# Using spin of $\tau$ to constrain hard interactions

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**Main Topics (please ask questions)**:

- **Universal and exact spin weight; semi factorization**
- **Typical approximations approximations**
- **Polarimetric vector from decay, properties**
- **Longitudinal spin, full spin**
- **Cases of missing information.**

## Formalism for $\tau^+\tau^-$

- To separate production and decay of the $\tau$ leptons we use the following relation

$$\frac{(\not p - m_\tau)_{\lambda\lambda'}}{p^2 - m_\tau^2 + i\epsilon} = \sum_\lambda \bar u(p,\lambda) u(p,\lambda') BW_\tau(p^2)$$

where $|BW_\tau(p^2)|^2 \to \delta(p^2 - m_\tau^2)$ in formula for cross section.

- Similar separation is not perfect for Z or W decays because ratio of $M/\Gamma$ is then only about 30. Approximation works at few % level only. Background diagrams contribute sizably etc. For $\tau$ of macroscopic lifetime there is no problem. Formula is perfect!

- We have for the process $f\bar f \to \tau^+\tau^- Y; \tau^+ \to X^+\bar\nu; \tau^- \to X^-\nu$ simplifies:

$$d\sigma = \sum_{spin} |M|^2 d\Omega = \sum_{spin} |\mathcal{M}|^2 d\Omega_{prod}\; d\Omega_{\tau^+}\; d\Omega_{\tau^-}$$

- Spin indices explicit, $\tau$ propagators lead to Dirac deltas and are in phase space:

$$M \cdot \frac{1}{BW_\tau(s_1)} \frac{1}{BW_\tau(s_2)} = \mathcal{M} = \sum_{\lambda_1\lambda_2=1}^{2} \mathcal{M}^{prod}_{\lambda_1\lambda_2}\, \mathcal{M}^{\tau^+}_{\lambda_1} \mathcal{M}^{\tau^-}_{\lambda_2}$$

- We square amplitudes

$$|\mathcal{M}|^2 = \Big(\sum_{\lambda_1\lambda_2=1}^{2} \mathcal{M}^{prod}_{\lambda_1\lambda_2}\, \mathcal{M}^{\tau^+}_{\lambda_1} \mathcal{M}^{\tau^-}_{\lambda_2}\Big)\Big(\sum_{\lambda_1\lambda_2=1}^{2} \bar{\mathcal{M}}^{prod}_{\bar\lambda_1\bar\lambda_2}\, \bar{\mathcal{M}}^{\tau^+}_{\bar\lambda_1} \bar{\mathcal{M}}^{\tau^-}_{\bar\lambda_2}\Big)$$

or

$$|\mathcal{M}|^2 = \sum_{spin} \mathcal{M}^{prod}_{\lambda_1\lambda_2}\, \bar{\mathcal{M}}^{prod}_{\bar\lambda_1\bar\lambda_2}\, \mathcal{M}^{\tau^+}_{\lambda_1} \mathcal{M}^{\tau^-}_{\lambda_2} \bar{\mathcal{M}}^{\tau^+}_{\bar\lambda_1} \bar{\mathcal{M}}^{\tau^-}_{\bar\lambda_2}$$

- We use completnes of sigma matrices ($\delta_{\lambda\lambda'}\delta_{\bar\lambda\bar\lambda'} = \sigma^{\mu}_{\lambda\bar\lambda}\sigma^{\mu}_{\lambda'\bar\lambda'}$)

$$|\mathcal{M}|^2 = \sum_{spin} \mathcal{M}^{prod}_{\lambda_1\lambda_2}\, \bar{\mathcal{M}}^{prod}_{\bar\lambda_1\bar\lambda_2}\, \delta_{\lambda_1\lambda_1'}\delta_{\bar\lambda_1\bar\lambda_1'}\delta_{\lambda_2\lambda_2'}\delta_{\bar\lambda_2\bar\lambda_2'} \mathcal{M}^{\tau^+}_{\lambda_1'} \mathcal{M}^{\tau^-}_{\lambda_2'}\, \bar{\mathcal{M}}^{\tau^+}_{\bar\lambda_1'} \bar{\mathcal{M}}^{\tau^-}_{\bar\lambda_2'}$$

to get

$$|\mathcal{M}|^2 = \Big(\sum_{spin} |\mathcal{M}^{prod}|^2\Big)\Big(\sum_{spin} |\mathcal{M}^{\tau^+}|^2\Big)\Big(\sum_{spin} |\mathcal{M}^{\tau^-}|^2\Big)\, R_{\mu^1\mu^2} h^{\mu^1}_{\tau^-} h^{\mu^2}_{\tau^+} = \sum_{\mu^1\mu^2}$$

$$\Big(\sum_{spin} \mathcal{M}^{prod}_{\lambda_1\lambda_2}\, \bar{\mathcal{M}}^{prod}_{\bar\lambda_1\bar\lambda_2}\, \sigma^{\mu^1}_{\lambda_1\bar\lambda_1}\sigma^{\mu^2}_{\lambda_2\bar\lambda_2}\Big)$$

$$\Big(\sum_{spin} \mathcal{M}^{\tau^+}_{\lambda_1'} \bar{\mathcal{M}}^{\tau^+}_{\bar\lambda_1'} \sigma^{\mu^1}_{\lambda_1'\bar\lambda_1'}\Big)\Big(\sum_{spin} \mathcal{M}^{\tau^-}_{\lambda_2'} \bar{\mathcal{M}}^{\tau^-}_{\bar\lambda_2'} \sigma^{\mu^2}_{\lambda_2'\bar\lambda_2'}\Big)$$

- Formula for the cross section can be re-written (without any loss of precision):

$$d\sigma = \Big(\sum_{spin} |\mathcal{M}^{prod}|^2\Big)\Big(\sum_{spin} |\mathcal{M}^{\tau^+}|^2\Big)\Big(\sum_{spin} |\mathcal{M}^{\tau^-}|^2\Big) wt\; d\Omega_{prod}\; d\Omega_{\tau^+}\; d\Omega_{\tau^-}$$

-

$$wt = \Big(\sum_{i,j=0,3} R_{ij} h^i h^j\Big)$$

$$R_{00} = 1, \quad < wt >= 1, \quad 0 \leq wt \leq 4.$$

$R_{ij}$ can be calculated from $\mathcal{M}_{\lambda_1 \lambda_2}$
and $h^i$, $h^j$ respectively from $\mathcal{M}^{\tau^+}$ and $\mathcal{M}^{\tau^-}$.

- the spin weight $wt$ depend only on angles $\theta$ and $\phi$ of overall spatial orientation of $\tau$ decay products with respect to production process.

- Bell inequalities tell us that it is impossible to re-write $wt$ in the following form

$$wt \neq \Big(\sum_{i,j=0,3} R_i^A h^i\Big)\Big(\sum_{i,j=0,3} R_j^B h^j\Big)$$

that means it is impossible to generate first $\tau^+$ and $\tau^-$ in some given ‘ quantum state’ and later perform separatelly decays of $\tau^+$ and $\tau^-$

- It can be done only if approximations are used !!!
- May be often reasonable, but nonetheless approximations.
- Important: $R_{ij}$ is of two indices running in rest-frames of $\tau^+$ and $\tau^-$.

# General formalism for semileptonic decays

- The differential partial width for the channel under consideration reads

$d\Gamma_X = G^2 \frac{v^2+a^2}{4M} d\mathrm{Lips}(P; q_i, N)(\omega + (H_\mu) s^\mu)$ The $s$ vector was obtained from production amplitude contracted with $\sigma$ matrices.

- The phase space distribution is given by the following expression where a compact notation with $q_5 = N$ and $q_i^2 = m_i^2$ is used

$$
\begin{aligned}
d\mathrm{Lips}(P; q_1, q_2, q_3, q_4, q_5) = & \frac{1}{2^{23}\pi^{11}} \int_{Q^2_{min}}^{Q^2_{max}} dQ^2 \int_{Q^2_{3,min}}^{Q^2_{3,max}} dQ_3^2 \\
\int_{Q^2_{2,min}}^{Q^2_{2,max}} dQ_2^2 \quad \times & \quad \int d\Omega_5 \frac{\sqrt{\lambda(M^2, Q^2, m_5^2)}}{M^2} \int d\Omega_4 \frac{\sqrt{\lambda(Q^2, Q_3^2, m_4^2)}}{Q^2} \\
\times & \quad \int d\Omega_3 \frac{\sqrt{\lambda(Q_3^2, Q_2^2, m_3^2)}}{Q_3^2} \int d\Omega_2 \frac{\sqrt{\lambda(Q_2^2, m_2^2, m_1^2)}}{Q_2^2}
\end{aligned}
$$

$$Q^2 = (q_1 + q_2 + q_3 + q_4)^2, \quad Q_3^2 = (q_1 + q_2 + q_3)^2, \quad Q_2^2 = (q_1 + q_2)^2$$

$$Q_{min} = m_1 + m_2 + m_3 + m_4, \quad Q_{max} = M - m_5 \; Q_{3,min} = m_1 + m_2 + m_3, \quad Q_{3,max} = Q - m_4$$

$$Q_{2,min} = m_1 + m_2, \qquad Q_{2,max} = Q_3 - m_3$$

# General formalism for semileptonic decays

- Matrix element used in TAUOLA for semileptonic decay

$$\tau(P,s) \rightarrow \nu_\tau(N)X$$
$$\mathcal{M} = \frac{G}{\sqrt{2}}\bar{u}(N)\gamma^\mu(v+a\gamma_5)u(P)J_\mu$$

- $J_\mu$ the current depends on the momenta of all hadrons

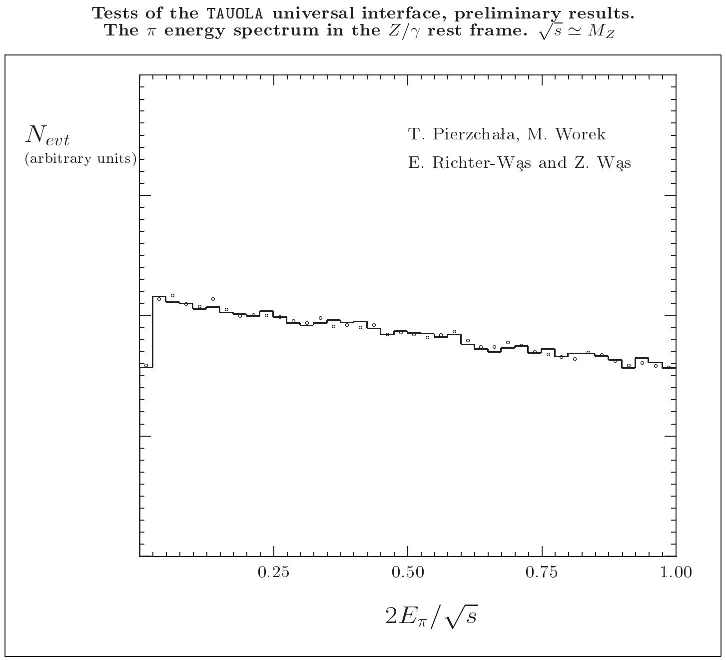
$$|\mathcal{M}|^2 = G^2\frac{v^2+a^2}{2}(\omega + H_\mu s^\mu)$$
$$\omega = P^\mu(\Pi_\mu - \gamma_{va}\Pi^5_\mu)$$
$$H_\mu = \frac{1}{M}(M^2\delta^\nu_\mu - P_\mu P^\nu)(\Pi^5_\nu - \gamma_{va}\Pi_\nu)$$
$$\Pi_\mu = 2[(J^*\cdot N)J_\mu + (J\cdot N)J^*_\mu - (J^*\cdot J)N_\mu]$$
$$\Pi^{5\mu} = 2\,\mathrm{Im}\,\epsilon^{\mu\nu\rho\sigma}J^*_\nu J_\rho N_\sigma$$
$$\gamma_{va} = -\frac{2va}{v^2+a^2}$$

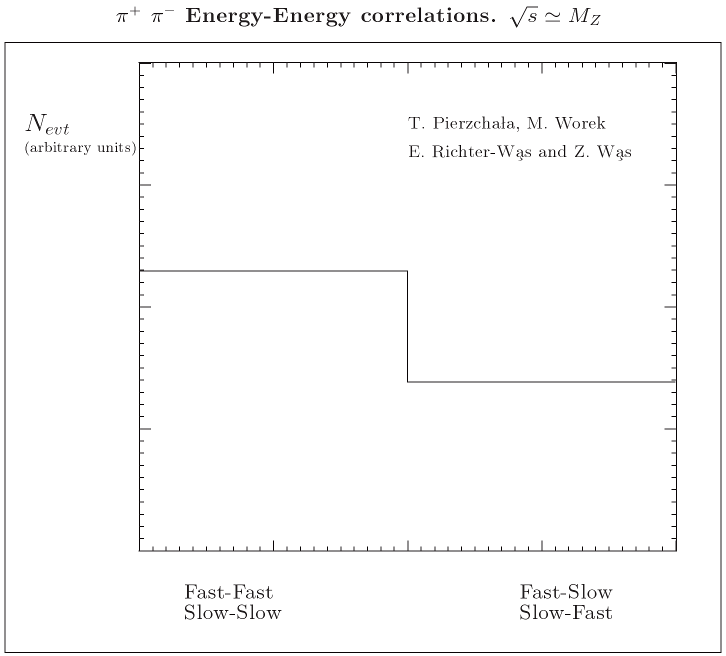
- vector $h$ because of coupling $\tau - -\nu_\tau - -W$ is parity sensitive. Left neutrino prefers to follow spin of the $\tau$ (if there is nothing else to carry spin like in case of $\tau \rightarrow \pi\nu_\tau$

# Measuring spin, means controlling $h$

- **The vector $h = H/H_0$ is a combination of $\tau$ decay products momenta in $\tau$ rest-frame.**
- **But we measure momenta in laboratory frame.**
- **Usually rest frame can not be reconstructed with the sufficient precision to control all of spin states.**
- **Longitudinal components are easiest, because they transtlate after boosts to energies in laboratory frame. Approximate reconstruction of $\nu_\tau$ momentum is sufficient.**
- **For longitudinal effects taken alone, statistical interpretation of LL RR LR RL $\tau$ states is OK. Powerful methods for work with experimental data.**
- **For transverse degrees things are more complicated.**

## longitudinal spin, energies.




**The z component of $R$, $R_{03} = R_{30}$ is nonzero in Z decays. To make spin effects large, we take sample of both $\tau$' decaying into $\pi\nu$. We must know enrgy of $\tau$.**

# $\tau^\pm \to \rho^\pm \nu$ decay

- Branching ratio 25%.
- In comparison to $\tau^\pm \to \pi^\pm \nu$ decay, its polarimetric force is more than a factor of 2 smaller but in many cases this can be improved if information on the $\rho$ decay products are used.
- The polarimetric vector is given by the formula

$$h^i = \mathcal{N}\Big(2(q\cdot N)q^i - q^2 N^i\Big)$$

where $q$ is the four-momentum of $\pi^\pm - \pi^0$ and $N$ is the four-momentum of $\nu_\tau$.

- In the $\tau$ lepton rest frame, when $m_{\pi^\pm} = m_{\pi^0}$ is assumed we have

$$q\cdot N = (E_{\pi^\pm} - E_{\pi^0})m_\tau.$$

- To exploit this part of the polarimetric vector, we need to know the difference of the $\pi^\pm$ and $\pi^0$ energies/momenta in their respective $\tau$ leptons rest frames.
- for leptonic decay channels and for decays into more particles things become more complicated but of the same principle.

# Why not spin of $\rho$ instead of spin of $\tau$? 

## Main References for Higgs Boson Parity At The Linear Collider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- G. R. Bower, T. Pierzchala, Z. Was and M. Worek, Phys. Lett. B **543** (2002) 227
- K. Desch, Z. Was and M. Worek, Eur. Phys. J. C **29** (2003) 491
- K. Desch, A. Imhof, Z. Was and M. Worek, Phys. Lett. B **579** (2004) 157

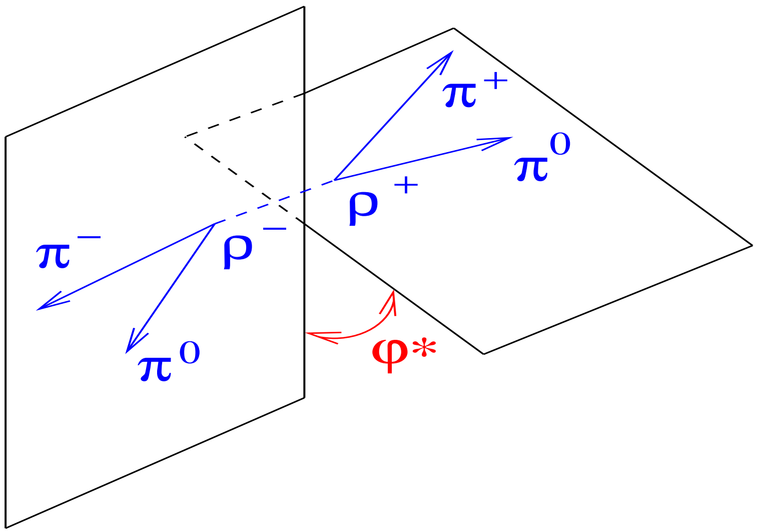
## Pure Scalar And Pseudoscalar Higgs Boson

- *Case of* $\tau \to \rho\nu_\tau$ *decay,* $\mathcal{BR}(\tau \to \rho\nu_\tau) = 25\%$
- *In def. of polarimeter vector* $h^i$ $q$ *denotes 4-vectors of* $\pi^\pm$ *minus* $\pi^0$ *and,* $N$ *of* $\nu_\tau$.

$$h^i = \mathcal{N}\Big(2(q\cdot N)q^i - q^2 N^i\Big)$$

$$q \cdot N = (E_{\pi^\pm} - E_{\pi^0})m_\tau$$

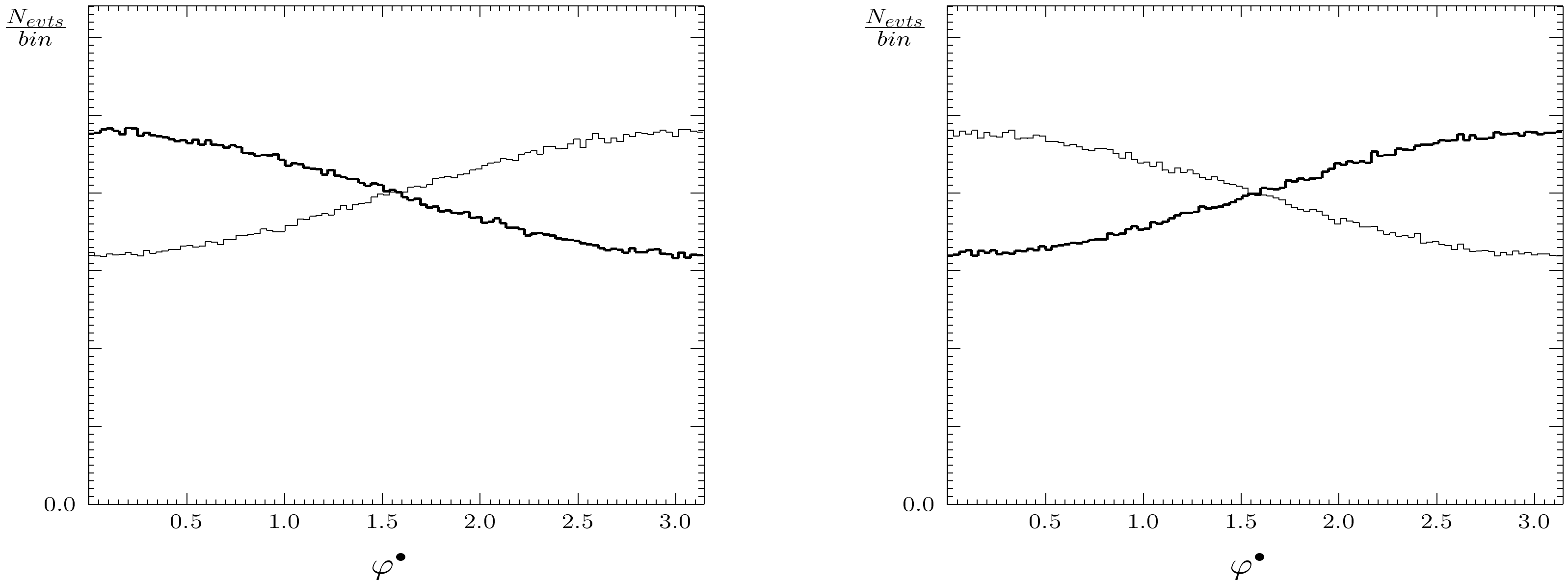
- *Acoplanarity of* $\rho^+$ *and* $\rho^-$ *decay prod. (in* $\rho^+\rho^-$ *r.f.) and events separation.*




$$y_1 y_2 > 0\,; \qquad y_1 y_2 < 0 \text{ (in } \tau^\pm \text{ r.f.'s)}$$

$$y_1 = \frac{E_{\pi^+} - E_{\pi^0}}{E_{\pi^+} + E_{\pi^0}}\,; \qquad y_2 = \frac{E_{\pi^-} - E_{\pi^0}}{E_{\pi^-} + E_{\pi^0}}.$$

## Results With Detector Effects




- *Gaussian spreads of the 'measured' quantities with respect to the generated.*
- *Resolutions verified with SIMDET. Replacement $\tau^{\pm}$ r.f.'s were used for $y_{1,2}$.*
- *Clearly distinguish the different parity states $-$ $3\sigma$ effect (0.5 ab$^{-1}$).*

$e^+e^- \longrightarrow ZH \rightarrow \mu^+\mu^- H$

$m_H = 120\ GeV$

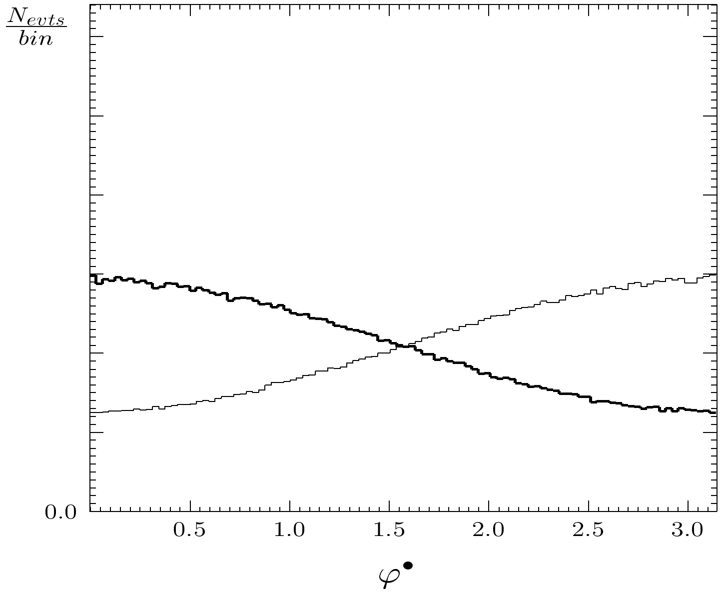
$\sqrt{s} = 500\ GeV$

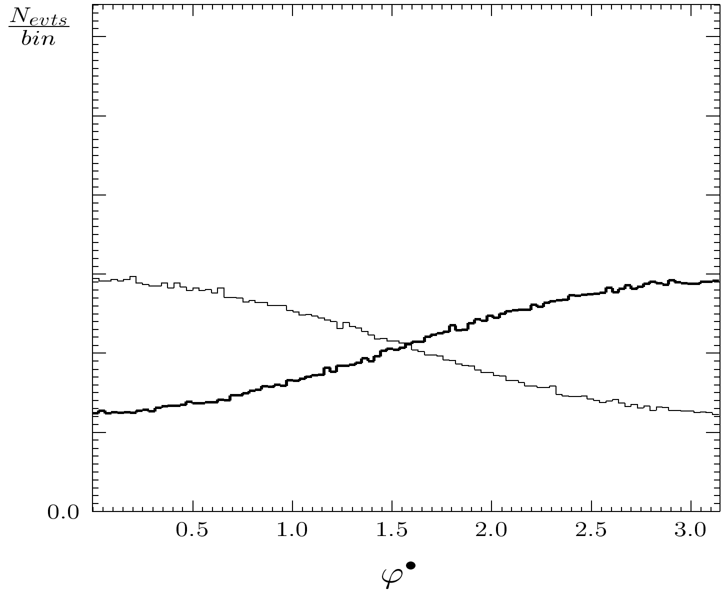
## *Replacement $\tau$ Rest Frame*

- Take just laboratory frame instead of $\tau^{\pm}$ r.f.'s.
- Invisibly better replacement $\tau$ rest frames:
  - In the restframe of $\rho^{+}\rho^{-}$ pair define $\tau^{\pm}$ momenta along direction of $\rho^{\pm}$,
  - For $\tau^{\pm}$ energies take half of the Higgs boson mass.
  - Boost replacement $\tau^{\pm}$ momenta to the lab frame.
- Many more, equally "good" options checked. The problem is that we can not determine direction of the $\nu_{\tau}$ because of Beamstrahlung.

*Here we used MC to understand observable build from 24 dimensions*

## *Results With $\tau$ Impact Parameter – Additional Cuts*

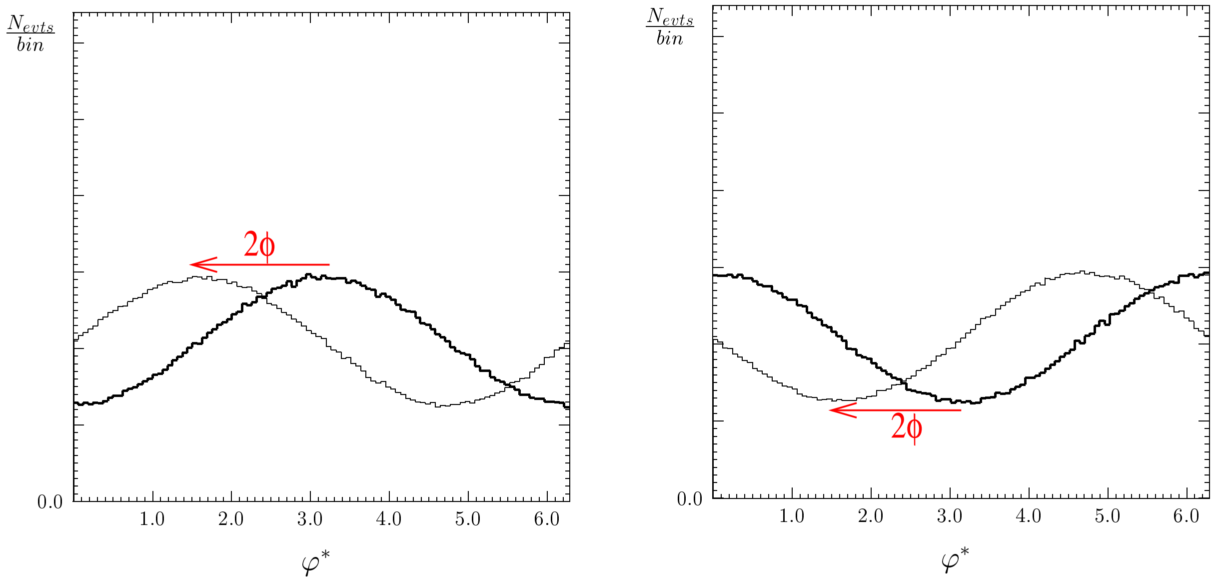



- *Only events where the signs of y1 and y2 are the same whether calculated using the method without or with the help of the $\tau$ impact parameter.*
- *Improvement* $\sim 107\%$.
- *Only* $\sim 52\%$ *events are accepted.*

*Improvement:* $\sim 4.5\sigma$

# Results For Mixed Scalar–Pseudoscalar Case




- *Only events where the signs of y1 and y2 are the same whether calculated using the method without or with the help of the $\tau$ impact parameter.*
- *Detector-like set-up is included (SIMDET).*
- *The thick line corresponds to a scalar Higgs boson, the thin line to a mixed one.*

*Precision on $\phi \sim 6^\circ$, for $1ab^{-1}$ and 350 GeV CMS.*

# Summary

- *We have reviewed spin effects.*
- *Theoretical aspects.*
- *Aspects of signatures buildup.*
- *How to negotiate less sensitive signatures but easier to detect.*
- *Use of any information.*
- *In other processes, eg. B decays similar optimalization may be of some use.*