

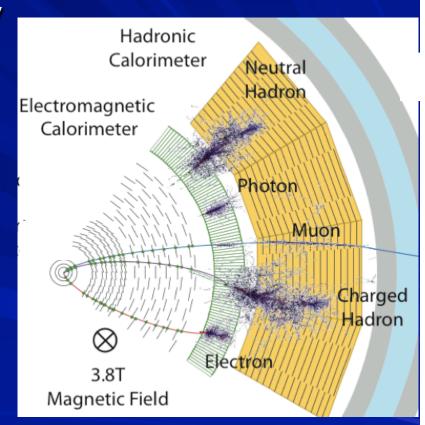
Detectors for Particle Physics

Calorimetry

D. Bortoletto Purdue University

What is a calorimeter?

- In nuclear and particle physics calorimetry refers to the detection of particles through total absorption in a block of matter
 - The measurement process is destructive for almost all particle
 - The exception are muons (and neutrinos) → identify muons easily since they penetrate a substantial amount of matter
- In the absorption, almost all particle's energy is eventually converted to heat → calorimeter
- Calorimeters are essential to measure neutral particles



Electromagnetic shower

- Dominant processes at high energies (E > few MeV):
- Photons: Pair production

$$\sigma_{pair} \approx \frac{7}{9} \left(4\alpha r_e^2 Z^2 \ln \frac{183}{Z^{1/3}} \right) = \frac{7}{9} \frac{A}{N_A X_0} \qquad \frac{dE}{dx} = 4\alpha N_A \frac{Z^2}{A} r_e^2 E \ln \frac{183}{Z^{1/3}} = \frac{E}{X_0}$$

$$I(x) = I_0 e^{-\mu x}$$
 $\mu = \frac{7}{9} \frac{\rho}{X_0}$

μ= attenuation coefficient X_0 = radiation length in [cm] or [g/cm²]

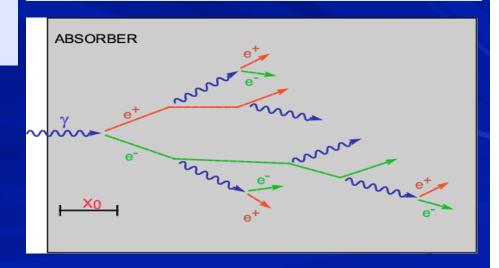
$$X_0 = \frac{A}{4\pi N_A Z^2 r_e^2 \ln \frac{183}{Z^{1/3}}}$$

Electrons: Bremsstrahlung

$$\frac{dE}{dx} = 4\alpha N_A \frac{Z^2}{A} r_e^2 E \ln \frac{183}{Z^{1/3}} = \frac{E}{X_0}$$

$$E = E_0 e^{-x/X_0}$$

After traversing $x=X_0$ the electron has only 1/e=37% of its initial energy

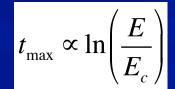


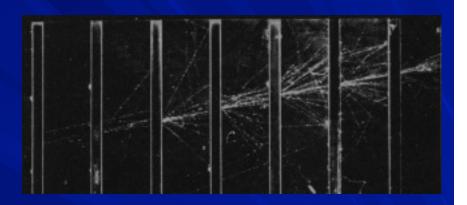
Analytic shower Model

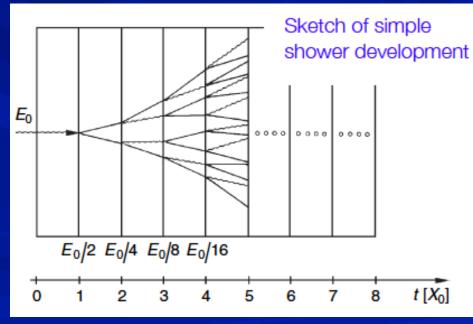
- Simplified model [Heitler]: shower development governed by X₀
 - e^{-1} loses [1 1/e] = 63% of energy in 1 X_0 (Brems.)
 - the mean free path of a γ is 9/7 X_0 (pair prod.)



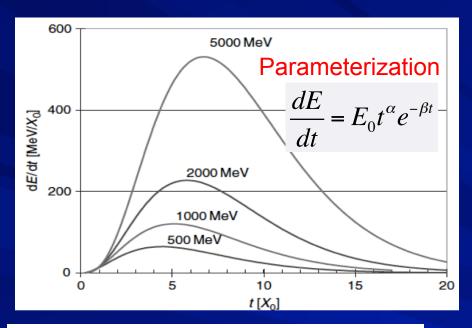
- E > E_c: no energy loss by ionization/excitation
- Simple shower model:
 - 2t particles after t [X₀]
 - each with energy E/2t
 - Stops if E < E_c
 - Number of particles N = E/E_c
 - Maximum at

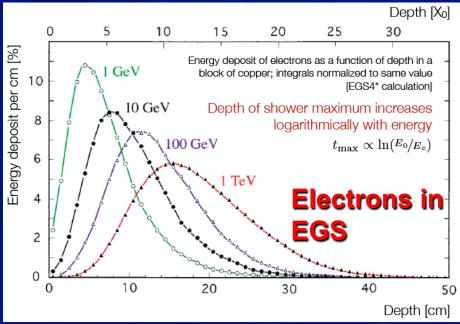


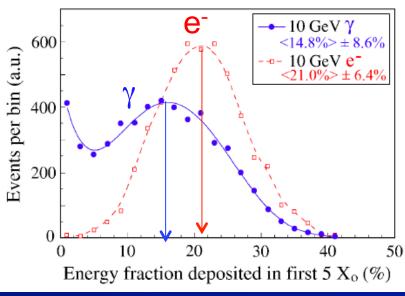




Longitudinal shower distribution







- Differences between electrons and photons generated showers
- Some photons penetrating (almost) the entire slab without interacting (peak at 0)

$$t_{\rm max} = \ln\!\left(\frac{E_0}{E_c}\right) + C_{e\gamma} \quad \begin{array}{l} {\rm C_{e\gamma}} = -0.5 \text{ for photons} \\ {\rm C_{e\gamma}} = -1 \text{ for electrons} \end{array}$$

Longitudinal containment

- Longitudinal shower distribution increases only logarithmically with the primary energy of the incident particle, i.e. calorimeters can be compact
- $L(95\%) = t_{max} + 0.08 Z + 9.6 [X_0]$

Number of particle in shower =
$$N_{\text{max}} = 2^{t_{\text{max}}} = \frac{E_0}{E_c}$$

Location of shower max =
$$t_{\text{max}} \approx \ln \left(\frac{E_0}{E_c} \right)$$

Longitudinal shower distribution =
$$L \approx \ln \left(\frac{E_0}{E_c} \right)$$

Transverse shower distribution

Example

$$E_C \approx 10 MeV$$
 $E_0 = 1 GeV$ $\Rightarrow t_{\text{max}} = \ln 100 \approx 4.6$ $N_{\text{max}} = 100$ $E_0 = 100 GeV$ $\Rightarrow t_{\text{max}} = \ln 10,000 \approx 9.2$ $N_{\text{max}} = 10,000$

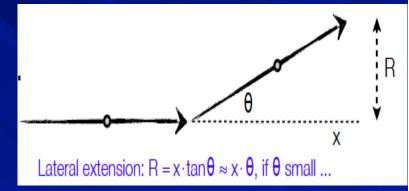
	Scint.	LAr	Fe	Pb	W
X ₀ (cm)	34	14	1.76	0.56	0.35

A 100 GeV electron is contained in 16 cm Fe or 5 cm Pb

Lateral development of EM shower

- Opening angle:
 - bremsstrahlung and pair production

$$\left\langle \theta^2 \right\rangle \approx \left(\frac{m_e c^2}{E_e} \right)^2 = \frac{1}{\gamma^2}$$



multiple coulomb scattering [Molière theory]

$$\langle \theta \rangle = \frac{E_s}{E_e} \sqrt{\frac{x}{X_0}}$$
 where $E_s = \sqrt{\frac{4\pi}{\alpha}} (m_e c^2) = 21.2 MeV$

- Main contribution from low energy electrons as <**0**> ~ 1/E_e, i.e. for electrons with $E = E_c$
- **Molière** Radius

$$R_M = \frac{E_s}{E_c} X_0 \approx \frac{21.2 MeV}{E_c} X_0$$

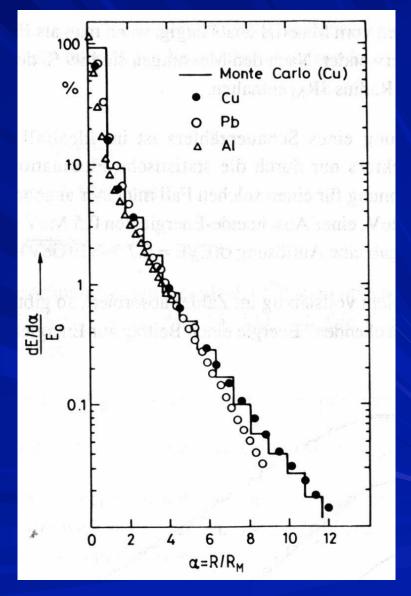
 $R_M = \frac{E_s}{E_c} X_0 \approx \frac{21.2 MeV}{E_c} X_0$ Assuming the approximate range of electrons to be X_0 yields $<\theta>\approx 21$ Me electrons to be X₀ yields <**6**>≈ 21 MeV/ E_e → lateral extension: R =<**θ**>X₀

Lateral development of EM shower

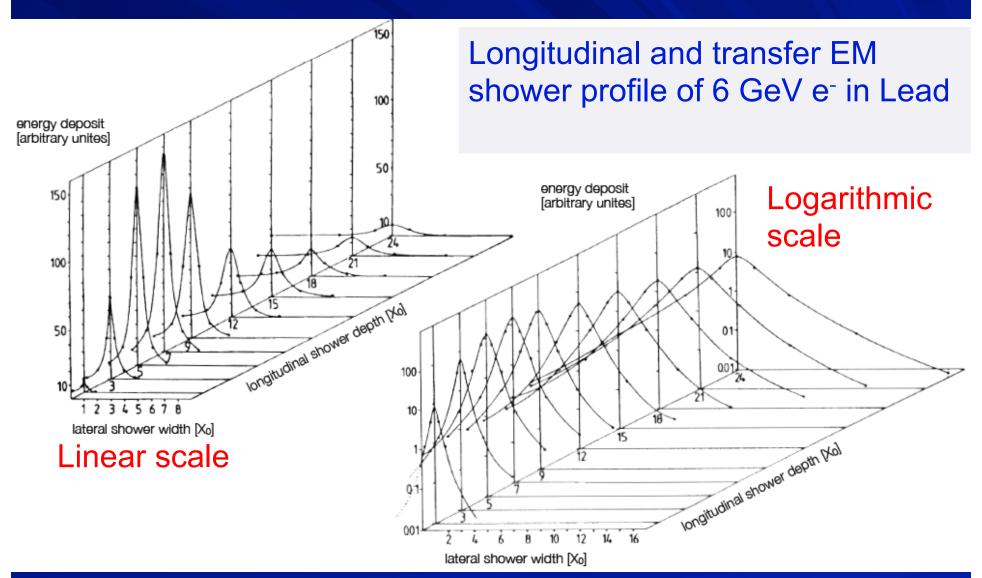
- Inner part is due to Coulomb's scattering of electron and positron
- Outer part is due to low energy photons produces in Compton's scattering, photo-electric effect etc.
 - Predominant part after shower max especially in high Z absorbers

$$\frac{dE}{dr} = \alpha e^{-r/RM} + \beta e^{-r/\lambda_{\min}}$$

- The shower gets wider at larger depth
- An infinite cylinder of radius 1 R_M contains 90% of the shower

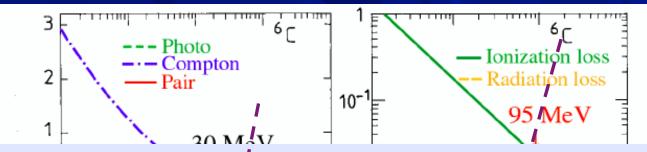


3D EM Shower development

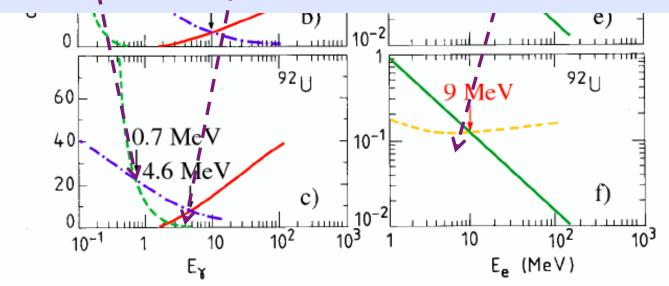


Material dependence

Z



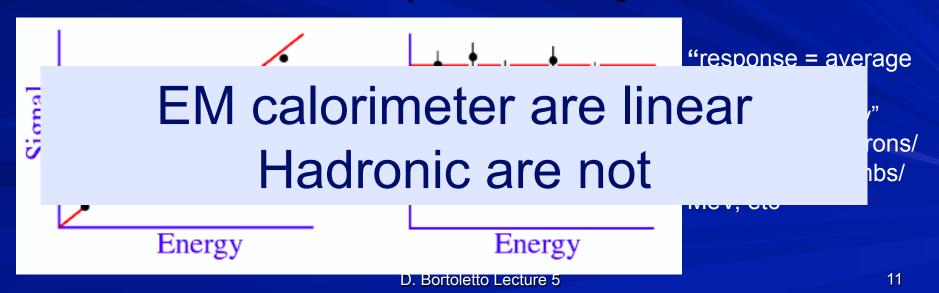
Even though calorimeters are intended to measure GeV, TeV energy deposits, their performance is determined by what happens at the MeV - keV - eV level



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Energy Measurement

- How we determine the energy of a particle from the shower?
 - Detector response → Linearity
 - The average calorimeter signal vs. the energy of the particle
 - Homogenous and sampling calorimeters
 - Compensation (for hadronic showers)
 - Detector resolution → Fluctuations
 - Event to event variations of the signal
 - What limits the accuracy at different energies?



Sources of Non Linearity

- Instrumental effects
 - Saturation of gas detectors, scintillators, photo-detectors, Electronics
- Response varies with something that varies with energy
- Examples:
 - Deposited energy "counts" differently, depending on depth
 - And depth increases with energy
- Leakage (increases with energy)

Signal linearity for electromagnetic showers

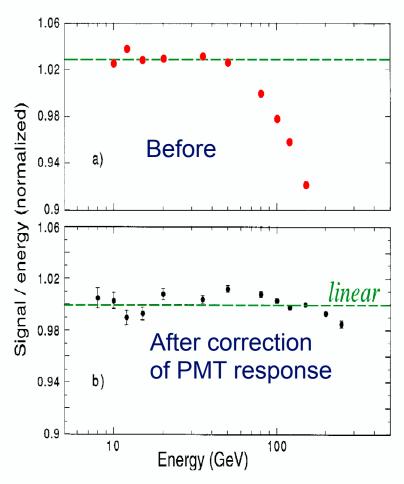


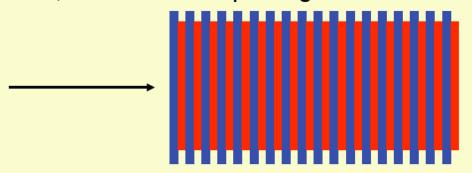
FIG. 3.1. The em calorimeter response as a function of energy, measured with the QFCAL calorimeter, before (a) and after (b) precautions were taken against PMT saturation effects. Data from [Akc 97].

Typical Calorimeter configurations

There are two general classes of calorimeter:

Sampling calorimeters:

Layers of passive absorber (such as Pb, or Cu) alternate with active detector layers such as Si, scintillator or liquid argon



Homogeneous calorimeters:

A single medium serves as both absorber and detector, eg: liquified Xe or Kr, dense crystal scintillators (BGO, PbWO₄), lead loaded glass.



Homogenous calorimeters

A block of material

Barrel: 62K 2.2x2.2x23 cm³ crystals

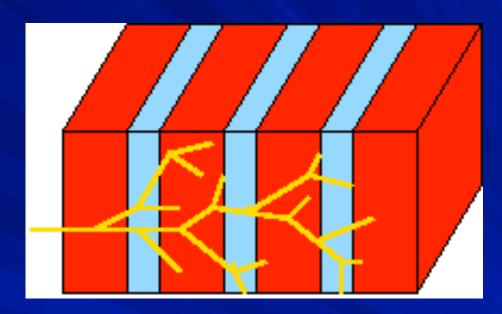
Endcap: 15K 3x3x22 cm³ crystals

Development of PbWO₄ radiation hard crystals

Sampling Calorimeters

- Use High density absorber Interleaved with active readout devices
 - sandwich structures
 - embedded fibres,
- Sampling fraction

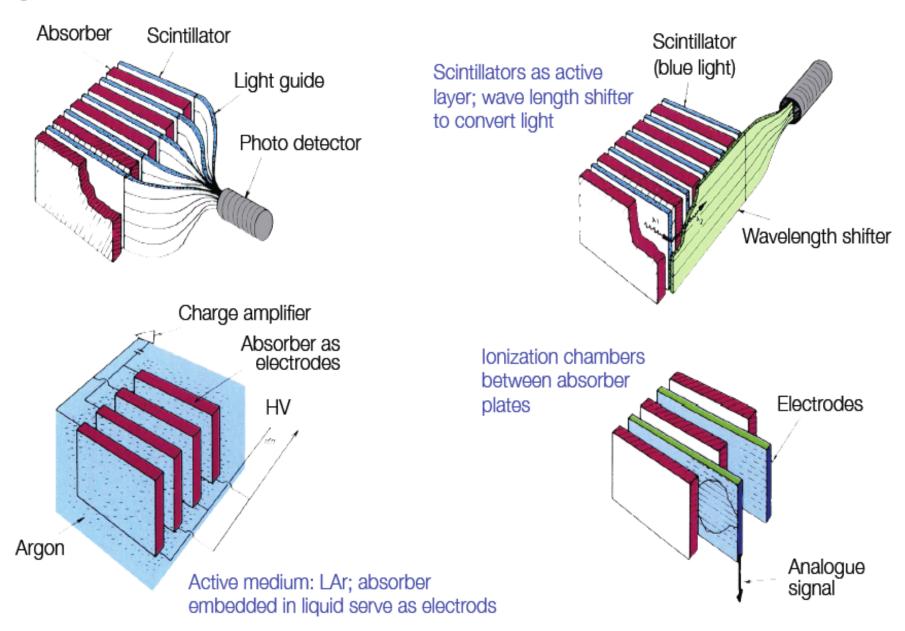
$$f_{sampling} = \frac{E_{visible}}{E_{deposited}}$$



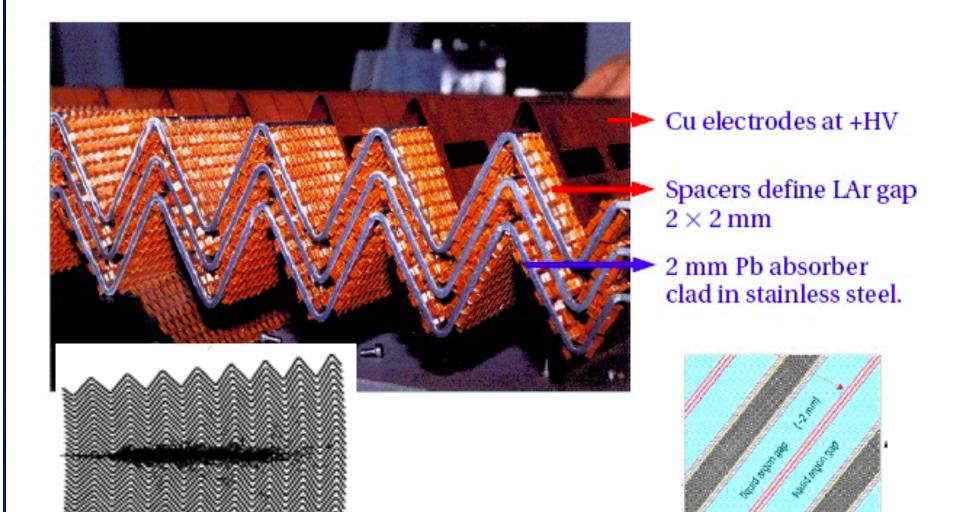
- Advantages
 - Cost, transverse and longitudinal segmentation
- Disadvantages:
 - Only part of shower seen, less precise
- Examples
 - ATLAS ECAL
 - Most HCALs

Possible setups

Scintillators as active layer; signal readout via photo multipliers



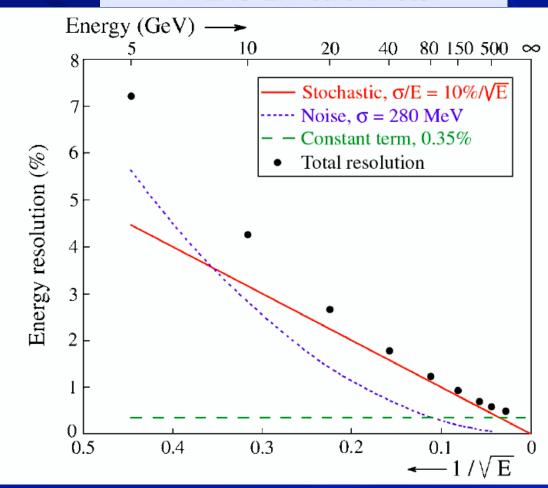
ATLAS Lar ECAL



Effects on energy resolution

- Different effects have different energy dependence
 - quantum, sampling
 fluctuations σ/E ~ E^{-1/2}
 - shower leakage σ/E ~
 E^{-1/4}
 - electronic noise $\sigma/E \sim E^{-1}$
 - structural nonuniformities:σ/E = constant
- $\sigma_{\text{tot}}^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2$

ATLAS EM calorimeter



Energy resolution

Ideally, if all shower particles counted:

$$E \propto N$$

$$\sigma_E \approx \sqrt{N} \approx \sqrt{E}$$

In practice

$$\sigma_E = a\sqrt{E} \oplus bE \oplus c$$

$$\frac{\sigma_E}{E} = \frac{a}{\sqrt{E}} \oplus b \oplus \frac{c}{E}$$

- a: stochastic term
 - intrinsic statistical shower fluctuations
 - sampling fluctuations
 - signal quantum fluctuations (e.g. photo-electron statistics)
- c: noise term
 - readout electronic noise
 - Radio-activity, pile-up fluctuations

- b: constant term
 - inhomogeneities (hardware or calibration)
 - imperfections in calorimeter construction (dimensional variations, etc.)
 - non-linearity of readout electronics
 - fluctuations in longitudinal energy containment (leakage can also be ~ E-1/4)
 - fluctuations in energy lost in dead material before or within the calorimeter

Resolution in Homogenous calorimeters

- Homogeneous calorimeters: signal = sum of all E deposited by charged particles with E>E_{threshold}
- If W is the mean energy required to produce a 'signal quantum' (eg an electron-ion pair in a noble liquid or a 'visible' photon in a crystal) the mean number of 'quanta' produced is ⟨n⟩ = E / W
- The intrinsic energy resolution is given by the fluctuations on n.

$$\frac{\sigma_E}{E} = \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{E/W}}$$

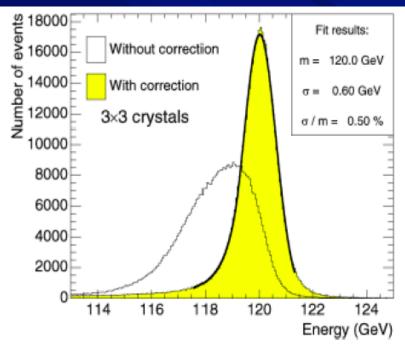
i.e. in a semiconductor crystals W \approx 3 eV (to produce e-hole pair) 1 MeV \mathbf{y} = 350000 electrons \rightarrow 1/ \sqrt{n} = 0.17% stochastic term

Fluctuations on n are reduced by correlation in the production of consecutive e-hole pairs: the Fano factor F

$$\frac{\sigma_E}{E} = \frac{1}{\sqrt{FE/W}}$$

The Fano factor depends on the material

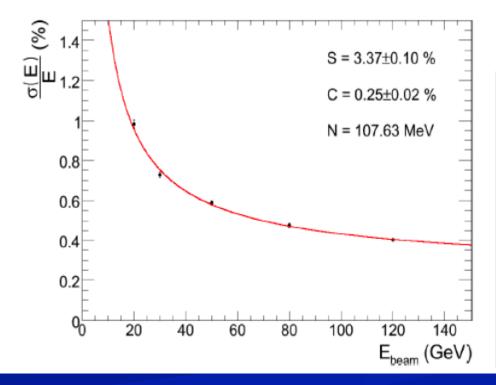
CMS ECAL resolution



Correction for radial loss

The sampling term is 3 times smaller than ATLAS; other terms are similar

$$\left(\frac{\sigma}{E}\right)^2 = \left(\frac{3.37\%}{\sqrt{E}}\right)^2 + \left(\frac{0.107}{E}\right)^2 + \left(0.25\%\right)^2$$
 stoch. noise const.



Resolution in Sampling calorimeters

- Main contribution: sampling fluctuations, from variations in the number of charged particles crossing the active layers.
- Increases linearly with incident energy and with the finess of the sampling.
- Thus:

$n_{ch} \propto E/t$ where (is the thickness of each absorber layer)

For statistically independent sampling the sampling contribution to the stochastic term is:

$$\frac{\sigma_{samp}}{E} = \frac{1}{\sqrt{n_{ch}}} \propto \sqrt{\frac{t}{E}}$$

- Thus the resolution improves as t is decreased.
- For EM calorimeters the 100 samplings required to approach the resolution of homogeneous devices is not feasible
- Typically

$$\frac{\sigma_{samp}}{E} = \frac{10\%}{\sqrt{E}}$$

Homogeneous vs Sampling

E in GeV

Technology (Experiment)	Depth	Energy resolution	Date
NaI(Tl) (Crystal Ball)	$20X_{0}$	$2.7\%/\mathrm{E}^{1/4}$	1983
$\mathrm{Bi_4Ge_3O_{12}}$ (BGO) (L3)	$22X_0$	$2\%/\sqrt{E} \oplus 0.7\%$	1993
CsI (KTeV)	$27X_0$	$2\%/\sqrt{E} \oplus 0.45\%$	1996
CsI(Tl) (BaBar)	$16-18X_0$	$2.3\%/E^{1/4} \oplus 1.4\%$	1999
CsI(Tl) (BELLE)	$16X_0$	1.7% for $E_{\gamma} > 3.5~{\rm GeV}$	1998
$PbWO_4$ (PWO) (CMS)	$25X_0$	$3\%/\sqrt{E} \oplus 0.5\% \oplus 0.2/E$	1997
Lead glass (OPAL)	$20.5X_0$	$5\%/\sqrt{E}$	1990
Liquid Kr (NA48)	$27X_{0}$	$3.2\%/\sqrt{E} \oplus 0.42\% \oplus 0.09/E$	1998
Scintillator/depleted U (ZEUS)	20-30X ₀	$18\%/\sqrt{E}$	1988
Scintillator/Pb (CDF)	$18X_0$	$13.5\%/\sqrt{E}$	1988
Scintillator fiber/Pb spaghetti (KLOE)	$15X_0$	$5.7\%/\sqrt{E} \oplus 0.6\%$	1995
Liquid Ar/Pb (NA31)	$27X_{0}$	$7.5\%/\sqrt{E} \oplus 0.5\% \oplus 0.1/E$	1988
Liquid Ar/Pb (SLD)	$21X_0$	$8\%/\sqrt{E}$	1993
Liquid Ar/Pb (H1)	$20 – 30X_0$	$12\%/\sqrt{E}\oplus 1\%$	1998
Liquid Ar/depl. U (DØ)	$20.5X_0$	$16\%/\sqrt{E} \oplus 0.3\% \oplus 0.3/E$	1993
Liquid Ar/Pb accordion (ATLAS)	$25X_{0}$	$10\%/\sqrt{E} \oplus 0.4\% \oplus 0.3/E$	1996

Hadron Showers

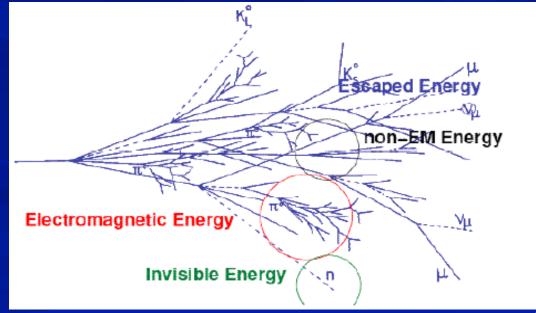
- Hadrons interact with detector material also through the strong interaction
- Hadron calorimeter measurement:
 - Charged hadrons: complementary to track measurement
 - Neutral hadrons: the only way to measure their energy
- In nuclear collisions many secondary particles are produced
 - Secondary, tertiary nuclear reactions → hadronic cascades
 - Electromagnetically decaying particles (π, η) initiate EM shower

Energy can also be absorbed as nuclear binding energy or target recoil

(Invisible energy)

■ Similar to EM showers, but more complex → need simulation tools (MC)

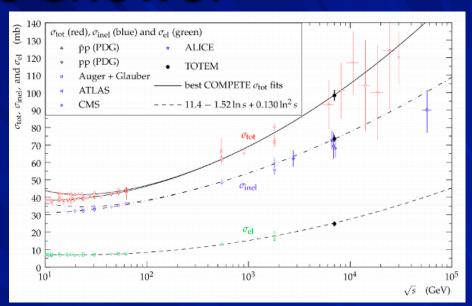
Characterized by the hadronic interaction length



Hadronic shower

Hadronic interaction Cross section

$$\sigma_{Tot} = \sigma_{el} + \sigma_{inel}$$
 $\sigma_{el} \approx 10mb$ $\sigma_{inel} \approx A^{2/3}$
 $\sigma_{Tot} = \sigma_{tot}(pp)A^{2/3}$
where: $\sigma_{tot}(pp)$ increases with \sqrt{s}



Hadronic interaction length

$$\lambda_{\text{int}} = \frac{1}{\sigma_{tot} \cdot n} = \frac{A\rho}{\sigma_{pp} A^{2/3} N_A} \approx \left(35g / cm^2\right) A^{1/3}$$

$$N(x) = N(0) e^{-x/\lambda_{\text{int}}}$$

λ_{int} characterizes both longitudinal and transverse shower profile

Rule of thumb argument: the geometric cross section goes as the square of the size of the nucleus, a_N^2 , and since the nuclear radius scales as $a_N \sim A^{1/3}$, the nuclear mean free path in gm/cm² units scales as $A^{1/3}$.

Hadronic vs EM showers

Hadronic vs. electromagnetic interaction length:

$$egin{aligned} X_0 \sim rac{A}{Z^2} \ \lambda_{
m int} \sim A^{1/3} \end{aligned} igg| iggrapha rac{\lambda_{
m int}}{X_0} \sim A^{4/3} \ \end{aligned}$$

$$\lambda_{
m int}\gg X_0$$
 [$\lambda_{
m int}/X_0>30$ possible; see below]

Typical

Longitudinal size: 6 ... 9 λ_{int} [95% containment]

Typical

Transverse size: one λ_{int}

[95% containment]

[EM: 15-20 X₀]

[EM: 2 R_M; compact]

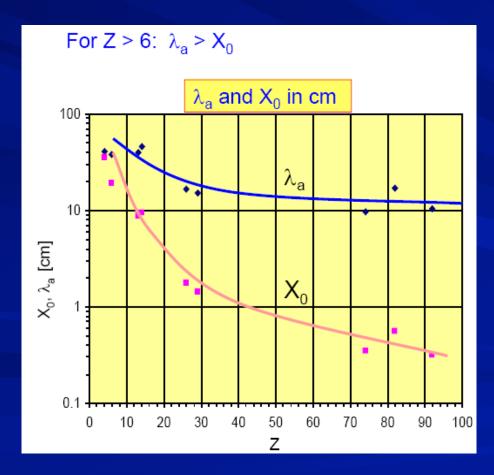
	λ _{int} [cm]	X ₀ [cm]
Szint.	79.4	42.2
LAr	83.7	14.0
Fe	16.8	1.76
Pb	17.1	0.56
U	10.5	0.32
C	38.1	18.8

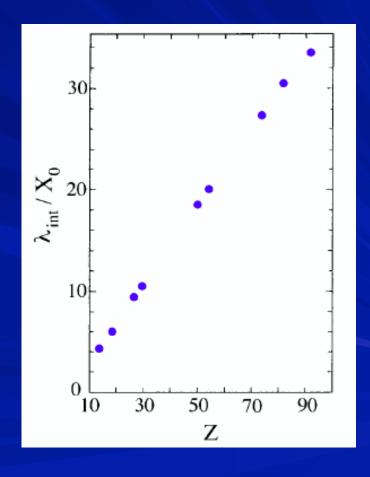
Hadronic calorimeter need more depth than electromagnetic calorimeter ...

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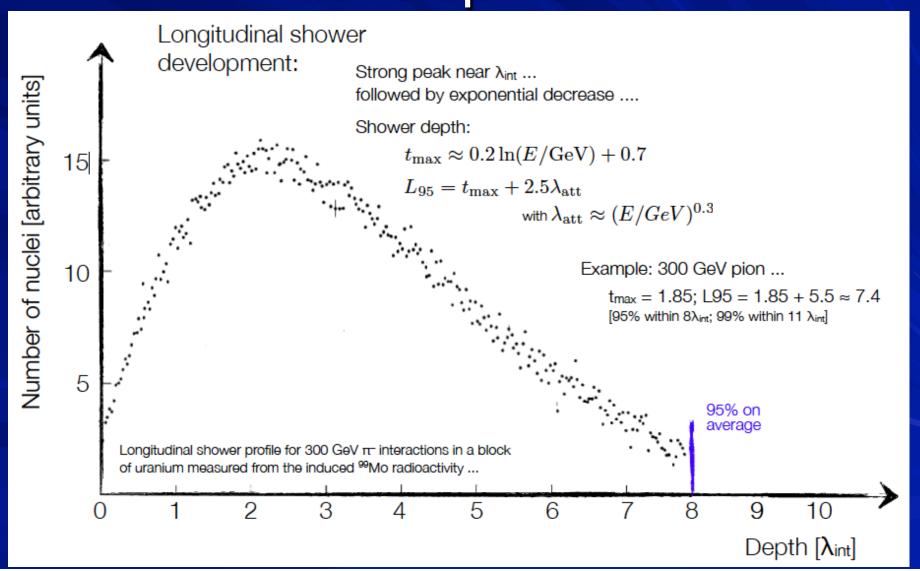
Material dependence

- λ_{int}: mean free path between nuclear collisions
- λ_{int} (g cm⁻²) \propto A^{1/3}
- Hadron showers are much longer than EM ones. Length depends on Z

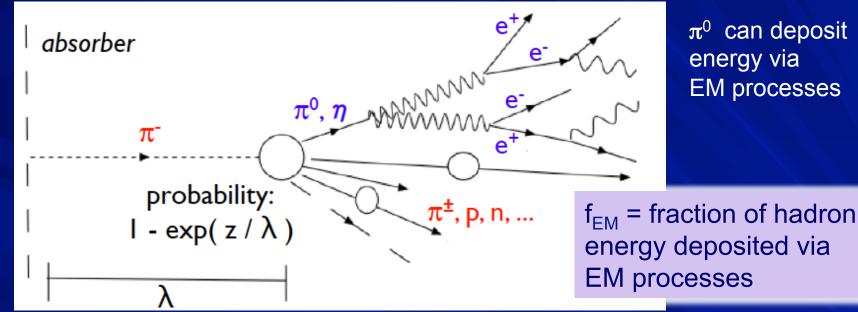




Hadronic shower: Longitudinal development



Hadronic Shower



- Electromagnetic
 - ionization, excitation (e±)
 - photo effect, scattering (γ)
- Hadronic
 - ionization ($\pi \pm$, p)
 - invisible energy (binding, recoil)

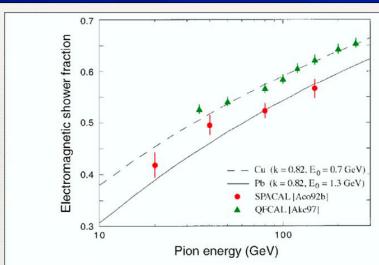


FIG. 2.22. Comparison between the experimental results on the em fraction of pion-induced showers in the (copper-based) QFCAL and (lead-based) SPACAL detectors. Data from [Akc 97] and [Aco 92b].

EM fraction in hadronic calorimeters

Charge conversion of $\pi^{+/-}$ produces electromagnetic component of hadronic shower (π^0)

- e = response to the EM shower component
- h = response to the non-EM component

$$\pi = f_{em} e + (1 - f_{em}) h$$

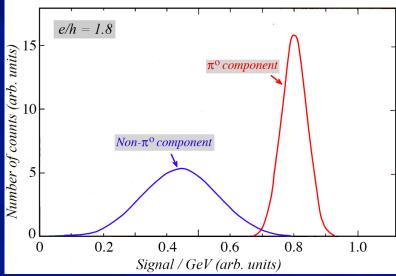
Comparing pion and electron showers:

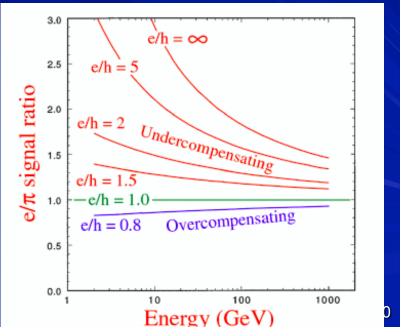
$$\frac{e}{\pi} = \frac{e}{f_{em}e + (1-f_{em})h} = \frac{e}{h} \frac{1}{1 + f_{em}(e/h-1)}$$

Calorimeters can be:

- Overcompensating e/h < 1
- Undercompensating e/h > 1
- Compensating e/h = 1

The origin of the non-compensation problems





Compensation

- Non-linearity determined by e/h value of the calorimeter
- Measurement of non-linearity is one of the methods to determine e/h
- Assuming linearity for EM showers, e(E1)=e(E2):

$$\frac{\pi(E_1)}{\pi(E_2)} = \frac{f_{em}(E_1) + [1 - f_{em}(E_1)] \cdot e/h}{f_{em}(E_2) + [1 - f_{em}(E_2)] \cdot e/h}$$

For e/h=1
$$\Rightarrow$$
 $\frac{\pi(E_1)}{\pi(E_2)} = 1$

 Response of calorimeters is usually higher for electromagnetic (e) than hadronic (h) energy deposits→e/h>1

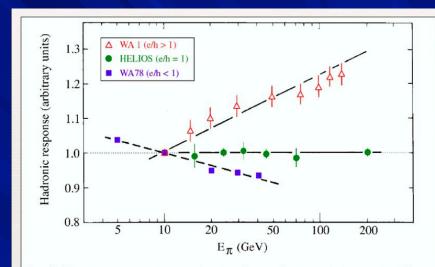
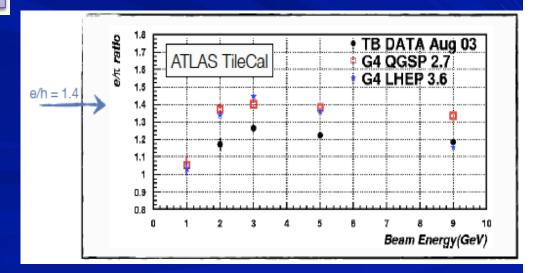
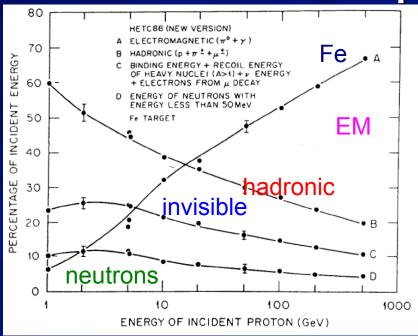
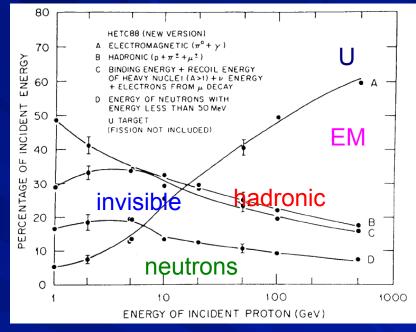


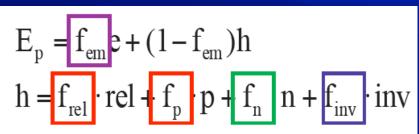
FIG. 3.14. The response to pions as a function of energy for three calorimeters with different e/h values: the WA1 calorimeter (e/h > 1, [Abr 81]), the HELIOS calorimeter ($e/h \approx 1$, [Ake 87]) and the WA78 calorimeter (e/h < 1, [Dev 86, Cat 87]). All data are normalized to the results for 10 GeV.



Compensation







Energy deposition mechanisms

- f_{relat}= Ionization by charged pions
- f_p=spallation protons
- f_n=neutrons evaporation
- f_{inv}=invisible energy by recoil nuclei

Compensation:

- Tuning the neutron response using hydrogenous active material (L3 Uranium/gas calorimeter)
- Compensation adjusting the sampling frequency

Compensation by tuning neutron response

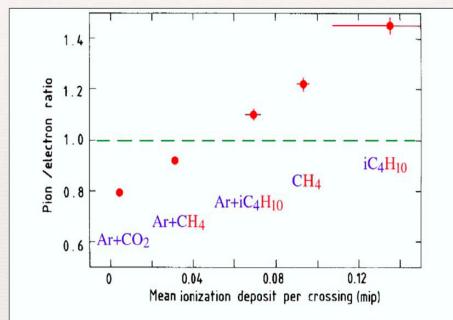
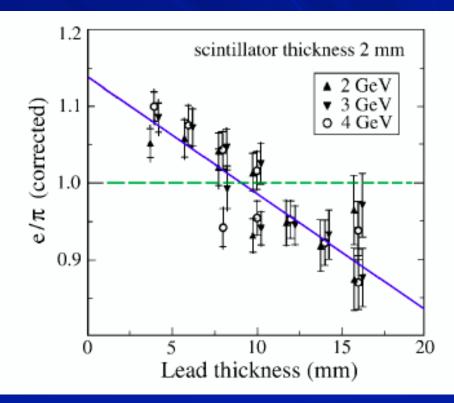
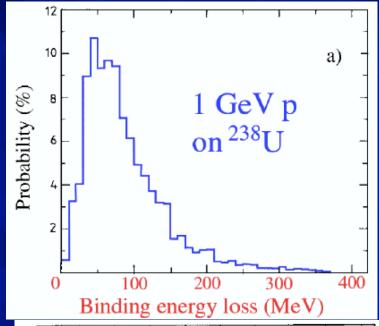


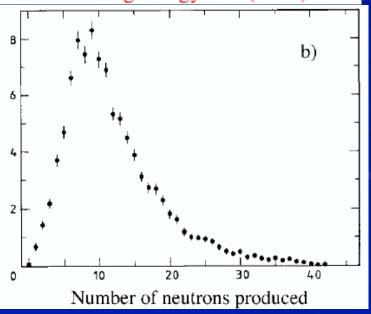
FIG. 3.32. The pion/electron signal ratio, averaged over the energy range 1.5 GeV, measured for different gas mixtures with the uranium/gas calorimeter of the L3 Collaboration. The horizontal scale gives the (calculated) average energy deposit in a chamber gap by slow neutrons [Gal 86].



Energy resolution of hadronic showers

- Fluctuations in visible energy (ultimate limit of hadronic energy resolution)
 - fluctuations of nuclear binding energy loss in high-Z materials ~15%
- Fluctuations in the EM shower fraction, f_{em}
 - Dominating effect in most hadron calorimeters (e/h >1)
 - Fluctuations are asymmetric in pion showers Differences between p, π induced showers
 - No leading π^0 in proton showers
- Sampling fluctuations sampling fluctuations only minor contribution to hadronic resolution in noncompensating calorimeter





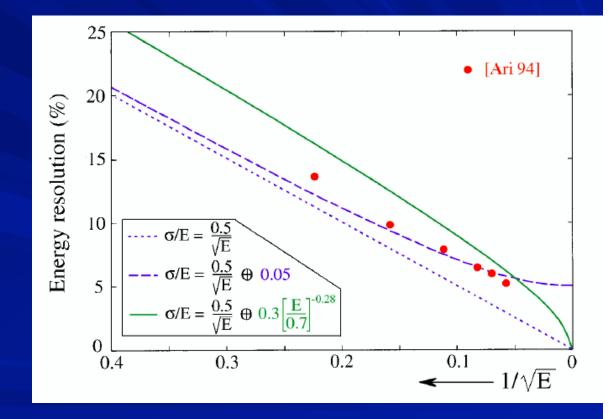
Energy resolution of hadron showers

■ Hadronic energy resolution of non-compensating calorimeters does not scale with 1/√E but as

$$\frac{\sigma_E}{E} = \frac{a}{\sqrt{E}} \oplus b \left(\frac{E}{E_0}\right)$$

But in practice we use

$$\frac{\sigma_E}{E} = \frac{a}{\sqrt{E}} \oplus b$$



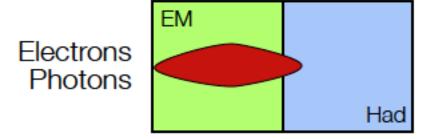
A realistic calorimetric system

Typical Calorimeter: two components ...

Schematic of a typical HEP calorimeter

Electromagnetic (EM) + Hadronic section (Had) ...

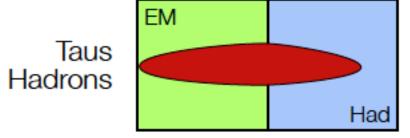
Different setups chosen for optimal energy resolution ...



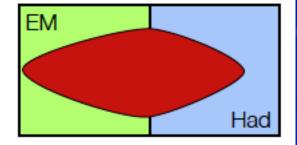
But:

Hadronic energy measured in both parts of calorimeter ...

Needs careful consideration of different response ...



Jets



LHC CALORIMETERS

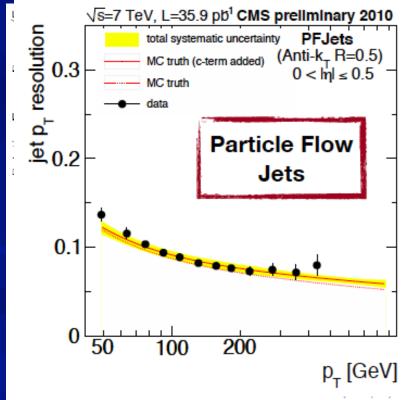


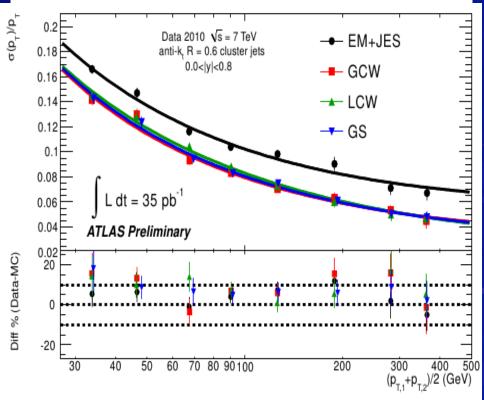
5 cm brass / 3.7 cm scint. Embedded fibres, HPD readout 14 mm iron / 3 mm scint. sci. fibres, read out by phototubes

Hadronic calorimeters resolution

- HCAL only $\sigma/E = (93.8 \pm 0.9)\%/\sqrt{E} \oplus (4.4 \pm 0.1)\%$
- ECAL+HCAL $\sigma/E = (82.6 \pm 0.6)\%/\sqrt{E} \oplus (4.5 \pm 0.1)\%$
- Improved resolution using full calorimetric system (ECAL+HCAL)

ATLAS LAr + Tile for pions:
$$\frac{\sigma(E)}{E} = \frac{42\%}{\sqrt{E}} \oplus 2\%$$



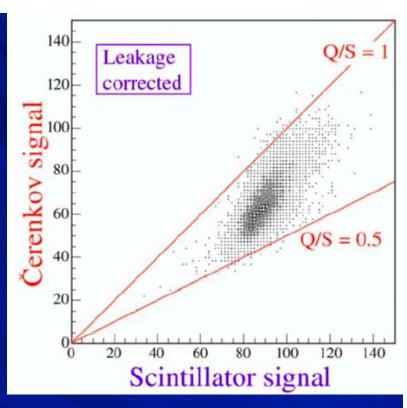


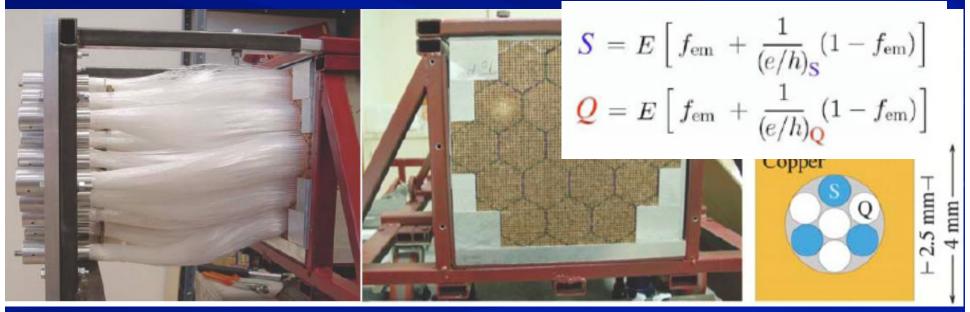
Future calorimeters

- Concentrate on improvement of jet energy resolution to match the requirement of the new physics expected in the next 30-50 years:
- Two approaches:
 - minimize the influence of the calorimeter and measure jets using the combination of all detectors → Particle Flow
 - measure the shower hadronic shower components in each event & weight directly access the source of fluctuations → Dual (Triple) Readout

DREAM

- Measure f_{EM} cell-by-cell by comparing Cherenkov and dE/dx signals
- Densely packed SPAgetti CALorimeter with interleaved Quartz (Cherenkov) and Scintillating Fibers
- Production of Cerenkov light only by emparticles (f_{EM})
- Aim at: σ_F/E ~ 15%/√E





Particle flow

Component	Detector	Fraction	Part. resolution	Jet Energy Res.
Charged (X [±])	Tracker	60%	10 ⁻⁴ E _x	negligible
Photons (γ)	ECAL	30%	0.1/√E _Y	.06/√E _{jet}
Neutral Hadrons (h)	E/HCAL	10%	0.5/√E _{had}	.16/√E _{jet}

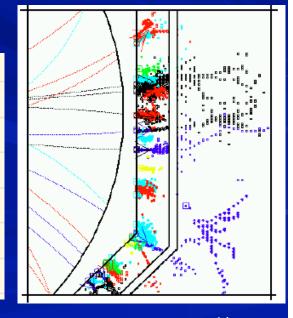
$$E_{JET} = E_{ECAL} + E_{HCAL}$$



- Granularity more important than resolution
- ECAL Tungsten silicon



- ECAL Tungsten scintillator strips
- ECAL Tungsten MAPS (Monolothic Active Pixel Sensors, CMOS technology) (DECAL)
- HCAL scintillator Tiles
- HCAL digital RPC or GEM (DHCAL)
- → HCAL semidigital gas (RPC or MICROMEGAS) device (SDGHCAL)
- TCMT : Scintillator/SiPM muon tagger



BACKUP

Summary

Radiation length:

$$X_0 = \frac{180A}{Z^2} \frac{\mathrm{g}}{\mathrm{cm}^2}$$

Problem:

Calculate how much Pb, Fe or Cu is needed to stop a 10 GeV electron.

Pb: Z=82, A=207, $\rho=11.34$ g/cm³ Fe : Z=26 , A=56 , $\rho=7.87$ g/cm³

Cu: Z=29, A=63, $\rho=8.92$ g/cm³

Critical energy:

$$E_c = \frac{550 \text{ MeV}}{Z}$$

[Attention: Definition of Rossi used]

Shower maximum:

$$t_{\rm max} = \ln \frac{E}{E_c} - \begin{cases} 1.0 & \text{e}^{-} \text{ induced shower} \\ 0.5 & \text{y induced shower} \end{cases}$$

Longitudinal

energy containment:

$$L(95\%) = t_{\text{max}} + 0.08Z + 9.6 [X_0]$$

Transverse

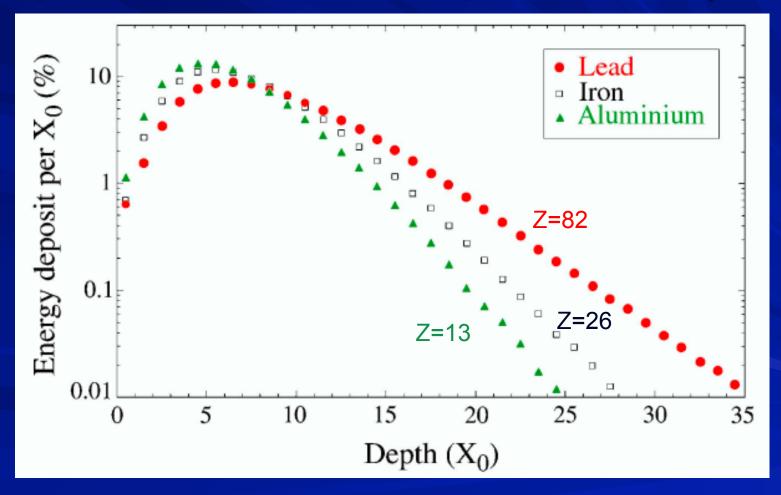
Energy containment:

$$R(90\%) = R_M$$

$$R(95\%) = 2R_M$$

Longitudinal development of EM

Shower decay: after the shower maximum the shower decays slowly through ionization and Compton scattering→ proportional to X₀

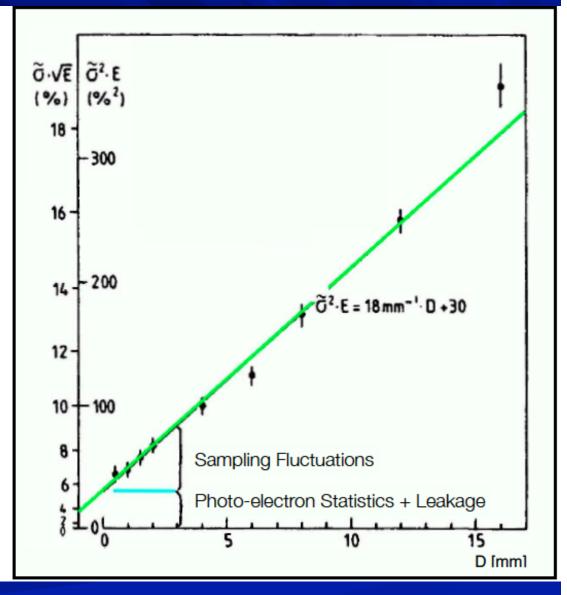


Dependence on sampling

Measure energy resolution of a sampling calorimeter for different absorber thicknesses

Sampling contribution:

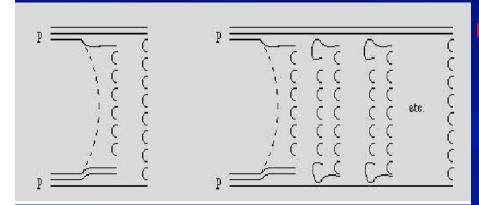
$$\frac{\sigma_E}{E} = 3.2\% \sqrt{\frac{E_c \,[\text{MeV}] \cdot t_{\text{abs}}}{F \cdot E \,[\text{GeV}]}}$$



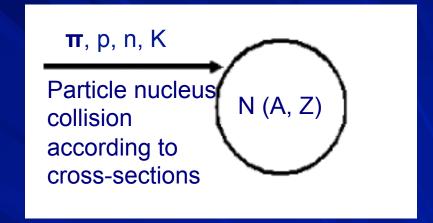
Hadronic interactions

1st stage: the hard collision

- pions travel 25-50% longer than protons (~2/3 smaller in size)
- a pion loses ~100-300 MeV by ionization (Z dependent)



Nucleon is split in quark di-quark Strings are formed String hadronisation (adding qqbar pair) fragmentation of damaged nucleus

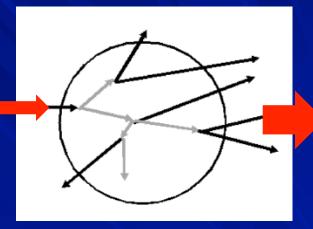


- Particle multiplication (string model)
 - average energy needed to produce a pion 0.7 (1.3) GeV in Cu (Pb)
 - Multiplicity scales with E and particle type
 - ~ 1/3 π⁰ → γγ produced in charge exchange processes: π⁺p → π⁰n and π⁻n → π⁰p
 - Leading particle effect: depends on incident hadron type e.g fewer π⁰ from protons, barion number conservation

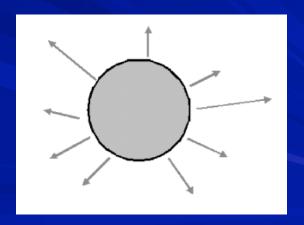
Hadronic interactions

2nd stage: spallation

- A fast hadron traversing the nucleus frees protons and neutrons in number proportional to their numerical presence in the nucleus.
- The nucleons involved in the cascade transfer energy to the nucleus which is left in an excited state
- Nuclear de-excitation
 - Evaporation of soft (~10 MeV) nucleons and α
 - fission for some materials
- The number of nucleons released depends on the binding E (7.9 MeV in Pb, 8.8 MeV in Fe)
- Mainly neutrons released by evaporation → protons are trapped by the Coulomb barrier (12 MeV in Pb, only 5 MeV in Fe)

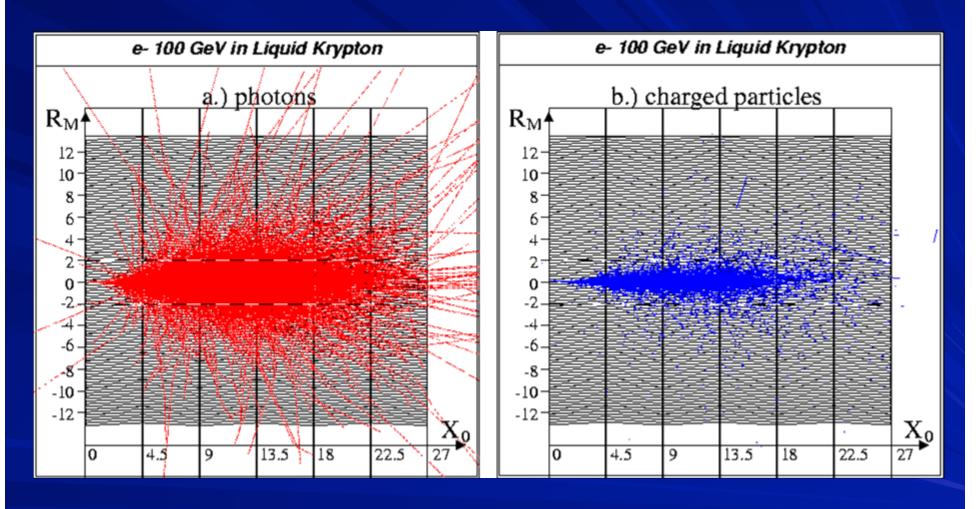


Dominating momentum component along incoming particle direction



isotropic process

EM shower development in liquid krypton (Z=36, A=84)



GEANT simulation of a 100 GeV electron shower in the NA48 liquid Krypton calorimeter (D.Schinzel)

Hadronic shower

Hadronic interaction:

Elastic:

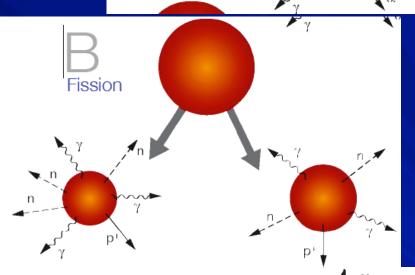
$$p + \text{Nucleus} \rightarrow p + \text{Nucleus}$$

Inelastic:

$$p + \text{Nucleus} \rightarrow \pi^+ + \pi^- + \pi^0 + \dots + \text{Nucleus}^*$$

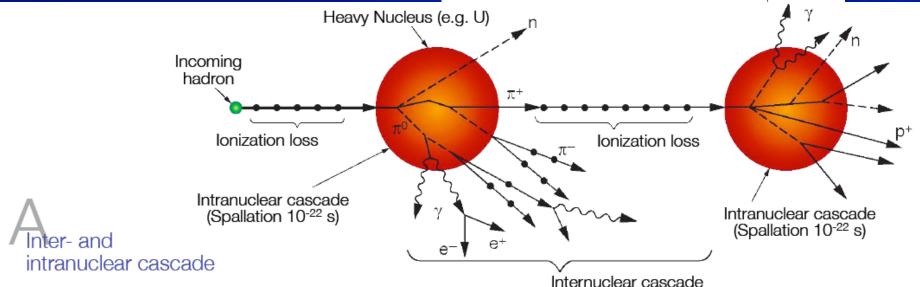
Nucleus*
$$\rightarrow$$
 Nucleus A + n, p, α , ... \rightarrow Nucleus B + 5p, n, π , ...

 \rightarrow Nuclear fission

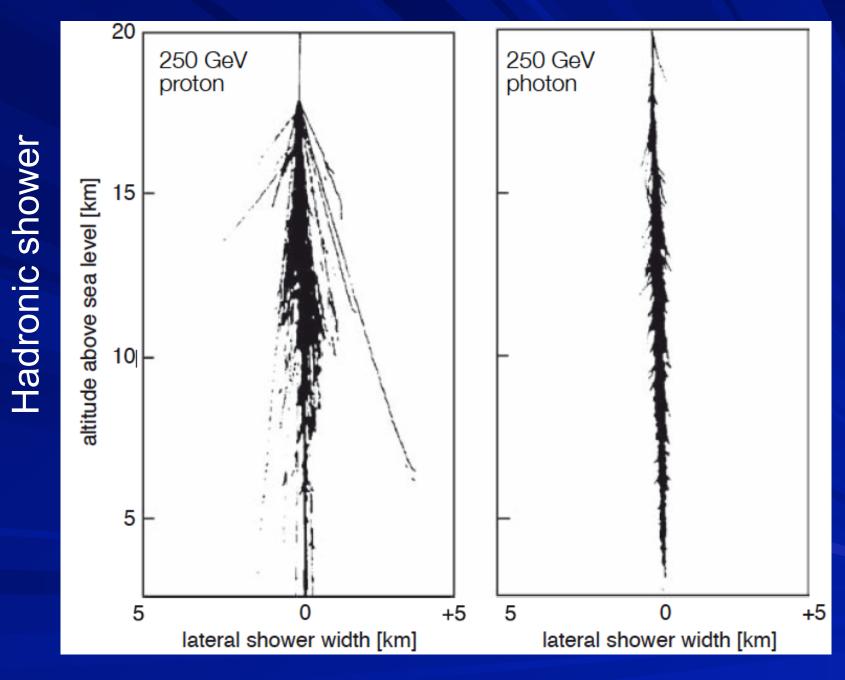


Nuclear

evaporation

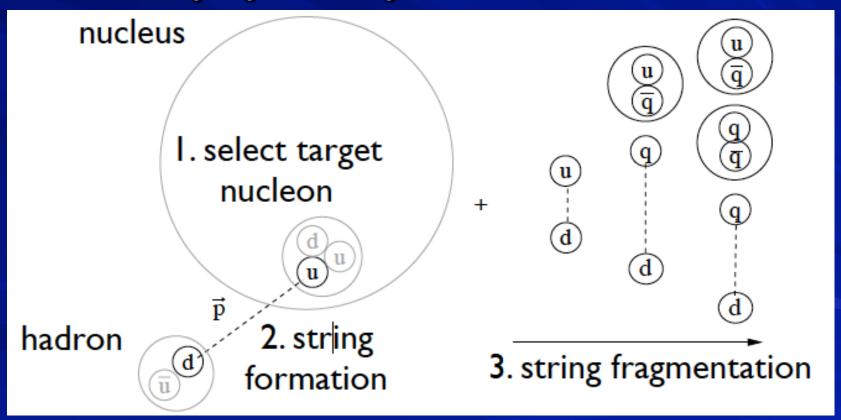






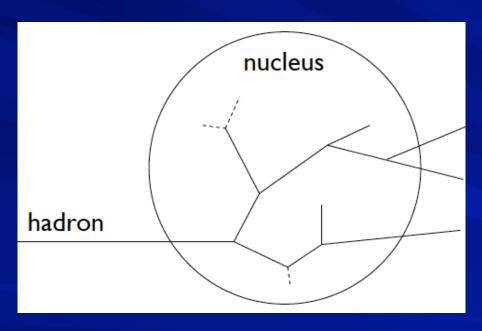
Simulation

- Interaction of hadrons with E > 10 GeV described by string models
 - projectile interacts with single nucleon (p,n)
 - a string is formed between quarks from interacting nucleons
 - the string fragmentation generates hadrons



Simulation

- Interaction of hadrons with 10 MeV < E < 10 GeV via intra-nuclear cascades</p>
- For E < 10 MeV only relevant are fission, photon emission, evaporation, ...</p>



Approximations

- λ_{deBroglie} ≤ d nucleon
- nucleus = Fermi gas (all nucleons included)
- Pauli exclusion: allow only secondaries above Fermi energy