BEAM-BEAM EFFECTS UNDER THE INFLUENCE OF EXTERNAL NOISE

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Abstract

Fast external noise, which gives fluctuation into beam orbit, is discussed in connection with beam-beam effect. Phase noise of the crab cavities and detection (position monitor) and kicker noise of the bunch by bunch feedback system are the sources. Beam-beam collision with offset fluctuating fast, turn by turn or several turns correlation, causes emittance growth and luminosity degradation. We discuss the tolerance of the noise amplitude for LHC and HL-LHC.

INTRODUCTION

Beam-beam effects under external noise is studied with the weak-strong model mainly in this paper. The strong beam is regarded as a target with a Gaussian charge distribution. In the model, an external noise is introduced into the transverse position of the strong beam at the collision point.

We first discuss an orbit (transverse position) shift of the strong beam given as follows,

$$\Delta x_{i+1} = (1 - 1/\tau)\Delta x_i + \delta x \hat{r}. \tag{1}$$

where Δx_i is the orbit shift at i-th turn. τ, δx and $\hat{r}$ are the damping time, a constant characterize the random fluctuation amplitude and Gaussian random number with the unit standard deviation, respectively. This is known as Ornstein-Uhlenbeck process. This type of noise is referred as 1st type later.

All particles in the weak beam experience the fluctuation of the strong beam, thus a transverse corective motion is induced. The corective motion results an emittance growth on smearing due to the nonlinear beam-beam force.

The stable amplitude of the fluctuation of the strong beam is given by

$$\Delta x^2 = \langle \Delta x^2_{n\to\infty} \rangle = \frac{\tau \delta x^2}{2}. \tag{2}$$

The correlation function between i-th and i+n-th turns is expressed by the damping time as follows,

$$\langle \Delta x_\ell \Delta x_{\ell+n} \rangle = \Delta x^2 e^{-|n|/\tau}. \tag{3}$$

The damping time is regarded as correlation time of the fluctuation. For turn by turn white noise, which corresponds to $\tau = 1$, the correlation function is expressed by

$$\langle \Delta x_\ell \Delta x_{\ell+n} \rangle = \Delta x^2 \delta_{n0} \tag{4}$$

where δ_{n0} is the Kronecker delta.

The beam oscillates with betatron frequency. We consider another (2nd) type of noise as follows,

$$\Delta x_{i+1} = (1 - 1/\tau)(\Delta x_i \cos\mu_o + \Delta p_i \sin\mu_o) + \delta x \hat{r} \tag{5}$$

$$\Delta p_{i+1} = (1 - 1/\tau)(-\Delta x_i \sin\mu_o + \Delta p_i \cos\mu_o) + \delta x \hat{r} \tag{6}$$

where x and p are coordinate and its canonical momentum normalized by beta function, so that $J = (x^2 + p^2)/2$. $\mu_o = 2\pi\nu_o$ is betaron tune multiplied by 2π of orbit oscillation. in collision offset induces an emittance growth The stable dipole oscillation amplitude is expressed by the same equation as Eq.(2). The correlation function containes betatron tune as follows,

$$\langle \Delta x_\ell \Delta x_{\ell+n} \rangle = \Delta x^2 e^{-|n|/\tau} \cos n\mu_o. \tag{7}$$

We discuss the effect of noise for LHC and High Luminosity-LHC The parameters are listed in Table 1. Basically the phenomena depend on the beam-beam parameter, noise amplitude normalized by the beam size and Piwinski angle.

EMITTANCE GROWTH DUE TO THE EXTERNAL NOISE

Nonlinear force due to the beam-beam interaction results an emittance growth under an external noise as is discussed by G. Stupakov, Y. Alexahin, T.Sen et al.,[1, 2, 3]. The previous works are reviewed in this section.

The beam-beam force (potential) with the bunch population (N_p) and the transverse size (σ_r) is expressed by

$$U(x) = \frac{N_p r_p}{\gamma_p} \int_0^\infty \frac{1 - e^{-x^2/(2\sigma_r^2 + q)}}{2\sigma_r^2 + q} dq \tag{8}$$

where r_p and γ_p are the classical radius of the proton and the relativistic factor of the (weak) beam, respectively. The potential is expanded by Fourier series as follows,

$$U(x) = \frac{N_p r_P}{\gamma_p} \sum_{k=0}^\infty U_k(a) \cos 2k\psi \tag{9}$$

where

$$U_k(a) = \int_0^a \left[\delta_{0k} - (2 - \delta_{0k})(-1)^k e^{-w} I_k(w)\right] \frac{dw}{w}, \tag{10}$$

and $a = \beta^* J/2\sigma_r^2 = J/2\varepsilon$. The change of J per one revolution is given by the derivative of the beam-beam potential for ψ as follows,

$$\Delta J = -\frac{\partial U}{\partial \psi} = \frac{N r_p}{\gamma} \sum_{k=0}^\infty 2k U_k \sin 2k\psi. \tag{11}$$

Table 1: Parameters for LHC and HL-LHC. Two parameter sets of HL-LHC are candidates for the bunch spacing 25 ns and 50 ns.

	LHC	HL-LHC(25ns)	HL-LHC(50ns)
Circumf. (L[m])		26,658	
Energy (E[TeV])		7	
Normalized Emittance (μm)	2.0	2.5	3.0
β^*	0.55	0.15	0.15
Bunch length (m)		0.0755	
Tune (x/y/z)		64.31/59.32/0.0019	
Bunch population (10^{11})	1.65	2.2	3.5
Number of bunches	1380	2808	1404
Beam-beam parameter/IP	0.0034	0.005-0.011	0.005-0.014

This change, which indicates a stable sinusoidal modulation of the betatron amplitude, does not induce emittance growth.

We consider the case in which the strong beam has a small offset (Δx), The beam-beam potential with the offset is expanded for Δx,

$$U(x+\Delta x) = U(x) + U'(x)\Delta x. \tag{12}$$

Δx is a random variable fluctuated by Eq.(1) or (6).

The potential with the offset is expanded by Fourier series,

$$U'(J,\psi) = \frac{\partial U}{\partial J}\frac{\partial J}{\partial x} + \frac{\partial U}{\partial \psi}\frac{\partial \psi}{\partial x} \tag{13}$$

$$= \frac{N_p r_p}{2\gamma\sigma_r}\sum_{k=0}^{\infty} G_k(a)\cos(2k+1)\psi. \tag{14}$$

Fourier coefficient as function of a is expressed by

$$G_k(a) = \sqrt{a}\left[U'_{k+1} + U'_k\right] + \frac{1}{\sqrt{a}}\left[(k+1)U_{k+1} - kU_k\right]. \tag{15}$$

where U'_k is derivative for a.

The diffusion of J^2 after N revolutions is given by

$$\langle \Delta J^2(N)\rangle = \sum_{\ell=1}^{N}\sum_{n=-\ell+1}^{N-\ell}\frac{\partial U'(\ell)}{\partial\psi}\frac{\partial U'(\ell+n)}{\partial\psi}\langle\Delta x_\ell \Delta x_{\ell+n}\rangle \tag{16}$$

For turn-by-turn white noise, the correlation function is replaced by the Kronecker delta, δ_{n0}. The diffusion of J is expressed by

$$\langle \Delta J^2\rangle = \frac{\langle \Delta J^2(N)\rangle}{N} \approx \frac{N_p^2 r_p^2}{8\gamma^2\sigma_r^2}\sum_{k=0}^{\infty}(2k+1)^2 G_k(a)^2. \tag{17}$$

The diffusion of J per revolution is given for the fluctuation in Eq.(1) by

$$\langle \Delta J^2\rangle \approx \frac{N_p^2 r_p^2}{8\gamma^2\sigma_r^2}$$

$$\sum_{n=-\infty}^{\infty}\sum_{k=0}^{\infty}(2k+1)^2 G_k^2\cos[(2k+1)n\mu_o]e^{-|n|/\tau}$$

$$\approx \frac{N_p^2 r_p^2}{8\gamma^2\sigma_r^2}\sum_{k=0}^{\infty}\frac{(2k+1)^2 G_k(a)^2\sinh 1/\tau}{\cosh 1/\tau - \cos(2k+1)\mu_o} \tag{18}$$

The diffusion of J for the 2nd type of noise (Eq.(6)) is given using the correlation of Eq.(7) as follows,

$$\langle \Delta J^2\rangle \approx \frac{N_p^2 r_p^2}{16\gamma^2\sigma_r^2}\sum_{k=0}^{\infty}(2k+1)^2 G_k(a)^2\sinh 1/\tau$$
$$\left[\frac{1}{\cosh 1/\tau - \cos(2k\mu - \delta\mu)} + \frac{1}{\cosh 1/\tau - \cos(2(k+1)\mu + \delta\mu)}\right]. \tag{19}$$

where $\delta\mu$ is tune difference between the weak and strong beam oscillations ($\delta\mu = \mu - \mu_o$).

Figure 1 shows the diffusion rate of J as function of J. The diffusion rate is proportional to the square of the fluctuation amplitude Δx and on square of the beam-beam parameter $\propto N_p$. The rate is normalized by the combined factor, $C = (N_p r_p \Delta x/\gamma\sigma_r)^2/8$ in the figure.

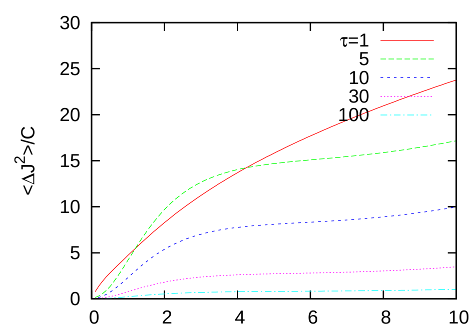

Figure 1: Diffusion rate given by Eq.(18). The rate is normalized by $C = (N_p r_p \Delta x/\gamma\sigma_r)^2/8$.

The emittance growth is evaluated by the diffusion rate, when the rate $\langle \Delta J^2 \rangle$ is proportional to J,

$$\frac{\Delta\varepsilon}{\varepsilon} = \frac{\langle \Delta J^2 \rangle}{2\varepsilon J} = \frac{1}{4\varepsilon^2}\frac{d\langle \Delta J^2 \rangle}{da}. \quad (20)$$

Actually Figure 1 shows the rate is proportional to J for small $J/2\varepsilon < 2$. The slope of $\langle \Delta J^2 \rangle$ for turn-by-turn noise ($\tau = 1$) gives

$$\frac{\langle \Delta J^2 \rangle}{a} = \frac{N_p^2 r_p^2}{8\gamma^2}\frac{\Delta x^2}{\sigma_r^2} \times 4.4. \quad (21)$$

The luminosity degradation rate per collision is estimated by the emittance growth rate as follows

$$\Delta L/L = \left(\xi \frac{\Delta x}{\sigma_r}\right)^2 \times 21.7. \quad (22)$$

For 2 IP, the formula is corrected by factor 2, $21.7 \rightarrow 10.8$ and replacement of $\xi \rightarrow \xi_{tot}$, The tolerance for the noise amplitude is given for one day luminosity life time $\Delta L/L = 10^{-9}$,

$$\xi_{tot}\frac{\Delta x}{\sigma_r} = 9.8 \times 10^{-6} \quad (23)$$

We now discuss second type of noise given by Eq.(6). Figure 2 shows the diffusion rates. Plots (a) and (b) are given for the beam-orbit oscillation with the same tune ($\delta\mu = 0$) and with difference of $\delta\mu = \xi = 0.01$, respectively. A strong enhancement of the diffusion is seen at small amplitude at a large correlation time in Plot (a). This behavior mainly comes from contribution of $k = 0$.

$$\langle \Delta J^2 \rangle \approx \frac{N_p^2 r_p^2}{16\gamma^2\sigma_r^2} G_0(a)^2 \tau \quad (24)$$

The strong beam modulation with the same tune gives an external force oscillation to the weak beam particles. For colliding beam, the assumption, in which beam-orbit oscillation have the same tune, is not clear. The diffusion rate for $\delta\mu = \xi$ in plot (b) may be better to represent the beam-beam system. The diffusion rate, which is saturated at $J/2\varepsilon = 1$, is similar as that of $\tau = 1$ on the whole. Therefore we study the diffusion rate for $\tau = 1$ in simulations later.

It may be better that the noise effects should be studied in the frame of strong-strong model, especially in the 2-nd type of noise. The noise induces σ mode, π mode or that with continuous frequency. σ mode does not contribute the emittance growth, or π does not also contribute because of no Landau damping. Emittance growth based on the strong-strong model had been discussed in Ref.[3]. The author (Y. Alexahin) discussed that 18% of the dipole motion induced by offset collision imparted into the mode with continuous frequency spectrum. The emittance increases during smearing the mode of the dipole motion. The growth rate is expressed by

$$\frac{\delta\varepsilon}{\varepsilon} \approx \frac{K}{\left(1 + \frac{1}{2\pi\tau|\xi|}\right)^2}\frac{\delta x^2}{\sigma_x^2} = \frac{2K}{\tau\left(1 + \frac{1}{2\pi\tau|\xi|}\right)^2}\frac{\Delta x^2}{\sigma_x^2} \quad (25)$$

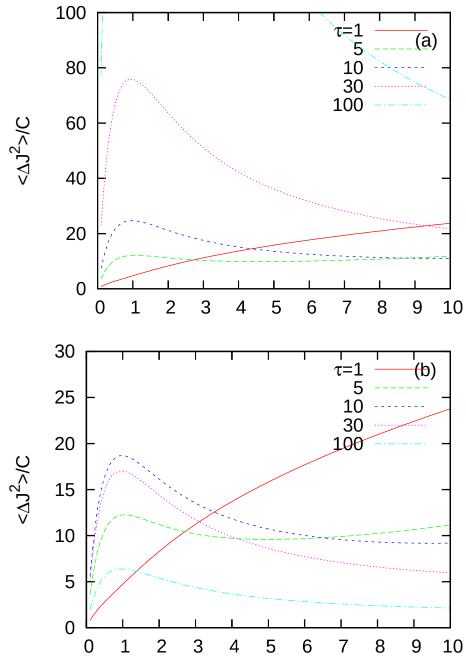

Figure 2: Diffusion rate given by Eq.(19). The rate is normalized by $C = (N_p r_p \Delta x/\gamma\sigma_r)^2/8$.

where $K = 0.089$ is a form factor for the emittance change induced by a dipole amplitude, and $1/\tau$ is damping rate of the coherent motion. The emittance growth rate is independent of the beam-beam tune shift, when $1/\tau \ll 2\pi|\xi|$, while the rate is proportional to the square of the beam-beam tune shift, when $1/\tau \gg 2\pi|\xi|$.

Figure 3 shows the emittance growth given by Eq.(25) and by a strong-strong beam-beam simulation [4], where the beam-beam tune shift is $\xi = 0.0034/IP$. The results agree fairly well. The strong-strong simulation suffers numerical noise related to the statistics of macro-particles. 1 million macro-particles are used in the simulation, thus 0.1% of the offset noise is induced by the statistics.

SIMULATION OF EXTERNAL NOISE

Study based on LHC

The analytic theory is based on the near solvable system far from resonances. There is no such limitation in beam-beam simulations, while simulation takes considerable computing time to evaluate a slow emittance growth. Simulations considering the external noise is straightforward: a modulation is applied to strong beam with Eq.(1) or (6). Effects of resonances, longitudinal motion and crossing angle are taken into account in simulations.

We solely discuss weak-strong simulation taking into account of an external noise. The weak beam is represented by 131,072 macro-particles. The particles are tracked 1M turns with interacting a strong beam located at two interaction points. The luminosity is calculated turn-by-turn, and

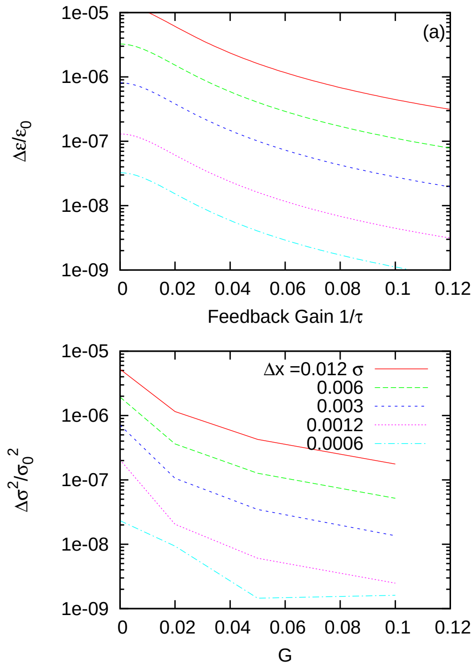

Figure 3: Emittance growth given by Eq.(25) and by a strong-strong beam-beam simulation [4].

averaged every 100 turns. Luminosity degradation is evaluated by fitting its evolution.

Figure 4 shows Luminosity degradation for collision without crossing angle. The degradation is plotted as function of the fluctuation amplitude for three total beam-beam parameters, $\xi_{tot} = 0.02$, 0.04 and 0.05. Three lines given by analytic formula Eq.(23 are drawn in the figure. The simulation results agree with the formula fairly well.

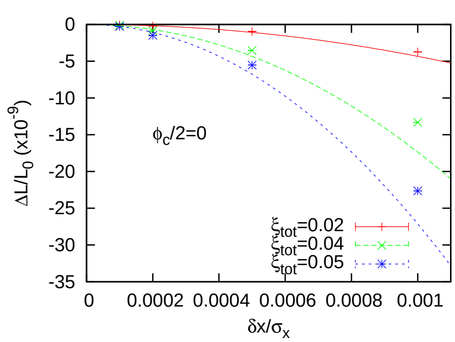

Figure 4: Diffusion rate given by weak-strong simulation and Eq.(23).

Luminosity degradation for collision with crossing angle ($\phi_c = 290\ \mu$rad) is shown in Figure 5. Piwinski angle is $\phi_c\sigma_z/2\sigma_r = 0.89$.

Figure 6 shows luminosity degradation as a function of the beam-beam parameter under an offset noise. The tune shift is reduced to 70% for the crossing collision. The luminosity degradation for noise is independent of the crossing angle. At higher beam-beam parameter > 0.05, the luminosity degradation due to crossing angle is dominant.

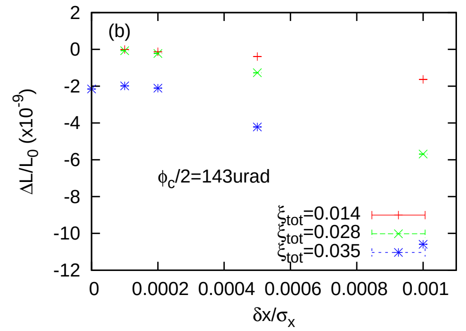

Figure 5: Diffusion rate for crossing collision given by weak-strong simulation.

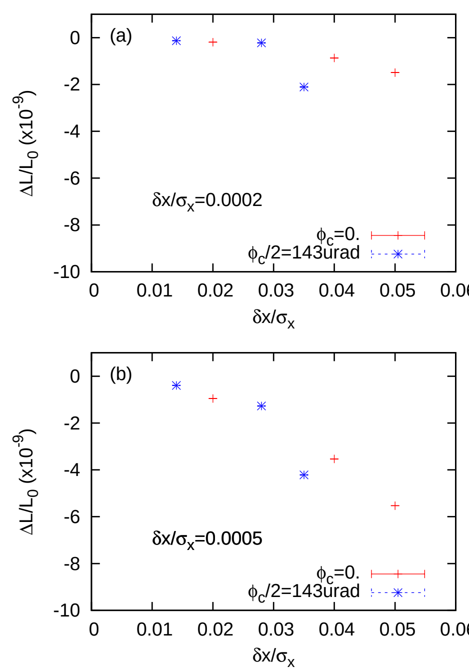

Figure 6: Luminosity degradation as a function of the beam-beam parameter under an offset noise.

There was no qualitative change from collision without crossing angle. For $\xi_{tot} = 0.035$, degradation due to crossing angle is seen, but clear cross-talk is not seen. The degradation of luminosity due to the fluctuation depends on ξ_{tot}, but little depends on existence of the crossing angle.

HL-LHC

In HL-LHC, higher luminosity is targeted by increasing bunch population and squeezing to smaller beta function. While pile up of collision event limits the luminosity $L/coll = 2.6 \times 10^{31}$ cm^{-2}s^{-1}. The luminosity at $\beta = 0.15$ m is $L/coll = 8.6$ or 18×10^{31} cm^{-2}s^{-1} for the bunch population of 2.2 or 3.5×10^{11}, respectively. Therefore luminosity leveling with keeping the luminosity $L/coll = 2.6 \times 10^{31}$ cm^{-2}s^{-1} is proposed. The leveling is done by controlling of crab cavity voltage or IP beta function. In the leveling with beta function, the total beam-beam parameter (2IP) is $0.011 \times 2 = 0.022$ (25ns) or $0.014 \times 2 = 0.028$ (50ns) at the early stage of the collision, where the beta function is 0.49 m or 1.02 m [6]. The results given by the previous subsection is applied for the beam-beam parameters: that is,

$$\frac{\Delta x}{\sigma_r} = 4.5 \times 10^{-4} \text{ or } 3.5 \times 10^{-4}. \qquad (26)$$

for 25 ns and 50 ns schemes, respectively.

In the leveling with crab cavity, the crab voltage increases to keep the luminosity for the beam current decrease. At the early stage of collision, the crab voltage is low and two beams collide with a large Piwinski angle, where $\phi_c\sigma_z/2\sigma_r = 3.14$ or 2.87 for 25 ns or 50 ns scheme, respectively. We actually study effects of noise for the collision with a large Piwinski angle in this subsection.

Figure 7 shows the luminosity degradation rate as function of the offset amplitude. The simulation is performed for 2 IP. The tune shift is 0.0015/0.0050 in crossing/perpendicular plane for the design bunch population $N_p = 2.2 \times 10^{11}$ (25ns). The tune shift is 0.0065 in both plane, because of the combination of the horizontal and vertical crossing. The fluctuation amplitude 0.2% is tolerable limit for $\Delta L/L_0 = 10^{-9}$ as shown in the figure. The simple formula Eq.(23) is satisfied in HL-LHC, $0.0065 \times 0.002 = 1.3 \times 10^{-5}$, 30% difference from the formula.

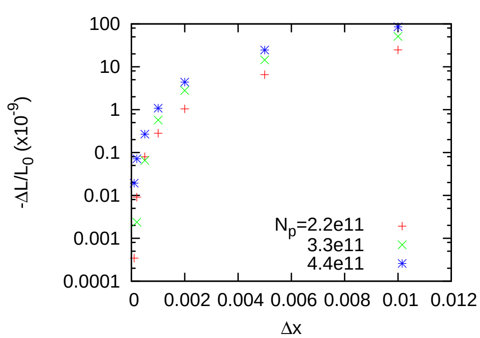

Figure 7: Luminosity degradation as function of noise amplitude.

Figure 8 shows the luminosity degradation for the correlation time. The luminosity degradation, which scales $1/\tau$, behaves in consistency with Eq.(18).

Effects of noise for large Piwinski angle collision is similar as those for the ordinary collision without crossing angle on the tolerance for noise and dependence on the correlation time. The luminosity degradation is the beam-beam parameter and the noise amplitude, but is little dependent of Piwinski angle.

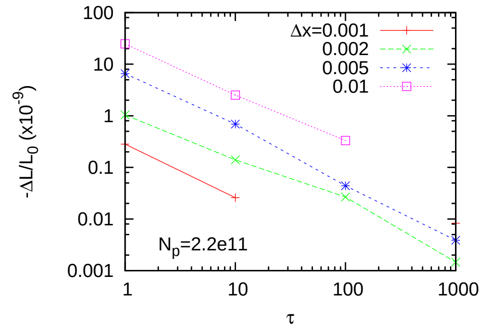

Figure 8: Luminosity degradation as function of the correlatoion time.

Tolerance for crab cavity phase noise in HL-LHC

Crab cavities are used to compensate the crossing angle ($\phi_c = 590$ μrad) at IP. The relation of the pase noise and collision offset is given by

$$\Delta\varphi_{cc} = \frac{\omega_{cc}}{c\phi_c/2}\Delta x, \qquad (27)$$

where $\Delta\varphi_{cc}$ and ω_{cc} are the phase fluctuation and frequency of crab cavity.

Beta function leveling the beam-beam parameter is very high, $\xi_{tot} = 0.022$ or 0.028 for 25 ns or 50 ns schemes, respectively. The tolerance of the noise amplitude is given Eq.(26). Corresponding phase error is $\Delta\varphi = 1.6 \times 10^{-4}$ or 2.3×10^{-4} rad.

In the crab cavity leveling, the beam-beam parameter is $\xi_{tot} = 0.0065$ The tolerance of the noise amplitude is $\Delta x/\sigma_r = 0.002$ and corresponding phase error is $\Delta\varphi = 4\times10^{-3}$, where the crab angle is 10% of the crossing angle, $\phi_c = 59$ μrad ($L/coll = 2.7 \times 10^{31}$ cm^{-2}s^{-1}).

The crab cavity noise was measured at KEKB, 1.7×10^{-4} rad above 1kHz ($\tau < 10$). The value is critical for the beta function leveling, because of the high beam-beam parameter. For using 4 crab cavities, the noise tolerance is twice more severe. While for the crab voltage leveling, the measured phase error is tolerable.

Incoherent noise due to intra-beam scattering

Emittance growth due to intra-beam scattering (IBS) is 105 h and 63 h for the horizontal and longitudinal, respectively, in the nominal LHC [13]. The transverse emittance and bunch population in the nominal are 5.0×10^{-10} and 1.15×10^{11}, respectively. The horizontal IBS growth rate is approximately proportional to the particle density in the six dimensional phase space. The growth time is 40 h for ξ_{tot}=0.02 in this paper ($\varepsilon = 2.7 \times 10^{-10}$ and $Np = 1.63\times10^{11}$). The fluctuation is $\delta x/\sigma_x = 5.5\times10^{-5}$ for ξ_{tot}=0.05 (16h). The luminosity degradation is determined by geometrical emittance growth $\delta L/L_0 = \delta x^2/\sigma_x^2$ for incoherent noise.

COHERENT BEAM-BEAM EFFECTS UNDER EXTERNAL NOISE

Effects of external noise in crab cavity were performed in KEKB during 2008 and 2009 [7]. Sinusoidal noise is applied into crab cavity RF system. Near the σ mode tune, a strong luminosity drop ($L = 0.2L_0$) was seen suddenly exceeding a threshold excitation amplitude. A weak luminosity drop ($L = 0.9L_0$) was seen near π mode freqency. Strong-strong simulation reproduced these luminosity drops. Systematic study using the strong-strong simulation showed that these was characteristic phenomena for coherent nonlinear beam-beam interactions. Similar phenomenon was observed in Ref.[8]. The detailed analysis is published in Ref.[7].

CONCLUSIONS

Fast noise of the collision offset degrades the luminosity performance in hadron colliders. The luminosity degradation depends on the product of the noise amplitude and the beam-beam parameter as shown in Eq.(23), while little depends on the crossing angle. The tolerance of the crab cavity phase error was obtained for HL-LHC.

The crab cavity noise was measured at KEKB, 1.7×10^{-4} rad above 1kHz ($\tau < 10$). The value is critical for the beta function leveling, because of the high beam-beam parameter. While for the crab voltage leveling, the measured phase error is tolerable because of the small beam-beam tune shift.

More studies related to beam-beam mode should be done using strong-strong model.

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