Status of NNLO t-tbar cross sections

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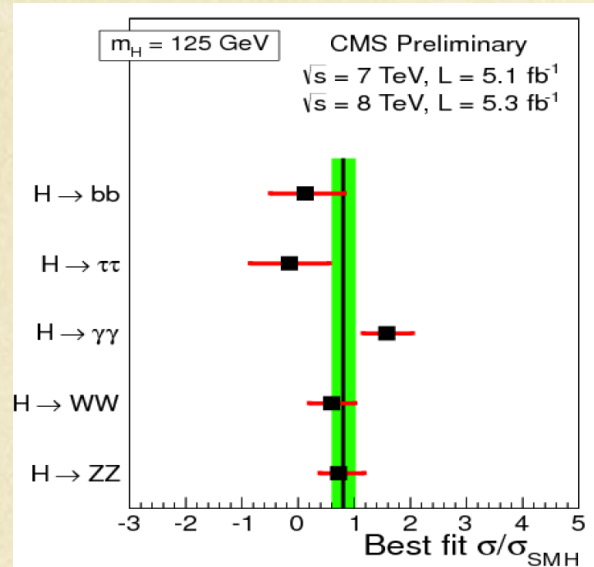

NNLO: the new wave in top physics
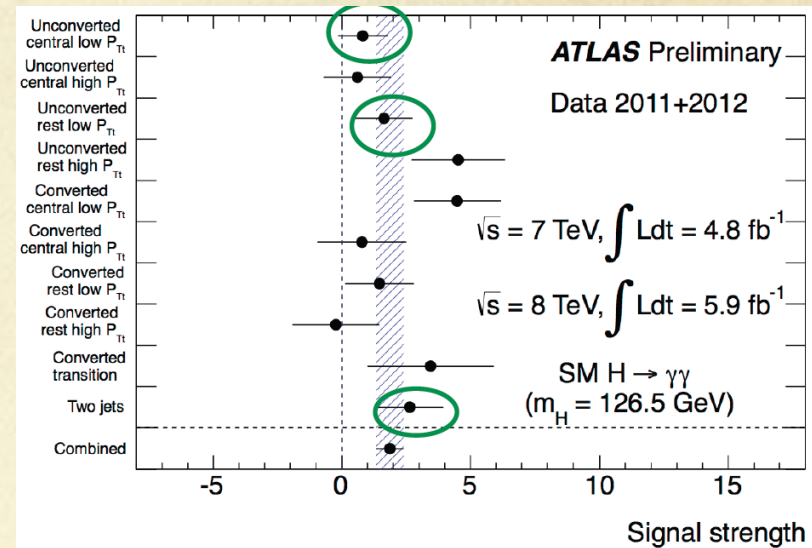
Why NNLO?

✓ Here is the best answer: discovery of the Higgs

Giovanni Petrucciani, talk at LHC2TSP

Alex Read, talk at LHC2TSP

✓ Precision is the confidence to answer the question:

    ✓ Is it the SM Higgs?
    ✓ Is it not → New Physics

<u>Back to top physics:</u>

σ_{TOT} is the test bed, a benchmark, for what can be achieved at the LHC over its lifetime

The quest for higher order corrections in top production: an engine for theoretical developments.

✓ Early NLO QCD results (inclusive, semi-inclusive)

Nason, Dawson, Ellis '88
Beenakker et al '89

✓ First fully differential NLO

Mangano, Nason, Ridolfi' 92

✓ Nowadays: *the industry* of the NLO revolution

Talks by K. Melnikov, P. Nason and Z. Trocsanyi

✓ 1990's: the rise of the soft gluon resummation at NLL

Catani, Mangano, Nason, Trentadue '96
Kidonakis, Sterman '97
Bonciani, Catani, Mangano, Nason `98

✓ NNLL resummation developed (and approximate NNLO approaches)

Beneke, Falgari, Schwinn '09
Czakon, Mitov, Sterman `09
Beneke, Czakon, Falgari, Mitov, Schwinn `09
Ahrens, Ferroglia, Neubert, Pecjak, Yang `10-`11

✓ Electroweak effects at NLO known (small ~ 1.5%)

Beenakker, Denner, Hollik, Mertig, Sack, Wackeroth `93
Hollik, Kollar `07
Kuhn, Scharf, Uwer `07

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Until 2 months ago σ_{TOT} analyzed in approximate NNLO QCD

Many groups:

Beneke, Falgari, Klein, Schwinn `09-`11

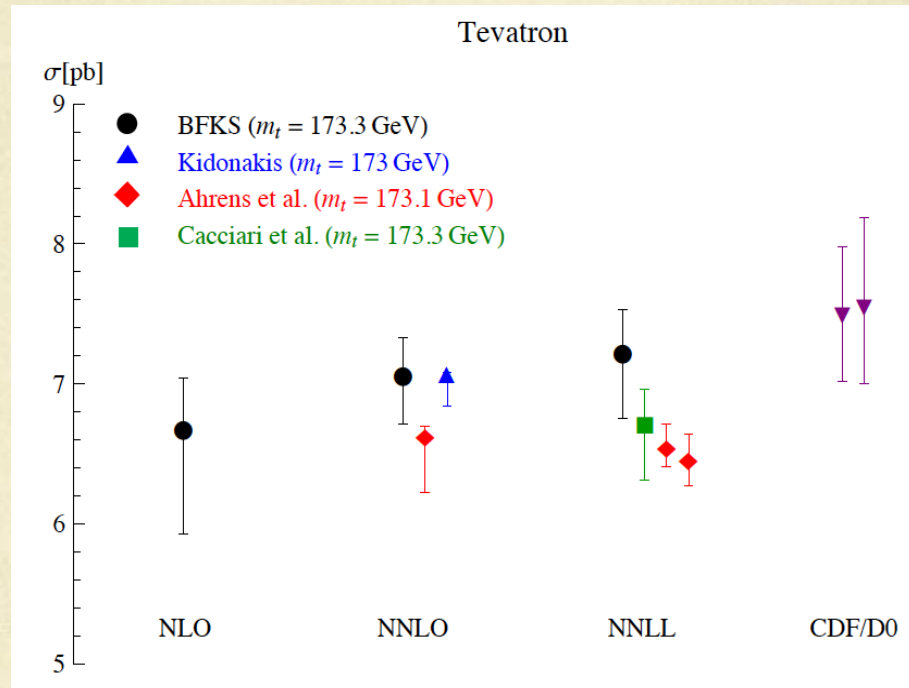Ahrens, Ferroglia, Neubert, Pecjak, Yang `10-`11

Kidonakis `04-`11

Aliev, Lacker, Langenfeld, Moch, Uwer, Wiedermann '10

Cacciari, Czakon, Mangano, Mitov, Nason `11

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Comparison between various NNLO$_{approx}$ groups shows:

Tevatron

σ[pb]

- ● BFKS ($m_t = 173.3$ GeV)
- ▲ Kidonakis ($m_t = 173$ GeV)
- ◆ Ahrens et al. ($m_t = 173.1$ GeV)
- ■ Cacciari et al. ($m_t = 173.3$ GeV)

NLO   NNLO   NNLL   CDF/D0

Beneke, Falgari, Klein, Schwinn `11

✓ Significant differences between various predictions

✓ Suggests the true uncertainty of approximate NNLO

✓ The differences are due to effects beyond the soft approximation. Must be tamed!

A step forward: first complete NNLO result

✓ First ever hadron collider calculation at NNLO with more than 2 colored partons.

✓ First ever NNLO hadron collider calculation with massive fermions.

➢ published qQ → tt +X

Bärnreuther, Czakon, Mitov `12

➢ Published all fermionic reactions (qq,qq',qQ')

Czakon, Mitov `12

➢ Work on the remaining reactions (qg, gg) progressing well.

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Total inclusive versus differential calculations

It is obvious which one is the winner ...

➢ But we have to consider the practical limitations.

➢ And the learning curve.

  ➢ Total inclusive x-sections: a big and relevant first step.

  ➢ Eventually, all must be differential, even at NNLO (and will be)

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$$\sigma = \frac{\alpha_s^2}{m_t^2} \sum_{ij} \int_0^{\beta_{\text{max}}} \mathcal{L}_{ij}(\beta)\hat{\sigma}(\beta)$$

$$\rho = \frac{4m_t^2}{s} \qquad \beta = \sqrt{1-\rho}$$

Relative velocity of tT

✓ The partonic cross-section computed numerically in 80 points. Then fitted.

✓ Many contributing partonic channels:

Computed. Dominant at Tevatron (~85%)

$$q\bar{q} \to t\bar{t} \qquad\qquad gg \to t\bar{t} \qquad\qquad qg \to t\bar{t}q$$
$$q\bar{q} \to t\bar{t}g \qquad\qquad gg \to t\bar{t}g \qquad\qquad qg \to t\bar{t}qg$$
$$q\bar{q} \to t\bar{t}gg \qquad\qquad gg \to t\bar{t}gg \qquad\qquad qq' \to t\bar{t}qq' \,, \quad q \neq \bar{q}'$$
$$q\bar{q} \to t\bar{t}q'\bar{q}' \,, \quad q \neq q' \quad gg \to t\bar{t}q\bar{q} \qquad\qquad q\bar{q} \to t\bar{t}q\bar{q}$$

All of the same complexity. No more conceptual challenges expected (just lots of CPU)

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What's needed for NNLO?

There are 3 principle contributions:

- ✓ 2-loop virtual corrections (V-V)

- ✓ 1-loop virtual with one extra parton (R-V)

- ✓ 2 extra emitted partons at tree level (R-R)
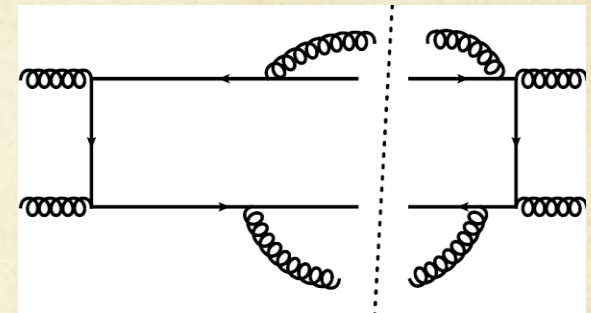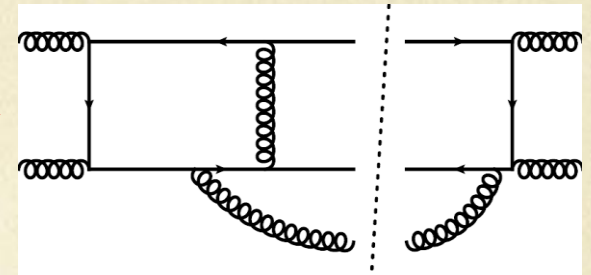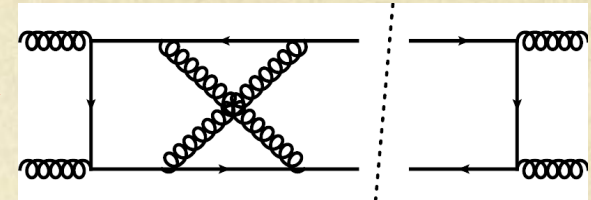
And 2 secondary contributions:

- ✓ Collinear subtraction for the initial state

- ✓ One-loop squared amplitudes (analytic)

Known, in principle. Done numerically.
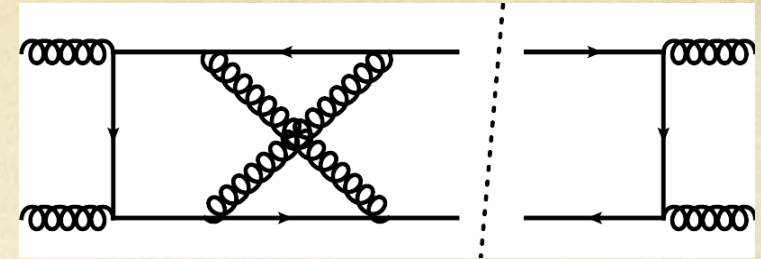
Korner, Merebashvili, Rogal `07

May be avoided?

Weinzierl `11

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What's needed for NNLO? V-V

Required are the two loop amplitudes:
qq → QQ and gg → QQ.

✓ Their high energy limits and their poles are known analytically.
Fermionic corrections, leading color, too.

Czakon, Mitov, Moch '07
Czakon, Mitov, Sterman '09
Ferroglia, Neubert, Pecjak, Yang '09
Bociani et al. `09-`11

✓ Directly used here: The qq→QQ amplitude is known numerically
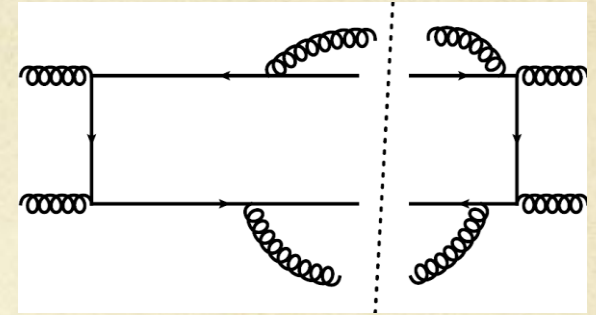
Czakon `07

✓ Numerical work underway for the gg→QQ

Czakon, Bärnreuther, to appear

What's the future here?

✓ Right now this is the biggest (and perhaps only) obstacle for NNLO
phenomenology on a mass scale

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What's needed for NNLO? R-R


✓ A wonderful result By M. Czakon

Czakon `10-11

✓ The method is general (also to other processes, differential kinematics, etc).

✓ Explicit contribution to the total cross-section given.

✓ Just been verified in an extremely non-trivial problem.

✓ Alternative method for the derivation of the all-fermionic corrections

Abelof and Gehrmann-De Ridder `11

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What's needed for NNLO? R-V
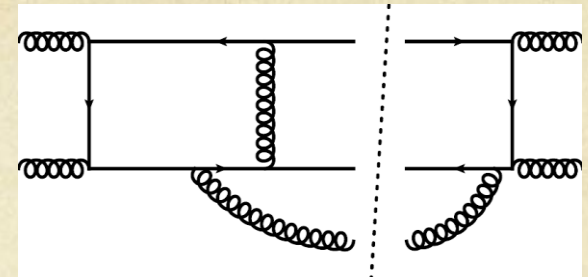
✓ Counterterms all known (i.e. all singular limits)

Bern, Del Duca, Kilgore, Schmidt '98-99
Catani, Grazzini '00
Bierenbaum, Czakon, Mitov '11

The finite piece of the one loop amplitude computed with a private code of Stefan Dittmaier.

Extremely fast code!

A great help!

Many thanks!

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✓ Guiding principle: do not try to combine all cuts into a single "finite" integration

   ✓ To have the flexibility to <u>somehow</u> compute each cut,

   ✓ Everything is done numerically. And in an independent approach.

✓ Glued together in STRIPPER subtraction scheme

<p align="right">Czakon `10</p>

✓ Inspired by FKS and Sector Decomposition

<p align="right">Frixione, Kunszt, Signer `96<br>Binoth, Heinrich `00
Anastasiou, Melnikov, Petriello `04</p>

"the subtraction terms are defined by the phase space, not us"

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Notable features:

✓ Small numerical errors
✓ Agrees with limits

Partonic cross-section through NNLO:

$$\sigma_{ij}\left(\beta, \frac{\mu^2}{m^2}\right) = \frac{\alpha_S^2}{m^2}\left\{\sigma_{ij}^{(0)} + \alpha_S\left[\sigma_{ij}^{(1)} + L\,\sigma_{ij}^{(1,1)}\right] + \right.$$

$$\left. \alpha_S^2\left[\sigma_{ij}^{(2)} + L\,\sigma_{ij}^{(2,1)} + L^2\sigma_{ij}^{(2,2)}\right] + \mathcal{O}(\alpha_S^3)\right\},$$
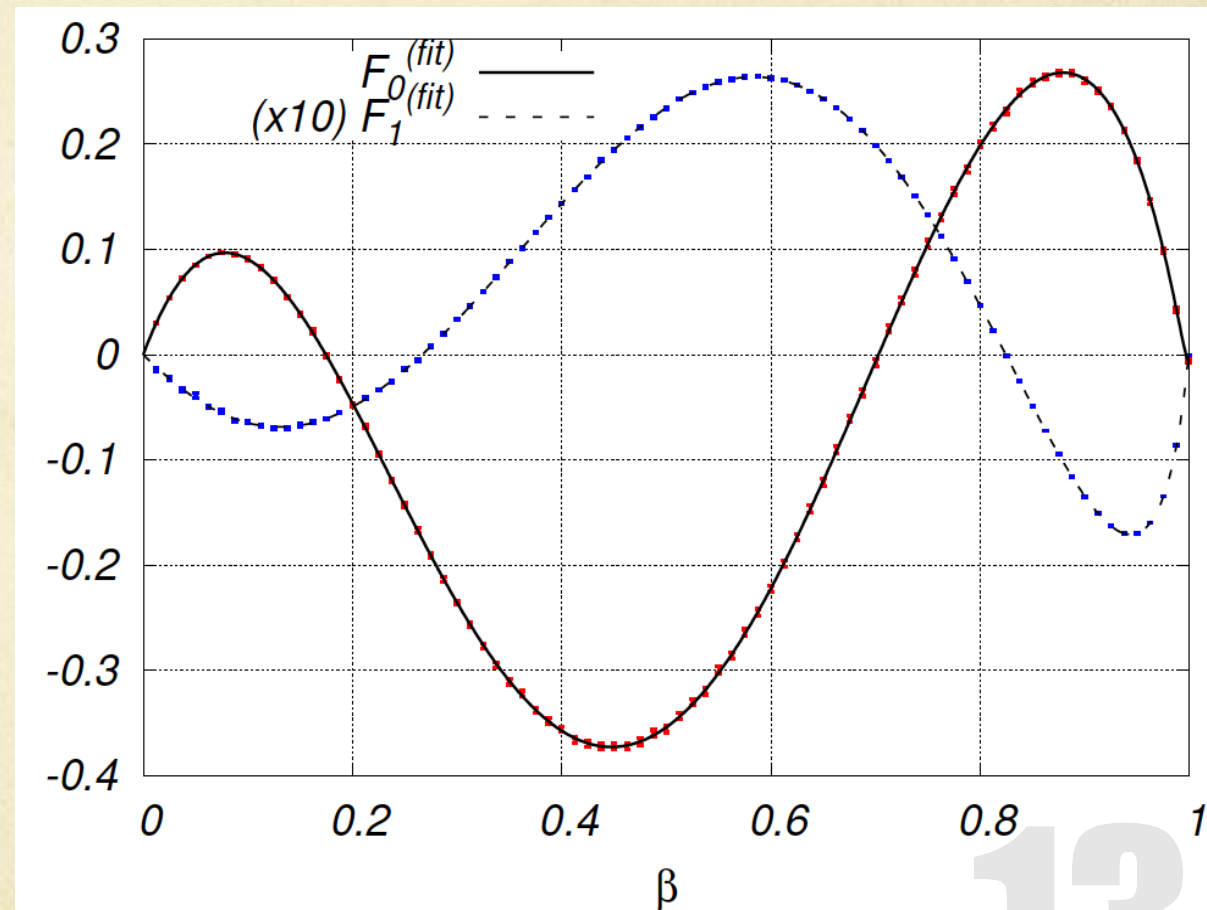
The NNLO term:

$$\sigma_{q\bar{q}}^{(2)}(\beta) = F_0(\beta) + F_1(\beta)N_L + F_2(\beta)N_L^2$$

Numeric   Analytic

$$F_i \equiv F_i^{(\beta)} + F_i^{(\text{fit})}, i = 0, 1$$
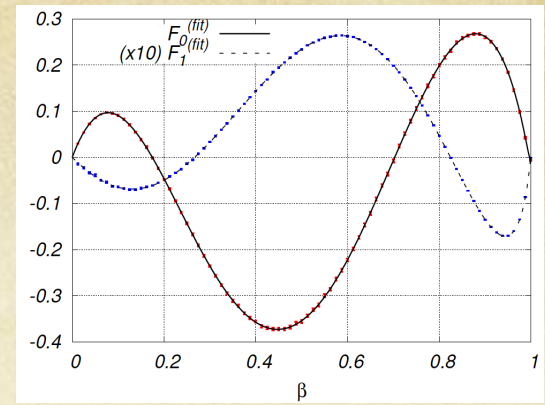
The known threshold approximation

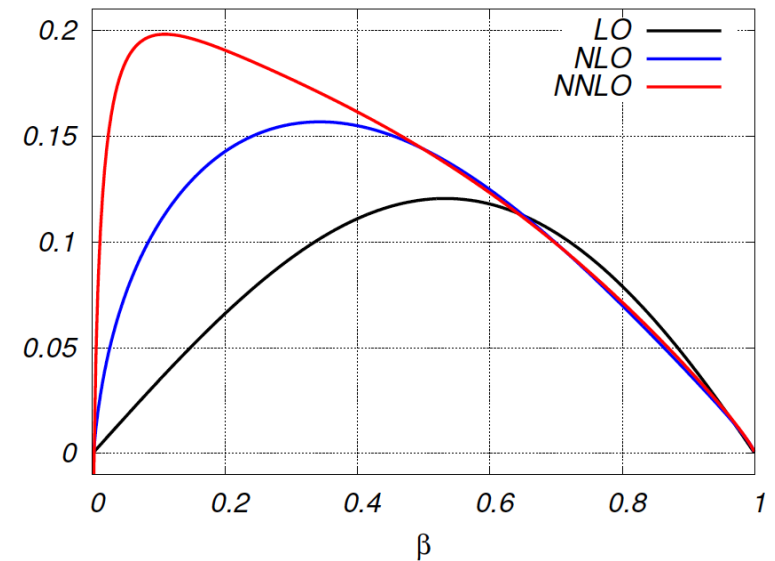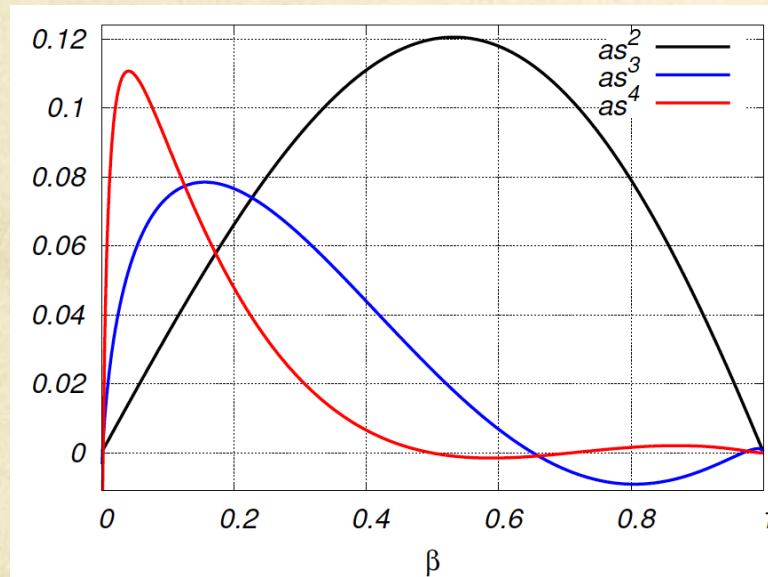Beneke, Czakon, Falgari, Mitov, Schwinn `09

P. Bärnreuther et al arXiv:1204.5201

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Results @ parton level: qqbar -> ttbar +X

$$\sigma_{ij}\left(\beta, \frac{\mu^2}{m^2}\right) = \frac{\alpha_S^2}{m^2}\left\{\sigma_{ij}^{(0)} + \alpha_S\left[\sigma_{ij}^{(1)} + L\,\sigma_{ij}^{(1,1)}\right] + \right.$$

$$\left. \alpha_S^2\left[\sigma_{ij}^{(2)} + L\,\sigma_{ij}^{(2,1)} + L^2\sigma_{ij}^{(2,2)}\right] + \mathcal{O}(\alpha_S^3)\right\},$$
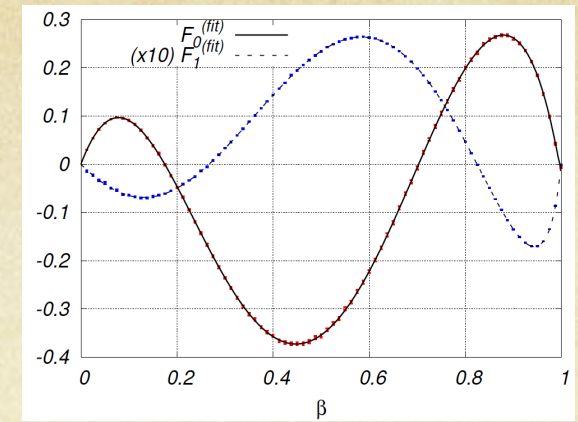
An alternative view of the partonic cross-sections:

$$\hat{\sigma}(\beta) = \frac{\alpha_S^2}{m^2}\left(\sigma^{(0)} + \alpha_S\sigma^{(1)} + \alpha_S^2\sigma^{(2)} + \dots\right) \equiv \frac{\alpha_S^2}{m^2}\left(f_{\alpha_S^2} + f_{\alpha_S^3} + f_{\alpha_S^4} + \dots\right)$$
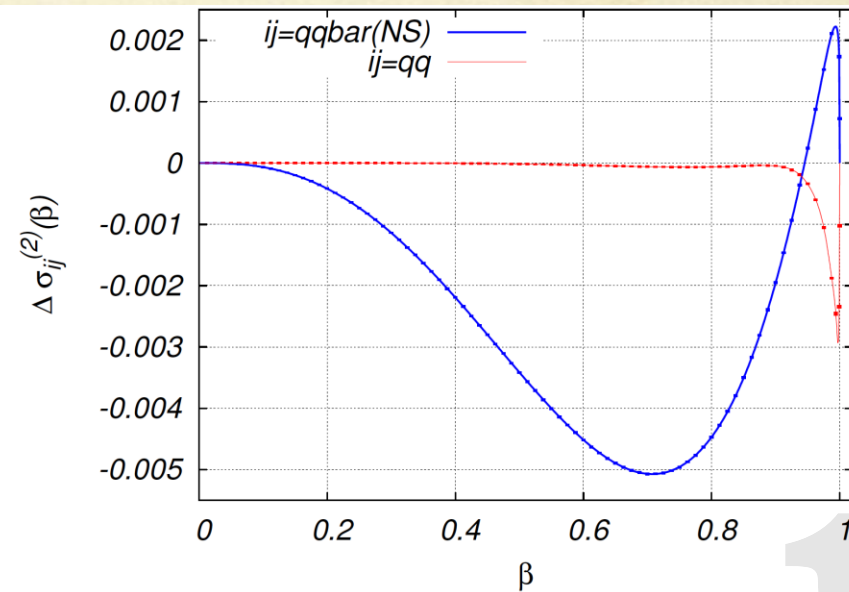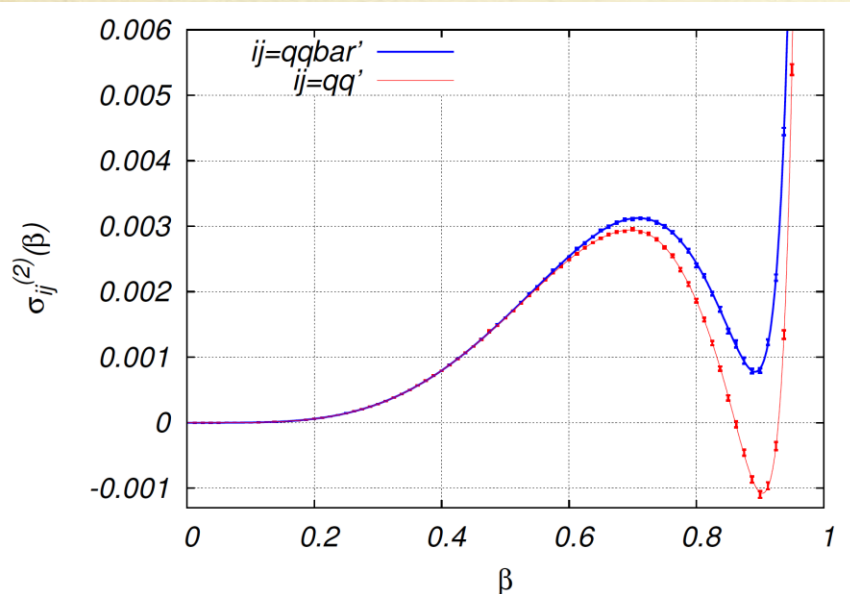
Results @ parton level:
The all-fermionic reactions

$$q\bar{q} \to t\bar{t} + q\bar{q}\big|_{\text{NS}} ,$$
$$q\bar{q}' \to t\bar{t} + q\bar{q}' ,$$
$$qq' \to t\bar{t} + qq' ,$$
$$qq \to t\bar{t} + qq .$$

Czakon, Mitov '12

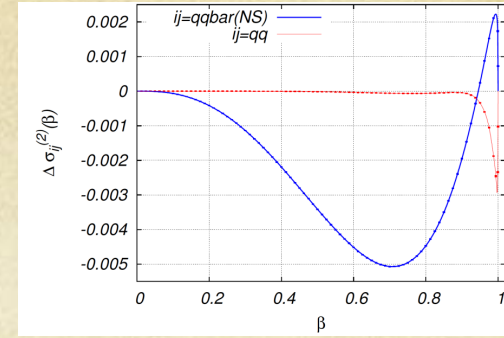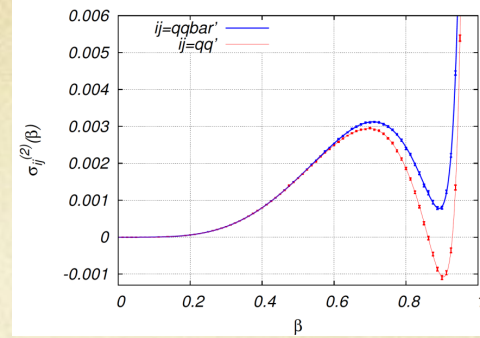P. Bärnreuther et al arXiv:1204.5201

These partonic cross-sections are very small.
Compare to the ones involving gluons!

✧ Had to compute up to beta=0.9999 to get the high-energy behavior right.

Results @ parton level:
The all-fermionic reactions

$$q\bar{q} \to t\bar{t} + q\bar{q}\big|_{\text{NS}},$$
$$q\bar{q}' \to t\bar{t} + q\bar{q}',$$
$$qq' \to t\bar{t} + qq',$$
$$qq \to t\bar{t} + qq.$$

The interesting feature: high-energy logarithmic rise:

$$\sigma^{(2)}_{f_1 f_2 \to t\bar{t} f_1 f_2}\bigg|_{\rho \to 0} \approx c_1 \ln(\rho) + c_0 + \mathcal{O}(\rho) \qquad \rho = \frac{4m_t^2}{s}$$

$$c_1 = -0.4768323995789214$$

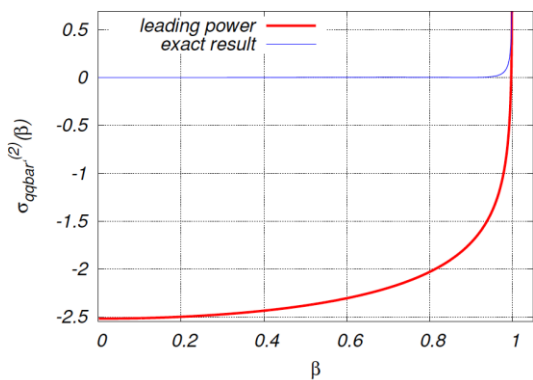Predicted in analytical form   Ball, Ellis `01

$$c_0 \text{ (from Eqs. (6.3, 6.4))} = \begin{cases} -2.5173 & \text{from} \quad \sigma^{(2)}_{q\bar{q}'} \\ -2.5186 & \text{from} \quad \sigma^{(2)}_{qq'} \end{cases}$$

❖ Direct extraction from the fits.
   5% uncertainty.   Czakon, Mitov `12

❖ Agrees with independent prediction.
   50% uncertainty.
                          Moch, Uwer, Vogt `12

High-energy expansion
non-convergent.

Applies only to the
high-energy limit.

	Tevatron	LHC 7 TeV	LHC 8 TeV	LHC 14 TeV
$\Delta\sigma_{q\bar{q},(\text{NS})}$ [pb]	-0.0020	-0.0097	-0.0124	-0.0299
$\sigma_{q\bar{q},(\text{NS})}$ [pb]	-0.0009	-0.0001	0.0021	0.0464
σ_{all} [pb]	0.0003	0.0970	0.1504	0.7885
σ_{tot} [pb]	7.0056	154.779	220.761	852.177

Czakon, Mitov `12

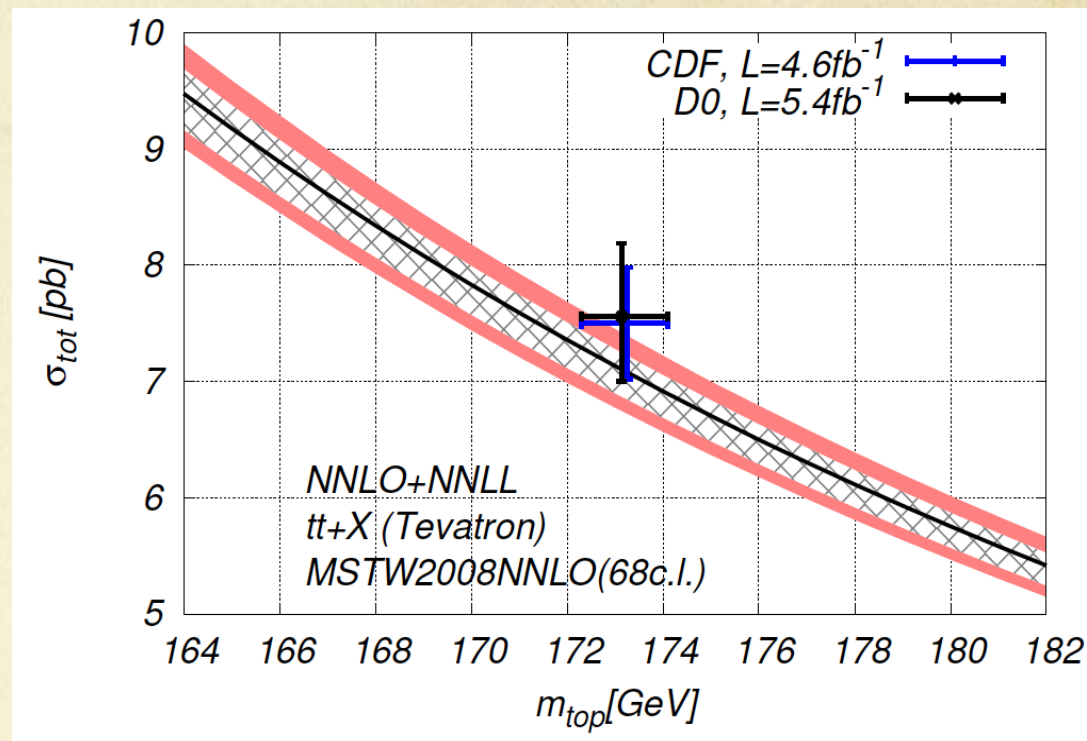# NNLO phenomenology at the Tevatron:

P. Bärnreuther et al arXiv:1204.5201

✓ Independent F/R scales
✓ MSTW2008NNLO
✓ mt=173.3

NNLO

$$\sigma_{\text{tot}}^{\text{NNLO}} = 7.005 \ ^{+0.202\,(2.9\%)}_{-0.310\,(4.4\%)} \ [\text{scales}] \ ^{+0.170\,(2.4\%)}_{-0.122\,(1.7\%)} \ [\text{pdf}]$$

$$\sigma_{\text{tot}}^{\text{res}} = 7.067 \ ^{+0.143\,(2.0\%)}_{-0.232\,(3.3\%)} \ [\text{scales}] \ ^{+0.186\,(2.6\%)}_{-0.122\,(1.7\%)} \ [\text{pdf}]$$

Best prediction at NNLO+NNLL

✓ Two loop hard matching coefficient (in qqbar) extracted and included

✓ Very week dependence on unknown parameters (sub 1%): gg NNLO, A, etc.

✓ ~ 50% scales reduction compared to the NLO+NNLL analysis of
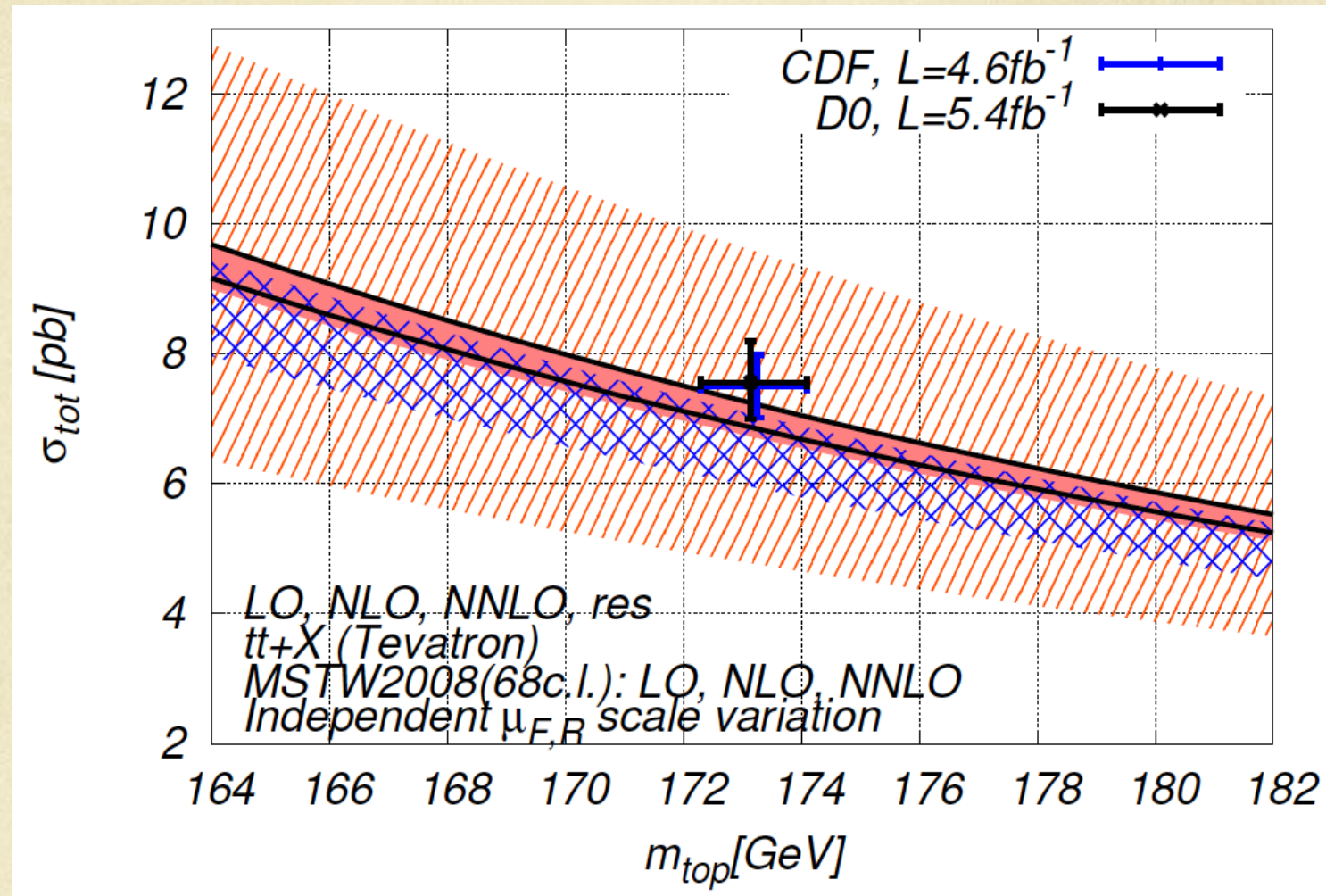
Cacciari, Czakon, Mangano, Mitov, Nason '11

$$6.722 \ ^{+0.238\,(3.5\%)}_{-0.410\,(6.1\%)} \ [\text{scales}] \ ^{+0.160\,(2.4\%)}_{-0.115\,(1.7\%)} \ [\text{PDF}]$$

Resumed (approximate NNLO)

Good perturbative convergence:

P. Bärnreuther et al arXiv:1204.5201

✓ Independent F/R scales
✓ mt=173.3

✓ Good overlap of various orders (LO, NLO, NNLO).
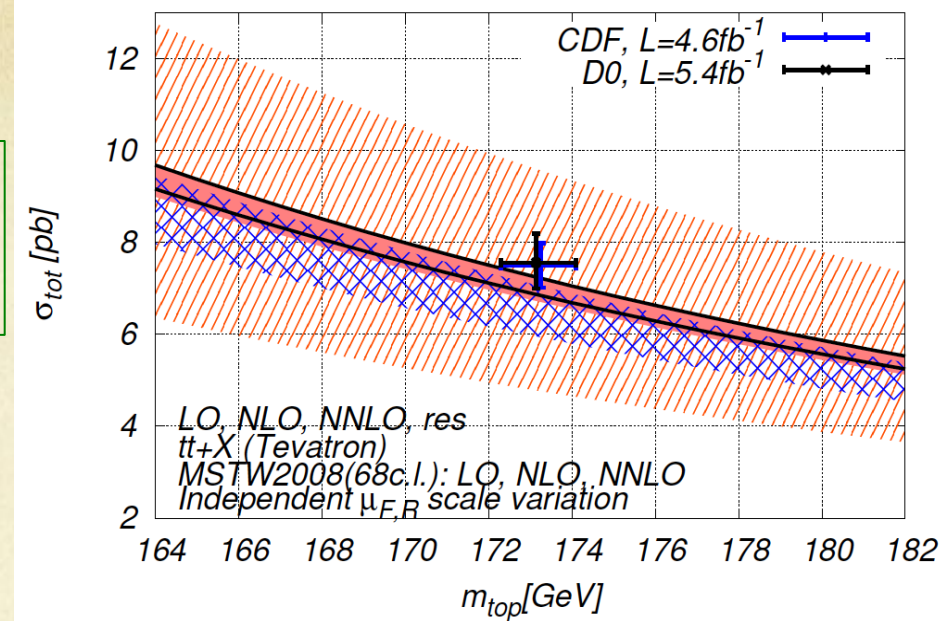✓ Suggests our (restricted) independent scale variation is good

K-factors. Computed with:

> - Numbers Computed with **Top++**
> - Current version 1.3
> - Includes all available NNLO results + resummation

Czakon, Mitov arXiv:1112.5675

> - New program just released: **Topixs**
> - Resummation in x-space

Beneke, Falgari, Klein, Piclum, Schwinn, Ubiali, Yan '12

σ^{NNLO} (NNLO pdf) $= 5.22059 + 1.23417 + 0.548064$

$(\alpha_S)^2$ $(\alpha_S)^3$ $(\alpha_S)^4$

Same K-factors with NLO pdf

$K_{NLO/LO} = 1.24$

$K_{NNLO/NLO} = 1.08$ ($K_{NNLO+NNLL/NLO} = 1.09$)

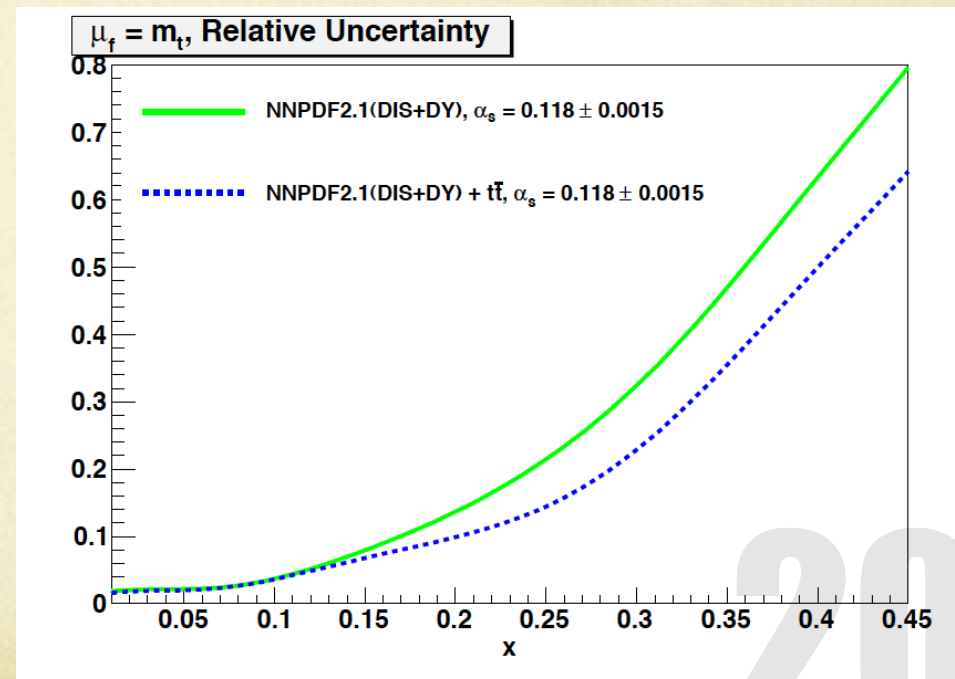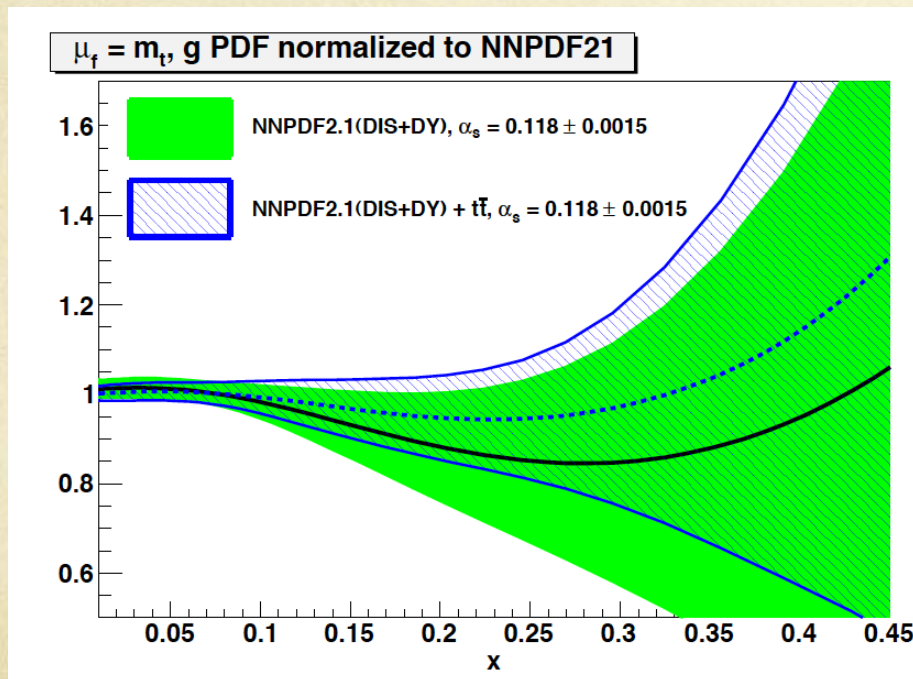K-factors alone not totally adequate without taking uncertainties into account

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Interesting applications for the NNLO top results

✓ Top mass extraction

$$m_t = 171.4^{+5.4}_{-5.7}\,\text{GeV}$$

Beneke, Falgari, Klein, Piclum, Schwinn, Ubiali, Yan '12

✓ Constraining the gluon pdf

Applications to the LHC

✓ The currently known reactions do not bring dramatic improvement at the LHC over NLO (the decrease in scale uncertainty is about +- 1%)

✓ But significant improvement (up to x3) can be expected with the inclusion of gg @ NNLO

✓ This is good news since LHC measurements are becoming systematics limited!

Beyond obvious top physics applications: Higgs physics and pdf's.

Cross section Ratios between 8 and 7 TeV

Cross section Ratios between 14 and 8 TeV
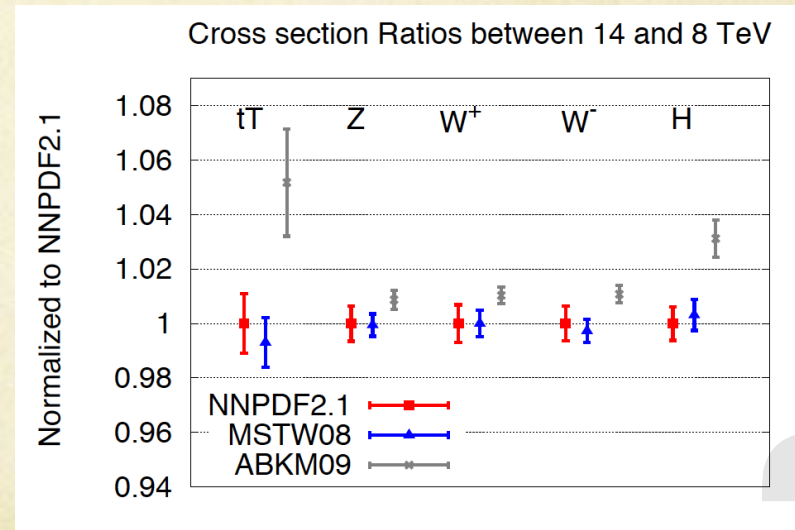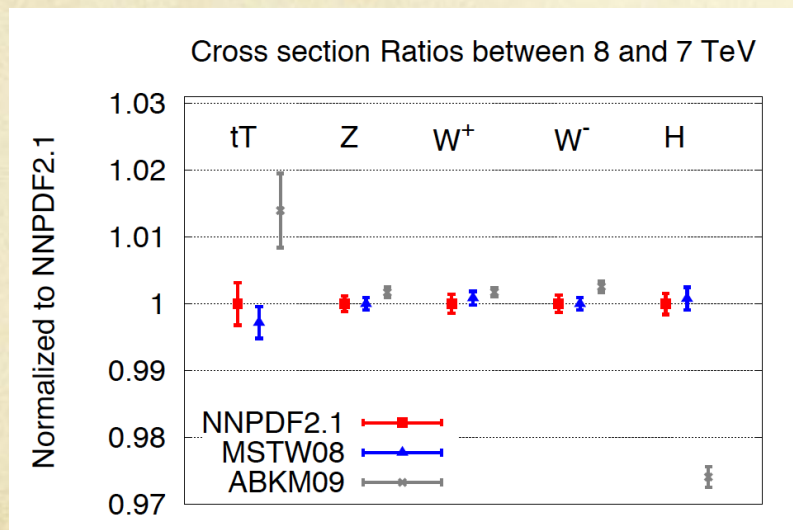
Pdf errors shown

Mangano, Rojo `12

Summary and Conclusions

❖ First NNLO calculations for heavy flavor production have appeared.

❖ The improvement over NLO, or approximate NNLO, is dramatic and in line with expectations

   ✓ Full NNLO result qq → tt *available*

   ✓ All all-fermionic reactions *available*

❖ at Tevatron: scale uncertainty cut in half compared to NLO+NNLL

❖ at LHC: can be expected up to x3 reduction in scale uncertainty.

❖ Methods are numeric, very flexible and with high precision.

Outlook

❖ Expect all partonic channels soon (qg and gg *remaining*)

❖ Fully differential partonic x-sections at NNLO will follow

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Backup

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✓ Example: VV

$$\mathcal{O}_{VV} \sim \int d^d \Phi_2 \, |M_{2\to2}|^2(\epsilon)$$

$$|M_{2\to2}|^2(\epsilon) \sim \sum_{i \leq 4} \frac{M_i}{\epsilon^i}$$

✓ Since the phase space integration is non-singular:

1. Expand phase-space and matrix element in Eps

1. Integrate each term separately (i.e. derive 5 results; 4 will cancel)

1. Amplitude is known numerically. But its poles are known analytically.

Ferroglia, Neubert, Pecjak, Yang '09

✓ The poles of any 2-loop amplitude (with masses too) can be predicted

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Mitov, Sterman, Sung '09-'10
Ferroglia, Neubert, Pecjak, Yang '09-'10

✓ Example RR : The basic logic is very simple:

1. Split the phase-space into sectors (algorithmic and process independent)

$$
1 \;=\;
\begin{aligned}
&+\theta_1(k_1)\theta_1(k_2) \\
&+\theta_2(k_1)\theta_2(k_2)
\end{aligned} \Bigg\} \text{ triple-collinear sector}
$$

$$
\begin{aligned}
&+\theta_1(k_1)\theta_2(k_2)(1-\theta_3(k_1,k_2)) \\
&+\theta_2(k_1)\theta_1(k_2)(1-\theta_3(k_1,k_2))
\end{aligned} \Bigg\} \text{ double-collinear sector}
$$

$$
+(\theta_1(k_1)\theta_2(k_2)+\theta_2(k_1)\theta_1(k_2))\theta_3(k_1,k_2) \; \} \text{ single-collinear sector .}
$$

1. Remap the phase-space integration variables in each sector (algorithmic)

1. The singularities are factored out explicitly (no counterterms needed)

$$
\mathcal{O}_S = \mathcal{N} \int_0^1 d\zeta\, d\eta_1 d\eta_2 d\xi_1 d\xi_2 \; \theta_1(k_1)\theta_1(k_2) \; \frac{1}{\eta_1^{1-b_1\epsilon}} \frac{1}{\eta_2^{1-b_2\epsilon}} \frac{1}{\xi_1^{1-b_3\epsilon}} \frac{1}{\xi_2^{1-b_4\epsilon}} \int d\Phi_n(Q)\, F_J\, \mathcal{M}_S
$$

2. Apply the usual identities:

$$
\frac{1}{\lambda^{1-b\epsilon}} = \frac{1}{b}\frac{\delta(\lambda)}{\epsilon} + \sum_{n=0}^{\infty} \frac{(b\epsilon)^n}{n!} \left[\frac{\ln^n(\lambda)}{\lambda}\right]_+
$$

$$
\int_0^1 d\lambda \left[\frac{\ln^n(\lambda)}{\lambda}\right]_+ f(\lambda) = \int_0^1 \frac{\ln^n(\lambda)}{\lambda}(f(\lambda)-f(0))
$$

All is driven by phase-space

Effective counterterm.
Known from singular limits.

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✓ Example RV : Similar to RR.

1. Less singular regions (one soft and/or one collinear)

1. Remap the phase-space integration variables in each sector (algorithmic)

1. The singularities are factored out explicitly (no counterterms needed)

1. Apply the usual identities:

$$\frac{1}{\lambda^{1-b\epsilon}} = \frac{1}{b}\frac{\delta(\lambda)}{\epsilon} + \sum_{n=0}^{\infty}\frac{(b\epsilon)^n}{n!}\left[\frac{\ln^n(\lambda)}{\lambda}\right]_+ \qquad \int_0^1 d\lambda \left[\frac{\ln^n(\lambda)}{\lambda}\right]_+ f(\lambda) = \int_0^1 \frac{\ln^n(\lambda)}{\lambda}(f(\lambda) - f(0))$$

> Effective counterterm.
> Known from singular limits.

2. The matrix elements are now divergent – no problem: expand and integrate

3. Counterterms more complicated, but known analytically.

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