Analysis of long range studies in the LHC - comparison with the model

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TRIUMF

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The model: analytical expression for the smear

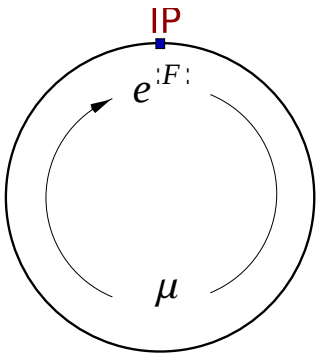
1. An analytical expression for the smear caused by beam-beam in the LHC (weak-strong)
   - arbitrary number head-on or long-range collisions
   - valid to first order
   - one dimension
2. used Lie-algebraic method to derive generalized Courant-Snyder invariant
3. applied to interpret
   - tracking results ($<$ 2009);
   - MD experiments (this talk).

sources used:

1. Alex J. Dragt, Oleg G. Jakubowicz, *Analysis of the Beam-Beam Interaction Using Transfer maps*, Proc of the Beam Beam Seminar, Stanford Ca, 1980, SLAC-R-541
2. A. Chao, *Lie Algebra Techniques for Nonlinear Dynamics*, `http:www.slac.stanford.edu/~achao`
3. J. Bengtsson and J. Irwin, *Analytical Calculation of Smear and Tune Shift*, SSC-232, February 1990 ($\rightarrow$ 2 ord.; the LHC code SODD)
4. E. Forest, Analytical Computation of the Smear, SSC-95.

Invariant h for a ring with single head-on IP, round beams

A. Chao, Lie Algebra Techniques for Nonlinear Dynamics



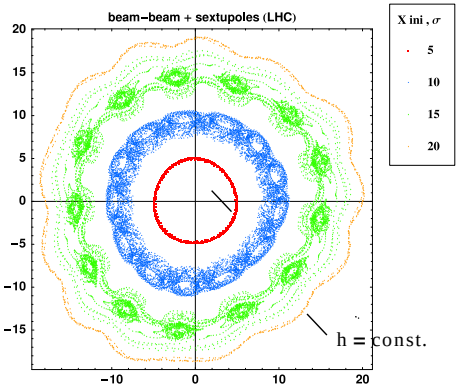


Figure: $h(\bar{x}, \bar{p}_x)$=const or $h(J, \phi)$=const

One-turn Lie map for the weak-beam particle in (x, p_x):

$$e^{:f_2:} e^{:F:} = e^{:h:}$$

For small perturbations, away from resonances, particle coord. are restricted on Poincare surface of sect.
$h = const.$

Find the effective Hamiltonian $-h/C$, i.e. the function $h(J, \phi)$ to first order in $\lambda = \frac{N_p r_p}{\gamma}$.

With J, ϕ action-angle:

$x = \sqrt{2J\beta^*} \sin\phi; \quad p_x = \sqrt{2J/\beta^*} \cos\phi$

and this beam-beam (kick) Hamiltonian (Fourier-expanded):

$$-H = F = \int_0^x dx' \, \frac{2\lambda}{x'} \left[1 - exp\left(\frac{x'^2}{2\sigma^2} \right) \right] = \sum_{n=-\infty}^{\infty} c_n(J) e^{in\phi}$$

γ – relativistic parameter; N_b – particles per bunch; r_0 – class.p. radius; $\sigma = \sqrt{\epsilon\beta^\star}$

- the solution is

$$h = -\mu J + \sum_{n=-\infty}^{\infty} c_n(A) \frac{n\mu/2}{\sin n\mu/2} e^{in\phi+\mu/2} ,$$

where $A = J/\epsilon$. The coefficients c_n are:

$$c_n(A) = \left\{ \begin{array}{ll} \mathrm{n\ odd} & 0, \\ \mathrm{n} = 0 & \lambda \int_0^{A/2} \frac{1}{t} \left[1 - e^{-t} I_0(t)\right] \, d\,t, \\ \mathrm{n\ even} & -\lambda \int_0^{A/2} \frac{e^{-t}}{t} I_{n/2}(t) \, d\,t. \end{array} \right\} .$$

I_k are modified Bessel; μ is phase adv. of the ring;
Note: h is real since $c_n \neq 0$ for even n only.

- away from resonances, the oscillating terms (with $m \neq 0$) can be removed via normal form transform – only c_0 remains;
- our goal here is different: use the osc. terms to construct a nonlinear invariant (generalized Courant-Snyder inv.) and extract smear [1]

[1] info about smear is contained in the normal form operator that transforms phase space circles into onion-like structures $e^{:g:}$ (Irwin) or $A = e^{:F_3:} A_2$ (Chao) (See the said Lect. – case of sextupole nonl.)

LHC case: Invariant h for multiple IPs arbitrary set of head-on and long-ranges in IR5 and IR1

Question: What shape to expect for the graph: smear(amplitude)?

The picture is:

- a weak-beam "test" particle in the horiz. plane sees two h.o. collisions and also in-plane (near IP5) and vertical (near IP1) l. r.collisions;
- for IR5 collisions, smear depends on the initial amplitude: how close it is to the strong-beam centroids – one expects sharp growth of smear near the entrance to the strong-beam core
- For IP1 (in the other, vertical plane), the l.r. should not contribute

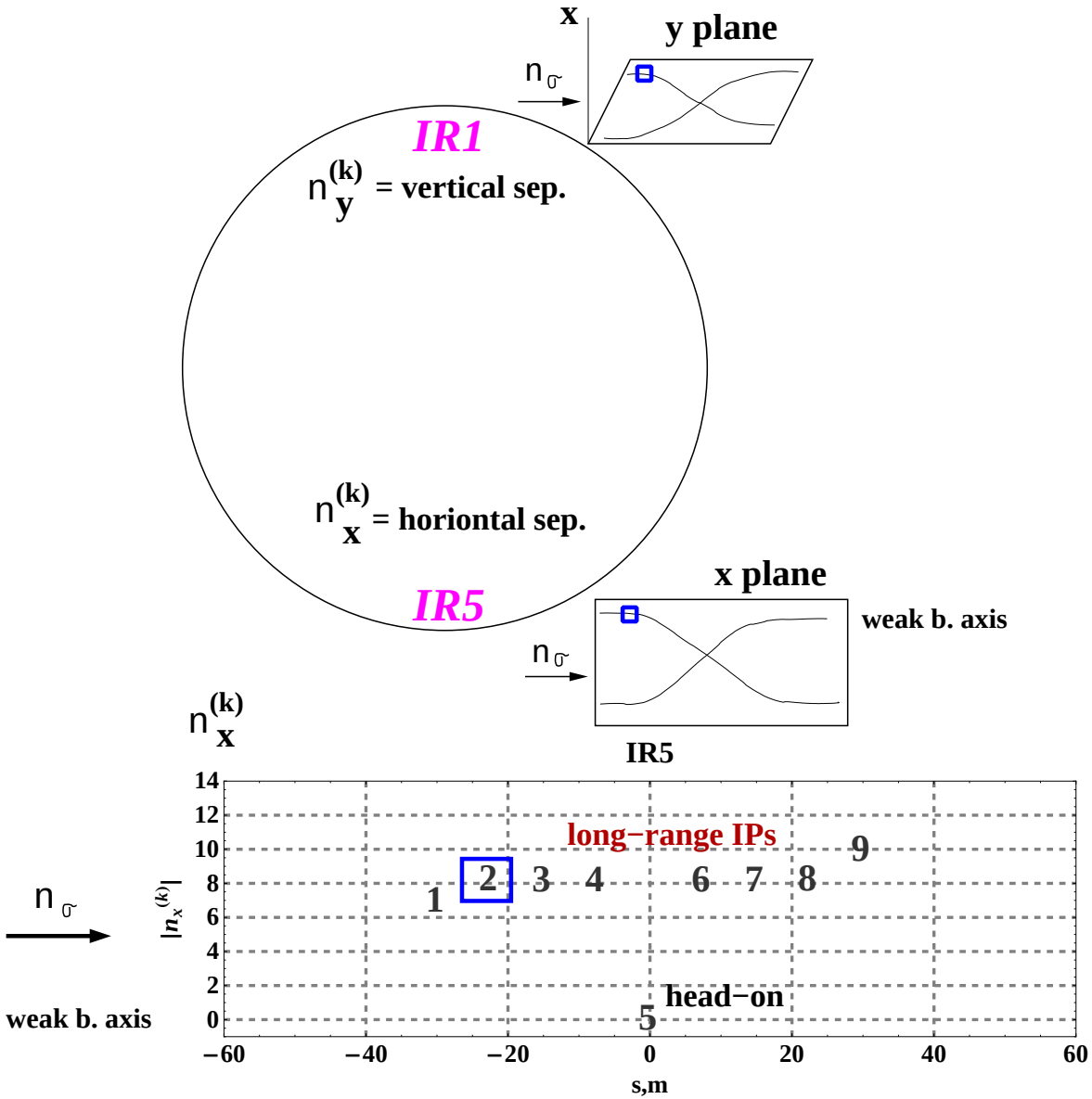


Figure: n_σ is weak-beam-particle amplitude; $n_{x,y}^{(k)}$ are normalized sep. (σ units). The case shown is 16 l.r. in both IP5 and IP1 (50 ns). Shown is the set $(s^{(k)}, n_x^{(k)})$ $k = (1, 17)$; s, m is distance to IP5.

Multiple IPs: Hamiltonian H for the kth collision

Also potential, or Lie factor $F = -H$ (in units of $\lambda \equiv \frac{N_b r_0}{\gamma}$).
Two ways to write it:

$$F(x) = -H(x) =$$
$$= \int_0^P (1 - e^{-\alpha}) \frac{d\alpha}{\alpha} \quad \text{(one way)}$$
$$= \quad \overline{\gamma} + \Gamma_0(P) + ln(P) \quad \text{(another way)}$$
$$P = P(x) = \frac{1}{2} \left[(n_x + \frac{x}{\sigma})^2 + n_y^2 \right] .$$

$\Gamma_s(P) \equiv \Gamma(s, P)$ is the upper incompl. gamma func. and $\overline{\gamma}$ = 0.577216 is the Euler's constant.

The Beam-beam Kick (in units of λ):

$$\Delta x' \equiv -\frac{d}{dx} H(x) = \frac{2(x + n_x\sigma)}{(x + n_x\sigma)^2 + (n_y\sigma)^2} \left(1 - e^{-\frac{(x+n_x\sigma)^2 + (n_y\sigma)^2}{2\sigma^2}} \right) .$$

Hamiltonian only depends on $n_{x,y}$ which take values from the above set $n_{x,y}^{(k)}$.
We have assumed **round beams** (a very good approximation), i.e.:

$$\sigma^{(k)} = \sqrt{\beta^{(k)}\epsilon} \quad (\beta_x^{(k)} = \beta_y^{(k)} \equiv \beta^{(k)})$$

Multiple IPs: Invariant (see also additional slides)

- for a set of IPs with sep. $n^{(k)}$ and phases $\phi^{(k)}$ $(k = 1, N)$:

$$h(J,\phi) = -\mu J - \sum_{k=1}^{N}\sum_{m=1}^{\infty}\left(\frac{m\mu/2\, c_m^{(k)}}{\sin(m\mu/2)} e^{im(\mu/2+\phi+\phi^{(k)})} + c.c.\right)$$

use numb. of σ: $n_\sigma = \sqrt{2J/\epsilon}$.

$$c_m = \frac{1}{2\pi}\int_0^{2\pi} e^{-im\phi} H_{\rm nonl}\, d\phi\,, \qquad H_{\rm nonl}(n_\sigma,\phi) = H_{\rm nonl}(x)|_{x\,\to n_\sigma\,\sigma\, sin(\phi)}\ ;$$

$$H_{\rm nonl} = H - H_{(1)} - H_{(2)}\,,\ \ H_{(1)} \sim x\,,\ \ H_{(2)} \sim x^2\,;$$

- Expressions for the coefficients [2]. For the full H with $\mathcal{C}_m \equiv \frac{1}{2\pi}\int_0^{2\pi} e^{-im\phi} H\, d\phi$

For $m = 0$: $\mathcal{C}_0 = \int_0^1 \frac{dt}{t}[1 - e^{-\frac{t}{2}n_x^2} e^{-\frac{t}{4}n_\sigma^2} \sum_{k=-\infty}^{\infty} I_{-2k}(t\, n_\sigma n_x) I_k(-\frac{t}{4}n_\sigma^2)]$.

For $m \neq 0$: $\mathcal{C}_m = -i^m \sum_{k=-\infty}^{\infty}\int_0^1 \frac{dt}{t} e^{-\frac{t}{2}n_x^2} e^{-\frac{t}{4}n_\sigma^2} I_{m-2k}(t\, n_\sigma n_x) I_k(-\frac{t}{4}n_\sigma^2)\,.$

Only the ones for $m \neq 0$ participate. Notice : $\mathcal{C}_m \sim i^m$.

For a head-on, they reduce to Chao's Lect. $n_x^{(k)} = n_y^{(k)} = 0 \ \Rightarrow\ \mathcal{C}_m = c_m^{\rm Chao\ Lect.}$

[2] D. Kaltchev, *Hamiltonian for Long-range Beam-Beam – Fourier coefficients*, TRIUMF note

Multiple IPs: 1-order smear(amplitude) $S(n_\sigma)$

For a particle with amplitude n_σ the inv curve passes through the init. point:

$$h(A,\phi) = h(A_0, \pi/2), \qquad A_0 \equiv J_0/\epsilon = n_\sigma^2/2$$

This implicitly defines A as a function of ϕ. We denote this funct. with $I(\phi)$. Divide by $\mu\epsilon$. The final expression is [3]:

$$I(\phi) = A_0 + \sum_{k=1}^{N} \left(dI^{(k)}(\phi) - dI^{(k)}(0) \right) ,$$

$$dI^{(k)}(\phi) = \frac{\lambda}{\epsilon} \sum_{m=1}^{M} \left(\frac{m\, c_m^{(k)}(A_0)}{2\sin(m\mu/2)} e^{im(\mu/2+\phi-\phi^{(k)}+\pi/2)} + c.c. \right)$$

$$M \sim 40$$

$dI^{(k)}(\phi) - dI^{(k)}(0)$ is the individual contribution to I of the kth IP.

Define smear(amplitude) $S(n_\sigma)$:

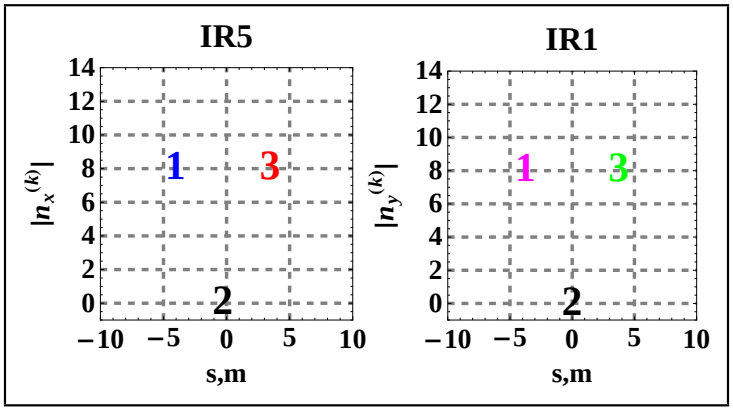
$$S(n_\sigma) = \sqrt{V}/\langle I \rangle ,$$

$$V = \frac{1}{2\pi}\int (I - \langle I \rangle)^2 d\phi , \quad \langle I \rangle = \frac{1}{2\pi}\int I d\phi .$$

[3] so that $I(0) = A_0$. On some plots below the origin may be shifted: $\phi \to \phi - \pi/2$.

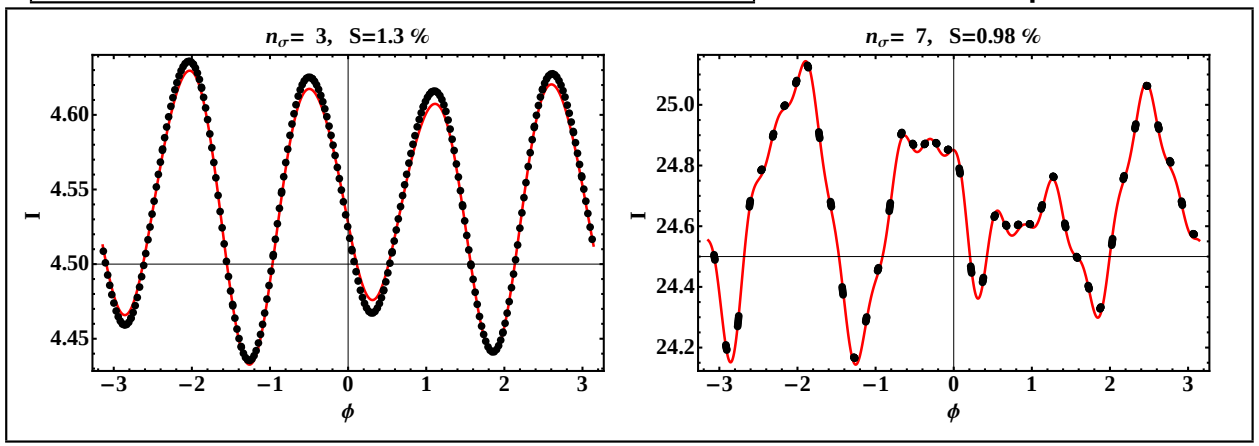
Tracking to verify of the invariant. Using simplest set of IPs.

Conditions: 3.5 TeV, $N_{\rm b} = 1.2\,10^{11}$ and $\epsilon_n = 2.5\,10^{-6}$.

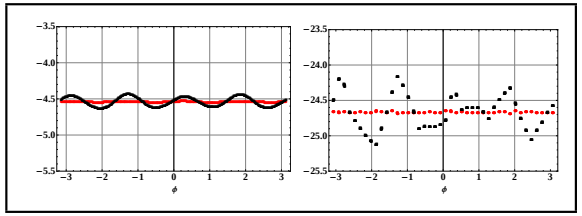


← using this simplest still symmetric set: 1 h.o.and 2 l.r. in each IR5 and IR1 and simple kick-matrix-kick- . . . model – uses the nonlin. part of the kick: $\Delta x'_{\rm nonl}$

Start 2 particles with $n_\sigma = 3$ and 7.

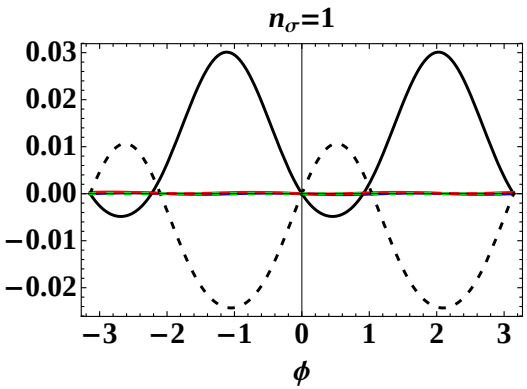


Shown are points (A_i, ϕ_i) **(black)** and the inv. curve $I(\phi)$ (it passes through the initial point $A_0 = n_\sigma^2/2$, $\phi = \pi/2$).

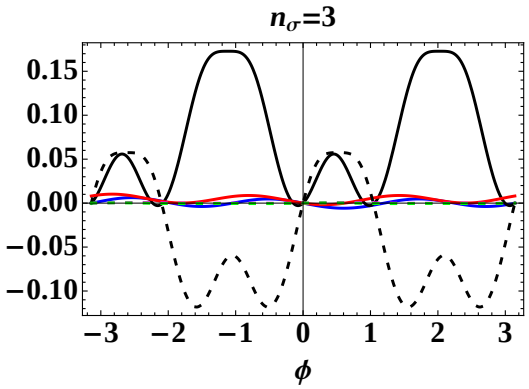


← another way to say it: h/μ is more constant than the ordinary Courant-Snyder inv. $-J$

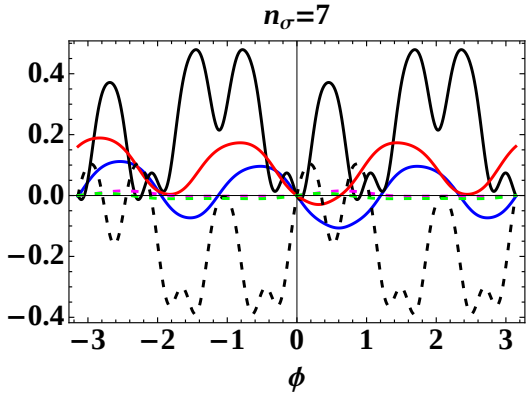
Individual contributions



$n_\sigma = 1$: h.o. of IR5 (solid) and IR1 (dashed) **nearly compensate**.

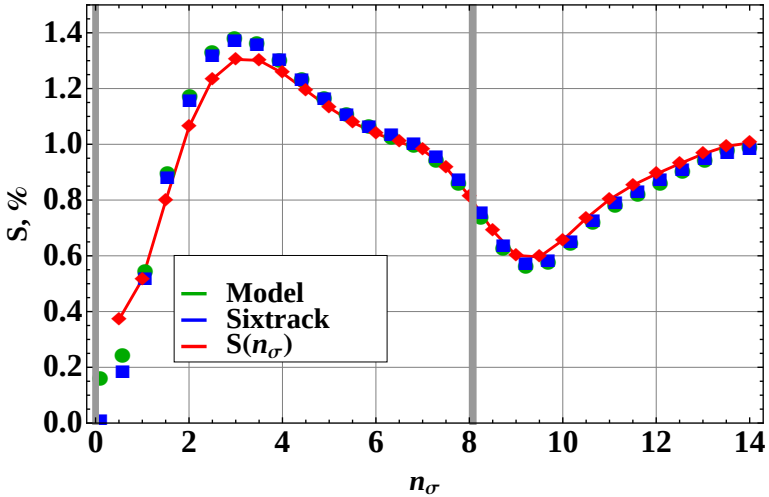


$n_\sigma = 3$: we begin to see l.r.



$n_\sigma = 7$: seen l.r. of IR5 and NO l.r. of IR1

Agreement of the first-order smear $S(n_\sigma)$ with Sixtrack [4]



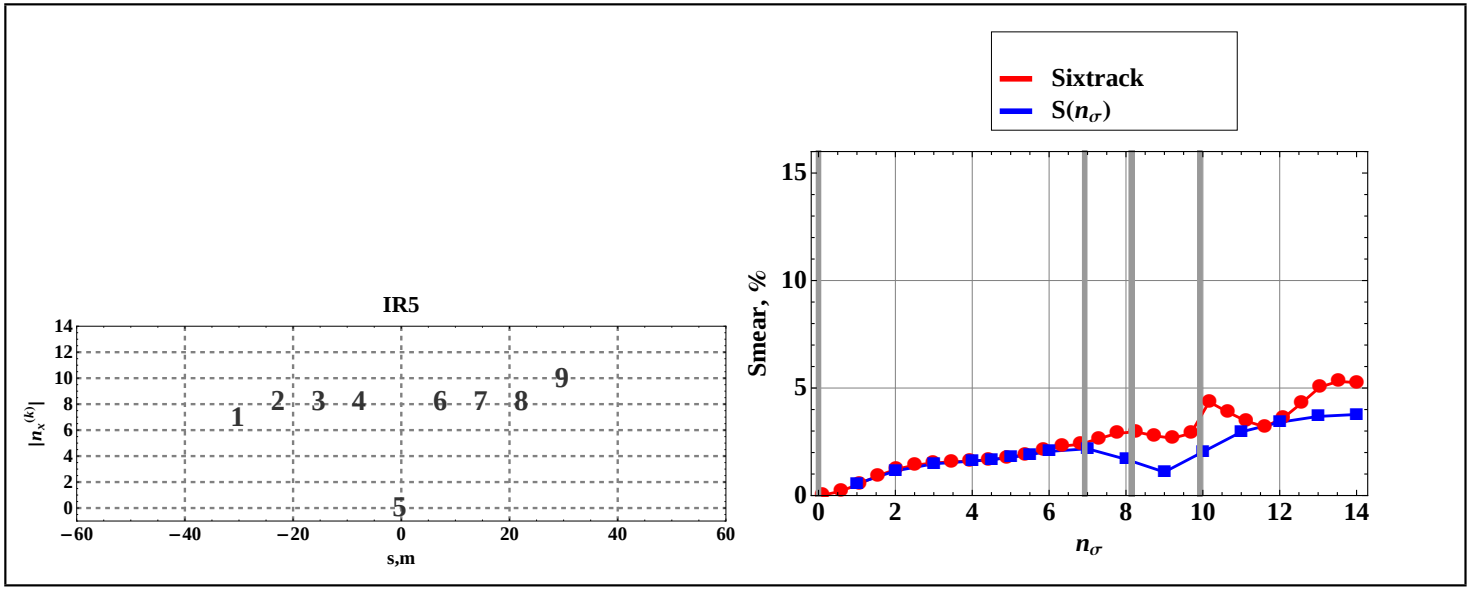
here first-order smear agrees with exact smear at all amplitudes: call it "linear regime".

Gray lines at 0 and ~ 8 represent $n_x^{(2)}$ and $n_x^{(1)}$.

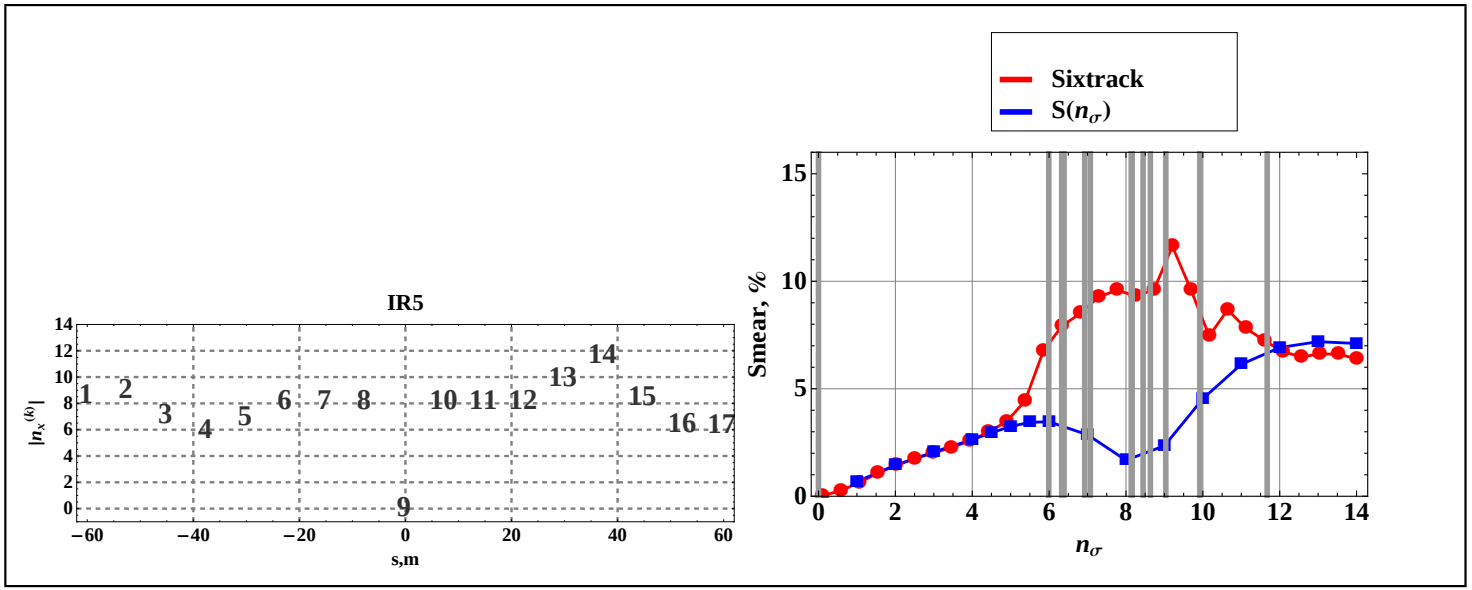
[4] F. Schmidt, Sixtrack, V.4.2 CERN/SL/94-56, Jul 2008.

Properties of $S(n_\sigma)$ – the maximum

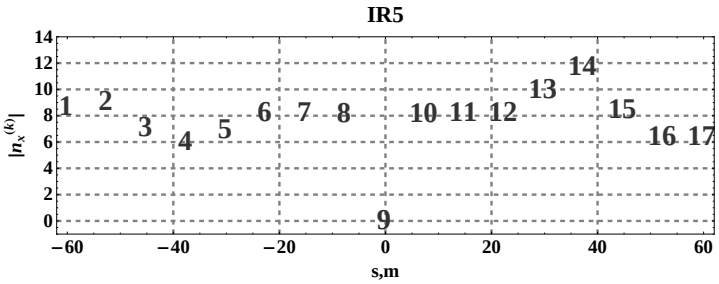
Agreement with Sixtrack breaks down near entrance of strong-beam core (vertical gray line). The 1-st order smear has a maximum nearby.

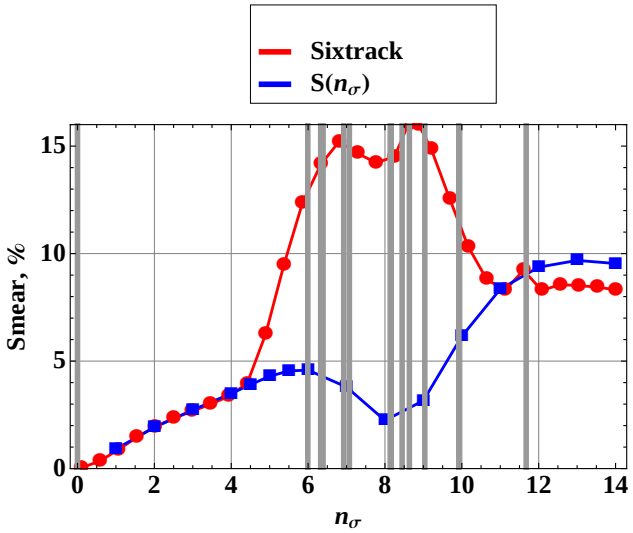


← now increasing number of l.r. to 16. ($N_b = 1.2\ 10^{11}$)

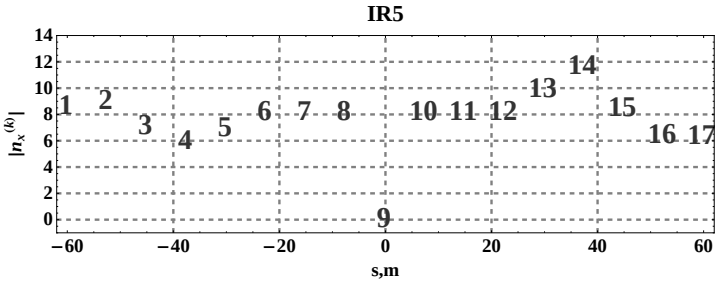


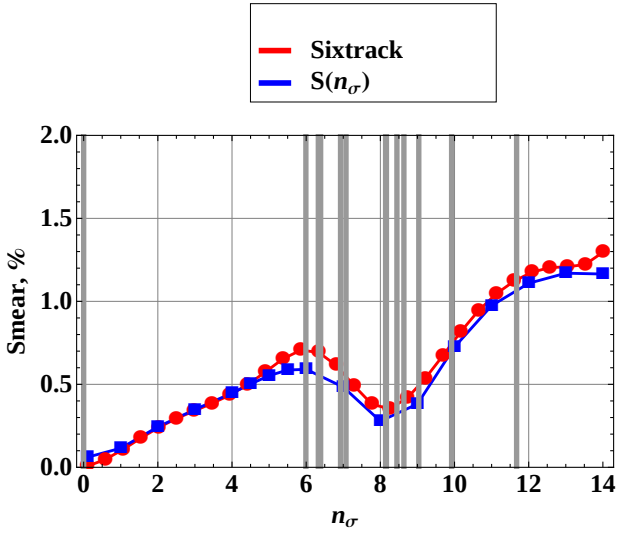
← Now increasing number of l.r. to maximum 32 ⇒ the agreement breaks down a little earlier ($N_b = 1.2\ 10^{11}$)





← keeping l.r. at 32, but N_b increased to 1.6 10^{11} ($N_b = 1.6\ 10^{11}$)





← decrease to 0.2 10^{11} ⇒ **return to linear regime.** ($N_b = 0.2\ 10^{11}$)

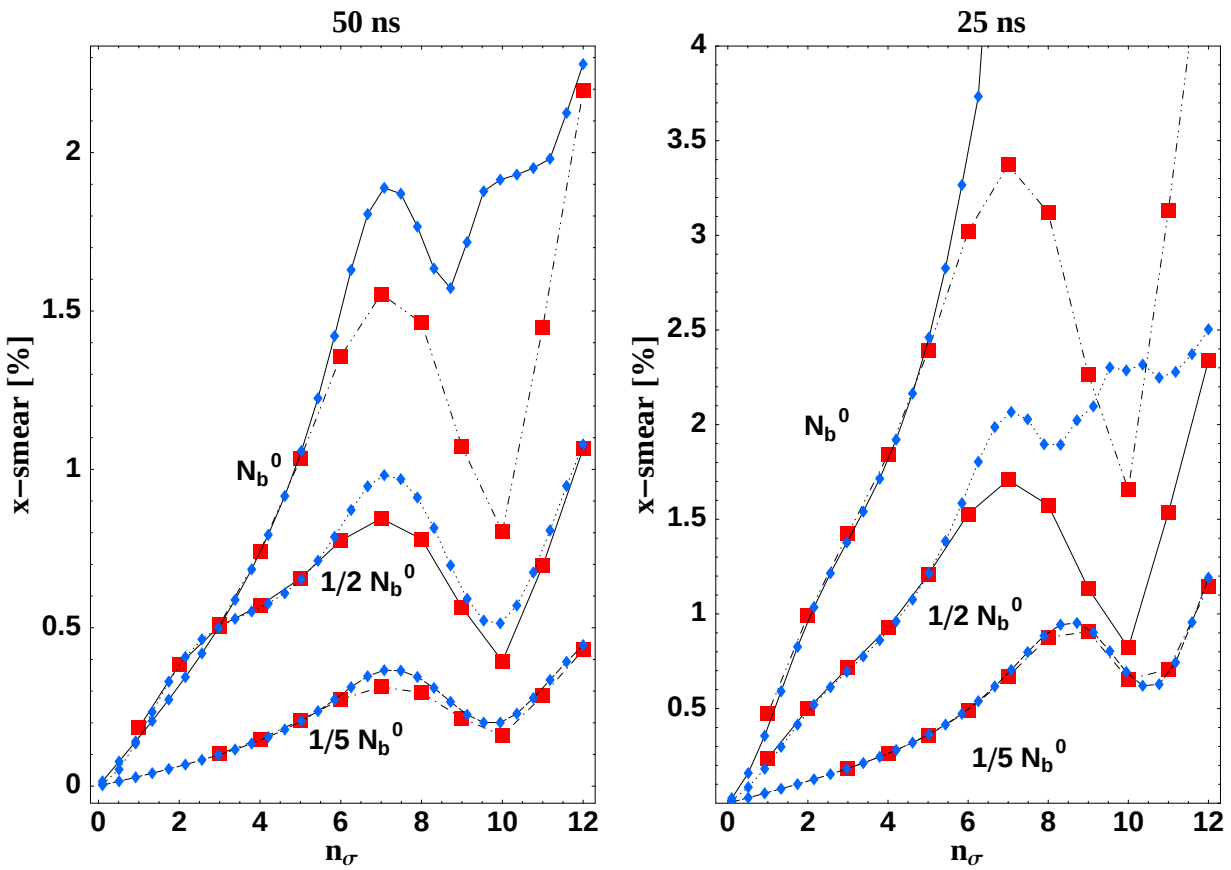
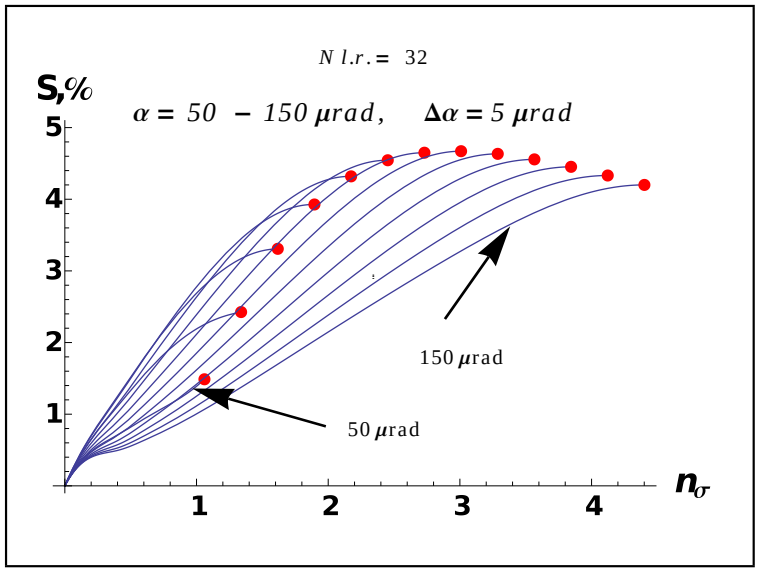


Figure: Reported previously at PAC09. The full LHC collision scheme is used. Agreement between Lie-algebra model (red) and Sixtrack (blue) for increasing bunch population N_b and two LHC collision schemes “50 ns” and “25 ns” (head-on is included). The curves correspond to $1, 1/2$ and $1/5$ times the nominal bunch population $N_b^0 = 1.15\ 10^{11}$.

Properties of $S(n_\sigma)$

- obviously proportional to N_b: $S(n_\sigma) \sim N_\mathrm{b}$
- dependence on 1/2-crossing angle α:

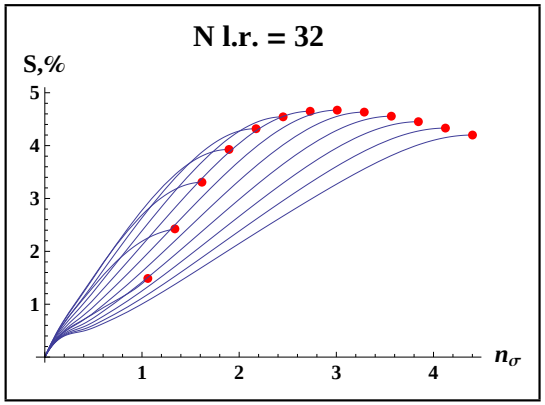


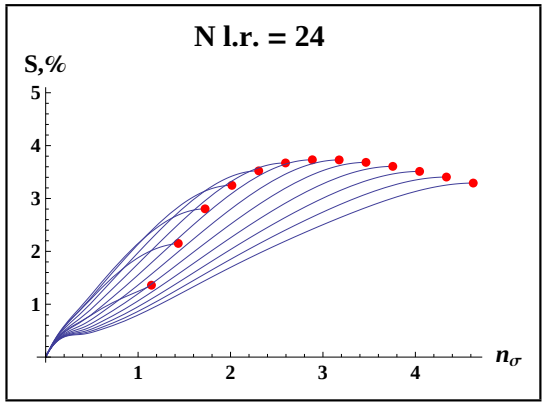
← smear(ampl.) for different crossing angles.

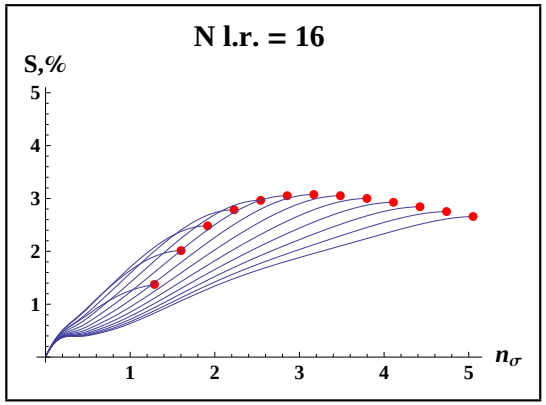
- shown only monotonic part to first maximum (red dot) where we know it is always true

conditions: $\beta = 0.6$ m (as on MD)
norm. emittance 2.5 μm

- dependence on N l.r.:



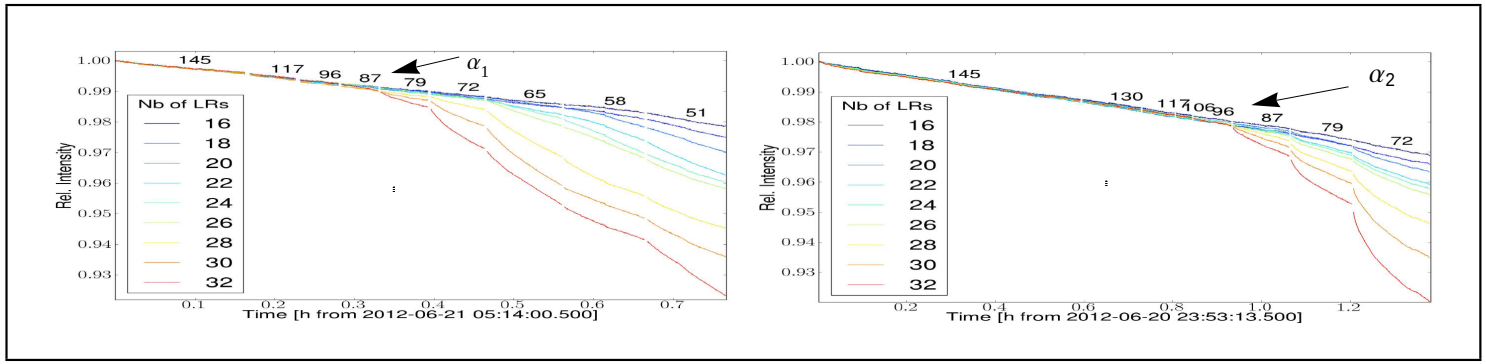




MD on beam-beam scaling with separation and intensity

MD title: "Beam-Beam Limits, long range, effect of separation" (1/12/2011)

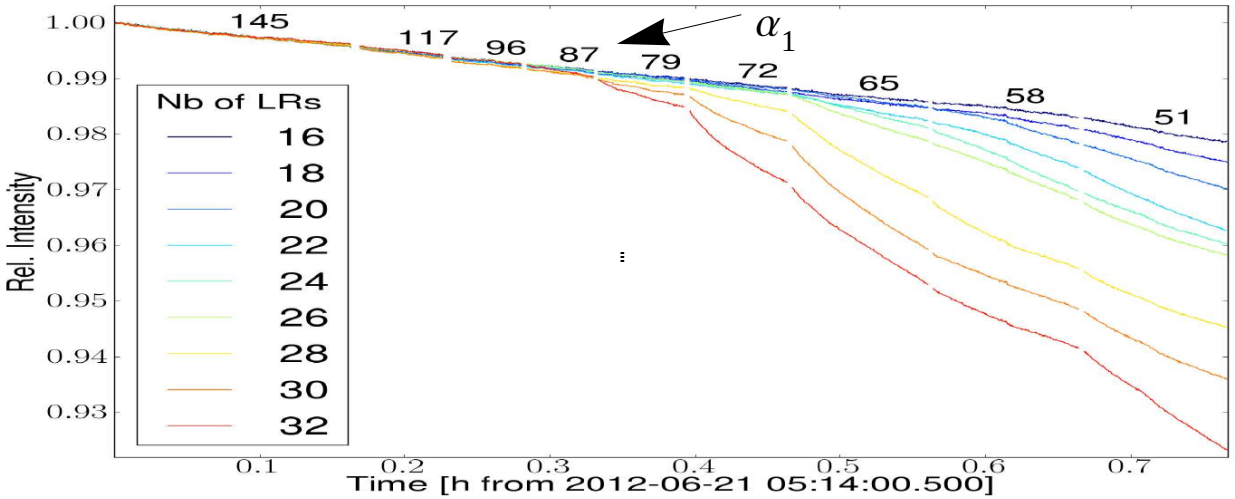
- two experiments: bunch population $1.2\,10^{11}$ and $1.6\,10^{11}$
- 1/2-cross. angle α decreased in steps: 145, 130, 117, . . . and intensity plotted in rel. units. This done for different weak beam bunch, ie. different N l.r. (for simpl. we use here 32, 24 and 16)



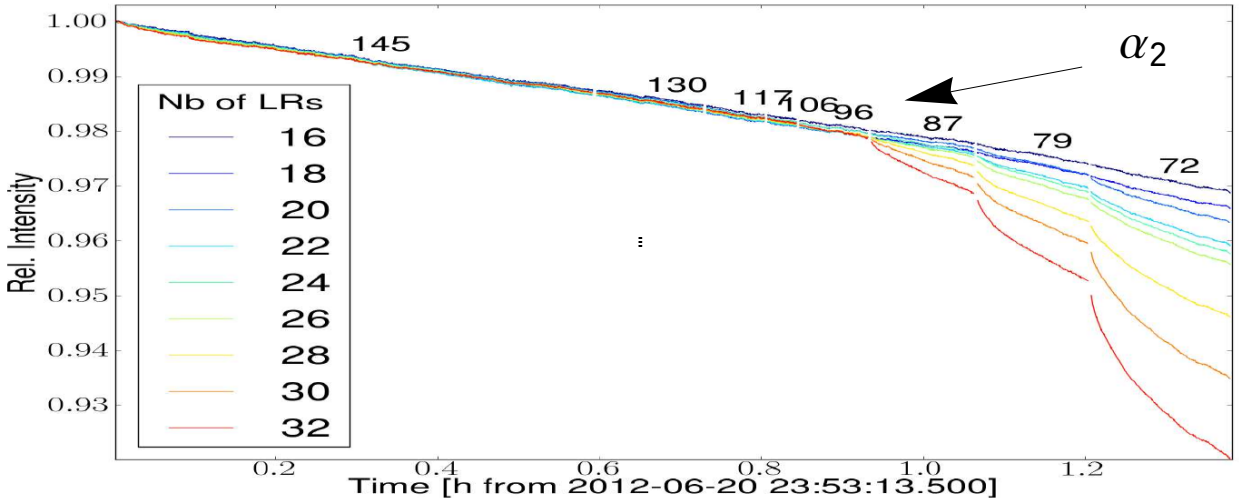
- *can we explain the critical α (corresponding to start of losses)*?

1. *The dark-brown (32) decay-curves: $\alpha_1 \approx 87$ and $\alpha_2 \approx 96$ (μrad)*
   – For $\alpha_2 = 96$, the theory gives $\alpha_1 \approx 86$, which corresponds to smear $\sim 3\%$ at 1.5 σ (both experiments). This is very close the observed value 87.
   – If we use as a constraint "smear 4% at 2 σ" we get $\alpha_1 \approx 84$ (also close)
   – for these two results knowing only first-order smear $S(n_\sigma)$ is sufficient

2. *Other departing points: say green (24) and black (16):*



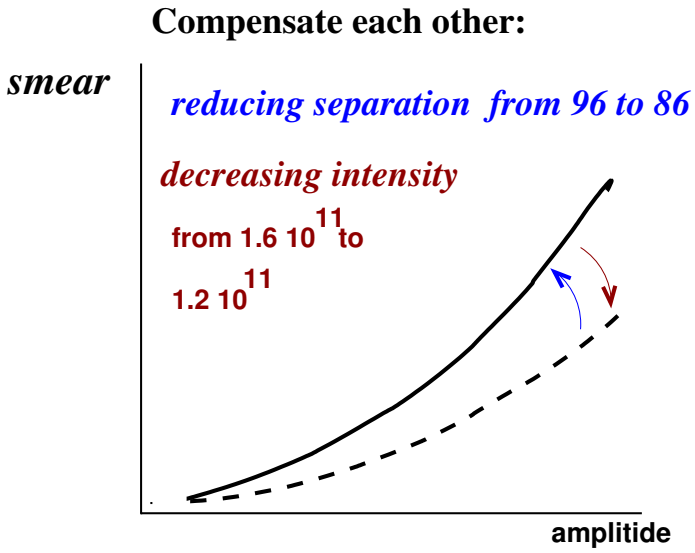
Second experiment: $N_b = 1.2\,10^{11}$, losses start at $\alpha = \alpha_1 \approx 87$? $\mu\mathrm{rad}$

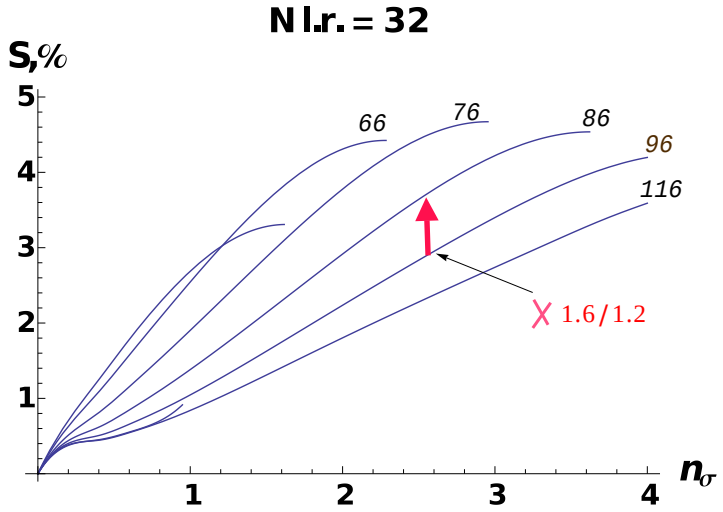


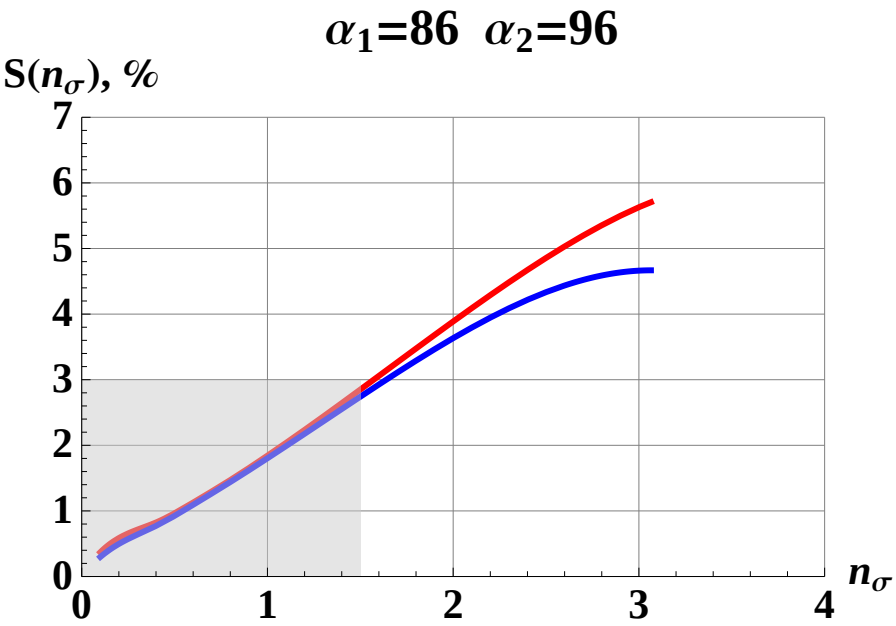
First experiment: $N_b = 1.6\,10^{11}$, losses start at $\alpha = \alpha_1 \approx 96\ \mu\mathrm{rad}$

(brown curves: 32 l.r.)

scaling up by a factor 1.6/1.2 does the same as switching from 86 to 96 μrad







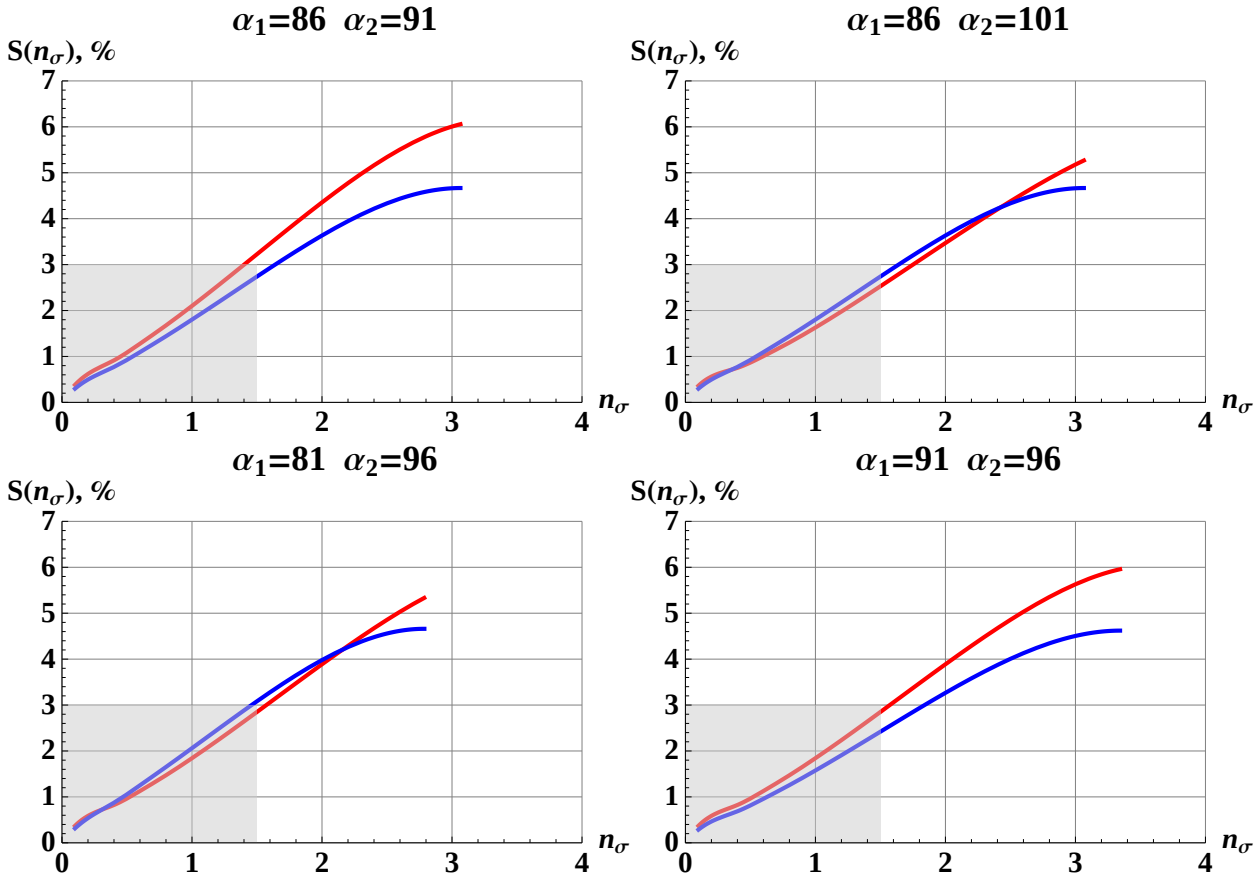
For these α_1 and α_2 the effects compensate: increasing intensity from $N_\mathrm{b} = 1.2\,10^{11}$ to $N_\mathrm{b} = 1.6\,10^{11}$ and reducing angle from α_2 to α_1 preserves the graph – the two curves overlap

$$\boxed{S(n_\sigma; 1.2\,10^{11}, 86) = S(n_\sigma; 1.6\,10^{11}, 96)}$$

over some amplitude range

$$0 < n_\sigma < 1.5.$$

if keeping one and vary the other, the overlap is less perfect, e.g. trying $\alpha_{1,2} \pm 5\ \mu\mathrm{rad}$:



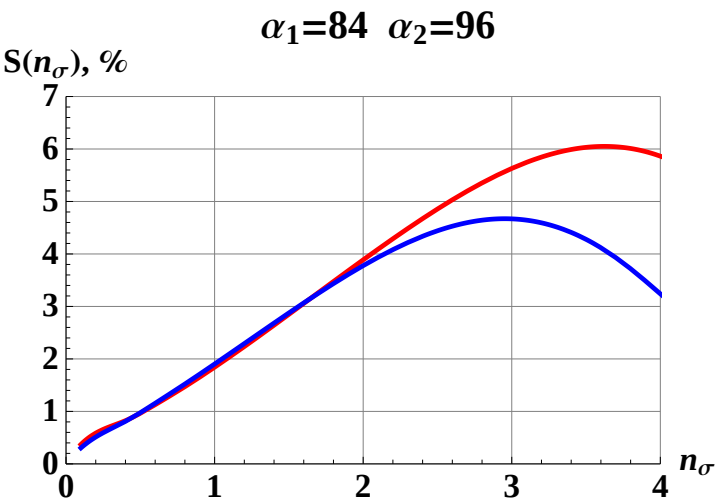
MD on scaling with separation and intensity N_b continues

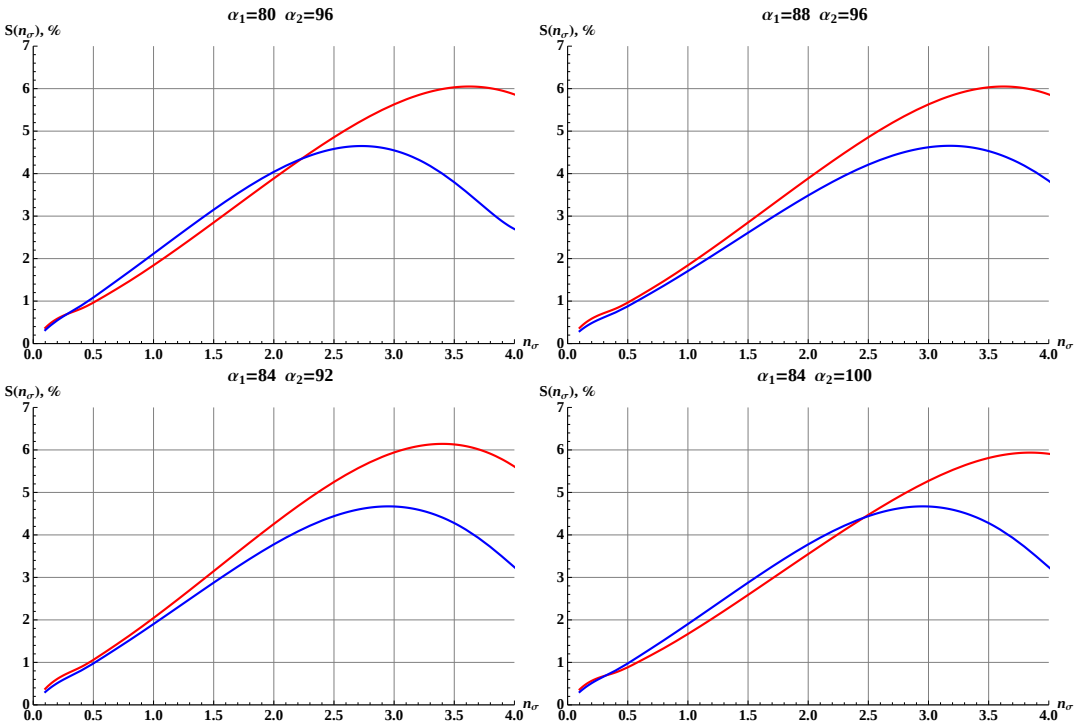
- Another solution (different by 2 $\mu\mathrm{rad}$):

$$\boxed{S(n_\sigma; 1.2\,10^{11}, 84) = S(n_\sigma; 1.6\,10^{11}, 96)}$$

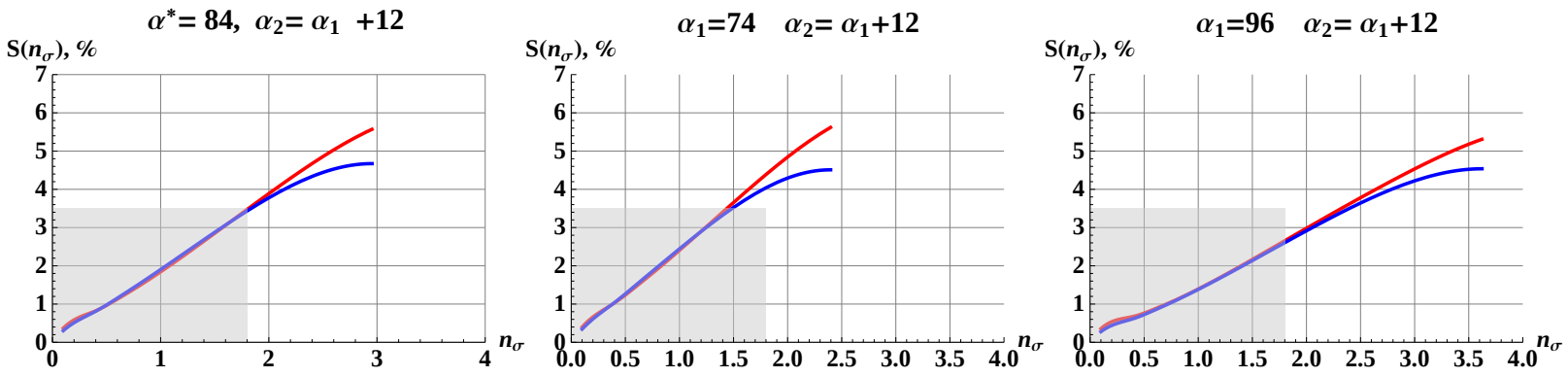
$$0 < n_\sigma < 2$$

trying $\alpha_{1,2} \pm 4$ $\mu\mathrm{rad}$:

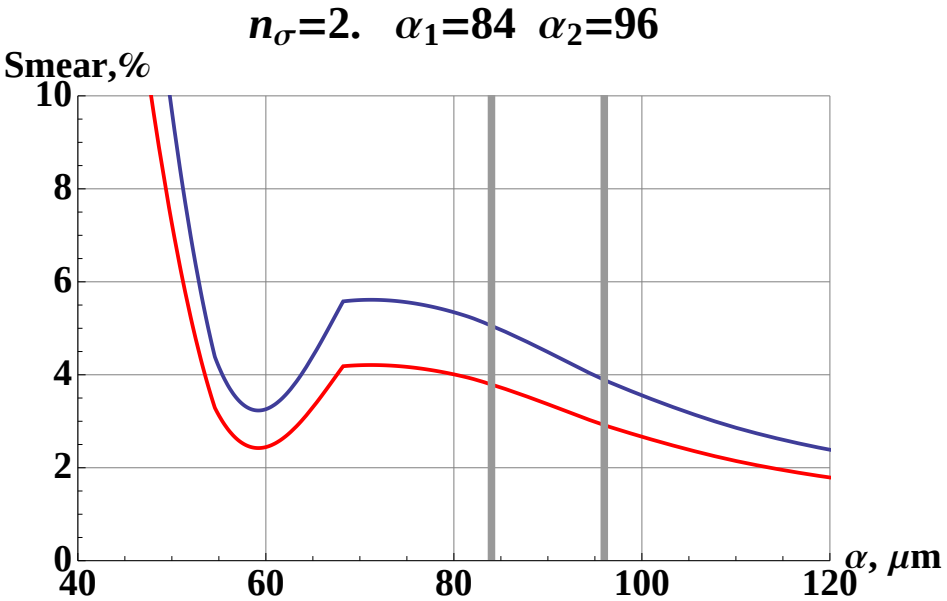


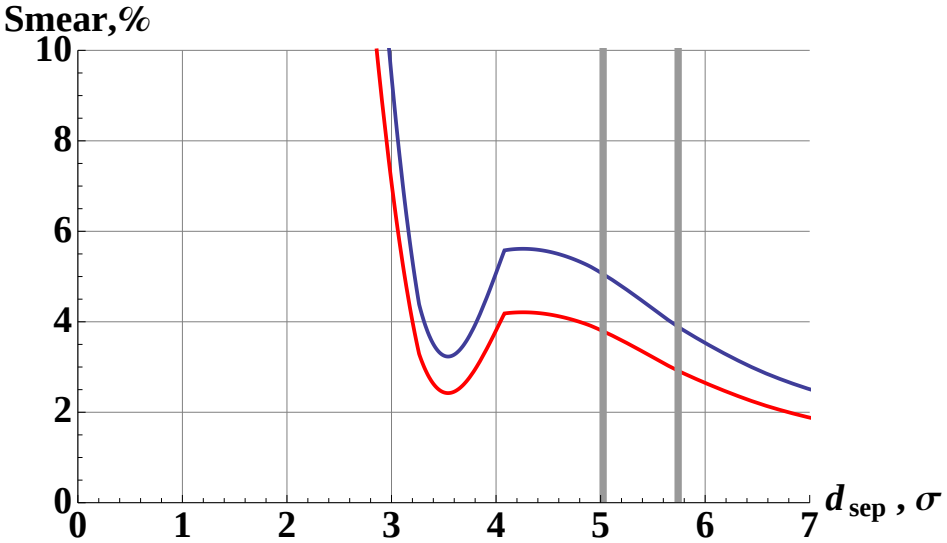


If we shift both angles, but keep the difference $\alpha_2 - \alpha_1$, then we change slope (see above) – both red and blue graphs rotate.
$\alpha_1 = \alpha^\star$ and $\alpha_2 = \alpha^\star + \Delta\alpha \quad \Delta\alpha = 12 \quad (\sim 0.7\sigma)$.



Fixed amplitude: $n_\sigma = 2$:





Alternative: fix $n_\sigma = 2$ and show $S(n_\sigma = 2, 1.2\,10^{11}; \alpha)$ and $S(n_\sigma = 2, 1.6\,10^{11}; \alpha)$ Both intersection points are at the same smear level.

In terms of separation: $\alpha_2 - alpha_1 \sim 0.8\ \sigma$.

In practice: solved the Eqn."smear to be X % at n_σ^X sigma"

Smear X % at n_σ^X sigma

means looking for angles that are the solution of

$$\boxed{S(n_\sigma^X; 1.2\,10^{11}, \alpha_1) = S(n_\sigma^X; 1.6\,10^{11}, \alpha_2) = X}$$

There are two constraints for the 2 unknowns.
The graphical way used above seems better.

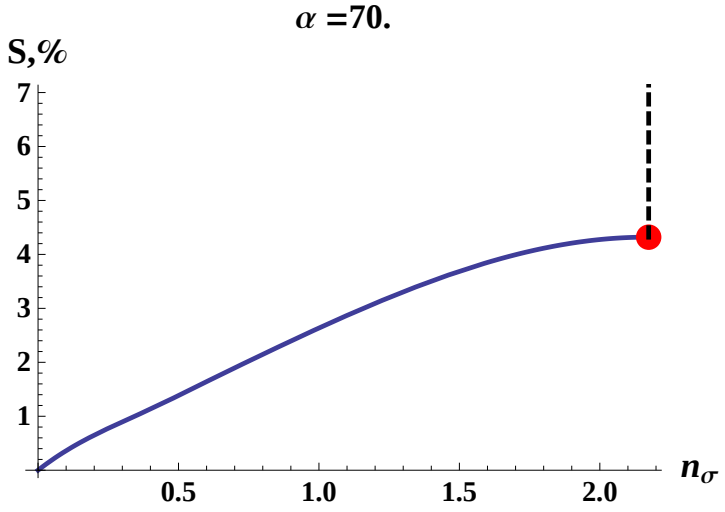
Case N l.r. =24 and 16 (green and black curves)

For $\alpha \gtrsim 60\ \mu\mathrm{rad}$ there is no need of approximations – first order smear is sufficient.

When the critical angle is very small: changing α no longer means (approximately) changing slope

and we need to make a guess for the shape of higher-order smear (use the comparisons with Sixtrack above):

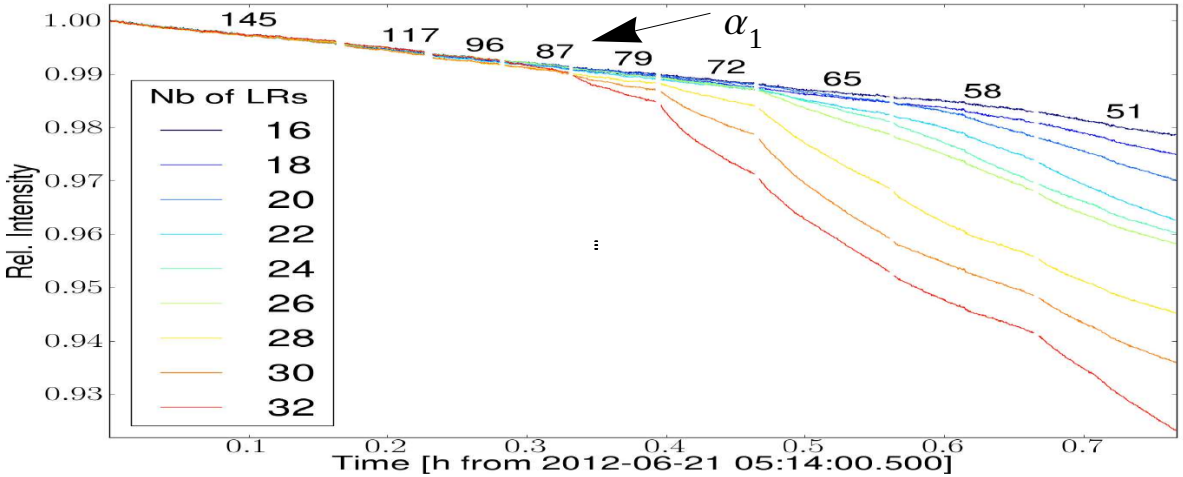
- below some threshold strength of beam-beam int. (intensity, numb. of l.r, and cross. angle), the shape $S(n_\sigma)$ works at all amplitudes.
- above this threshold, we approximate it with a step, like this:



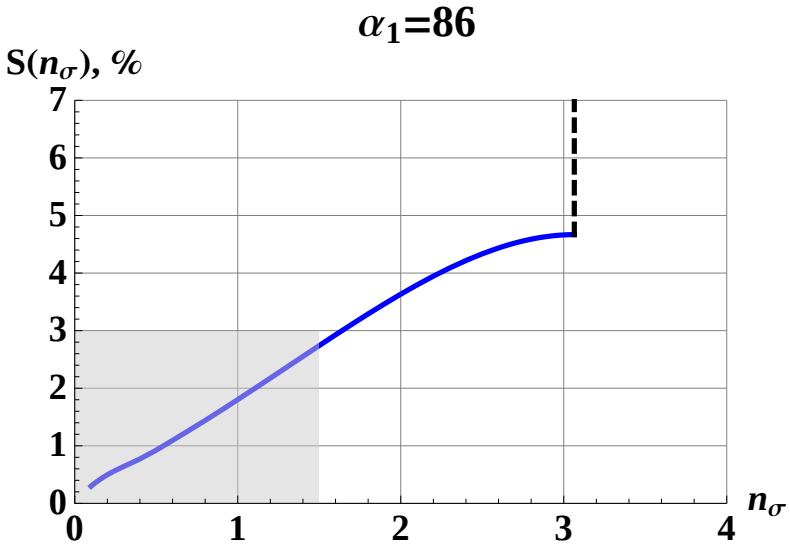
where we know analytically the position of the maximum (red dot)

Green and black decay curves for $N_b = 1.2\,10^{11}$

angle solutions for smear 3% at 1.5 σ

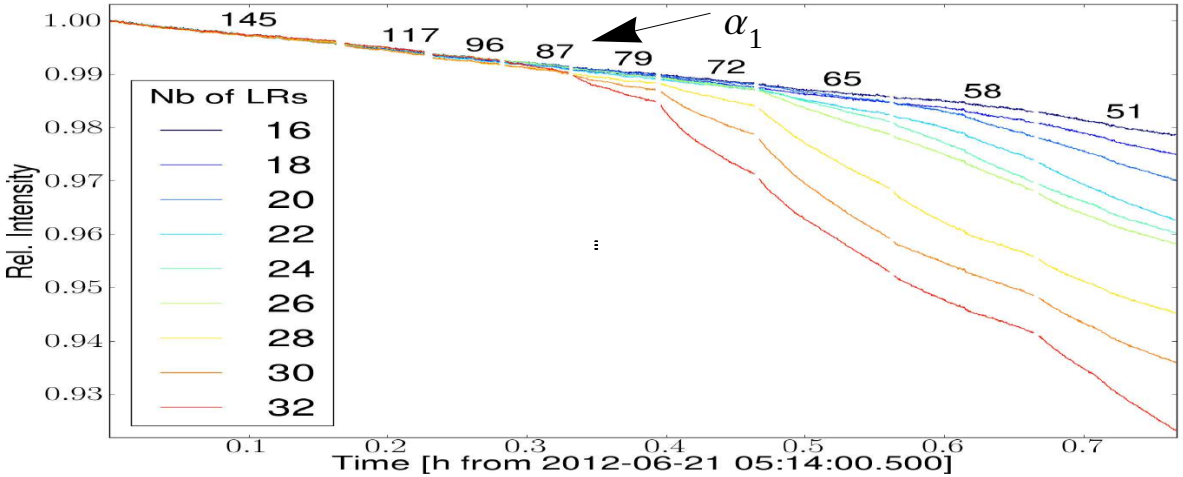


predicted angles: 86,, 65, 53 (requires appr.)

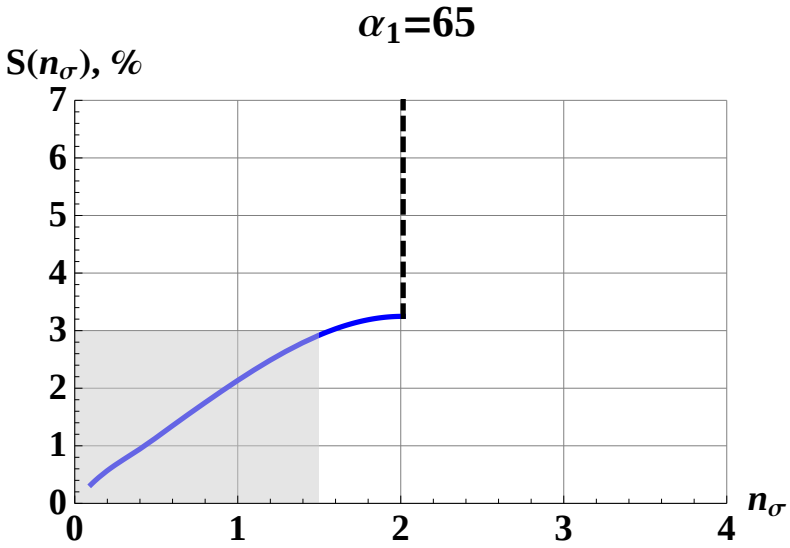


Green and black decay curves for $N_b = 1.2\,10^{11}$

angle solutions for smear 3% at 1.5 σ

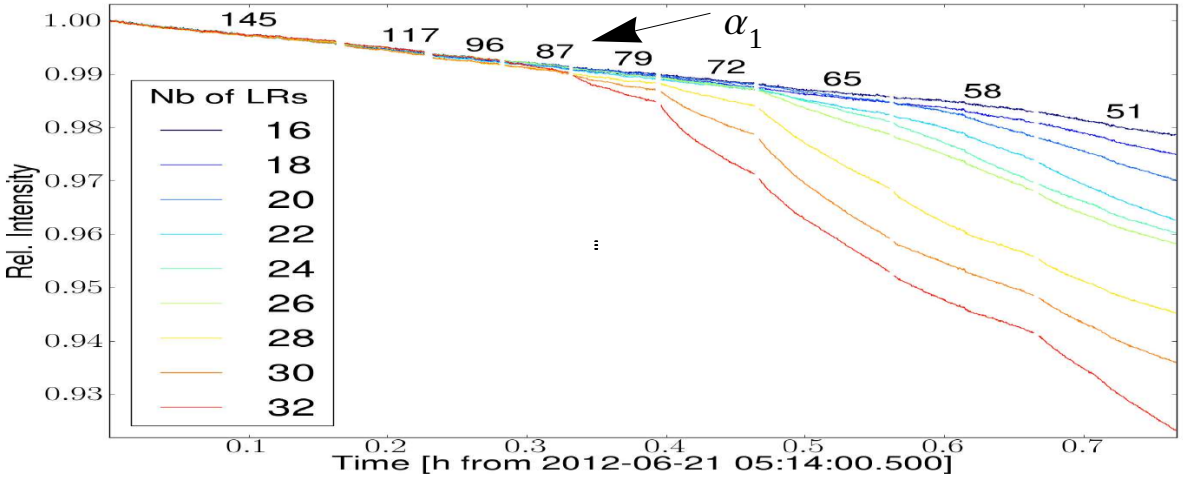


predicted angles: 86,, 65, 53 (requires appr.)

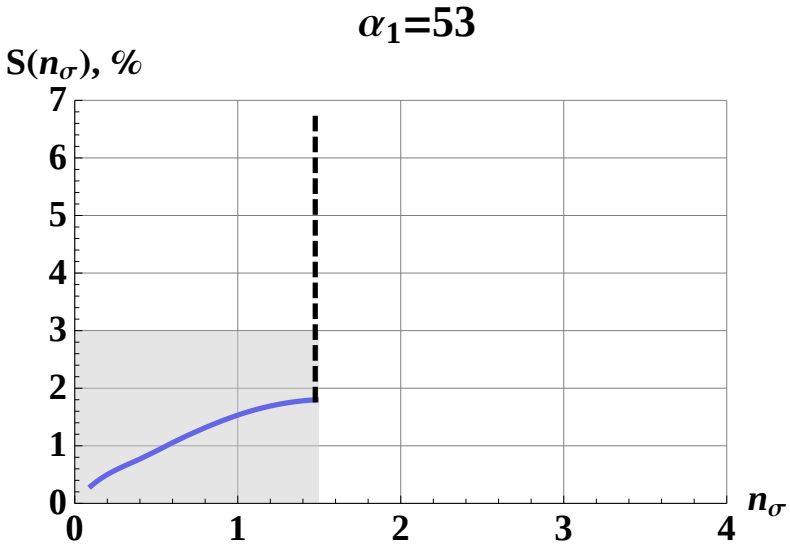


Green and black decay curves for $N_b = 1.2\,10^{11}$

angle solutions for smear 3% at 1.5 σ

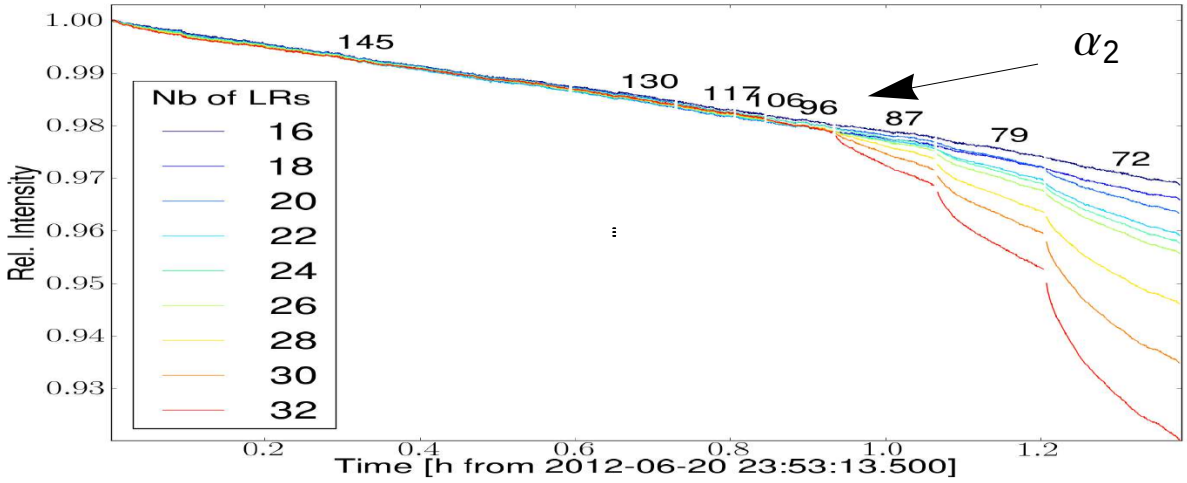


predicted angles: 86,, 65, 53 (requires appr.)

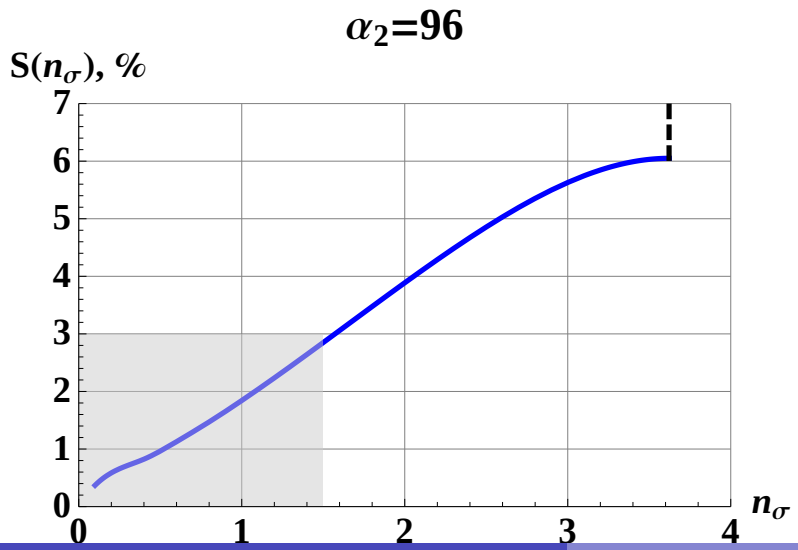


Green and black decay curves for $N_b = 1.6\,10^{11}$

angle solutions for smear 3% at 1.5 σ

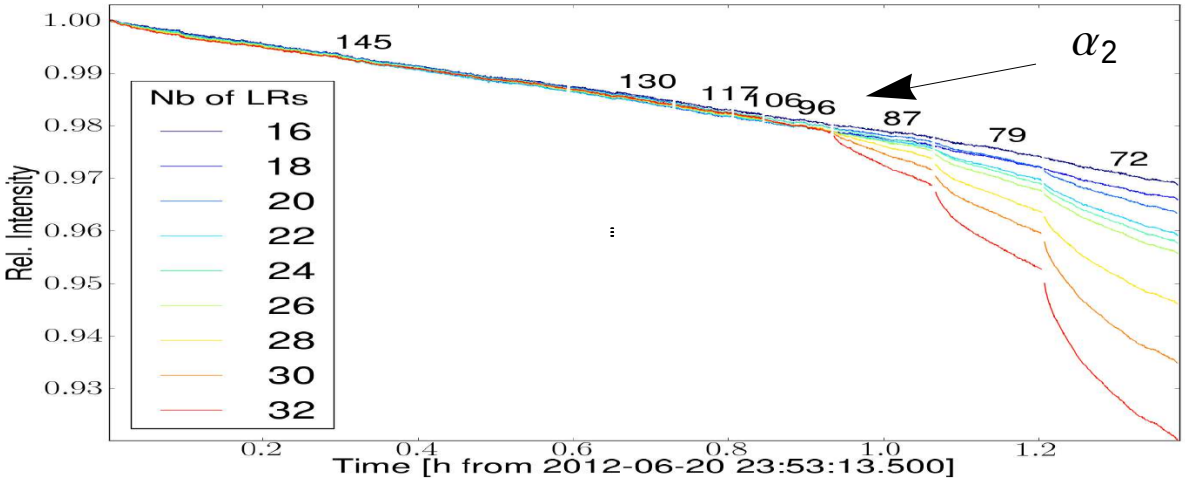


predicted angles: 96,, 83, 72

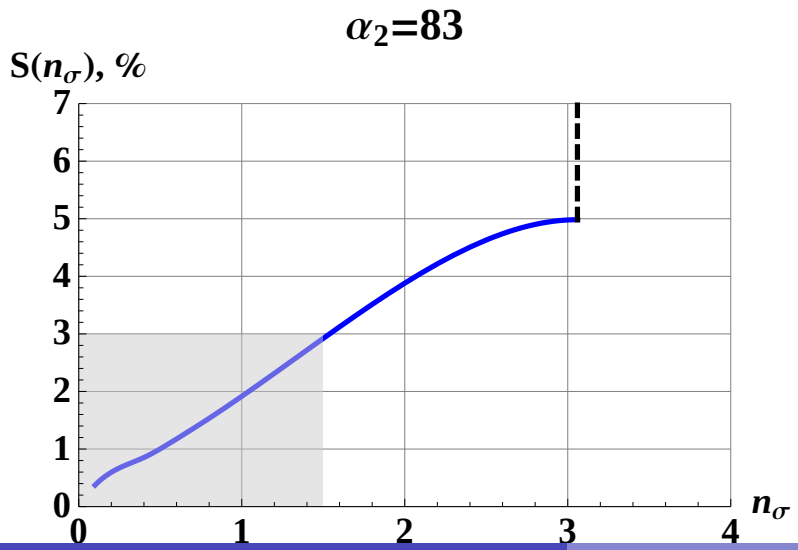


Green and black decay curves for $N_b = 1.6\,10^{11}$

angle solutions for smear 3% at 1.5 σ

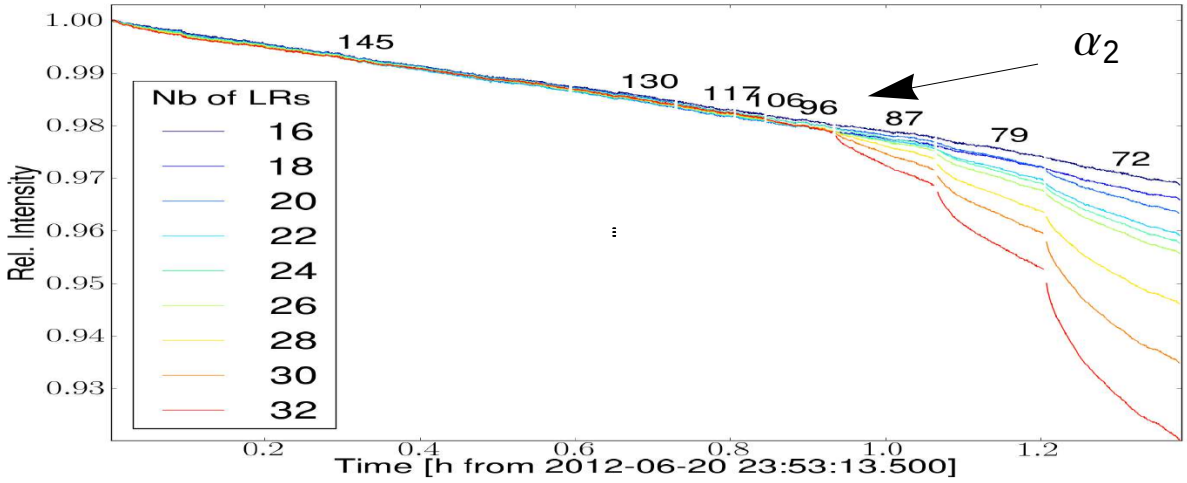


predicted angles: 96,, 83, 72

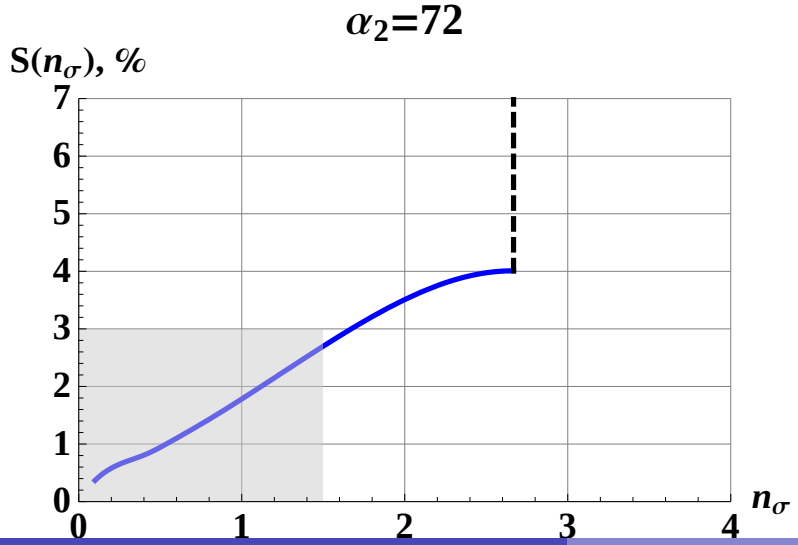


Green and black decay curves for $N_b = 1.6\,10^{11}$

angle solutions for smear 3% at 1.5 σ



predicted angles: 96,, 83, 72



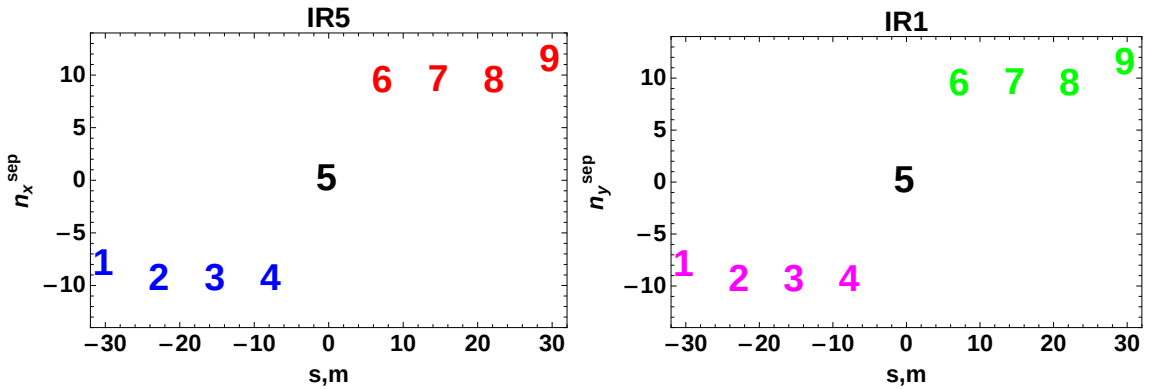
Summary

- the observed critical angles (start of losses) can be explained with the analytical smear(amplitude) formula
- accuracy $\pm(3-4)\ \mu\mathrm{rad}$
- first-order formula is sufficient in all cases except one: the smallest angle ($\alpha \approx 50\ \mu\mathrm{rad}$)
- these angles consistent with smear $\sim 3\%$ at 1.5 σ
- can be used (for after LS1 + HL-LHC)
  – to compare different configurations (without tracking!)
  - to extrapolate 50 ns to 25 ns (N l.r. doubled)
- The scaling with the normalized separation allows to assess the effect of different energies and emittances. An optimization of collision scenarios should be much easier using this technique.

Additional slides

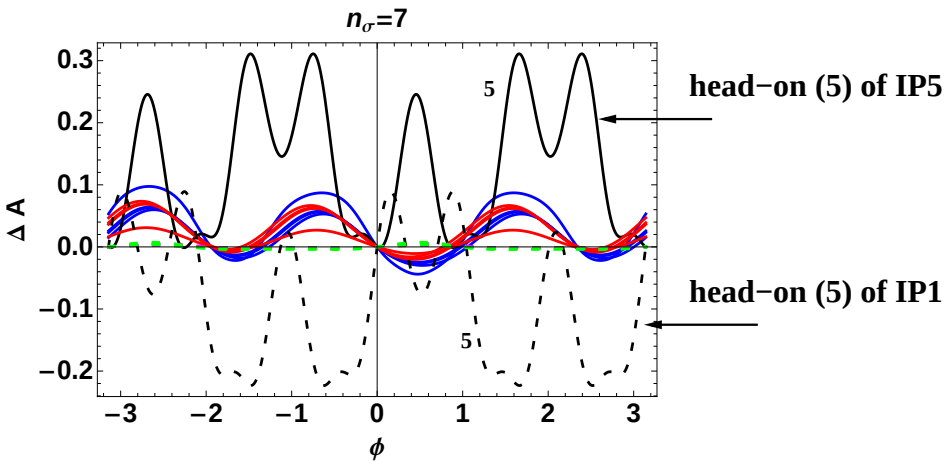
Smear maximum explained with l.r. contrib. changing sign

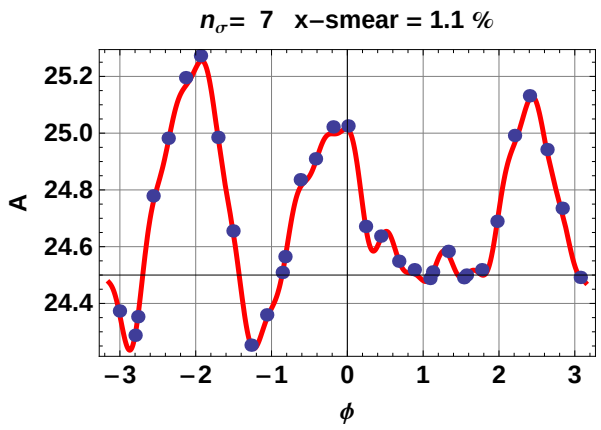
- some sample set of h.o. and l.r. (7 TeV):



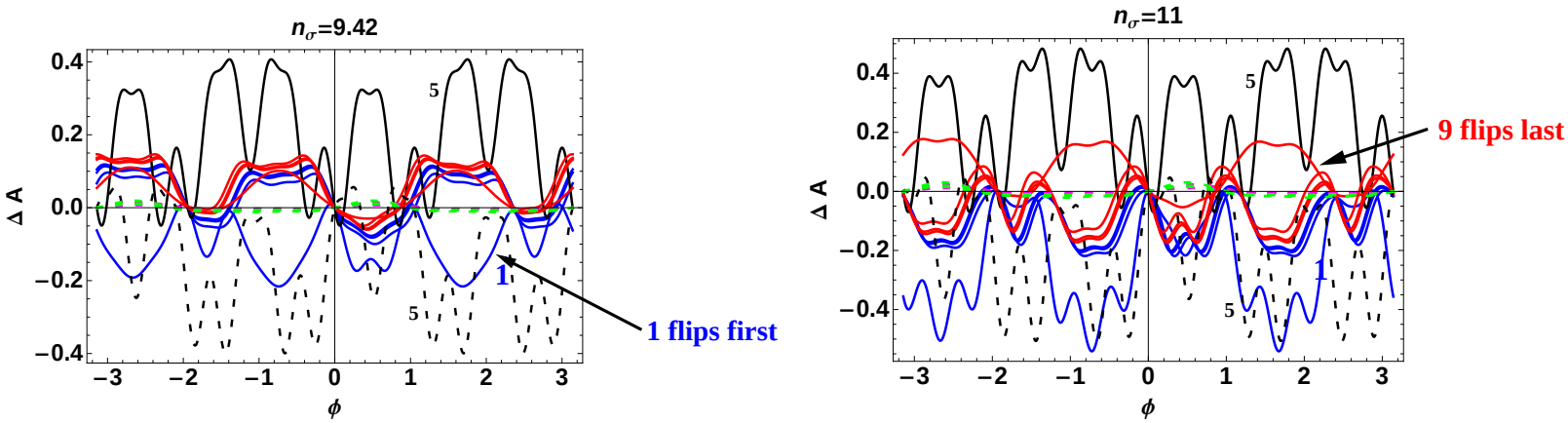
2-4 and 6-8 are at $\pm$ 9.4 σ
1 is at -8 σ
9 is at 11.5 σ

- $n_\sigma = 7$ ($\sim$2.4 sigma from the core): long ranges in IP5 contrib. nearly equally. no contrib from IR1

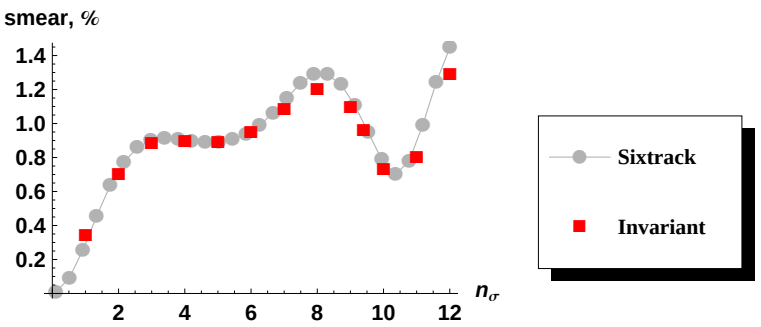




- Next, increase starting amplitude n_σ. Observe that, as the test particle passes the strong-beam core, the long-range contrib. change sign or flip about x-axis.



- During this flipping the smear stops growing $\Rightarrow S(n_\sigma)$ goes through a maximum:



Click to see animation: animation.gif, or animation.avi

Multiple IPs: Nonlinear Hamiltonian

$$F_{\rm nonl} = F - F_{(1)} - F_{(2)} \,,$$

goto action-angle coordinates J-ϕ:

$$x = \sqrt{2J\beta}\sin\phi = n_\sigma \sigma \sin\phi$$

$$\boxed{F_{\rm nonl}(n_\sigma, \phi) = F_{\rm nonl}(x)|_{x \to n_\sigma\, \sigma\, sin(\phi)}}$$

Use this kick for test-tracking:

$$\boxed{\Delta x'_{\rm nonl} \equiv \frac{d}{dx} F_{\rm nonl}(x)}$$

the exact expression:

$$F_{\rm nonl}(n_\sigma, \phi) = \gamma + \Gamma_0(P) + ln(P) - F_{(1)} - F_{(2)} \,,$$

$$P = \frac{1}{2}\left((n_x + n_\sigma \sin\phi)^2 + n_y^2\right) \,,$$

$$F_{(1)} = \frac{2n_x}{(n_x^2+n_y^2)}\left(1 - e^{-\frac{n_x^2+n_y^2}{2}}\right) n_\sigma \sin\phi \,,$$

$$F_{(2)} = \frac{-n_x^2+n_y^2+e^{-\frac{n_x^2}{2}-\frac{n_y^2}{2}}\left(n_x^2+n_x^4-n_y^2+n_x^2 n_y^2\right)}{\left(n_x^2+n_y^2\right)^2} n_\sigma^2 \sin^2\phi \,.$$

Weak-beam particle amplitude in number of sigmas:

$$n_\sigma = \sqrt{2J/\epsilon}$$

also

$$A = \frac{\beta^* J}{\sigma^2} = \frac{J}{\epsilon} = \frac{n_\sigma^2}{2} \,.$$

Multiple IPs: Lie-algebraic derivation

Lie Map M for a chain of factor maps – linear elem. M_k alternating with non-linear kicks (used same procedure as for sextupoles).

- with $f^{(k)}(x) \equiv F^{(k)}_{\rm nonl}(x)$ (order reversed so as all act on the same x):

$$M = M_1 e^{:f^{(1)}:} M_2 e^{:f^{(2)}:} \dots e^{:f^{(N-1)}:} M_N e^{:f^{(N)}:} M_{N+1}$$

- after rotating all kicks $f^{(k)}$ to the end of the lattice:

$$M = e^{:\widetilde{f}^{(1)}:} e^{:\widetilde{f}^{(2)}:} \dots e^{:\widetilde{f}^{(N)}:} R; \qquad R = \begin{pmatrix} \cos\mu & \sin\mu \\ -\sin\mu & \cos\mu \end{pmatrix},$$

$$\boxed{\widetilde{f}^{(k)}(n_\sigma, \phi) = F^{(k)}_{\rm nonl}(x)\Big|_{x \to n_\sigma\, \sigma\, sin(\phi+\phi^{(k)})}}$$

then to first order

$$\boxed{M \approx e^{:f:} R = e^{:h:}, \qquad f \equiv \sum_{k=1}^{N} \widetilde{f}^{(k)}}$$

$$h(J,\phi) = -\mu J + \frac{:f_2:}{1 - e^{-:f_2:}} f^\star =$$

$$= \quad -\mu J - \lambda \sum_{k=1}^{N} \sum_{m=1}^{\infty} \left(\frac{m\mu c_m^{(k)}}{2\sin(m\mu/2)} e^{im(\mu/2+\phi+\phi^{(k)})} + c.c. \right)$$

Two points:

1) **just add a phase shift**

$x \to n_\sigma\, \sigma\, sin(\phi+\phi^{(k)})$

2) **use only the oscillating part** $f^\star$ of f.

There is no $m = 0$ term.

more details

operator representing the beam-beam map is $e^{:F:}$
$e^{:f_2:}$ with

$$: f_2 : = -\mu J$$

acts in the same way as the linear-ring matrix $R = R(\nu, \beta^*)$, where
$\nu \equiv Q_x = \mu/(2\pi)$ is the ring tune.
For an arbitrary function G, we have:

$$G(: f_2 :)e^{in\phi} = G(i\, n\, \mu\,)e^{in\phi}$$

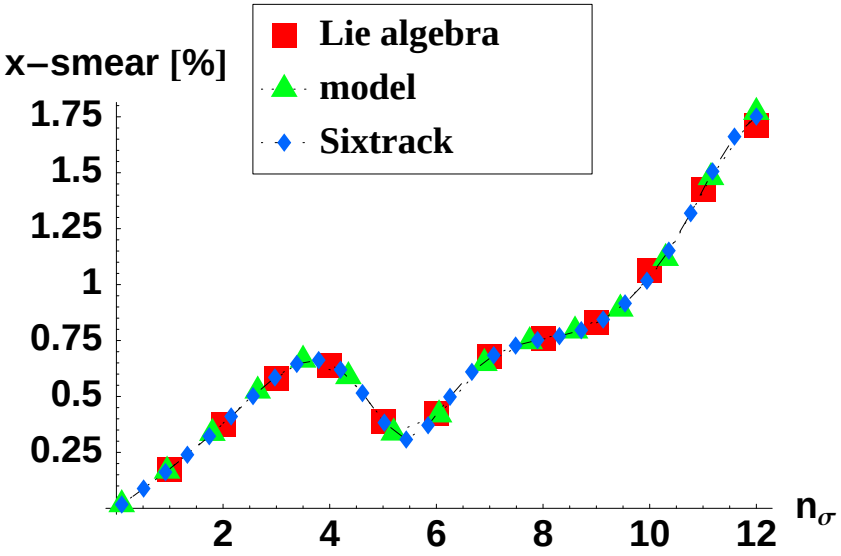
Choose $G(: f_2 :) \equiv \frac{:f_2:}{1-e^{:f_2:}}$. To first order in F one has (BCH theorem):

$$h \approx f_2 + \frac{: f_2 :}{1 - e^{:f_2:}} F = f_2 + \frac{: f_2 :}{1 - e^{:f_2:}} \sum_{n=-\infty}^{\infty} c_n(J) e^{in\phi},$$

also using $: f_2 : = -\mu J, \quad : f_2 : e^{in\phi} = i\, n\, \mu\, e^{in\phi}$ and $G(: f_2 :)e^{in\phi} = G(i\, n\, \mu\,)e^{in\phi}$

Tracking to verify of the invariant: Old result: PAC09

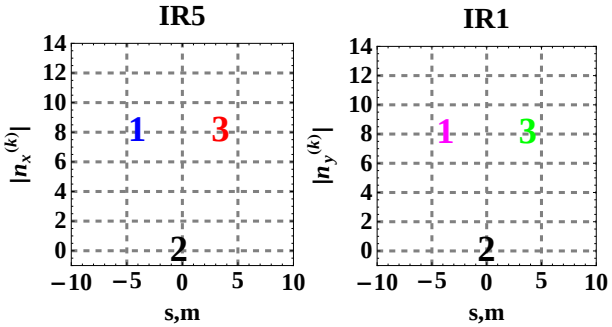
A simple tracking model based on the kick $(\Delta x')_{\mathrm{nonl}}$

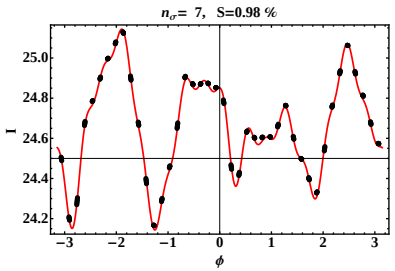


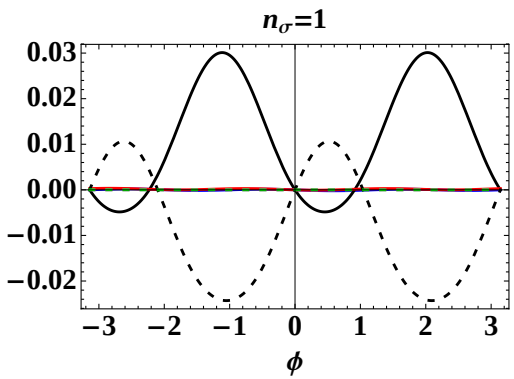
Individual contributions $dI^{(k)}$ to the invariant

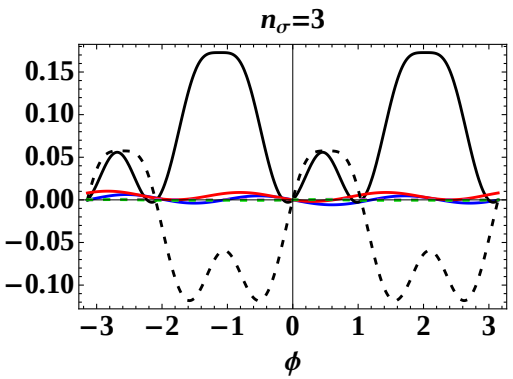
Assigned color to each collision:

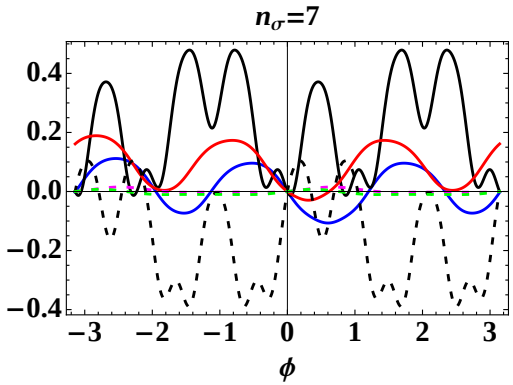
head-ons are in black – solid for IR5 and dashed for IR1.









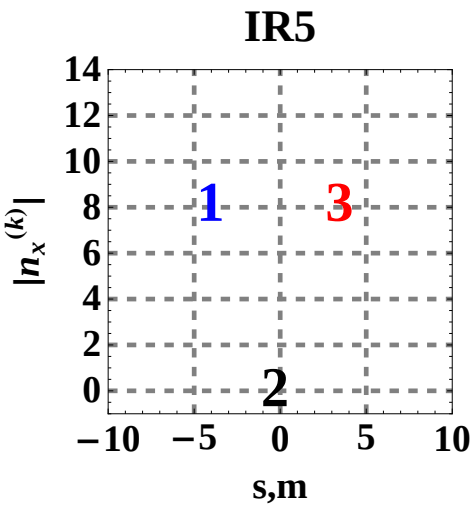


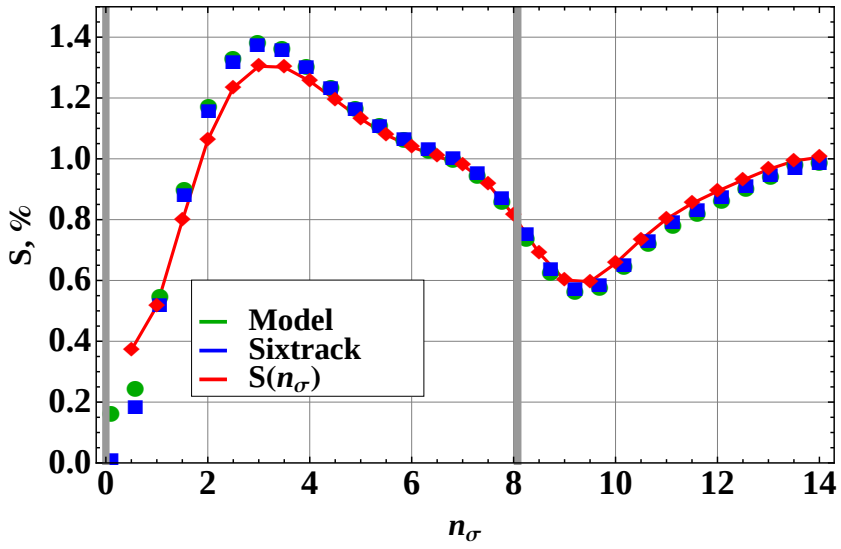
(a) Small amplitudes $\rightarrow$ head-ons in IR5 (solid) and IR1 (dashed) nearly compensate each other.

(b) At $n_\sigma = 3$ we begin to see l.r.

(c) Near-compensation is no longer true

Tracking to verify of the invariant - agreement with Sixtrack[5]



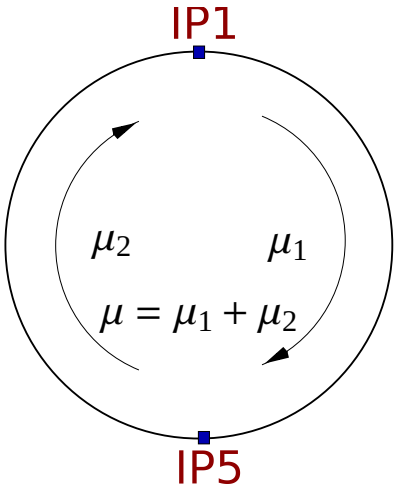


The conditions are: minimum set, energy 3.5 TeV, $N_b = 1.2\,10^{11}$ and $\epsilon_n = 2.5\,10^{-6}$. $\alpha = 145\ \mu\mathrm{rad}\ \beta^\star = 0.6$ m

Gray lines at (0 and $\sim 8\ \sigma$) represent the norm. sep.
The first order smear agrees with exact smear at all amplitudes: call this a "linear regime"

[5] F. Schmidt, Sixtrack, V.4.2 Single Particle Tracking Code Treating Transverse Motion with Synchrotron Oscillations in a Symplectic Manner, CERN/SL/94-56, Jul 2008.

Case of two head-on IPs: phase = $\pi/2 \times (\mathrm{integer})$ cancels resonances $\nu \approx \frac{N}{M}$ for which M is odd



Take $\nu \approx \frac{N}{M}$, N,M integers. Take an IP positioned at zero phase and IP at phase μ_1. Then the second IP is at $\mu_1 = \pi/2 \times (\mathrm{integer})$ and n is divisible by 4, the n th term is resonant (∞ as the denominator $\rightarrow$ 0). However terms with $n = 2 \times (\mathrm{odd\ integer})$ are finite because the factor $\cos(n\mu_1/2)$ cancels the zero denominator. In particular, if M is an odd number, then the corresponding resonance is canceled.

In this sum below IP1-IP5 betatronic phase $\mu_1 = \pi/2 \times (\mathrm{integer})$ cancels those resonance terms for which n is not divisible by 4.

$$h_{two} = -\mu J + dI(\mu, \mu_1) + dI(\mu, 0) =$$

$$= -\mu J + \sum_{n=-\infty}^{\infty} \frac{n\ \mu\ c_n(J)}{2sin(n\mu/2)} \left[e^{i\ n\ (\phi+\mu/2+\mu_1)} + e^{i\ n\ (\phi+\mu/2)} \right] =$$

$$= -\mu J + 2\ c_0(J) + \sum_{n=2,4,\ldots}^{\infty} \frac{2\ n\ \mu\ c_n(J)}{sin(n\mu/2)} \cos\left[n(\phi + \mu/2 + \mu_1/2)\right] \cos(n\mu_1/2)$$

Old result: SixTrack tune-scans show two dips in the hor. plane. Of these two resonances $\nu = 4/13$ can be compensated in this way, while $\nu = 5/16$ can not.

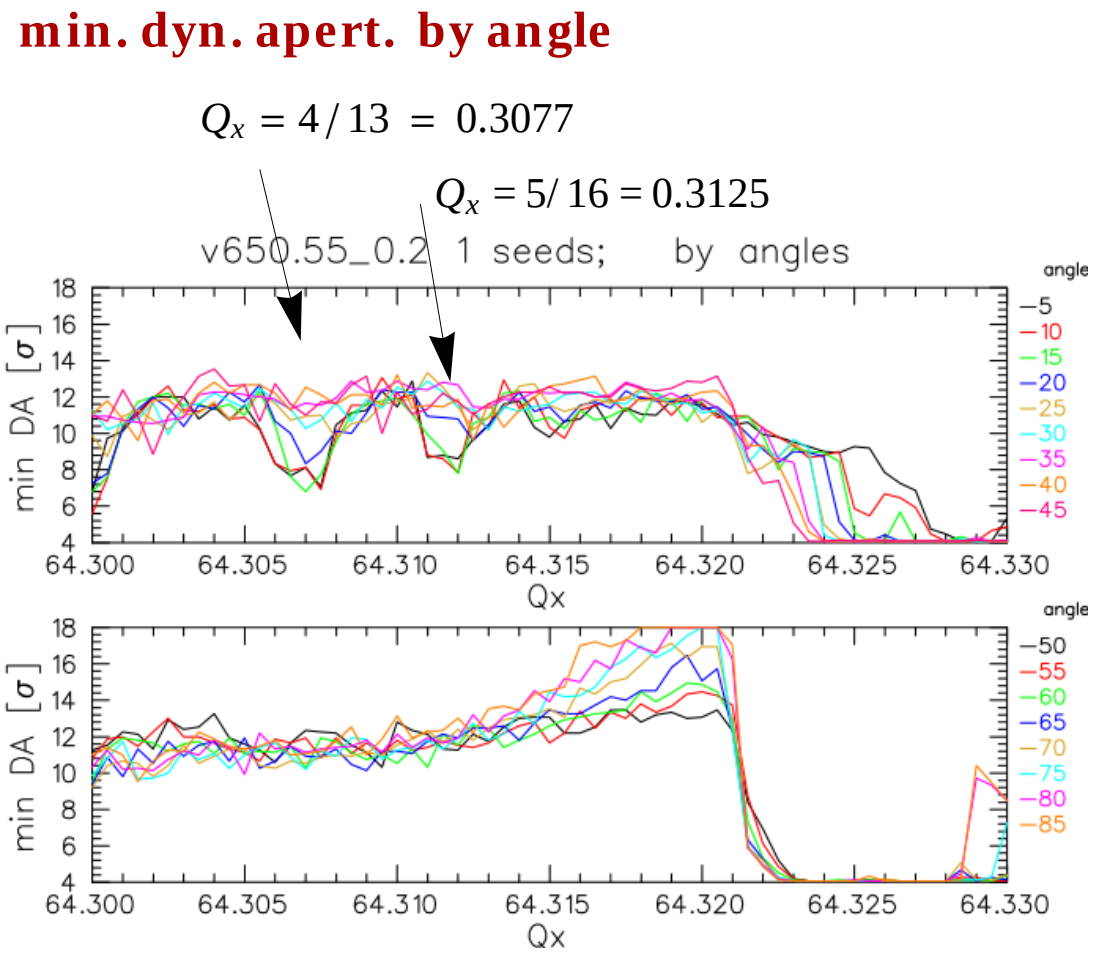


Figure: Minimum dynamic aperture and two dips on both sides of the nominal working point near $Q_x = 4/13 = 0.3077$ and $Q_x = 5/16 = 0.3125$