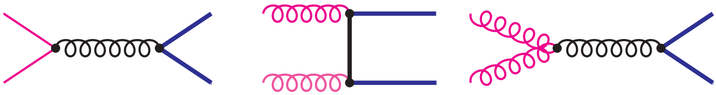
# Status of (N)NLO+NNLL Cross Section

Jan Piclum

Tools for precision and discovery physics with top quarks
CERN, 17 July 2012



$$\sigma_{t\bar{t}}(s) = \sum_{i,j} \int_{4m_t^2}^{s} \mathrm{d}\hat{s}\; \mathcal{L}_{ij}(s, \hat{s}, \mu_f)\; \hat{\sigma}_{ij}(\hat{s}, \mu_f, \mu_r)$$

two ways to compute cross section:

- fixed order → talk by A. Mitov
- sum dominant contributions to all orders

# Threshold Kinematics

production threshold: $\beta = \sqrt{1 - 4m_t^2/\hat{s}} \quad \to \quad 0$

pair-invariant mass: $1 - M_{t\bar{t}}^2/\hat{s} \quad \to \quad 0$

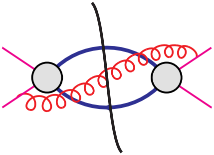
single-particle inclusive: $\hat{s} + \hat{t}_1 + \hat{u}_1 \quad \to \quad 0$

- dominant terms are singular in the respective limit
- real gluons are soft in each limit $\rightsquigarrow$ soft-gluon resummation
- $\beta \to 0$ limit also contains Coulomb singularities

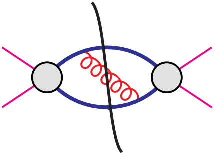
# Dominant Terms

consider threshold limit: $\beta \to 0$

Soft corrections:

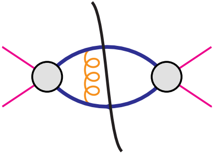


$\sim \alpha_s \ln^2 \beta$



$\sim \alpha_s \ln \beta$

Coulomb corrections:

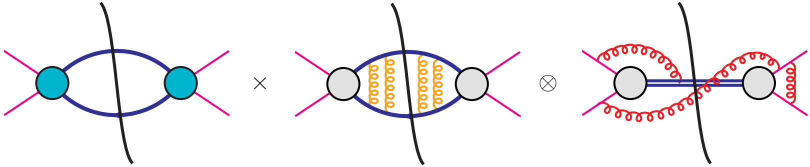


$\sim \alpha_s / \beta$

Power counting: $\alpha_s/\beta \sim \alpha_s \ln \beta \sim 1$

Soft and Coulomb resummation: Beneke, Falgari, Schwinn 2009,2010

$$\hat{\sigma}_{ij} = \sum_R H^R_{ij} \int \mathrm{d}\omega J^R(E - \omega/2)\, W^R(\omega)$$



characteristic scales: hard $\mu_h \sim m_t$, Coulomb $\mu_C \sim m_t\beta$, soft $\mu_s \sim m_t\beta^2$

$$\hat{\sigma} \quad \propto \quad \hat{\sigma}^0 \sum_k \left(\frac{\alpha_s}{\beta}\right)^k \exp[\underbrace{\ln\beta\, g_0(\alpha_s \ln\beta)}_{\text{LL}} + \underbrace{g_1(\alpha_s\ln\beta)}_{\text{NLL}} + \underbrace{\alpha_s g_2(\alpha_s \ln\beta)}_{\text{NNLL}} + \ldots]$$
$$\times \{1\,(\text{LL, NLL})\,; \alpha_s, \beta\,(\text{NNLL})\,; \ldots\}$$

# Resummation in Mellin Space

Sterman 1987; Catani, Trentadue 1989; Kidonakis, Sterman 1997; Bonciani, Catani, Mangano, Nason 1998; ...

convolution of luminosity and partonic cross section becomes product:

$$\sigma_{t\bar{t}}(N) = \int_0^1 \mathrm{d}\rho \, \rho^{N-1} \, \sigma_{t\bar{t}}(\rho) = \sum_{i,j} \mathcal{L}_{ij}(N) \, \hat{\sigma}_{ij}(N) \qquad \rho = \frac{\hat{s}}{4m_t^2}$$

partonic cross section factorises in threshold limit $N \to \infty$:

$$\hat{\sigma}_{ij}(N) = \sum_R \hat{\sigma}^{\text{hard}}_{ij,R} \, \hat{\sigma}^{\text{Coul}}_{ij,R}(N) \, \Delta_{ij,R}(N)$$

Cacciari, Czakon, Mangano, Mitov, Nason 2011

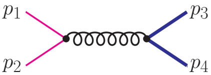
- Coulomb terms included at fixed order only
- inverse Mellin transform leads to Landau pole
- use minimal or Borel prescription

Catani, Mangano, Nason, Trentadue 1996
Forte, Ridolfi, Rojo, Ubiali 2006; Abbate, Forte, Ridolfi 2007

# PIM and 1PI Kinematics

$$1 - M_{t\bar{t}}^2/\hat{s} \quad \to \quad 0$$

$$\hat{s} + \hat{t}_1 + \hat{u}_1 \quad \to \quad 0$$




$$\hat{t}_1 = (p_1 - p_3)^2 - m_t^2$$
$$\hat{u}_1 = (p_2 - p_3)^2 - m_t^2$$

cross section factorises in convolution of hard and soft function

resum large logarithms in differential cross section:

$\frac{d\sigma_{t\bar{t}}}{dM_{t\bar{t}}}$ Ahrens, Ferroglia, Neubert, Pecjak, Yang 2010

$\frac{d\sigma_{t+X}}{dp_T}$ Kidonakis 2011; Ahrens et al. 2011

- no Coulomb corrections, since top quarks are relativistic
- soft function depends on kinematic invariants
- total cross section from integration over phase space
- include some higher-order terms in $\beta$

fixed soft scale: Becher, Neubert, Xu 2007

- minimises relative fixed-order 1-loop soft correction to $\sigma_{t\bar{t}}$
- resums logarithms in hadronic cross section
- does not predict partonic cross section

# Fixed vs. Running Soft Scale

fixed soft scale: Becher, Neubert, Xu 2007

- minimises relative fixed-order 1-loop soft correction to $\sigma_{t\bar{t}}$
- resums logarithms in hadronic cross section
- does not predict partonic cross section

running soft scale: Beneke, Falgari, Klein, Schwinn 2011

- divide $\beta$ integration into two regions
- $\beta < \beta_{\text{cut}}$: small ambiguities, $\mu_s = 2m_t\beta_{\text{cut}}^2$
- $\beta > \beta_{\text{cut}}$: no large logarithms, $\mu_s = 2m_t\beta^2$
- Tevatron: $\beta_{\text{cut}} = 0.35$; LHC: $\beta_{\text{cut}} = 0.54$

# Matching to Fixed Order Cross Section

- $t\bar{t}$ production at Tevatron and LHC is not close to threshold: $\beta \approx 0.4$
- NLO: threshold expansion gives reasonable estimate of integral over $\beta$
- match to fixed order result to improve behaviour at large $\beta$

$$\sigma^{\text{NLO+NNLL}} = \sigma^{\text{NNLL}} - \sigma^{\text{NNLL}}\Big|_{\text{NLO}} + \sigma^{\text{NLO}}$$

# Matching to Fixed Order Cross Section

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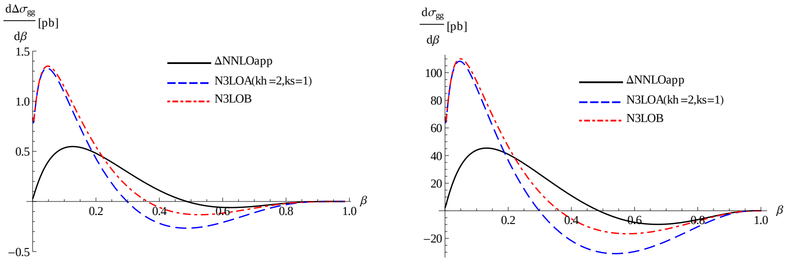
$$\sigma^{\text{NLO+NNLL}} = \sigma^{\text{NNLL}} - \sigma^{\text{NNLL}}\Big|_{\text{NLO}} + \sigma^{\text{NLO}}$$

$$\sigma^{\text{NNLO}_{\text{app}}+\text{NNLL}} = \sigma^{\text{NNLL}} - \sigma^{\text{NNLL}}\Big|_{\text{NNLO}} + \sigma^{\text{NLO}} + \sigma^{\text{NNLO}_{\text{app}}}$$

$\sigma^{\text{NNLO}_{\text{app}}}$ contains all singular terms in $\beta$ at NNLO

# Necessity of Resummation

$$\frac{\mathrm{d}\Delta\sigma_{gg}}{\mathrm{d}\beta} = \frac{8\beta m_t^2}{s(1-\beta^2)^2}\, \mathcal{L}_{gg}(\beta)\, \Delta\hat{\sigma}_{gg}(\beta)$$




Beneke, Falgari, Klein, Schwinn 2011

- potentially large corrections at $\mathrm{N}^3\mathrm{LO}$
- $1/\beta^4$ term at $\mathrm{N}^4\mathrm{LO}$

$\rightsquigarrow$ resummation is necessary

# Results at NNLL

total cross section: $\sigma_{t\bar{t}}$

distributions: $\dfrac{d\sigma_{t\bar{t}}}{dM_{t\bar{t}}}, \dfrac{d\sigma_{t+X}}{dp_T}, \ldots$

## $\text{NNLO}_{\text{app}}$

Moch, Uwer 2008; Langenfeld, Moch, Uwer 2010

Beneke, Czakon, Falgari, Mitov, Schwinn 2009

Ahrens, Ferroglia, Neubert, Pecjak, Yang 2010, 2011

Kidonakis 2010

## NNLL soft gluon resummation

Cacciari, Czakon, Mangano, Mitov, Nason 2011

Ahrens, Ferroglia, Neubert, Pecjak, Yang 2010, 2011

## NNLL soft + Coulomb resummation

Beneke, Falgari, Klein, Schwinn 2011

# Different Implementations

- different methods:
  momentum vs. Mellin space
  threshold vs. PIM vs. 1PI
- inclusion/resummation of Coulomb terms
- inclusion of bound states
- matching to fixed order result:
  NLO vs. $\text{NNLO}_{\text{app}}$
- treatment of soft scale:
  fixed vs. running
- uncertainty due to unknown NNLO correction

# Programs

- HATHOR — Aliev, Lacker, Langenfeld, Moch, Uwer, Wiedermann 2010

  NNLO$_{\text{app}}$ with full scale dependence, exact $q\bar{q}$[1], high-energy limit[2]

[1] NNLO correction to $q\bar{q}$ channel — Bärnreuther, Czakon, Mitov 2012

[2] NNLO high-energy limit for all channels — Moch, Uwer, Vogt 2012

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- `TopNNLO` — Ahrens, Ferroglia, Neubert, Pecjak, Yang 2011
  $\text{NNLO}_{\text{app}}$ from distributions in PIM and 1PI kinematics

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  NNLO$_{\text{app}}$ from distributions in PIM and 1PI kinematics
- Top++ — Czakon, Mitov 2011
  NNLO$_{\text{app}}$ with exact $q\bar{q}$[1,3]
  NNLL with soft-gluon resummation, Coulomb corrections

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[3] NNLO corrections to fermion-pair initiated channels — Czakon, Mitov 2012

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  NNLL with soft-gluon resummation, Coulomb corrections
- TOPIXS — Beneke, Falgari, Klein, JP, Schwinn, Ubiali, Yan 2012
  NNLO$_{\text{app}}$ with exact $q\bar{q}$[1]
  NNLL with soft-gluon and Coulomb resummation, bound states

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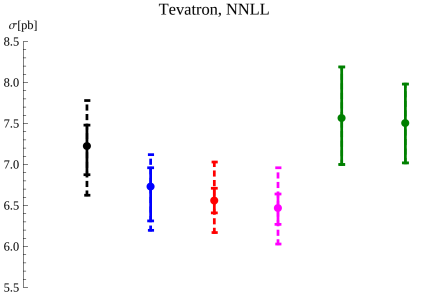
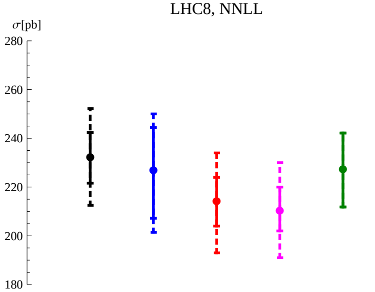
# Comparison of NNLL I



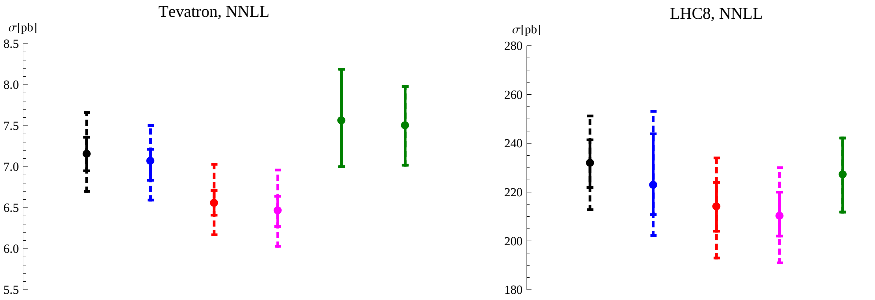




Beneke, Falgari, Klein, Schwinn 2011; `TOPIXS 1.0` ($m_t = 173.3$ GeV)
Cacciari, Czakon, Mangano, Mitov, Nason 2011 ($m_t = 173.3$ GeV)
Ahrens, Ferroglia, Neubert, Pecjak, Yang 2011 (1PI, $m_t = 173.1$ GeV)
Ahrens, Ferroglia, Neubert, Pecjak, Yang 2011 (PIM, $m_t = 173.1$ GeV)
D0 2011; CDF 2009; CMS result from ICHEP 2012

error bars: solid – theory, dashed – PDF($+\alpha_s$)

PDF set: MSTW 2008 NNLO

# Comparison of NNLL II




Beneke, Falgari, Klein, JP, Schwinn, Ubiali, Yan 2012 ($m_t = 173.3$ GeV)
top++ 1.3 ($m_t = 173.3$ GeV)
Ahrens, Ferroglia, Neubert, Pecjak, Yang 2011 (1PI, $m_t = 173.1$ GeV)
Ahrens, Ferroglia, Neubert, Pecjak, Yang 2011 (PIM, $m_t = 173.1$ GeV)
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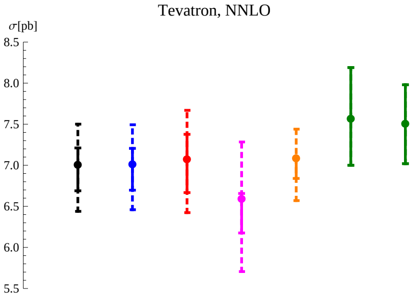
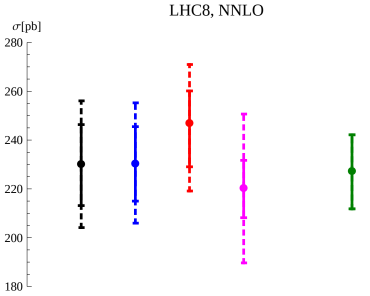
PDF set: MSTW 2008 NNLO

# Comparison of NNLO$_{(app)}$

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top++ 1.3 ($m_t = 173.3$ GeV)
HATHOR 1.3 ($m_t = 173.3$ GeV)
TopNNLO ($m_t = 173.3$ GeV)
Kidonakis 2010 ($m_t = 173$ GeV)
D0 2011; CDF 2009; CMS result from ICHEP 2012

error bars: solid – theory, dashed – PDF($+\alpha_s$)

PDF set: MSTW 2008 NNLO

# Summary

- several implementations exist at NNLL
- good agreement between results in threshold limit
- public programs can be used for analyses

# Open Issues and Questions

- different methods:
  momentum vs. Mellin space
  threshold vs. PIM vs. 1PI
- inclusion/resummation of Coulomb terms
- inclusion of bound states
- matching to fixed order result:
  NLO vs. $\mathrm{NNLO}_{\mathrm{app}}$
- treatment of soft scale:
  fixed vs. running
- uncertainty due to unknown NNLO correction
- theory uncertainty

# Kinematic Ambiguity in Fixed Soft Scale

cross section depends on energy $E = \sqrt{\hat{s}} - 2m_t$

threshold limit: $E \approx m_t\beta^2 + \ldots$

$\rightsquigarrow$ soft corrections can be expressed in terms of $\ln(E/\mu_s)$ or $\ln(m_t\beta^2/\mu_s)$

expressions agree in threshold limit, but lead to large differences at Tevatron and LHC energies:

| | $\ln(E/\mu_s)$ | $\ln(m_t\beta^2/\mu_s)$ |
|---|---|---|
| Tevatron | $\mu_s = 52$ GeV | $\mu_s = 35$ GeV |
| LHC (8 TeV) | $\mu_s = 103$ GeV | $\mu_s = 60$ GeV |

| | Tevatron | LHC8 |
|---|---|---|
| Beneke et al. 2011 | $7.22\,^{+0.26}_{-0.34}\,^{+0.30}_{-0.25}$ | $231.9\,^{+10.5}_{-10.3}\,^{+9.8}_{-9.1}$ |
| Beneke et al. 2012 | $7.15\,^{+0.21}_{-0.20}\,^{+0.30}_{-0.25}$ | $231.8\,^{+9.6}_{-9.9}\,^{+9.8}_{-9.1}$ |
| Cacciari et al. 2011 | $6.72\,^{+0.24}_{-0.41}\,^{+0.16}_{-0.16}$ | $226.6\,^{+17.8}_{-19.4}\,^{+5.6}_{-5.8}$ |
| `top++ 1.3` | $7.06\,^{+0.15}_{-0.23}\,^{+0.29}_{-0.24}$ | $222.7\,^{+21.2}_{-11.9}\,^{+9.2}_{-8.6}$ |
| Ahrens et al. 2011, 1PI | $6.55\,^{+0.16}_{-0.14}\,^{+0.32}_{-0.24}$ | $214\,^{+10}_{-10}\,^{+10}_{-11}$ |
| Ahrens et al. 2011, PIM | $6.46\,^{+0.18}_{-0.19}\,^{+0.32}_{-0.24}$ | $210\,^{+10}_{-8}\,^{+10}_{-11}$ |
| D0 2011 | $7.56\,^{+0.63}_{-0.56}$ | |
| CDF 2009 | $7.50\,^{+0.48}_{-0.48}$ | |
| CMS 2012 | | $227^{+15}_{-15}$ |

# Comparison of NNLO — Numbers

| | Tevatron | LHC8 |
|---|---|---|
| TOPIXS 1.0 | $7.00\,^{+0.21}_{-0.31}\,^{+0.29}_{-0.25}$ | $229.8\,^{+16.5}_{-16.7}\,^{+9.7}_{-9.0}$ |
| top++ 1.3 | $7.00\,^{+0.20}_{-0.31}\,^{+0.29}_{-0.24}$ | $230.2\,^{+15.3}_{-15.2}\,^{+9.8}_{-9.0}$ |
| HATHOR 1.3 | $7.07\,^{+0.31}_{-0.40}\,^{+0.29}_{-0.24}$ | $246.8\,^{+13.4}_{-17.7}\,^{+10.8}_{-9.9}$ |
| TopNNLO | $6.59\,^{+0.07}_{-0.41}\,^{+0.63}_{-0.47}$ | $220.0\,^{+11.7}_{-11.8}\,^{+19.0}_{-18.5}$ |
| Kidonakis 2010 | $7.08\,^{+0.00}_{-0.24}\,^{+0.36}_{-0.27}$ | |
| D0 2011 | $7.56\,^{+0.63}_{-0.56}$ | |
| CDF 2009 | $7.50\,^{+0.48}_{-0.48}$ | |
| CMS 2012 | | $227^{+15}_{-15}$ |