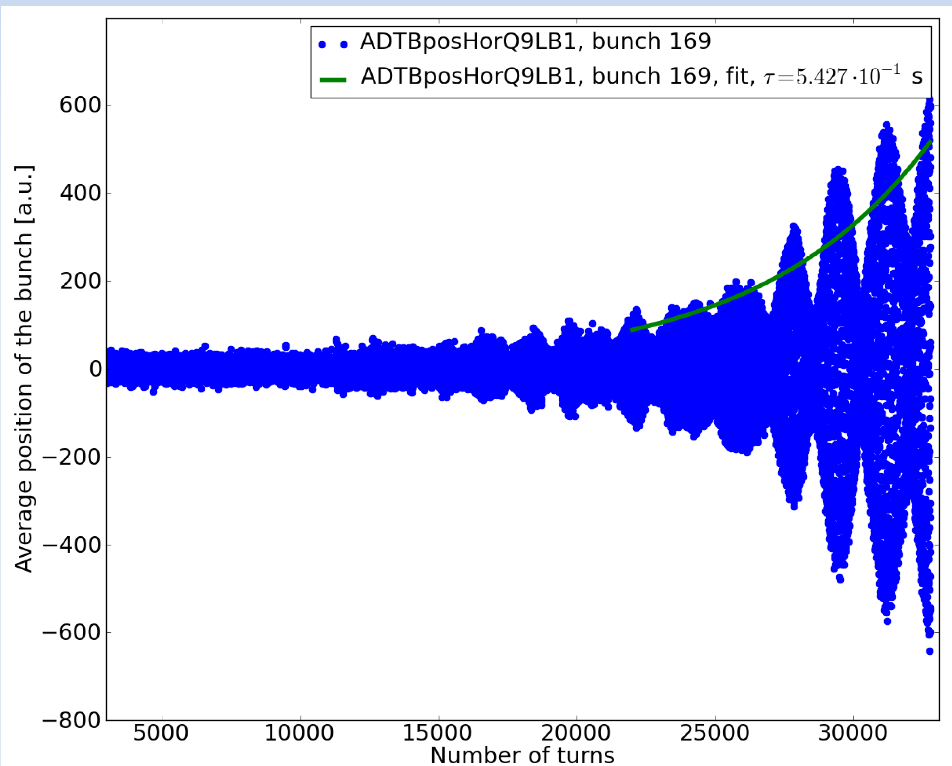
Intrabunch motion (bunch profile for subsequent turns)



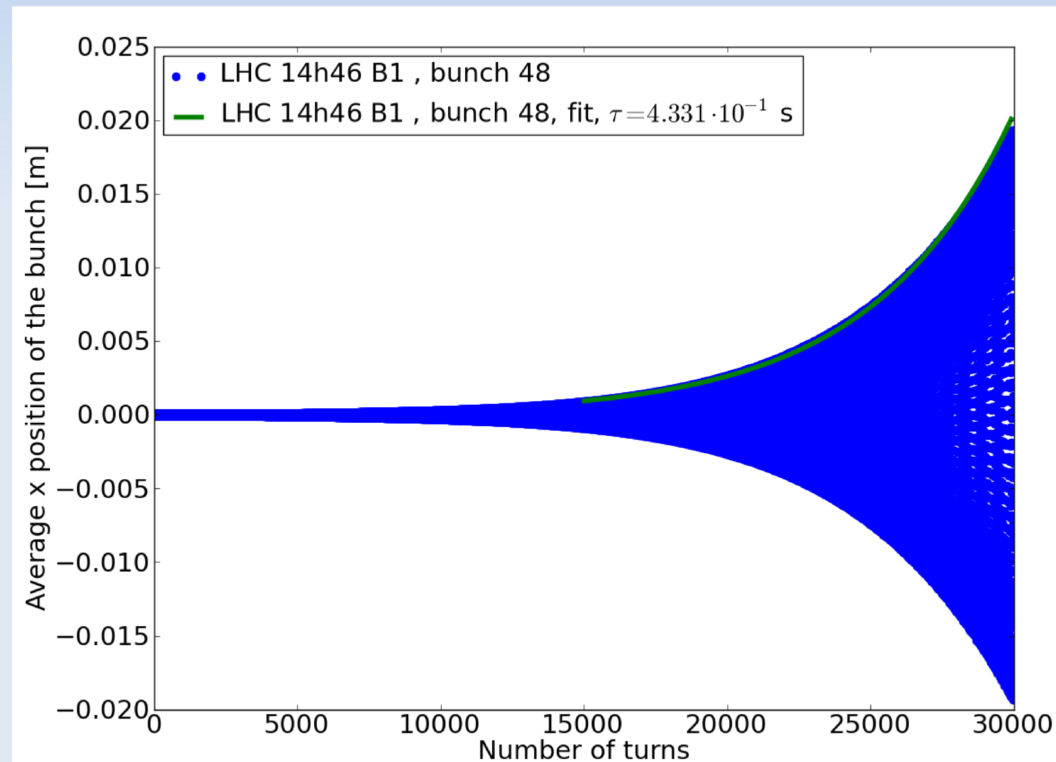
Oscillation pattern along bunch train → coupled-bunch nature of the instability

Comparisons between simulations and beam-based impedance measurements

- At 450 GeV/*c*, 12+36 bunches, switched off feedback for 2.5 s, with Q'_x=0.4 → coupled-bunch instability: here for the last bunch of the train



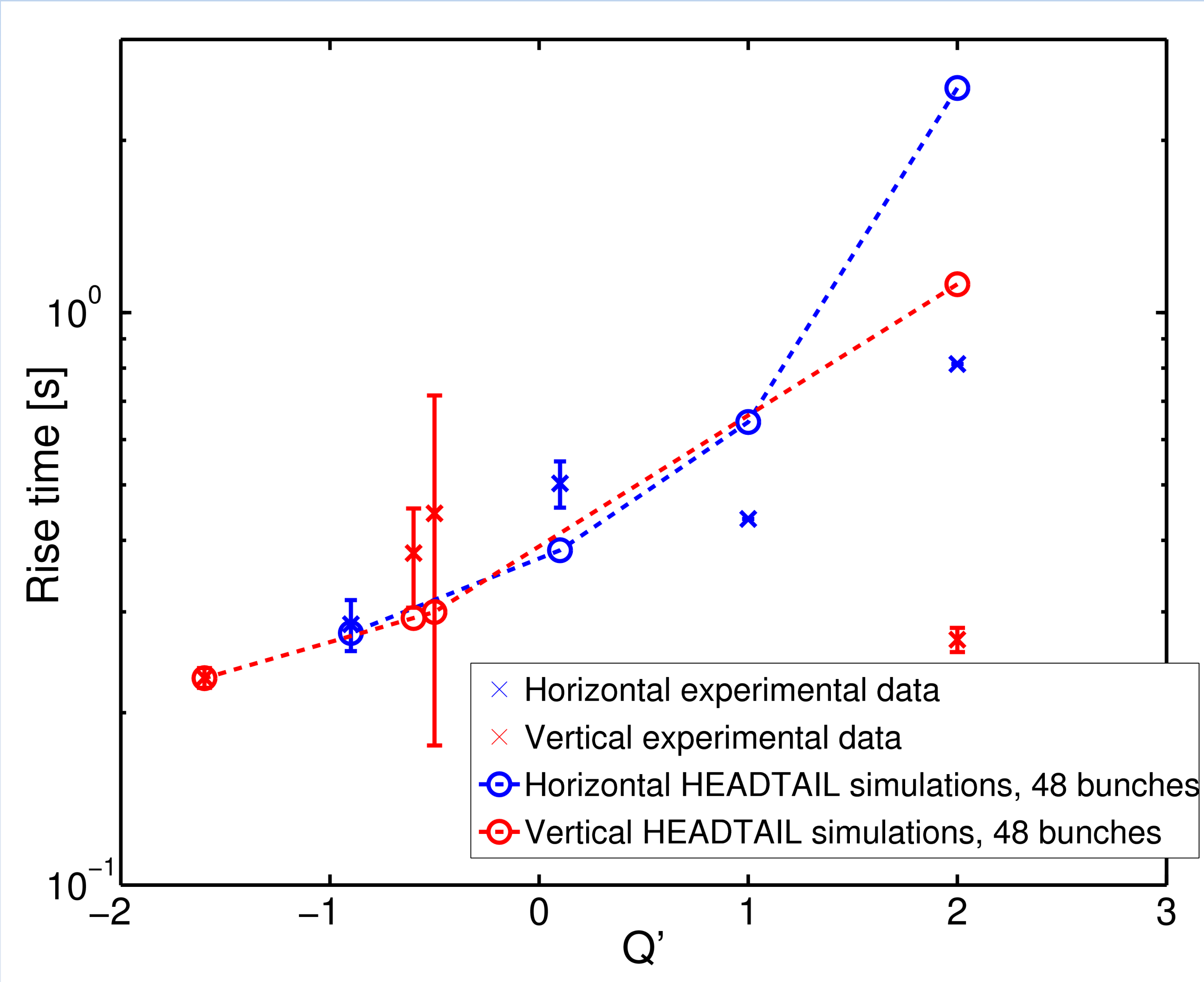
Measurement



Simulation

Comparisons between simulations and beam-based impedance measurements

- 12+36 bunches at 450GeV/*c*, coupled-bunch instability rise times measured vs. simulations (beam 2)

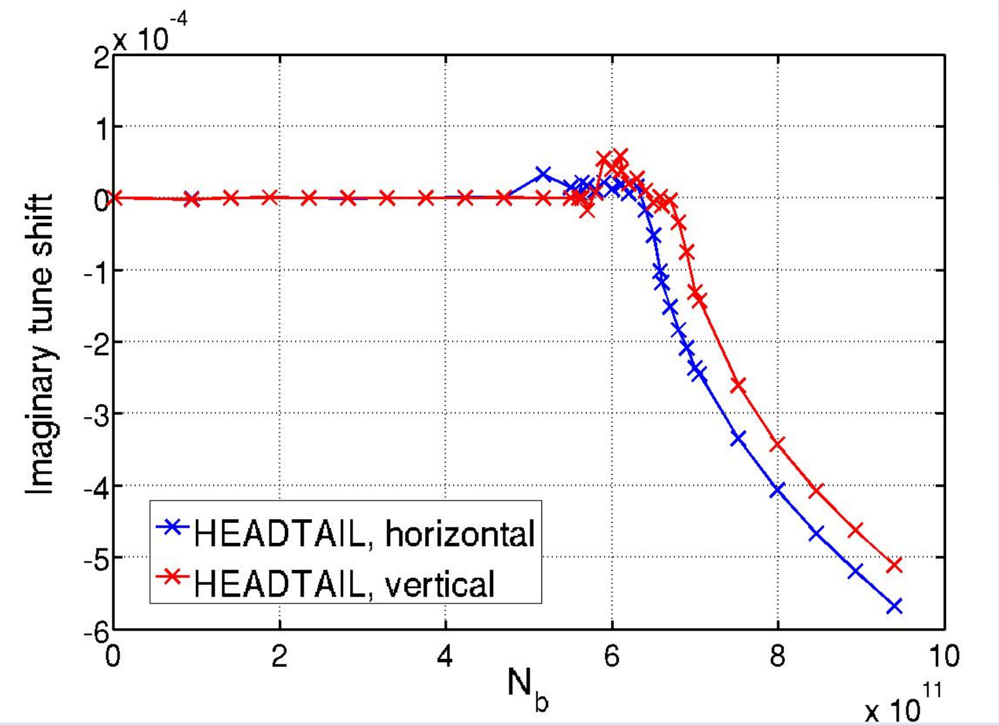


→ at this energy, measured rise times well reproduced by the model.

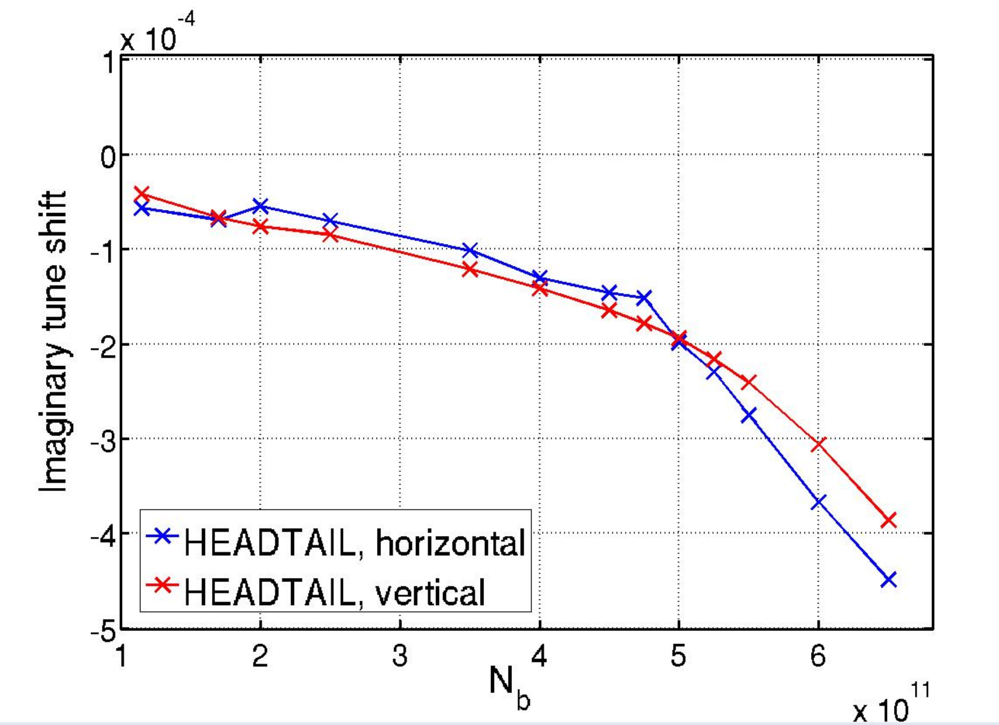
Note: at 3.5 TeV/c, measured rise times at a factor 2-3 from the model.

Predictions for the future operation of the LHC at 7 TeV/*c* : multibunch TMCI

- Transverse mode coupling instability (TMCI) intensity threshold can now be evaluated in coupled-bunch regime: at 7 TeV/c (50ns)



Single-bunch
Threshold ~ 6.4 10^{11} protons/bunch



Coupled-bunch (1404 bunches)
Threshold ~ 5 10^{11} protons/bunch

⇒ Coupled-bunch TMCI around 20% more critical than single-bunch one.

Summary

- Beam-coupling impedances and wake functions:
  - Impedance theories on an axisymmetric multilayer chamber and flat multilayer chamber → the most general to date, considering any velocity, any frequency and any azimuthal mode number *m*. New terms exhibited.
  - New algorithm to compute Fourier integrals of analytical functions, fast and accurate, useful in particular for wake functions.

  ⇒ Implemented in codes, also used for other machines (e.g. SPS, CLIC).
- Updated impedance and wake-function model for the LHC.
- Multibunch extension of an existing beam dynamics simulation code.
- Results concerning the LHC transverse coupled-bunch instability:
  - case of small train of bunches vs. fully filled machine,
  - coupled-bunch instabilities with intrabunch motion,
  - comparison between measurements and simulations,
  - transverse mode coupling threshold in coupled-bunch regime.

What else can still be done ?

- About beam-coupling impedances theories:

  → study in the same way other geometries: elliptical (e.g. for PS-Booster beam pipe), general 2D geometry (LHC beam screen weld).

- About the impedance model of the LHC:

  → add other contributors (kicker magnets, RF cavities),

  → better models (e.g. 3D) for contr. already taken into account (collimators).

- About the HEADTAIL beam dynamics simulation code:

  → implement other sources of nonlinearities (space-charge, beam-beam force at the collision point).

- About the LHC transverse coupled-bunch instability:
  - study of the impact of the second derivative of the tune (on going),
  - many other comparisons between measurements and simulations.

Thank you for your attention !