Beam-Beam, Impedance and Damper

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FNAL-LARP

many thanks to

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ICFA BB2013 Workshop, March 19/2013

Contents

- Damper theorem: statement and proof
- Damper theorem: simple model illustration
- Nested Head-Tail (NHT) Vlasov Solver
  - NHT main blocks
  - Damper theorem: NHT illustration
- Beam-beam-beam effect in the LHC?

Flat Response Functions

- Any mechanism of linear response to a beam perturbation can be described by a proper response function.

- *Definition*: A response is **flat** if its gradients over a bunch length can be neglected.

- LHC examples of flat response functions:
  - Damper;
  - Inter-bunch wake (except HOM if any);
  - Coherent beam-beam when $\beta \gg \sigma_z$.

- All flat responses can be described in a similar way.

- The net result of all the flat responses is very sensitive to one which is in our hands – to the **damper**.

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Damper Theorem:

- **If the gain is high enough, all other flat responses can be neglected.**

- *Proof :*

  Flat response is a response to bunch centroids.

  At high gain, bunch centroids do not move, being frozen by the damper. Thus, there is no input signal for any flat response, making it turned off.

- *Question:*

  What does it mean: “the gain is high enough”?

To answer, let’s consider a proof #2…

Proof #2: Simple Model

Let $A_{1,2}$ be amplitudes of HT eigenmodes in beam 1 and 2. Due to BB, they become coupled:

$$\dot{A}_1 = -i\omega_c \dot{A}_1 - d\alpha A_1 - iq\alpha A_2 \ ;$$
$$\dot{A}_2 = -i\omega_c \dot{A}_2 - d\alpha A_2 - iq\alpha A_1 \ .$$

Here d and q are the damping rate and beam-beam tune shifts correspondingly, the parameter α reflects a weight of the center of mass in the amplitudes A. For mode=0, at chroma=0, $\alpha = 1$.

From here, the pi and sigma tune shifts follow:

$$\Omega = \omega_c - id\alpha \pm q\alpha \ .$$

Since the mode is unstable (otherwise we do not care) $d\alpha < \mathrm{Im}(\omega_c)$.

Thus the coherent BB tune shift is as small as

$$q\alpha < \mathrm{Im}(\omega_c) q / d.$$

Proof #2: Simple Model (2)

- Since the stability diagram is normally 3-10 times less sensitive to the real tune shift than to the imaginary, this gives a conservative estimate for the CBB role in the instability threshold:

$$\frac{\Delta I_{\mathrm{CBB}}}{I} \lesssim 0.3\frac{q}{d}\,.$$

- Note that this CBB destabilizing is reduced by the incoherent BB in case the latter does not conflict with Landau octupoles.

- This estimate assumes sufficiently smooth dependence of the threshold on the machine and beam parameters (more details follow).

- Same formula is correct for the coupled-bunch flat contribution, with q as the couple-bunch tune shift, computed for bunches as macroparticles.

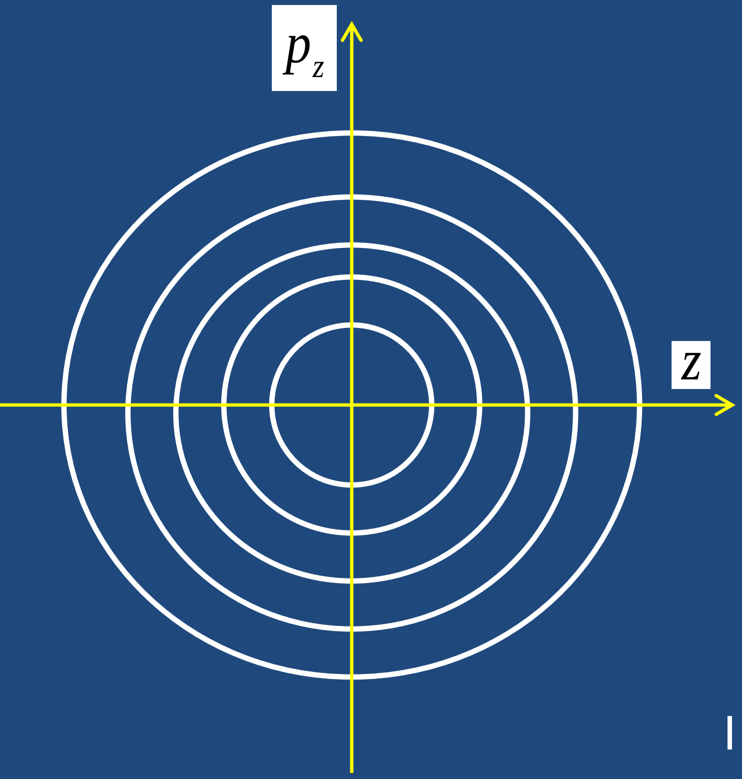
Nested Head-Tail Vlasov Solver (AP Forum 4/12/12)

- NHT is my Mathematica-based program for LHC-type beam stability analysis. It accepts the following external data:
  - Inter- and intra-bunch wakes (arbitrary functions);
  - Damper with provided gain frequency profile;
  - Beam-beam collision scheme;
  - Octupoles and beam-beam nonlinearity;
  - Bunch distribution function.
- The code computes:
  - Azimuthal, radial, coupled-bunch and beam-beam modes;
  - Beam stability thresholds.

- NHT was cross-checked with BeamBeam3D tracking simulations (S. White), showing a full agreement.

- Generation of all the results for given gain and chromaticity takes ~1 second at my 3 year old laptop.

Nested Head-Tail Basis



$$\psi_{l\alpha} \propto \exp(il\phi + i\chi_\alpha \cos\phi - i\omega_b t)\,;$$

$$\chi_\alpha = \frac{Q'\omega_0 r_\alpha}{c\eta}\;;$$

I am using n_r equally populated rings which radii r_α are chosen to reflect the phase space density.

NHT, single bunch

- In the air-bag single bunch approximation, beam equations of motion can be presented as in Ref [A. Chao, Eq. 6.183]:

$$\dot{X} = \hat{S} \cdot X + \hat{Z} \cdot X + \hat{D} \cdot X$$

where X is a vector of the HT mode amplitudes,

$$(\hat{S} + \hat{Z})_{lm\alpha\beta} = -il\delta_{lm}\delta_{\alpha\beta} - i^{l-m}\frac{\kappa}{n_r}\int_{-\infty}^{\infty} d\omega Z(\omega) J_l(\omega\tau_\alpha - \chi_\alpha) J_m(\omega\tau_\beta - \chi_\beta)$$

$$\hat{D}_{lm\alpha\beta} = -i^{m-l}\frac{d}{n_r} J_l(\chi_\alpha) J_m(\chi_\beta)$$

d is the damper gain in units of the damping rate, $\kappa = \dfrac{N_b r_0 R_0}{8\pi^2 \gamma Q_b Q_s}$

time is in units of the angular synchrotron frequency.

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NHT, Coupled Bunches

Main idea:

For LHC, wake field of preceding bunches can be taken as flat within the bunch length.

The only difference between the bunches is CB mode phase advance, otherwise they are all identical.

Thus, the CB kick felt by any bunch is proportional to its own offset, so the CB matrix $\hat{C}$ has the same structure as the damper matrix $\hat{D}$:

$$\dot{X} = \hat{S} \cdot X + \hat{Z} \cdot X + \hat{D} \cdot X + \hat{C} \cdot X ;$$

$$\hat{D}_{lm\alpha\beta} = -i^{m-l} \frac{d_\mu}{n_r} J_l(\chi_\alpha) J_m(\chi_\beta); \qquad \hat{C} = 2\pi i \kappa W(\varphi_\mu) \hat{D} / d_\mu ;$$

$$W(\varphi_\mu) = \sum_{k=1}^{\infty} W(-ks_0) \exp(-ik\varphi_\mu); \quad \varphi_\mu = \frac{2\pi(\mu + \{Q_x\})}{M_b}; \quad 0 \le \mu \le M_b - 1 .$$

Wake and impedance are determined according to A. Chao book.

NHT, Coherent Beam-Beam

Main assumption: bunch length << beta-function. For transversely dipolar modes, CBB is a cross-talk of bunch CM – thus, intra-bunch matrix structure is similar to the ADT and CB:

$$
\begin{aligned}
\dot{X}_1 &= \hat{S}\cdot X_1 + \hat{Z}\cdot X_1 + \hat{D}\cdot X_1 + \hat{C}\cdot X_1 + b_{12}\hat{B}\cdot X_2;\\
\dot{X}_2 &= \hat{S}\cdot X_2 + \hat{Z}\cdot X_2 + \hat{D}\cdot X_2 + \hat{C}\cdot X_2 + b_{21}\hat{B}\cdot X_1;\\
\hat{B} &= -i\Delta\omega_{\mathrm{bb}}(\hat{D}/d_\mu)\sum_{k=-K}^{K}\frac{\beta_k}{\rho_k^2}\cos(k\phi_\mu)\Big/\sum_{k=-K}^{K}\frac{\beta_k}{\rho_k^2};\\
b_{12} &= b_{21}^{*} = 1-\exp(-i\psi).
\end{aligned}
$$

Here 2 identical opposite IRs are assumed (IR1 and IR5 for LHC) with $2K+1$ LR collisions for each, every one with its beta-function and separation β_k,ρ_k .

Alternating x/y collision for IR1/IR5 is assumed with ψ as a difference between the two phase advances, while $\Delta\omega_{\mathrm{bb}}$ is the incoherent beam-beam tune shift per IR.

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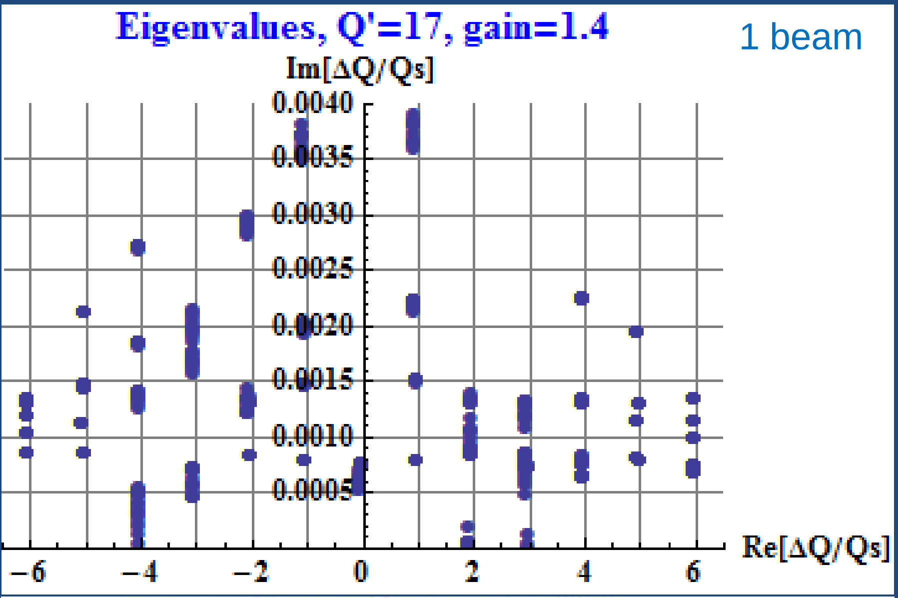
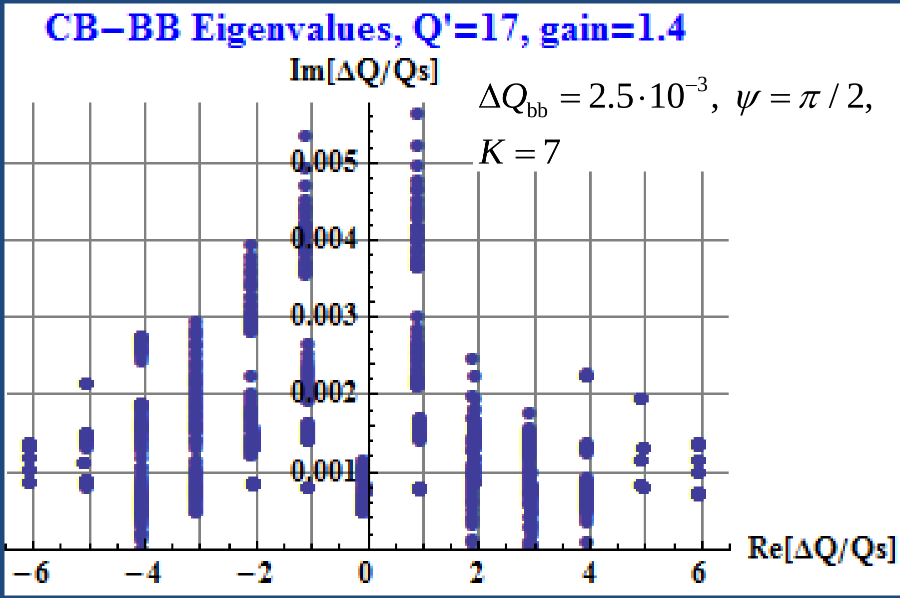
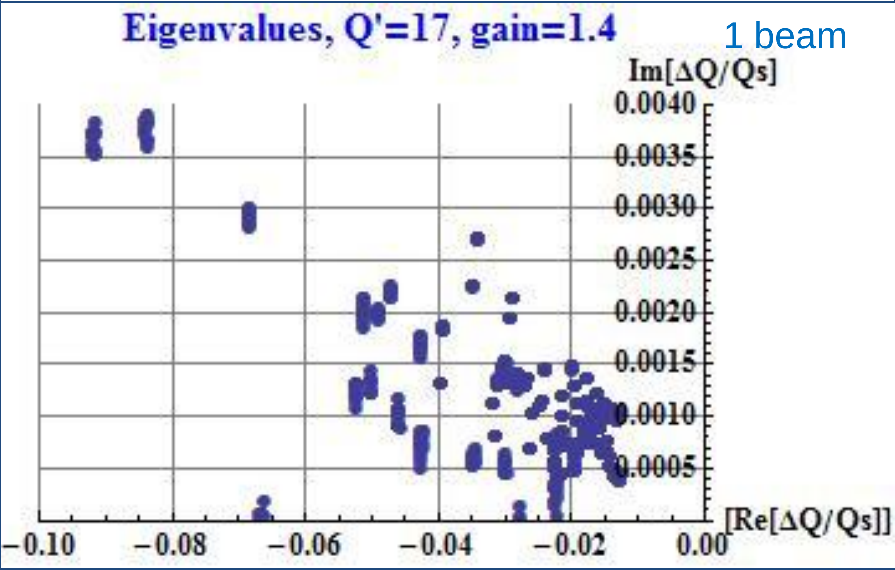
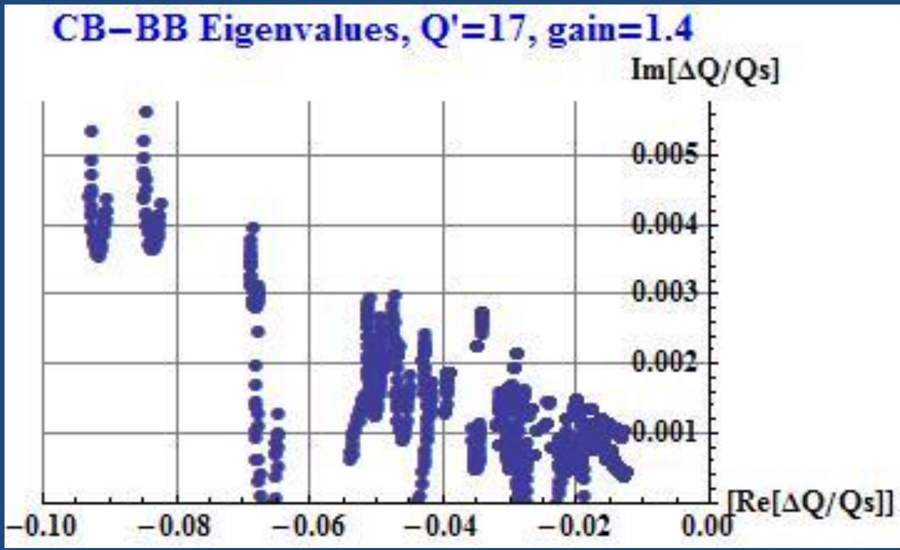
NHT, Coherent BB: effect ~30%









Impedance = 2*(N. Mounet model)

NHT, Stability Diagram

Stability diagram (SD) is defined as a map of real axes Ω on the complex plane:

$$\mathrm{D} = \left(-\int \frac{J_x \partial F / \partial J_x}{\Omega - l\omega_s - \delta\omega_x + io} d\Gamma \right)^{-1}$$

$$\delta\omega_x = \omega_0 \delta Q_x$$

$$\mathrm{D} = \Omega_c - l\bar{\omega}_s \equiv \omega_0 \Delta Q$$

To be stable, the coherent tune shift has to be inside the SD, determined by the octupoles and beam-beam:

$$\delta\mathbf{Q} \equiv \left(\delta Q_x, \delta Q_y\right)^{\mathrm{T}}; \qquad \mathbf{J} \equiv (J_x, J_y)^{\mathrm{T}};$$
$$\delta\mathbf{Q} = (\hat{\mathbf{A}}^{\mathrm{oct}} + \hat{\mathbf{A}}^{\mathrm{bb}}) \cdot \mathbf{J} / \varepsilon.$$

LHC, Landau octupoles (LO):
(E.Metral, N.Mounet, B.Salvant, 2010)

$$\hat{\mathbf{A}}^{\mathrm{oct}} = Q_s \frac{I_{\mathrm{oct}}}{100\mathrm{A}} \frac{\varepsilon}{2\mu\mathrm{m}} \begin{pmatrix} a_{xx} & a_{xy} \\ a_{yx} & a_{yy} \end{pmatrix};$$
$$a_{xx} = a_{yy} = 1.8 \cdot 10^{-2};$$
$$a_{xy} = a_{yx} = -1.3 \cdot 10^{-2};$$

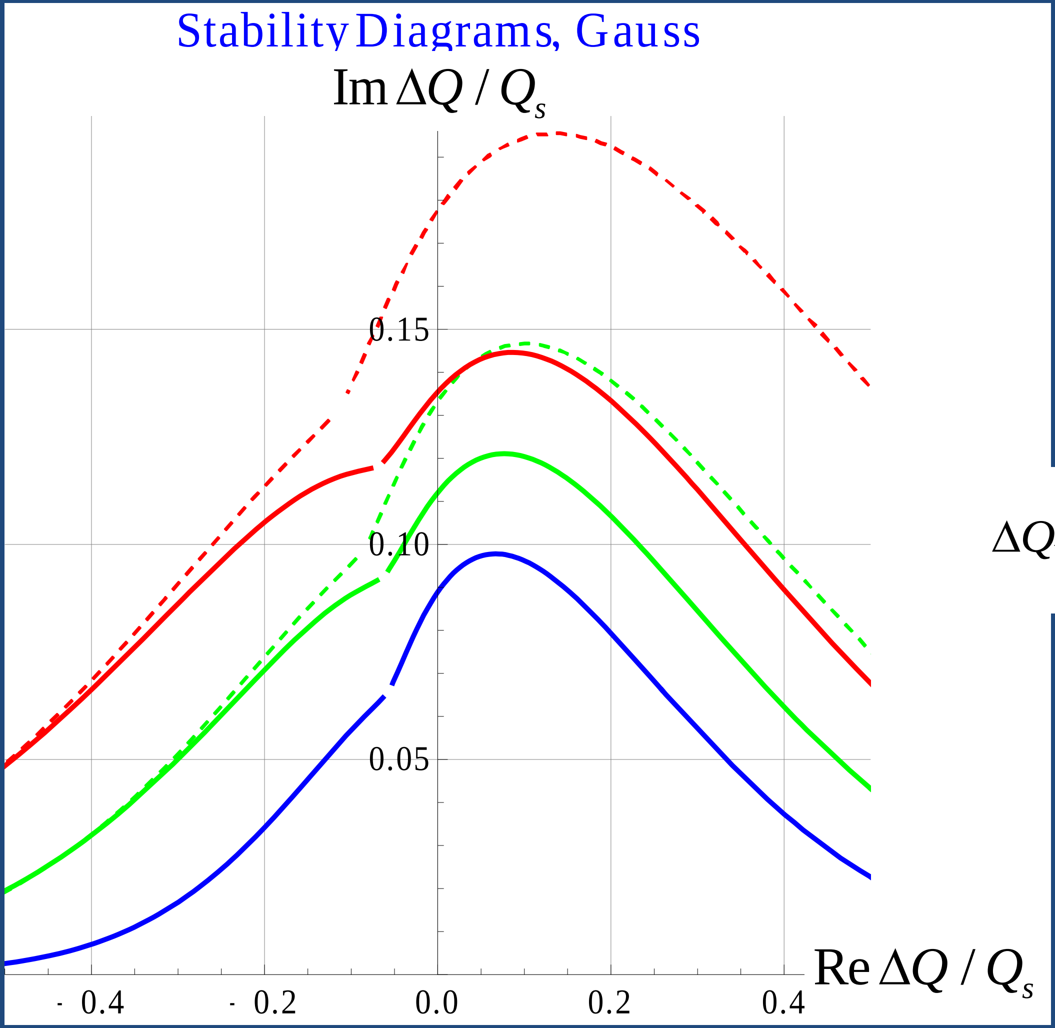
LHC, long-range beam-beam (LRBB), IR1+IR5, round-beam approximation:

$$\hat{\mathbf{A}}^{\mathrm{bb}} = \frac{3\Delta Q_{\mathrm{bb}}}{r^2} \begin{pmatrix} 1 & -2 \\ -2 & 1 \end{pmatrix};$$

ΔQ_{bb} - quadrupolar tune shift per IR

r - normalized LRBB separation

Stability Diagrams, End-Squeeze



LO=500A, no beam-beam

LO=500A , pacman LRBB (solid)

LO=750A , no beam-beam (dashed)

LO=500A, full LRBB (solid)

LO=1000A, no beam-beam (dashed)

$$\Delta Q_{\mathrm{bb}} = \begin{cases} 2.5\cdot 10^{-3}, & \text{full LRBB} \\ 1.25\cdot 10^{-3}, & \text{pacman bunch} \end{cases}; \quad r = 9.5$$

At the negative side, full LRBB is equivalent to additional 500A of the Landau octupoles - **it is a big number (disagreement with W. Herr and X. Buffat talks).**

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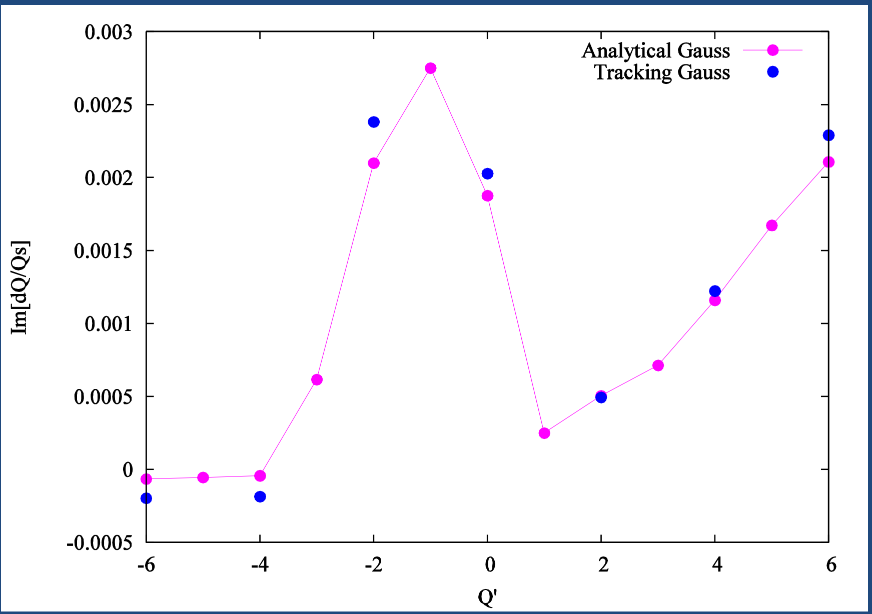
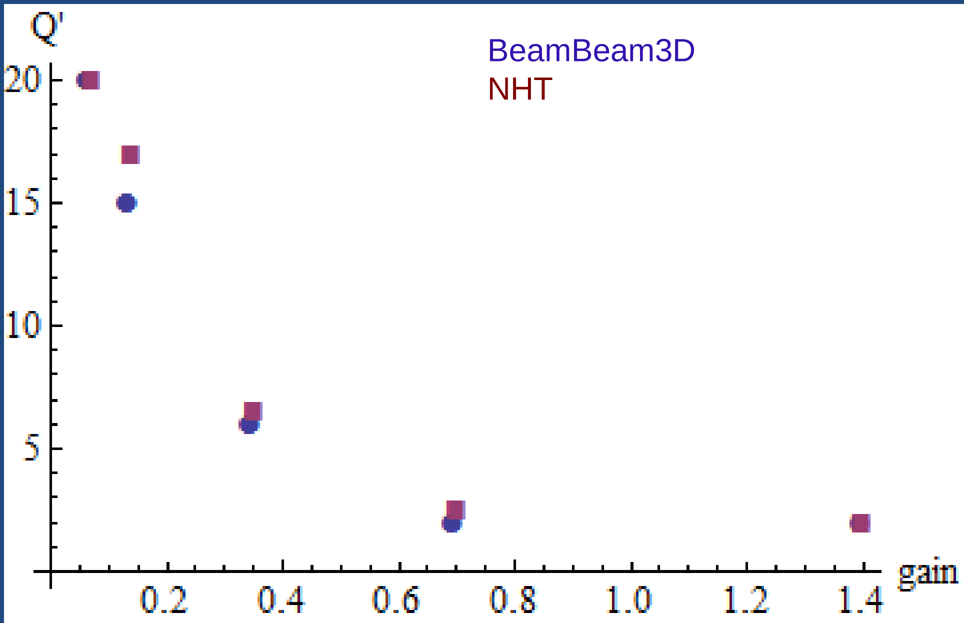
NHT vs BeamBeam3D (S. White)





Highest growth rates for

single beam, single bunch,
maximal gain and nominal impedance

Threshold chromaticity vs gain for

two single-bunch LR-colliding beams,
end of the squeeze parameters,
no octupoles.

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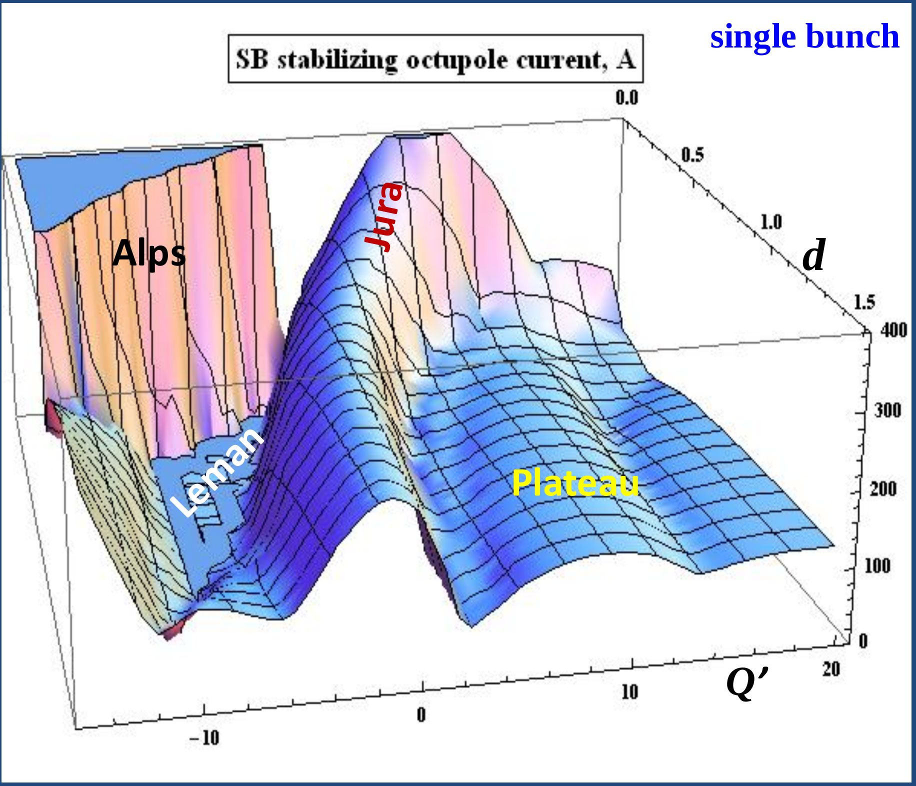
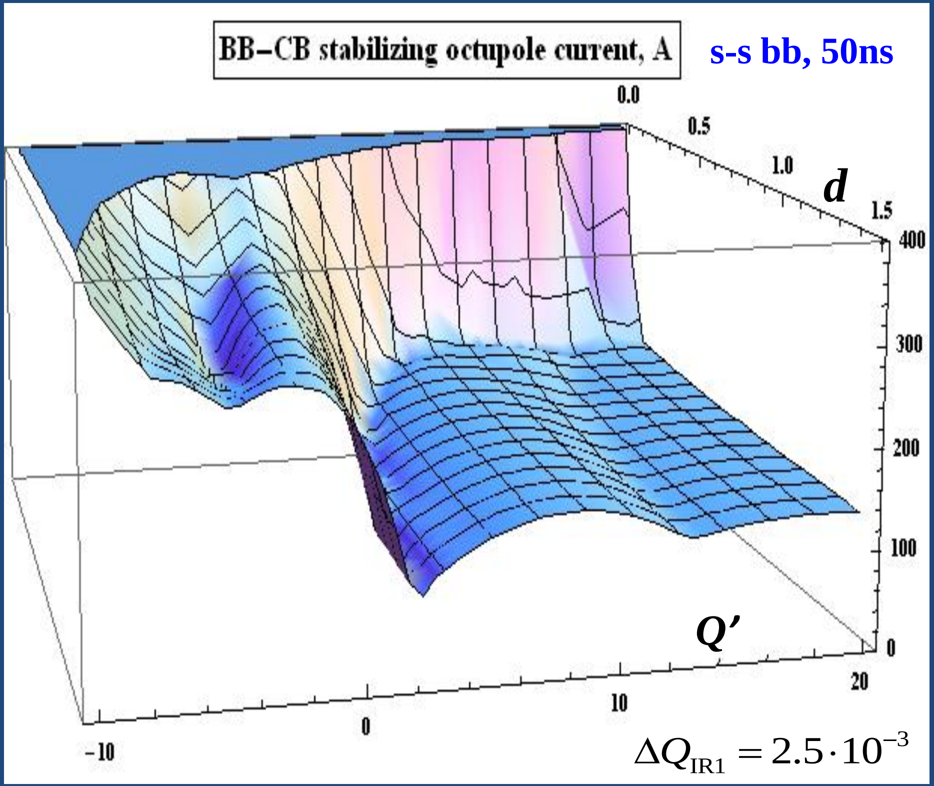
Proof #3: NHT for LHC (end of squeeze)





At the Plateau, the two cases are almost identical at *q/d*=1.
For pacman bunches, incoherent BB contributes ~230A =>

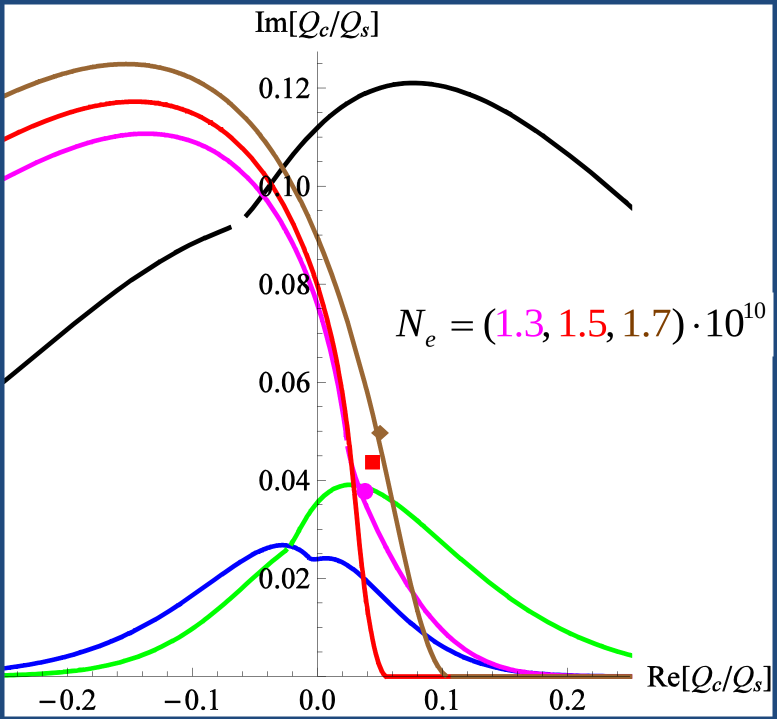
No octupoles should be needed at Plateau at all…
In the reality: 530A of LO are not yet sufficient at the end of squeeze.
Thus, CBB has nothing to do with the instability.

LO+, 2*Imp, bbb ADT. Nominal impedance – N. Mounet PhD.

End of the Squeeze Instability in LHC

- In the LHC at flat top, a single beam is stabilized at 200A of the octupoles.
- A regular transverse instability was observed at the end of the squeeze, when the octupoles were at maximal current of ~530A and the incoherent beam-beam effectively added ~230A, making in total ~800A, or 4 times above the single-beam threshold.
- NHT showed that beam-beam coupling cannot be a cause of this instability (ICE meeting 31/10/12). Confirmed a month later at 2-beam impedance (“cogging”) MD (S. Fartoukh et al).
- A model of three-beam instability was suggested: 2 LHC beams plus e-cloud in the high-beta area of IR1 and IR5.

Beam-Beam-Beam Effect in LHC



LO=200A – computed threshold

(Pacman) BB only, LO=0

BB and LO=500A

BB, LO=500A, dQe0=6.0E-4

BB, LO=500A, dQe0=8.0E-4

BB, LO=500A, dQe0=1.0E-3

Markers – MUMs, colors correspond

Instability is driven by e-cloud attracted by 2 beams in the high-beta area of IR1&5.

It happens due to a right-collapse of the SD + low-frequency e-wake with positive coherent tune shifts.

Electron wake:

$$W(\tau) \simeq W_0 \sin(\omega_e \tau) \exp\left(\omega_e \tau / 2Q\right);$$

$$W_0 = \frac{N_e r_e c}{4\sigma_\perp^4 \omega_e}, \quad Q \sim 3-5, \ \tau < 0$$

$$\psi_e \equiv \omega_e \sigma_z / c = \begin{cases} 6.5 \text{ rad} & \text{for } \beta\text{=300m} \\ 1.4 \text{ rad} & \text{for } \beta\text{=4km} \end{cases}$$

Summary

- Damper theorem

  If the gain is high enough, all other flat responses can be neglected -

  is proved by three ways: qualitative, simple model, NHT.

- Its application for LHC showed that end-of-the-squeeze instability cannot be explained by collective beam-beam coupling.

- Beam-beam-beam effect / three-beam instability is suggested as an explanation.

Many Thanks!