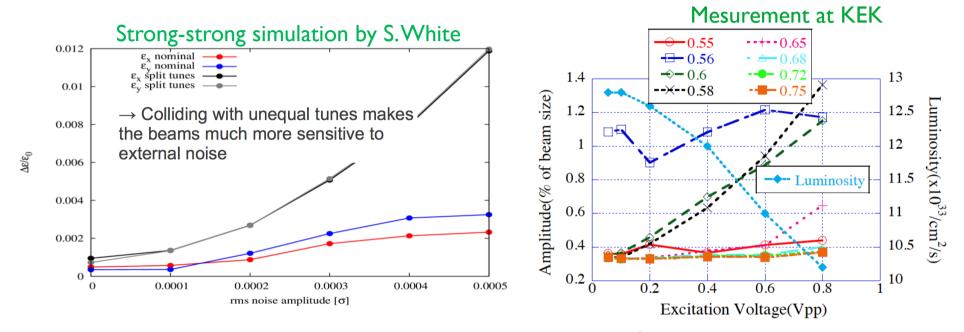
# Comments on BBNoise and E-lens instability

K. Ohmi (KEK) BB2013

#### Alternative understanding in noise effect



- Unequal tune result seems to fit weak-strong picture. Quadratic increase for noise amplitude.
- In equal tune, white noise induces σ mode, then the amplitude does not contribute emittance growth, results in non quadratic increase.

#### Heat-tail instability due to e-lens.

#### Tracking simulation

- Solve both equations of beam and electrons simultaneously and self consistently.
- Electrons are produced and tracked with the exact initial condition.

A typical ep instability simulation method, K. Ohmi et al., PRST 5, I 14402(2002).

$$\frac{d^{2}\mathbf{x}_{p,a}}{ds^{2}} + K(s)\mathbf{x}_{p,a} = \frac{2r_{p}}{\gamma} \sum_{j=1}^{N_{i}} \mathbf{F}_{G}(\mathbf{x}_{p,a} - \mathbf{x}_{e,j}; \sigma(s))\delta(s - s_{j})$$

$$\frac{d^{2}\mathbf{x}_{e,j}}{dt^{2}} = 2N_{p}r_{e}c^{2}\mathbf{F}_{G}(\mathbf{x}_{e,j} - \mathbf{x}_{p,a}; \sigma(s))\delta(t - t(s_{e,j})) + \frac{e}{m_{e}}\frac{dx_{e}}{dt} \times \mathbf{B} - 2r_{e}c^{2}\frac{\partial\phi}{\partial\mathbf{x}},$$

$$\uparrow \mathbf{p}^{+} \text{ beam} \qquad \sim 10-100\text{m}$$
Electron cloud

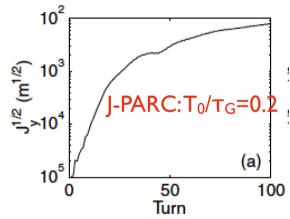
## Frequency range of the wake

$$\omega_e = \frac{eB}{m_e} = 2\pi \times 6.6 \times 10^{12} \text{ Hz}$$
  $B = 6 \text{ T}$ 

$$\omega_e \sigma_z / c = 1550 \quad \sigma_z = 0.44 \text{ m}$$

Stability

$$U = \frac{\sqrt{3}}{\Delta\omega\tau_G} = 1$$



$$\Delta\omega = \omega_e \eta \sigma_{\Delta p/p} = \frac{2\pi\nu_s \omega_e \sigma_z/c}{T_0} = \frac{3.5}{T_0} \qquad \nu_s = 0.00036$$

• Unstable  $T_0/T_G > 2$ 

Higher field -> stronger damping

#### Analytical formula for the threshold

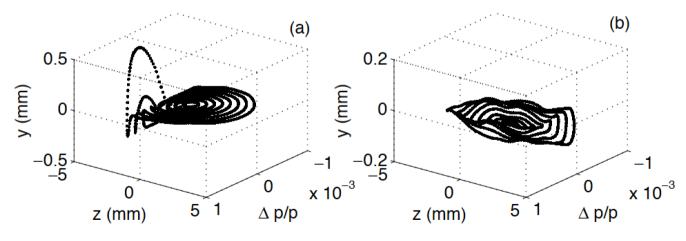
$$\begin{split} W_{1}(z) &= c \frac{R_{S}}{Q} \frac{\omega_{e}}{\tilde{\omega}} \exp \left(\frac{\alpha}{c}z\right) \sin \left(\frac{\tilde{\omega}}{c}z\right), & cR_{S}/Q = \frac{\lambda_{e}}{\lambda_{p}} \frac{L}{(\sigma_{x} + \sigma_{y})\sigma_{y}} \frac{\omega_{e}}{c}. \\ Z_{1}(\omega) &= \frac{c}{\omega} \frac{R_{S}}{1 + iQ\left(\frac{\omega_{e}}{\omega} - \frac{\omega}{\omega_{e}}\right)} \\ &= \frac{\lambda_{e}}{\lambda_{p}} \frac{L}{\sigma_{y}(\sigma_{x} + \sigma_{y})} \frac{\omega_{e}}{\omega} \frac{Z_{0}}{4\pi} \frac{Q}{1 + iQ\left(\frac{\omega_{e}}{\omega} - \frac{\omega}{\omega_{e}}\right)}, \end{split}$$

The shape of the wake is different, but the stability should be similar in the order of magnitude.

$$U \equiv \frac{\sqrt{3}\lambda_p r_p \beta \omega_0}{\gamma \omega_e \eta \sigma_{\delta E/E}} \frac{|Z_1(\omega_e)|}{Z_0}$$

### Another method

- Simulation using multi-circular distribution in longitudinal phase space.
- Landau damping due to the slippage is taken into account automatically. But I feel the result is inaccurate for high  $\omega_e\sigma_z/c=1550$

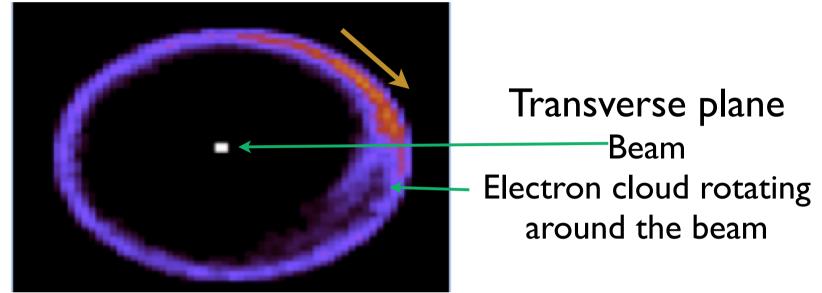


K. Ohmi, F. Zimmermann, PRL85, 3821 (2000)

## Complete PIC like PEHTS/HEADTAIL

• Very hard for  $\omega_e \sigma_z/c = 1550$ 

#### Slow mode related to EB drift



• Frequency depend on macroscopic radius of electron motion  $\bar{r}$ .

Rotate around beam

$$\omega = \frac{r_e \lambda_p c^2}{\omega_e \bar{r}^2}$$

• Can the motion be coherent?