Standard Model

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$SM - III - IV - V$

Different interactions and their symmetries

1. Take ^a look once again at the table of quarks and leptons and interactions among them.

2. Connection between Symmetries and conservation laws.

3. Discuss differences among different interactions, viz. different conservation laws they obey.

4. Summarise experimentally observed features of Weak Interactions in weak decays of nuclei, strange particles and $V-A$ theory.

5. Quark mixing and Glashow Model.

Electromagnetic interactions: known for a long time, QED constructed first, best tested. \Rightarrow $(U(1)_{em})$

Nuclear strong interaction (α decay), weak interaction (β decay) known simultaneously. $\Rightarrow (SU(2)_L)$

Development of gauge theory of strong and weak interactions happened simultaneously, but QCD $(SU(3)_C)$ now completely established.

EW still has one piece to be completely settled.

Force between two particles can be understood either in terms of the field or in terms of exchange of ^a field quantum!

The properties of the quantum such as spin, mass govern the nature of the force such as range, dependence on \vec{r} between the particles etc.

Analyse differences between the strong, weak and electromagnetic interactions, to appreciate differences between g, W^{\pm}, Z^0 and γ . All are gauge interactions but have different conservation laws.

Both cases mass of the gauge boson is zero. But the behaviour of the potential is very different. For QED it is $(1/r)$. For QCD it is $\propto r(\log r)$ at long distances and $1/r$ at short distances!

This is technically due to the fact that gluons themselves have ^a colour charge but photons do not. The effective charges have different energy dependence in QED and QCD.

Calculating inter quark potential in QCD is a nonperturbative problem. Solved on Lattice. Calculations do seem to give rise to a confining potential. Very much ^a case of work in progress.

$$
\alpha_s(Q^2) = \frac{g_s^2(Q^2)}{4\pi} = \frac{12\pi}{(33-2N_f)\log(Q^2/\Lambda_{QCD}^2)}
$$

Along with QFT another tool that Particle Physicists used heavily was the connection between conserved quantities and symmetries. (Noether charges: lectures by Prof. Servant)

Space-time symmetries:

The form of laws of physics can not depend on the choice of coordinate axes or origin..

$$
\vec{F} = m\vec{a} = m\frac{d^2\vec{x}}{dt^2}
$$

In particular it HAS to be invariant under ^a coordinate translation or time translation. Choice of origin for coordinates or time.

 $\vec{x}' = \vec{x} + \vec{a}$, and $t' = t + t_0$

Such an invariance corresponds to conservation of linear momentum and energy. (In classical mechanics we construct integrals of motion)

Form of this equation remains the same i.e. is covariant with respect to ^a simple coordinate rotation.

$$
\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}
$$

If now $\vec F(\vec x) = \vec F(r)$ where $r = \sqrt{(x^2+y^2+z^2)}$ then we say the system is invariant under ^a rotation of coordinate system. We know that ^a central force \Rightarrow conservation of orbital Angular Momentum (\bar{L})

Rotational symmetry implies Conservation pf total angular momentum $\bar{J}=\bar{L}+\bar{S}$, \bar{S} being the spin angular momentum.

Consider two frames which move at ^a constant velocity with respect to each other.

A static charge q will exert only electrostatic field, where as a charge in motion (current) will also produce magnetic field.

I.e. the values of electric and magnetic fields depend on the frame where you study the motion, but laws of motion of course do not. Expression for the Lorentz force

$$
\vec{F} = (q\vec{E} + \frac{q}{c}\vec{v} \times \vec{B})
$$

no matter in which frame you do the experiments.

In fact Maxwell's equations can be written in ^a form that is 'covariant' under Lorentz transformations.

Maxwell's equations implied that velocity of electromagnetic waves is the same in all frames of references.

Further, one knows from electromagnetism that this constant velocity c is in vacuum is given by:

$$
c = \frac{1}{4\pi\sqrt{\epsilon_0\mu_0}}
$$

where ϵ_0, μ_0 are the values of dielectric constant and permeability of vacuum.

Unification of Electricity and Magnetism \Rightarrow value of c in terms of ϵ_0, μ_0 measured in laboratory.

We will like to see counterpart of such predictions for Electroweak Unification.

To describe behaviour of elementary particles like electrons, we need to use both Quantum Mechanics and special theory of relativity. I.e. the equations of quantum mechanics describing the electron HAVE to be 'covariant' under a Lorentz transformation.

This was achieved by Dirac in the famous 'Dirac' equation

$$
\left(\sum_{\mu=1,4} i\hbar \partial_{\mu} (\gamma^{\mu})_{\alpha\beta} + m \delta_{\alpha\beta}\right) \psi_{\alpha} = 0
$$

This equation predicted

1) Gyromagnetic ratio for electron is 2

2)A particle with the same mass and spin as the electron BUT opposite electric charge must exist.

I.e. QM and Special theory of relativity predicts existence of ^a positron.

Confirmed by Anderson in 1932.

This thus proved that a description of electromagnetic interactions of electron based on a picture consistent with Relativity and Quantum Mechanics is perhaps correct.

The studies led theoretical physicists further to $\text{QED}: U(1)_{em}$ Gauge Theory. (Surely will be discussed by G. Servant)

Antiproton was found in 1956.!

FIG. 1. A 63 million volt positron $(IIp=2.1\times10^6$ gauss-cm) passing through a 6 mm lead plate
and emerging as a 23 million volt positron ($IIp=7.5\times10^6$ gauss-cm). The length of this latter path
is at least ten times gre

This whole idea led to a realisation of the symmetry of laws governing fundamental particles, called charge conjugation \mathcal{C} .

Consider electromagnetism:

 $|particle>\rightarrow |(anti)particle>$ AND change signs of \vec{E},\vec{B} Nothing will change.

Call this operation Charge Conjugation : C . Looking at relationships between vector potential $A_\mu = (\phi , \vec{A})$ and \vec{E}, \vec{B} one can then conclude that the photon described by the field A_{μ} is odd under \mathcal{C} .

 $|e^{-} \rangle \mathcal{L}_{\rightarrow} |e^{+}\rangle$, $|p \rangle \mathcal{L}_{\rightarrow} |\bar{p}\rangle$,

 e^{\pm} can not be eigenstates of C

On the other hand,

 $|\gamma\rangle \mathcal{L}_{\rightarrow}$ (-) $|\gamma\rangle$.

Eigenvalues of C can be only ± 1 (strictly speaking phase).

But not all electrically neutral particles are eigenstates of C.

For spin $1/2$ neutral particles, fermion number changes sign, i.e. neutron too has an anti-particle and for baryons it will change the sign of the baryon number. For strange particles C will change sign of strangeness quantum number.

For a quantum system the fact that nothing changes under charge conjugation will mean C commutes with the Hamiltonian for electromagnetic interactions.

We know that if an operator commutes with the Hamiltonian then states can be classified according to the eigen value of this operator.

I.e. if the operation of charge conjugation is a symmetry of the system

If an interaction is invariant under charge conjugation, processes mediated by that interaction will conserve the charge conjugation quantum number, called C parity.

Parity P:

Consider space reflection:

 $\vec{r} \rightarrow -\vec{r}$, $t \rightarrow t$, $\vec{E} \rightarrow -\vec{E}$, $\vec{B} \rightarrow +\vec{B}$, $\vec{J} \rightarrow \vec{J}$.

We will find:

 $\vec{F} = m\vec{a} \rightarrow -\vec{F} = m(-\vec{a})$

In the Quantum context if the Hamiltonian is invariant under this operation, states can be classified as having even or odd parity. (Atomic states : $(-1)^{l}$)

Matter fermions are even under parity (convention). Anti-fermions HAVE to be then odd under parity Theoretical prediction. Necessary that experiments confirm this and they did!

An important concept required in discussing behaviour of weak interaction Hamiltonian under Parity transformation, is that of helicity.

I am using words helicity and handedness interchangeably, strictly true only when particles are relativistic.

All the particle states thus can be labeled by the mass, and spin.

Two states of spin 1/2 electron, with $S_z = \pm 1/2$.

If we choose z as the axis of motion of the particle are called these states with helicity ± 1 .

Helicity $+1$: right handed Helicity -1 : left handed.

 $|f, h = 1 > \frac{\mathcal{P}}{\rightarrow}|f, h = -1 >$

For a massive spin 1/2 particle, relativistic invariance says that states with BOTH helicity MUST exist, as we can change the direction of momentum by going to ^a different frame of reference. A massless particle, in principle, could exist in ONLY one helicity state, but not if Parity is conserved.

These arguments just tell us that invariances (symmetries) indeed are very constraining.

Most importantly Weak interactions were seen to violate parity and that too maximally.

C.N. Yang and T.D. Lee: Phys. Rev. 104, 254, October ¹⁹⁵⁶ : suggested in ^a theory paper, looking at so called τ – θ puzzle.

C.S.Wu: Tested it experimentally. Phys. Rev. 105, 1413, February 1957 *i.e.* The e^- which is emitted in β decay is left handed and the e^+ is right handed.

 ν emitted in β decay was always left handed and $\bar{\nu}$ is always right handed (Maurice Goldhaber et al,Phys. Rev. 109, ¹⁰¹⁵ (1958)).

In fact since β decay seemed to produce left handed fermions and right handed antifermions and the two transform into each other under the combined operation of \mathcal{CP} it looked like CP was conserved by Weak Interactions even if P was violated.

Somewhat later it was seen that Weak Interactions violate CP as well. Christenson, Cronin, Fitch and Turlay Phys. Rev. 140, B74(1965)

There exist two K –meson states very close in mass but with life times differing by three orders of magnitude.

In case of shorter lived K_S ($\sim 10^{-10}$ sec) the final state has two π . For the longer lived it is K_L ($\sim 10^{-7}$ sec) the final state has three π (CP parity of a π is -1).

K-decays always via Weak Interactions.

Pais: K_L and K_S have CP parity -1 and $+1$, decay interaction conserves CP, forcing K_L to decay into three π . Smaller phase space thus making it longer lived.

Experiment looked whether K_L ever decays into two pions. (Suggested by Pais)

Indeed tiny fraction of K_L decays has two pions in the final state.

Strength of the \mathcal{CP} violation: 1 part in 1000, The strength of \mathcal{CP} violating part responsible for the decay weaker by ^a factor 1000.

Is this yet another new, weaker interaction? (Wolfenstein: Superweak!)

A gauge theory of Weak Interactions (standard model), has all the necessary ingredient to automatically give rise to the observed level of CP violation.

In terms of Cabbibo-Kobayshi-Masakawa(CKM) mixing): Kobayashi-Masakawa Nobel prize 2008.

Not only the three interactions strong, electromagnetic and weak differ in their strengths, More importantly they have different symmetry properties, particularly under the discrete symmetry operations and follow different conservation laws.

The weak interactions, responsible for the radioactive decays and also the decays of strange particles, involve initial and final states of differing total strangeness, whereas the strong and electromagnetic interactions conserve strangeness.

Weak interactions seem to break strangeness conservation but in a very definite manner, i.e., : $\Delta S = \Delta Q$.

The violate parity maximally and CP by a tiny amount ! Of course have a short range! Act on ν 's, leptons and quarks.

Hypothesis: Real ampiltude with a very heavy boson approximated by current-current interaction. Good agreement of the predictions of this model with data meant that IVB was necessarily heavy. IVB: Internediate Vector Boson W (Klein/Schwinger)

$$
\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} (J^W_\mu J^{W\mu\dagger}) = \frac{G_F}{\sqrt{2}} (\bar{\psi}_3 \gamma_\mu \psi_1) (\bar{\psi}_4 \gamma^\mu \psi_2); \quad \frac{G_F}{\sqrt{2}} \propto \frac{g^2}{M^2}
$$

Roughly speaking: ψ wave function of the fermion as given by Dirac!

V-A Theory.

E.C.G. Sudarshan, R.E. Marshak Phys. Rev. ^D 109, 1860, ¹⁹⁵⁸ (Feynman, Gell mann : later)

The original Fermi hypothesis 'almost' correct, but replace all the ψ with ψ_L . describe correctly the handedness of fermions observed in all weak processes.

 $\Delta S = 0$ $\Delta S = 1$ Pure leptonic.

The basic unit of charged current weak interactions is ^a doublet of left handed fermions.

 f and f' differ in electromagnetic charge by one unit.

$$
f_L = d_L, f'_L = u_L
$$
 OR $f_L = s_L, f_L = u_L$

For strange quark case $\Delta S = 1$

Called Charged Current J_{μ}^{CC} .

Cabbibo's important observation Phys. rev. Lett. 10, ⁵³¹ (1963)

Pairs of leptons, quarks differing by unit electromagnetic charge connected by emission of W^{\pm} .

If the strength for pure leptonic case is (say) g then, for the $u-d$ transition it was found to be $g \cos \theta_c$ and for $u-s$ transition it was g sin θ_c with $\theta_c = 12^\circ$. (Cabbibo angle).

I.e. the unit for ^W interactions is doublets:

$$
\mathcal{L}_1 = \left(\begin{array}{c} \nu_e \\ e^- \end{array}\right)_L, \qquad \mathcal{Q}_1 = \left(\begin{array}{c} u \\ d\cos\theta_c + s\sin\theta_c \end{array}\right)_L
$$

Interaction eigenstate: $D_L = (d \cos \theta_c + s \sin \theta_c)_L$

Bjorken and Glashow (1964) Postulated a new quark c , with same quantum numbers as the u quark, which forms a doublet with orthogonal combination $S_L = s \cos \theta_c - d \sin \theta_c$.

i.e c_L will couple with $(s \cos \theta_c - d \sin \theta_c)_L$ by an emission of a W.

Symmetry between Leptons and Quarks !

$$
\mathcal{L}_2 = \left(\begin{array}{c} \nu_\mu \\ \mu^- \end{array}\right)_L, \qquad \mathcal{Q}_2 = \left(\begin{array}{c} c \\ s \cos \theta_c - d \sin \theta_c \end{array}\right)_L
$$

We will come back tomorrow to this quark-lepton symmetry!

Much more 'charm' to the charm quark than just this symmetry.

 $f_L = D_L, S_L, f'_L = u_L, c_L$

Obvious similarity: Spin one state of initial and final state fermions, ie. spin 1 nature of the exchanged boson.

Differences: $M_W >> M_\gamma = 0$

Only (left) handed fermions take part in interactions with the W . The electromagnetic interactions treat f_L, f_R equally.

Next questions: Electromagnetic and Weak interctions: similar yet different. How to give an unified description? (note QED was already developed and understood)

Devleopments to be discussed next:

 $SU(2)_L \times U(1)_Y$ model, prediction of a new interaction (weak neutral currents) and masses of the weak gauge bosons in terms of Neutral Current (NC) couplings of quarks and leptons.

Glashow:

(Two) Doublets of quarks \mathcal{Q}_i and leptons \mathcal{L}_i , $i = 1, 2$:

Postulated that Left handed quarks and lepton form ^a doublet of ^a $SU(2)_L$ the L implying that only the left handed fermions have a nonzero charge, i.e. interactions with the W^{\pm}

The two fermions connected to each other in the charged current J_{μ}^{CC} then are a doublet with "spin" up and down in a hypothetical space called 'Weak Isospin' space.

The two members of the pair have $I_W^3 = \pm 1/2$

The universality of coupling of ALL the $f\bar{f}'$ pairs to a W^{\pm} means that they all have the 'SAME' weak charge.

Glashow's observation:

 $Q = I_W^3 + Y/2$ (Q is the electromagnetic charge in units of |e|, where ^e is electron charge and negative.)

where $Y = 1/3$ for quark doublets and $Y = -1$ for lepton doublet. Thus Y is the hyper charge of the $U(1)$ symmetry group.

Right handed fermions will have $I_W^3 = 0$ and hence a different Y_R value than the left handed fermions.

Thus the net symmetry is $SU(2)_L \times U(1)_Y$. g_2 coupling constant for $SU(2)$ and q_1 for $U(1)_Y$.

 $SU(2)_L$: Three gauge bosons W^1, W^2, W^3 : all couple only to left handed fermions. $W^{\pm} = 1/\sqrt{2}(W^1 \mp iW^2)$ couple to J_{μ}^{CC} and W^3 a neutral boson coupling to $\bar{f}_L \gamma_\mu f_L$, all with the same strength g_2 .

 $U(1)_Y$: B couples to all fermions (LH and RH).

 $J_{\mu}^{CC} \equiv J_{\mu}^{W^{\pm}} = g_2 \bar{f}'_L \gamma_{\mu} f_L$; $f = u, c, f' = D, L$. Pair of fermions differring in E.mag. charge by one unit couples to W^{\pm}

 $J_{\mu}^{W^3} = g_2 \bar{f}_L \gamma_{\mu} f_L$; $(f = d, s, u, e^-)$. W³ can not be the γ . It couples ONLY to left handed particles.

$$
J^Y_\mu = g_1 ~ \frac{Y_L}{2} ~ \bar{f}_L \gamma_\mu f_R
$$
 , $g_1 ~ \frac{Y_R}{2} ~ \bar{f}_R \gamma_\mu f_R$; couples to $B.$

Can one fine combination of of J_{μ}^{Y} and $J_{\mu}^{W^{3}}$ which is

 $J_{\mu}^{em} = q_f |e| \bar{f}_L \gamma_{\mu} f_L + q_f |e| \bar{f}_R \gamma_{\mu} f_R$

 $J_{\mu}^{Y}B^{\mu}+J_{\mu}^{W^{3}}W_{\mu}^{3}\equiv J_{\mu}^{em}A_{\mu}+J_{\mu}^{Z}Z_{\mu}$

The boson to which J_{μ}^{em} couples HAS to γ which is mixture of B and Z .

The orhtogonal combination of the currents will be a new NEUTRAL Current J_{μ}^{Z} among fermions of the same electromagnetic charge, but which is different from electromagnetism because this will treat f_L and f_R differently. This current couples to a new boson state.

B and W^3 mix, giving one zero mass eigenstate γ . Identify other with a new Neutral vector boson called Z.

 $A_{\mu} = cos\theta_W B_{\mu} + sin\theta_W W_{\mu}^3$; $Z_{\mu} = -sin\theta_W B_{\mu} + cos\theta_W W_{\mu}^3$

$$
e = g_2 \sin \theta_W = g_1 \cos \theta_W; \quad \frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8M_W^2} = \frac{e^2}{8M_W^2 \sin^2 \theta_W}
$$

 $M_{\gamma} = 0$ gives abiltiy to calculate M_W, M_Z both in terms of sin θ_W

 $SU(2)_L \times U(1)$ model predicts

1)New particles: Z , charm quark c .

2)Weak Neutral current mediated by Z analogous to Weak Charged Current mediated by the W .

3)Mass predicted in terms of one parameter θ_W , which will decide the couplings of the various particles to the Z.

 $4)WWZ$ coupling possible and predicted This part can be discussed only when we talk about formal $SU(2)$ gauge theory.

5)A sort of unification: Unless sin θ_W unnaturally small (then the whole idea is not sensible) e, g_1, g_2 all of similar order. The difference in strengths of interactions only apparent due to large M_W .

6)No Higgs yet!

Like the photon the Z couples f to f and NOT f to f'. No flavour change.

Unlike the photon Z couples differently to the left and right handed fermions.

Like the W , Z coupling too violates parity, but unlike the W not necessarily maximally.

First attempt to see weak neutral current : D. H. Perkins, Veltman. Failed.

Discovery of neutral currents: In an experiment at CERN with a Bubble Chamber. (Photograph Courtesy CERN). Experiments with ν_{μ} beams obtained from π decays.

Current eigenstates that couple to the W are two doublets:

$$
\begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix} = \begin{pmatrix} u \\ D \end{pmatrix}, \qquad \begin{pmatrix} c \\ -d\sin\theta_c + s\cos\theta_c \end{pmatrix} = \begin{pmatrix} c \\ S \end{pmatrix}
$$

Quark mixing in two generations can then be represented by

$$
\begin{pmatrix}\nD \\
S\n\end{pmatrix} \equiv\n\begin{pmatrix}\nD_1 \\
D_2\n\end{pmatrix} =\n\begin{pmatrix}\n\cos \theta_c & \sin \theta_c \\
-\sin \theta_c & \cos \theta_c\n\end{pmatrix}\n\begin{pmatrix}\nd \\
s\n\end{pmatrix}
$$

$$
J_{\mu}^{CC} = \bar{D}\gamma_{\mu}u_L + \bar{S}\gamma_{\mu}c_L = \bar{D}_1\gamma_{\mu}u_L + \bar{D}_2\gamma_{\mu}c_L
$$

Charm is postulated. This does the trick of making Flavor Changing Neutral Current vanish at least by making sure that a vertex $d\bar{s}Z$ does not exist. Can more complicated diagrams produce FCNC?

W eak CC

W eak NC

 K^+ , $\bar{s}u$ bound state. $K^+ \rightarrow \mu^+ \nu_\mu$: weak charged current decay, $\Delta S = 1$

K⁰ is a $\bar{s}d$ bound state. If a weak neutral current with $\Delta S \neq 0$ (Flavor Changing Neutral Current: FCNC) were to exist with the same strength as the weak charged current it would cause problems.

Why? $K^0_L \to \mu^+ \mu^-$ happens very rarely (one part in 10⁸ among all K_L decays)

Once we have two quark doublets, tree level FCNC vanishes automatically.

 $\sum_{i=1,2} \bar{D}_i D_i = \bar{d}d + \bar{s}s.$ \odot

Is Rabi's question answered? We somehow show that we need a new quark. Can we say FCNC ordered a new quark?

Sorry, no, still no answer to Rabi's question 'who ordered the μ

But what this tells is that the left handed fermions have to appear as doublets. Hence we need the ^c.

Something we will not discuss: Quantum properties of the $SU(2)_L \times$ $U(1)$ gauge theory imply that the number of generations should be equal for quarks and leptons.

What happens with loops?

If charm contribution is absent the prediction for this flavour changing decay will be much too big compared to data.

Absence of Flavour Changing Neutral Currents is granted in the EW theory ONLY IF CHARM exists. Will be exactly zero if $m_c = m_u$. July 4 - July 10, 2012. CERN Summer Student Program.

For any physics beyond SM this is always a constraint that HAS to be satisfied.

This cancellation is an example of the Glashow-Iliopoulos-Maiani (GIM) cancellation mechanism.

Neutral meson oscillations:

Weak interactions cause mixing between K_0 and \bar{K}_0 Two states with $S = \pm 1$. Strong interaction eigenstates, the weak interactions are a perturbation, cause these to mix.

 $\Delta S = 2$ effect, higher order in Weak Interactions.

Mass difference measured experimentally through oscillations:

 $\Delta M_K = M_{K_L} - M_{K_S} = 7 \times 10^{-15} \times M_K \sim 3.5 \times 10^{-12} MeV$

Can be computed in EW theory.

Gaillard and Lee (Phys. Rev 10, 1974, 897) used the experimentally measured value of this mass difference to calculate m_c before Charmonium $(c\bar{c})$ bound state was found in 1974.

Recall CP violation in the neutral meson sector.

Can be calculated similarly. BUT is non zero ONLY with three generations.

Why?

CP violation technically means we need the Hamiltonian to have complex couplings and this means rotation matrix describing quark mixing should have at least one nonzero phase.

Cabbibo mixing is a two by two matrix. It can not accomodate a free phase. Need at least three generations.

Kobayashi, Masakawa extended Cabbibo's idea to a three generation mixing.

$$
\begin{pmatrix}\nD \\
S \\
B\n\end{pmatrix} \equiv \begin{pmatrix}\nD_1 \\
D_2 \\
D_3\n\end{pmatrix} = \begin{pmatrix}\nV_{11} & V_{12} & V_{13} \\
V_{21} & V_{22} & V_{23} \\
V_{31} & V_{32} & V_{33}\n\end{pmatrix} \begin{pmatrix}\nd \\
s \\
c\n\end{pmatrix}
$$

The measurements of meson mixing and CP violation in this system, gives constraints on elements of mixing among quark states (CKM)

CKM matrix is the 3×3 version of the mixing matrix we wrote before. This matrix should be unitary. Testing the unitarity is an achievement of the last decade.

Current status of measurement of the CKM matrix.

Constituents we covered here are W, Z and $c. \tau$ was discovered accidentally, then was b. But then t and ν_{τ} were hunted for!