

# *Fundamental Concepts in high energy / Particle Physics*

Lecture I :  
Introduction

Géraldine SERVANT  
CERN-Th



# Plan

Lecture 1: intro to QFT: Relativity, kinematics, symmetries

Lecture 2: Towards Gauge theories

Lecture 3: Towards the Standard Model

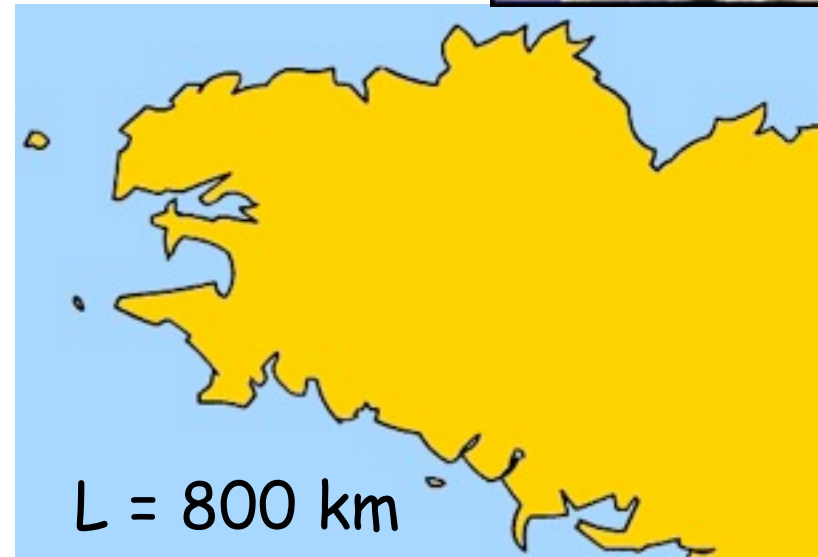
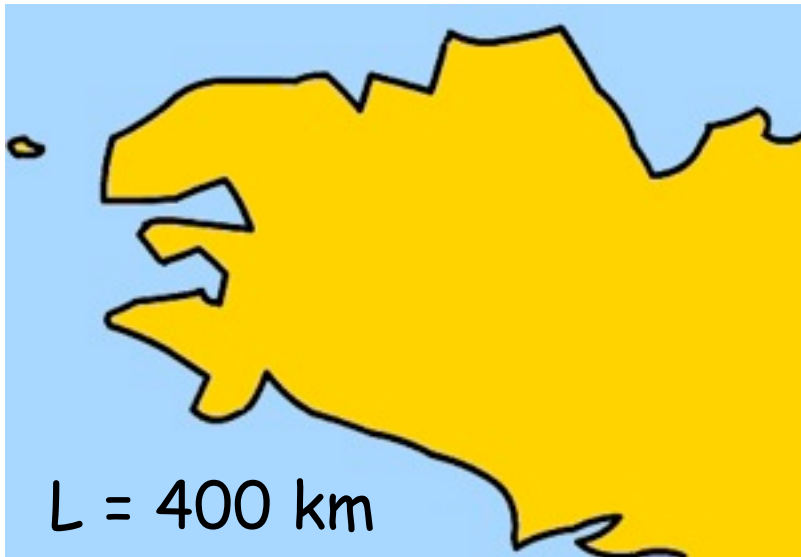
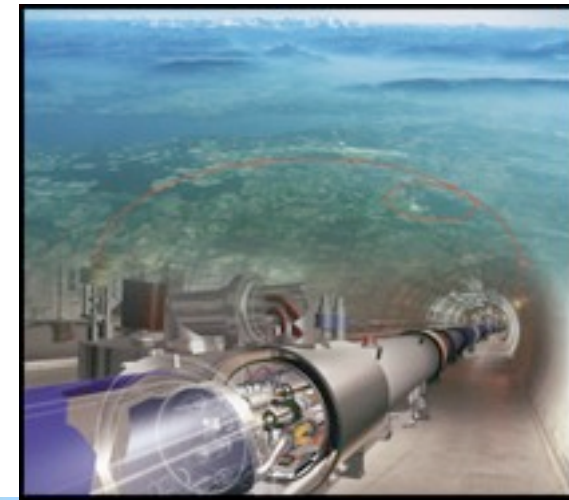
Lecture 4: Towards beyond the Standard Model

# why high energies

## The Large Hadron Collider

→ The LHC: the most gigantic microscope ever built

Going to higher energies  $\Rightarrow$  allows to study finer details



Particle Physics:  
study of short distances

*resolution limited by the de Broglie  
wavelength  $\lambda = h/p$*



High energy physics

*high resolution  
necessitates large  $p$*

$p \gg m$ :  
relativistic regime

# What is a particle?

*A small, quantum and fast-moving object*

## Quantum Mechanics

➔ duality  
wave-particle

➔ Heisenberg  
inequalities

energy non-conservation  
on time intervals  $\Delta t$  energy fluctuations  $\Delta E$

$$\Delta t \times \Delta E \sim \hbar$$

## Special Relativity

➔ space-time

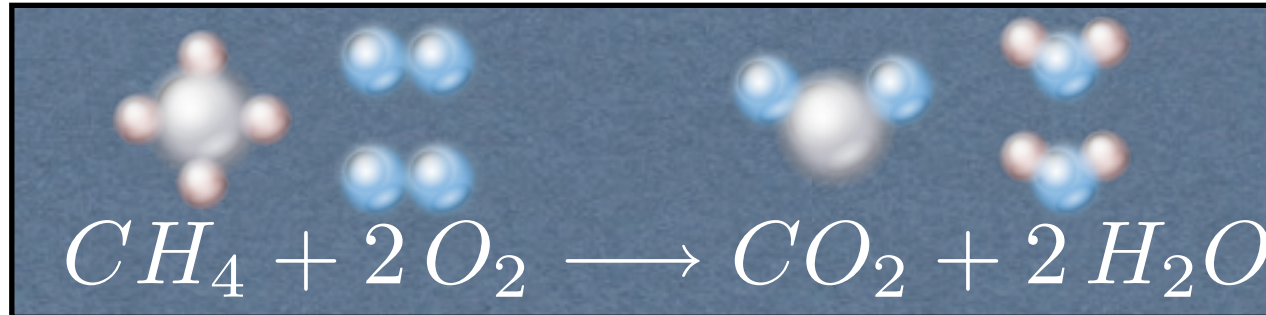
➔ energy = mass

➔ particle number  
non constant

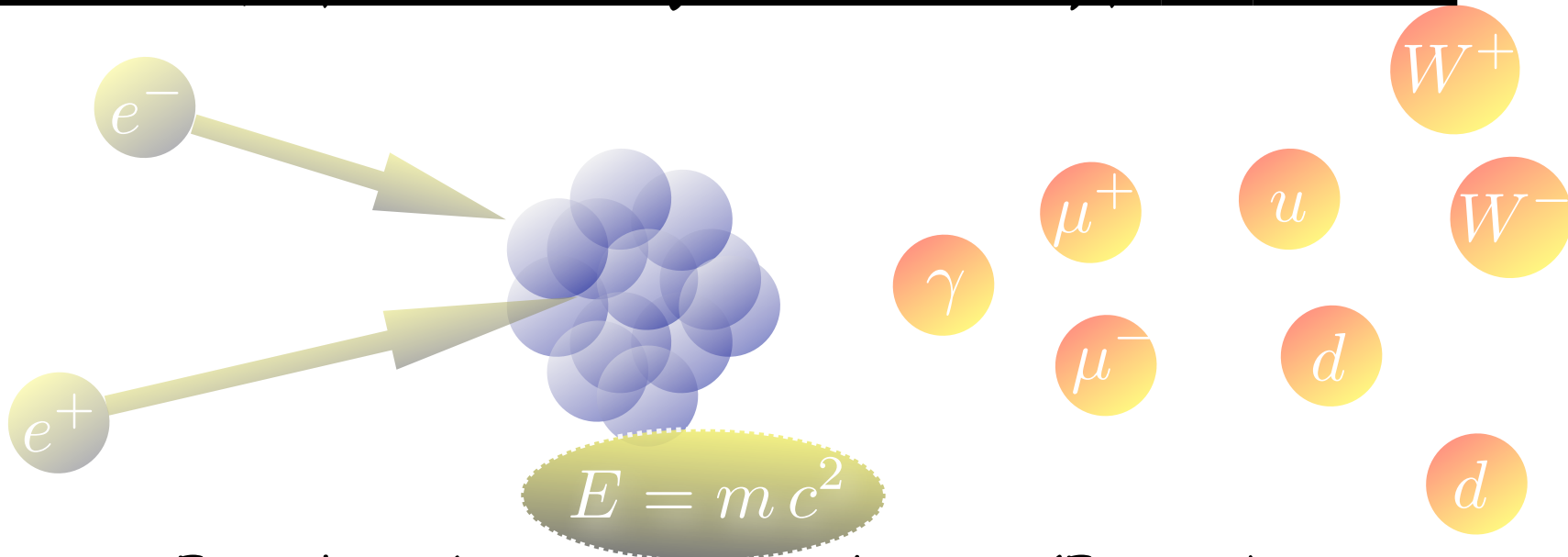
# Creation of matter from energy

## Chemistry : rearrangement of matter

the different constituents of matter reorganize themselves



## Particle physics : transformation energy $\leftrightarrow$ matter



Equivalence between mass and energy (Einstein's idea) plays a very fundamental role in particle physics

## Natural units in high energy physics

The fundamental units have dimension of length (L), mass (M) and time (T).  
All other units are derived from these.

The two universal constants in SI units

$$\hbar = 1.055 \times 10^{-34} \text{ J s} = 1.055 \times 10^{-34} \text{ kg m}^2/\text{s} \quad \text{and} \quad c = 3 \times 10^8 \text{ m/s}$$

In particle physics we work with units  $\hbar = c = 1$

Thus, velocity of particle is measured in units of the speed of light, very natural in particle physics where  $0 \leq v < 1$  for massive particles and  $v = 1$  for massless particles

In the  $c=1$  unit:

[velocity]=pure number

[energy]=[momentum]=[mass]

notation: dimension of quantity P is [P]

## Natural units in high energy physics

$\hbar$  has dimension of [Energy]×[time].

$\hbar/mc \sim \text{length}$  (from uncertainty principle  $\Delta p \Delta x \geq \hbar/2$  )  
or de Broglie's formula  $\lambda = h/p$

$$\hbar = 1 \quad \text{--->} \quad [\text{length}] = [\text{mass}]^{-1}$$

Thus all physical quantities can be expressed as powers of mass or of length.

e.g. energy density,  $E/L^3 \sim M^4$

$$\alpha = \frac{e^2}{4\pi\hbar c} \quad \text{pure number}$$

We specify one more unit taken as that of the energy, the GeV.

$$\text{mass unit: } M \quad c^2/c^2 = 1 \text{ GeV}$$

$$\text{length unit: } \hbar c/M \quad c^2 = 1 \text{ GeV}^{-1} = 0.1975 \text{ fm}$$

$$\text{time unit: } \hbar c/M \quad c^3 = 1 \text{ GeV}^{-1} = 6.59 \cdot 10^{-25} \text{ s.}$$

## Natural units in high energy physics

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

$$\rightarrow \hbar c = 1.055 \times 10^{-34} \text{ J s} \times 3 \times 10^8 \text{ m/s} = 1.978 \times 10^{-7} \text{ eV m}$$

$$\text{Using } 1 \text{ fm} = 10^{-15} \text{ m and } 1 \text{ MeV} = 10^6 \text{ eV: } \quad \hbar c = 197.8 \text{ MeV fm}$$

$$\text{So in natural units: } \quad 1 \text{ fm} \approx 1 / (200 \text{ MeV})$$

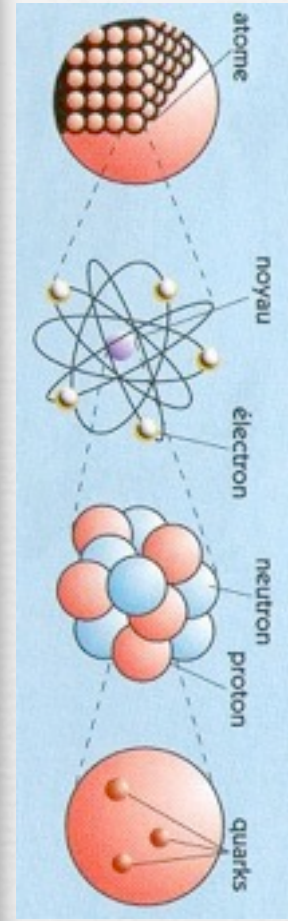
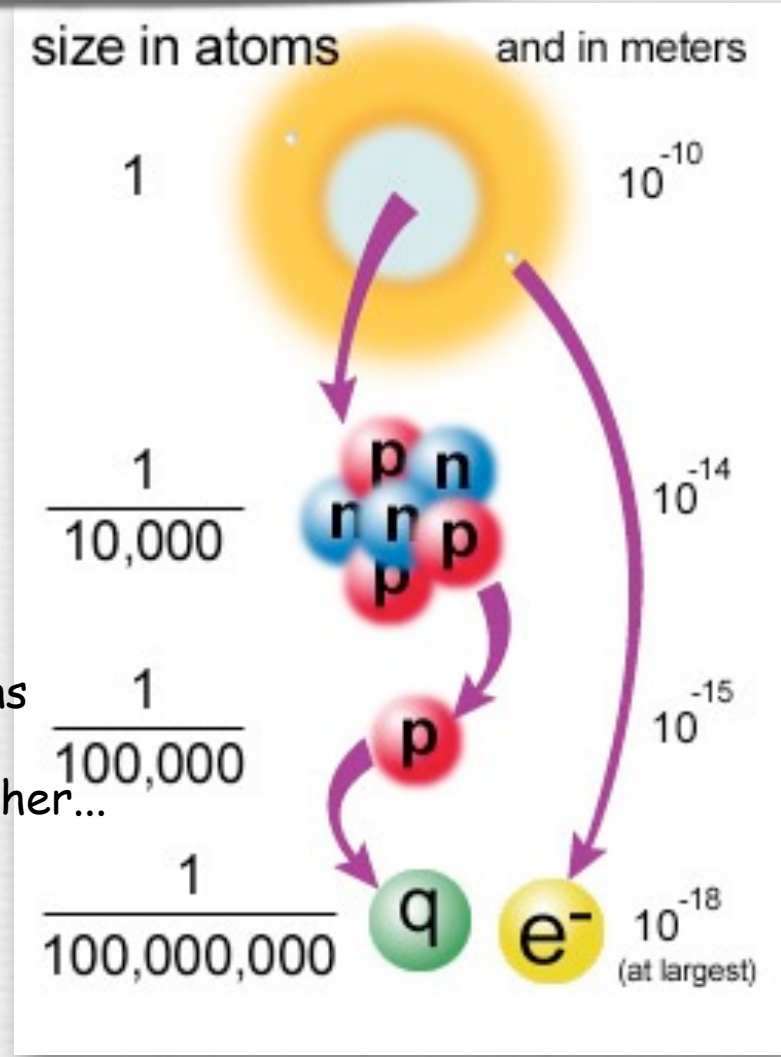
$$\text{also, } c=1 \rightarrow 1 \text{ fm} \sim 3 \times 10^{-24} \text{ s} \rightarrow \text{GeV}^{-1} \sim 6 \times 10^{-25} \text{ s}^{-1}$$

$$\hbar = 1.055 \times 10^{-34} \text{ kg m}^2/\text{s} \rightarrow \text{GeV} \sim 1.8 \times 10^{-27} \text{ kg}$$



# The elementary blocks of matter

- Matter is made of molecules
- Molecules are built out of atoms
- Atoms are made of nuclei and electrons
- Nuclei are assemblies of protons and neutrons
- Protons and neutrons are quarks bound together...



The volume of an atom corresponds to  $10^{24}$  times the volume of an electron

Classically, matter contains a lot of void

Quantum mechanically, this void is populated by pairs of virtual particles

$$1 \text{ TeV} = 10^{12} \text{ eV}$$

1 electron volt  
(eV) =

*The energy of an electron accelerated by an electric potential difference of 1 volt. One eV is thus equal to ...  $1.6 \cdot 10^{-19} \text{ J}$*

1 kg of sugar = 4000 kCalories = 17 millions of Joules  $\approx 10^{14} \text{ TeV}$

but 1 kg sugar  $\approx 10^{27}$  protons  $\rightarrow 0.1 \text{ eV} / \text{protons}$

To accelerate each proton contained in 1 kg of matter at 14 TeV, we would need the energy of of  $10^{14} \text{ kg}$  of sugar\* i.e. 1% of the world energy production

\*world annual production of sugar=150 millions of tons  $\approx 10^{11} \text{ kg}$

*How impressive is this?*

energies involved at CERN:  $1 \text{ TeV} = 1000 \text{ billions of eV} = 10^{-24} \text{ kg}$

*compared with the kinetic energy of a mosquito*  $10^{-3} \text{ J} \sim 10^{16} \text{ eV} \sim 10^4 \text{ TeV}$

*... however, in terms of energy density... this corresponds to the mass of the Earth concentrated in a  $1 \text{ mm}^3$  cube!*

# Classical versus quantum Collision

Compton wave length

$$\lambda = h/mc$$



strawberry :  $m \sim 30 \text{ g} \sim 10^{25} \text{ GeV}/c^2 \rightarrow \lambda \sim 10^{-40} \text{ m}$

classical:  $\lambda \ll R$

$e^-$  :  $m \sim 9.1 \times 10^{-31} \text{ kg} \sim 0.5 \text{ MeV}/c^2 \rightarrow \lambda \sim 4 \times 10^{-13} \text{ m}$

$p$  :  $m \sim 1.6 \times 10^{-27} \text{ kg} \sim 1 \text{ GeV}/c^2 \rightarrow \lambda \sim 10^{-16} \text{ m}$

quantum :  $\lambda \gg R$

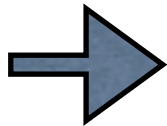
## Why relativity

Particle physics is all about creating and annihilating particles. This can only occur if we can convert mass to energy and vice-versa, which requires relativistic kinematics

A bit of history

Contemplating the unusual invariance of Maxwell's equations under Lorentz transformation, Einstein stated that Lorentz invariance must be the invariance of our space and time.

-> completely changed our view of space and time, so intertwined that it is now called spacetime, leading to exotic phenomena such as

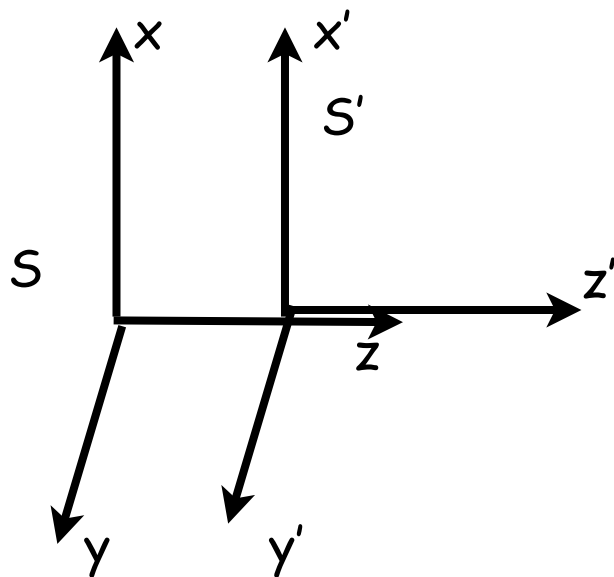


- time dilation
- length contraction
- prediction of antimatter when special relativity is married with quantum mechanics

# Relativistic transformations

## The two postulates of Special relativity

- Speed of light is the same in all reference frames
- Laws of physics are unchanged under a galilean transformation, i.e. in all reference frames moving at constant velocity with respect to each other



Look for coordinate transformations that satisfy these requirements

Unique choice:  
Lorentz transformations

$$\begin{pmatrix} ct' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} ct \\ z \end{pmatrix}$$

$$\beta = \frac{v}{c}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

# Implications of Lorentz transformation

$$\begin{pmatrix} ct' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} ct \\ z \end{pmatrix} \quad \beta = \frac{v}{c} \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

## Time dilation

consider time interval  $\tau = t'_2 - t'_1$  in  $S'$ , the rest frame of a particle located at  $z'_1 = z'_2 = 0$ .

then in frame  $S$  where the particle is moving:  $t_2 - t_1 = \gamma\tau$

-->The observed lifetime of a particle is  $\gamma \times \tau$

so it can travel over a distance  $\beta c \gamma \tau$

-->muons which have a lifetime  $\tau \sim 2 \times 10^{-6}$  s produced by reaction of cosmic rays with atmosphere at 15-20 km altitude can reach the surface

## length contraction

an object at rest in  $S'$  has length  $L_0 = z'_2 - z'_1$

It measures in  $S$   $z_2 - z_1 = L_0/\gamma$

--> densities increase  $\rho_0 = \Delta n / (\Delta x' \Delta y' \Delta z')$   $\rho = \Delta n / (\Delta x \Delta y \Delta z) = \gamma \rho_0$

$\Delta x \Delta y \Delta z \Delta t$  is invariant

# 4-vectors

Time and space get mixed-up under Lorentz transformations. They are considered as different components of a single object, a four-component spacetime vector:

$$x^\mu = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix} = \begin{pmatrix} x^0 \\ \vec{x} \end{pmatrix}$$

By construction:  $x_\mu x^\mu = \mathbf{x}^2 = x^{0^2} - \vec{x}^2$  is invariant  
 $d\tau = \sqrt{dt^2 - d\vec{x}^2}$  is invariant

Lorentz invariant action built with the proper time  $d\tau$

$$S = -m \int d\tau = \int \mathcal{L} dt$$

$$\left. \begin{aligned} \mathcal{L} &= -m\sqrt{1 - \dot{x}^2} \\ \vec{p} &= \frac{\partial \mathcal{L}}{\partial \dot{x}} = m\gamma \vec{\beta} \\ E &= \vec{p} \cdot \dot{x} - \mathcal{L} = m\gamma \end{aligned} \right\} \vec{p} = E \vec{\beta}$$

## Energy-momentum four-vector

so we find:  $m^2 = E^2 - \vec{p}^2$

This suggests to define the four-vector  $p^\mu = \left( \frac{E}{c}, p_x, p_y, p_z \right)$

$$m = 0 \rightarrow \beta = 1 \rightarrow \gamma = \infty \rightarrow \tau = \infty$$

a massless particle cannot decay



# Conservation of energy-momentum

Consider collision between A and B

Define center of mass (CM) frame as where  $\vec{p}_A + \vec{p}_B = 0$

Energy available in center of mass frame  
is an invariant:  $\sqrt{s} = E_* = E_A + E_B$

$$\mathbf{P}_{tot}^2 = E_*^2$$

## 1) Collision on fixed target

B is at rest in lab frame,  $E_B = m_B$  and  $E_A$  is energy of incident particle

$$E_*^2 = m_A^2 + m_B^2 + 2m_B E_A$$

## 2) Colliding beams A and B travel in opposite directions

$$E_*^2 = m_A^2 + m_B^2 + 2(E_A E_B + |p_A||p_B|) \approx 4E_A E_B$$

if  $m_A, m_B \ll E_A, E_B$

So for fixed target machine  $E_* \sim \sqrt{2m_B E_A}$

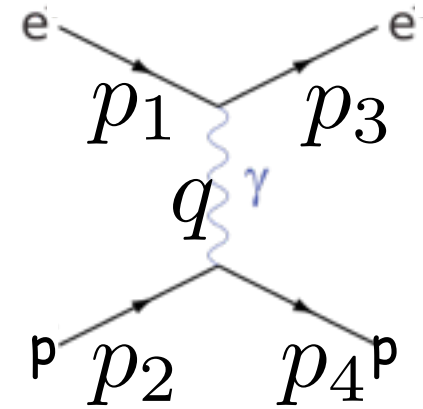
while for colliding beam accelerators  $E_* \sim 2E$

To obtain 2 TeV in the CM with a fixed proton target accelerator  
the energy of a proton beam would need to be 2000 TeV!

# Conservation of energy-momentum

Consider the interaction  $e + p \rightarrow e + p$  due to exchange of electromagnetic field

a photon is massless, however, for a short amount of time, an "exchanged" photon  $\gamma^*$  (virtual photon) can have a mass (Heisenberg inequalities)



$$p_1 + p_2 = p_3 + p_4$$

$$q = p_1 - p_3 = p_4 - p_2$$

$q$  is the transfer energy-momentum four-vector

$q^2 = m_{\gamma^*}^2$  : invariant, can be computed in any frame

in frame where proton is at rest:  $p_2 = \begin{pmatrix} m \\ \vec{0} \end{pmatrix}$   $p_4 = \begin{pmatrix} E_4 \\ \vec{p}_4 \end{pmatrix}$   $E_4 = m + T$   
kinetic energy  
 $T \approx p^2 / 2m$

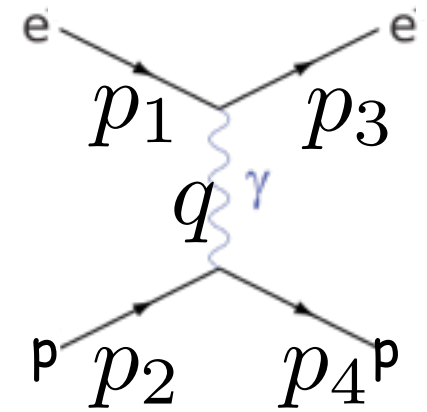
$$q^2 = (p_4 - p_2)^2 = p_4^2 + p_2^2 - 2p_4 \cdot p_2$$

$$q^2 = m^2 + m^2 - 2E_4 m = \underbrace{-2mT}_{< 0} \text{ virtual photon}$$

## Range of an interaction

$$R = \frac{\hbar c}{|m_*|}$$

reminder:  $\hbar c \sim 200 \text{ MeV fm}$



To probe the proton, we need

$$R \ll R_{proton} \sim 1 \text{ fm}^{-1}$$

$$|m_*| \gg 200 \text{ MeV}$$

## Next step : marry quantum mechanics and relativity

non relativistic

(Schrodinger's equation cannot account for creation/annihilation of particles)

Schrödinger Equation (1926):

$$\left( i\hbar \frac{\partial}{\partial t} + \frac{\hbar^2}{2m} \Delta - V \right) \Phi = 0$$

$$E = \frac{p^2}{2m} + V$$

classical  $\leftrightarrow$  quantum  
correspondance

$$E \rightarrow i\hbar \frac{\partial}{\partial t} \quad \& \quad p \rightarrow i\hbar \frac{\partial}{\partial x}$$



relativistic

Klein-Gordon Equation (1927):

$$\left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \Delta + \frac{m^2 c^2}{\hbar^2} \right) \Phi = 0$$

$$\frac{E^2}{c^2} = p^2 + m^2 c^2$$

negative energies  $E = \pm (p^2 + m^2)^{1/2}$

and

does not admit a positive probability density  
and does not describe fermions

# Antimatter and Dirac equation

Dirac Equation (1928):

$$\left( i\gamma^\mu \partial_\mu - \frac{mc}{\hbar} \right) \Psi = 0$$

$$E = \begin{cases} +\sqrt{p^2 c^2 + m^2 c^4} & \text{matter} \\ -\sqrt{p^2 c^2 + m^2 c^4} & \text{antimatter} \end{cases}$$

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$$

plane wave solution  $\Psi(x, t) = u(p) e^{i(p \cdot x - Et) / \hbar}$

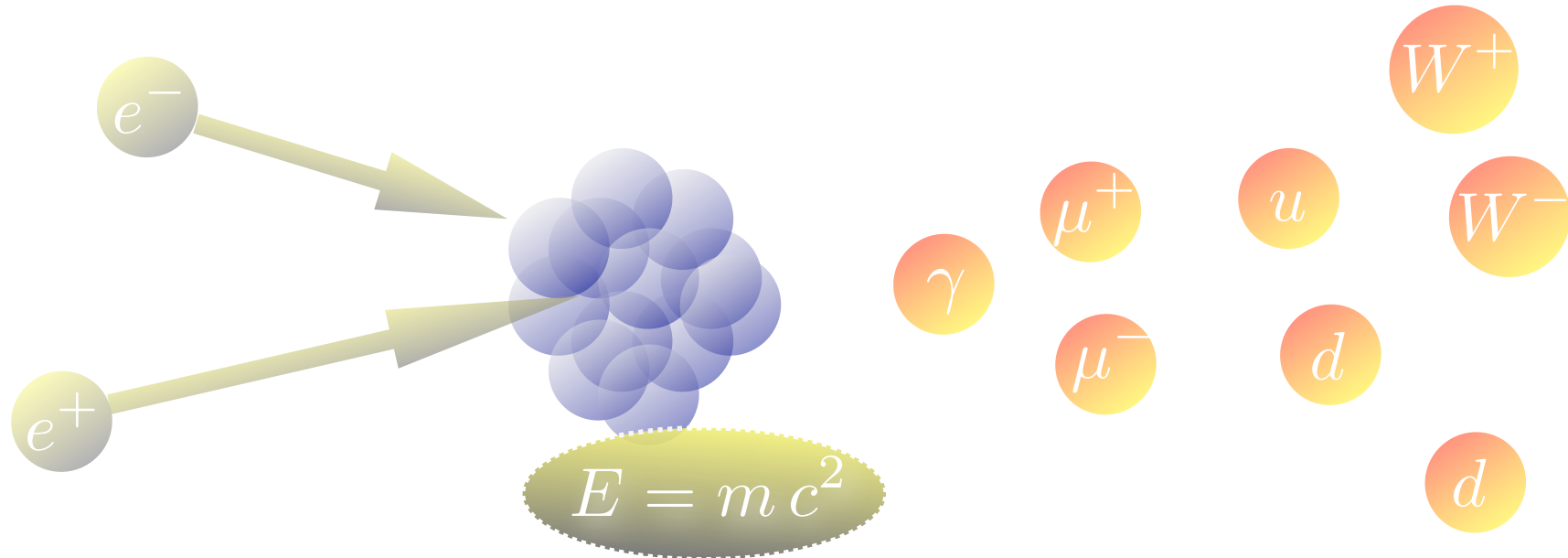
a particle of energy  $-E$  travelling backward in time  $\rightarrow$  antiparticle

positron ( $e^+$ ) discovered by C. Anderson in 1932

conservation of fermion number: +1 for particles and -1 for antiparticles. fermions can only be created or destroyed in pairs

## The necessity to introduce fields for a multiparticle description

Relativistic processes cannot be explained in terms of a single particle. Even if there is not enough energy for creating several particles, they can still exist for a short amount of time because of uncertainty principle



We need a theory that can account for processes in which the number and type of particles changes like in most nuclear and particle reactions

quantization of a single relativistic particles does not work, we need  
quantization of fields -> Quantum Field Theory (QFT)

# Quantum Field Theory

We want to describe  $A \rightarrow C_1 + C_2$  or  $A+B \rightarrow C_1 + C_2 + \dots$

1) Associate a field to a particle

2) Write action 
$$S = \int d^4x \mathcal{L}(\phi_i, \partial_\mu \phi_i)$$

3)  $\mathcal{L}$  invariant under Poincaré (Lorentz+translations) transformations and internal symmetries

The symmetries of the lagrangian specify the interactions

4) Quantization of the fields

# Symmetries and conservation laws: the backbone of particle physics

Noether's theorem (from classical field theory) :

A continuous symmetry of the system  $\leftrightarrow$  a conserved quantity

## I- Continuous global space-time symmetries:

translation invariance in space  $\leftrightarrow$  momentum conservation

translation invariance in time  $\leftrightarrow$  energy conservation

rotational invariance  $\leftrightarrow$  angular momentum conservation

Fields are classified according to their transformation properties under Lorentz group:

$$x^\mu \rightarrow x'^\mu = \Lambda^\mu_\nu x^\nu \quad \phi(x) \rightarrow \phi'(x')$$

$$\phi'(x) = \phi(x) \quad \text{scalar}$$

$$V^\mu \rightarrow \Lambda^\mu_\nu V^\nu \quad \text{vector}$$

$$\psi(x) \rightarrow \exp\left(-\frac{i}{2}\omega_{\mu\nu} J^{\mu\nu}\right)\psi(x) \quad \text{spinor}$$

The true meaning of spin arises in the context of a fully Lorentz-invariant theory (while it is introduced adhoc in non-relativistic quantum mechanics)



A field transforms under the Lorentz transformations in a particular way.

Picking a particular representation of the Lorentz transformation specifies the spin.

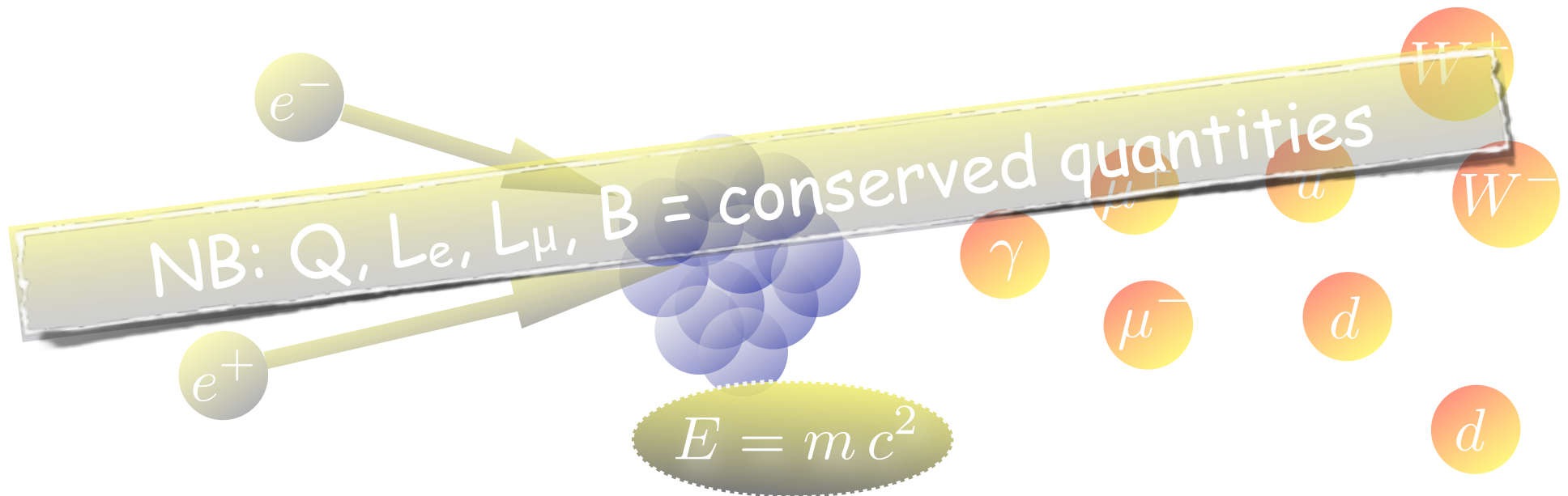
After quantizing the field, you find that the field operator can create or annihilate a particle of definite spin

The spin is part of the field

## II- Global (continuous) internal symmetries:

acting only on fields

conservation of baryon number and lepton number



## Quantum numbers and Conservation laws

When the positron was discovered, it raised a naive question:  
why can't a proton decay into a positron and a photon  $p \rightarrow e^+ \gamma$  ?

This process would conserve momentum, energy, angular momentum, electric charge and even parity

This can be understood if we impose conservation of baryon number

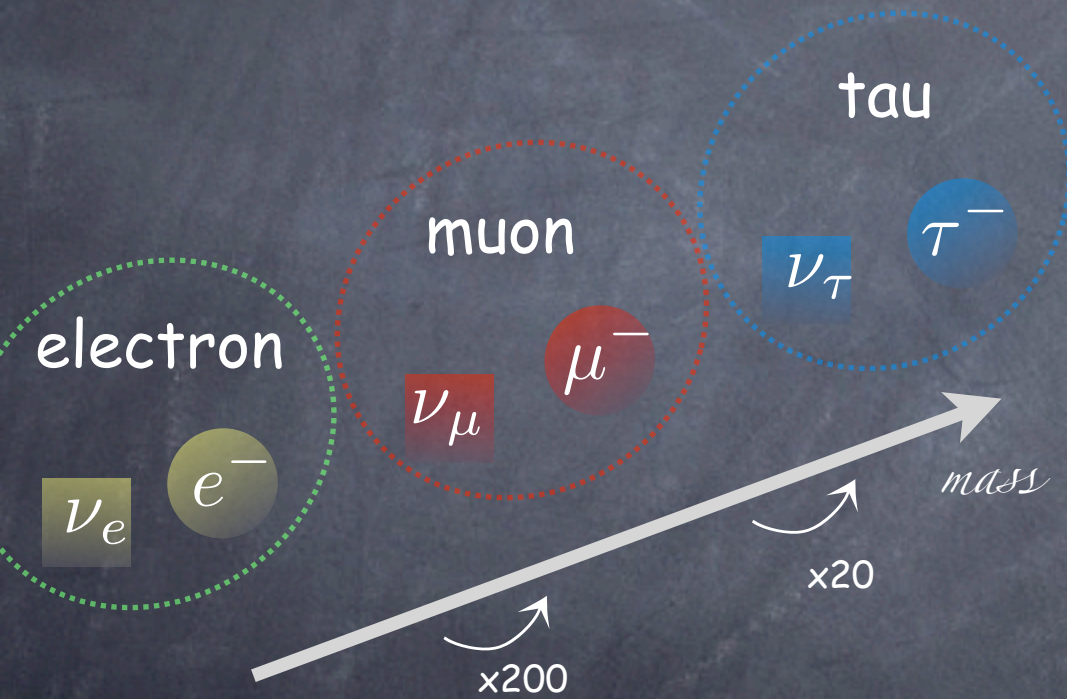
Similarly, when the muon was discovered, it raised the question:  
why doesn't a muon decay as  $\mu^- \rightarrow e^- \gamma$  ?

This led to propose another quantum number: lepton family number

# The Standard Model: matter

the elementary blocks:

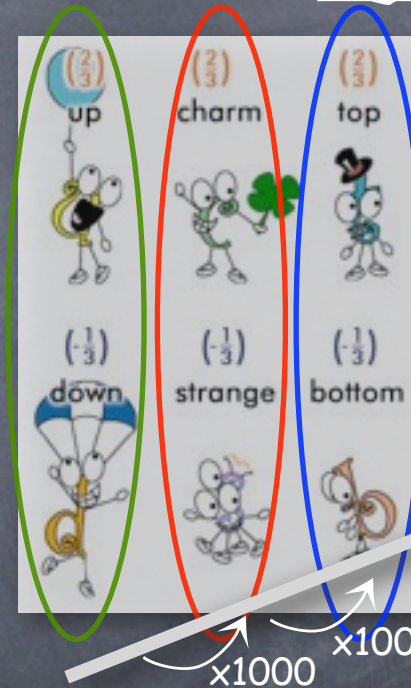
## LEPTONS



no composite states  
made of leptons

+ antiparticles

## QUARKS



each of the 6  
quarks  
exists in three  
colors

composite states (white objects)

0 baryons

proton  $p = (u, u, d)$

neutron  $n = (u, d, d)$

0 mesons

The following processes have not been seen.  
Explain which conservation law forbids each of them

$$n \rightarrow p\mu^- \bar{\nu}_\mu$$

$$\mu^- \rightarrow e^- e^- e^+$$

$$n \rightarrow p\nu_e \bar{\nu}_e$$

$$p \rightarrow e^+ \pi^0$$

$$\tau^- \rightarrow \mu\gamma$$

$$K^0 \rightarrow \mu^+ e^-$$

$$\mu^- \rightarrow \pi^- \nu_\mu$$

The following processes have not been seen.  
Explain which conservation law forbids each of them

$$n \rightarrow p\mu^- \bar{\nu}_\mu$$

energy

$$\mu^- \rightarrow e^- e^- e^+$$

muon number or electron number

$$n \rightarrow p\nu_e \bar{\nu}_e$$

electric charge

$$p \rightarrow e^+ \pi^0$$

baryon number or electron number

$$\tau^- \rightarrow \mu\gamma$$

tau number or muon number

$$K^0 \rightarrow \mu^+ e^-$$

muon number or electron number

$$\mu^- \rightarrow \pi^- \nu_\mu$$

energy