LUND UNIVERSITY
Faculty of Science

CERN Summer Student Lecture
Part 2, 20 July 2012

Introduction to Monte Carlo Techniques in High Energy Physics

Torbjörn Sjöstrand

How are complicated multiparticle events created?
How can such events be simulated with a computer?

Lectures Overview

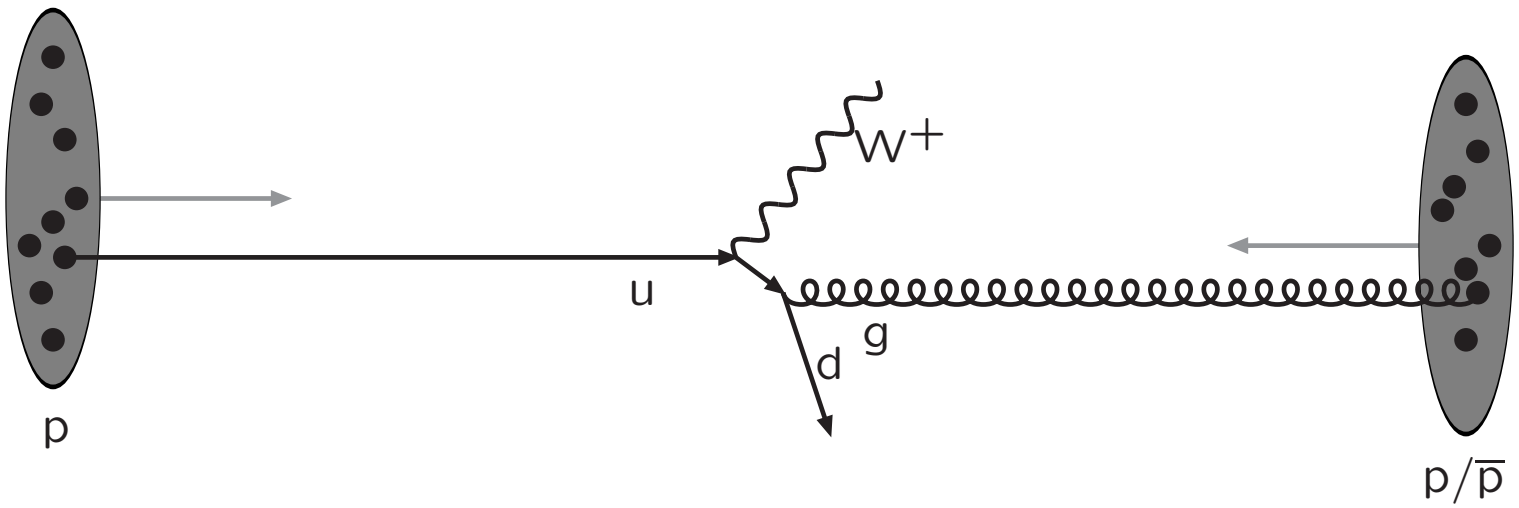
yesterday:	Introduction	the Standard Model Quantum Mechanics the role of Event Generators
	Monte Carlo	random numbers integration simulation
today:	Physics	hard interactions parton showers multiparton interactions hadronization
	Generators	HERWIG, PYTHIA, SHERPA MADGRAPH, ALPGEN, ... common standards

The Structure of an Event – 1

Warning: schematic only, everything simplified, nothing to scale, . . .

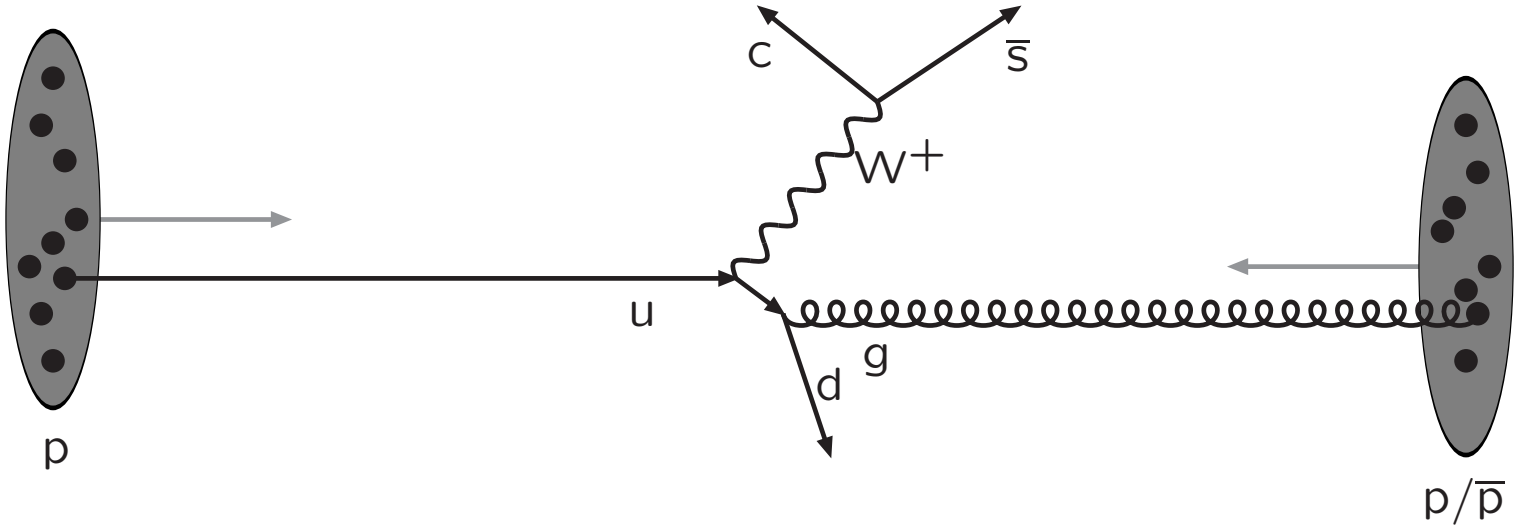
Incoming beams: parton densities

The Structure of an Event – 2



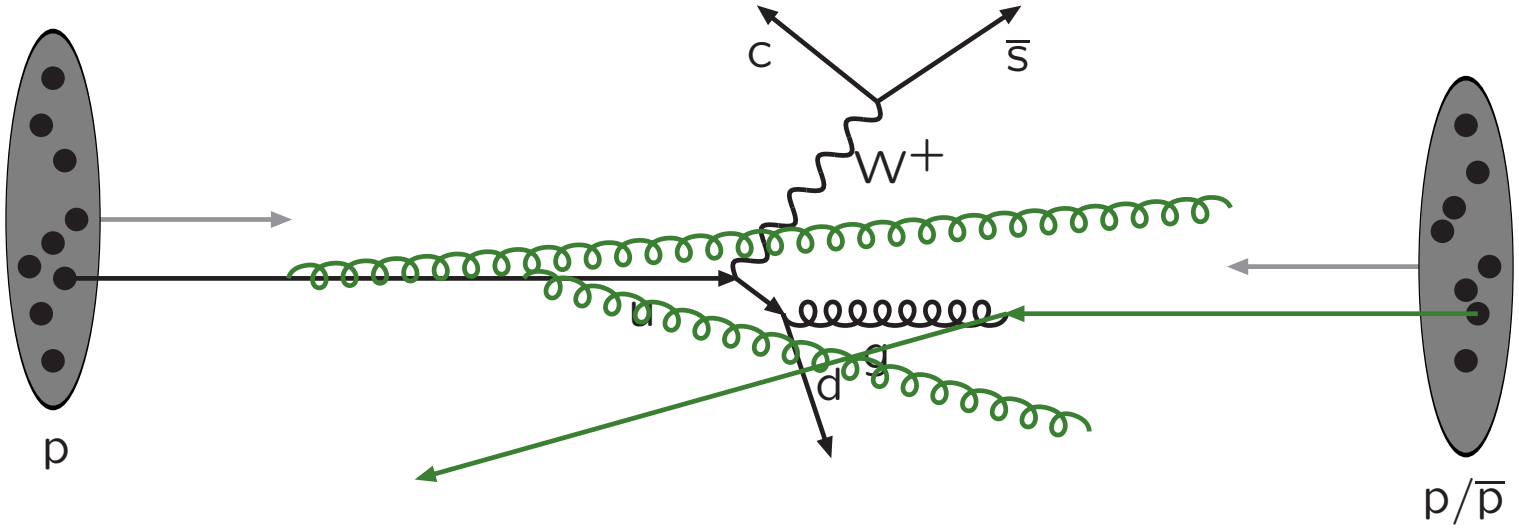
Hard subprocess: described by matrix elements

The Structure of an Event – 3



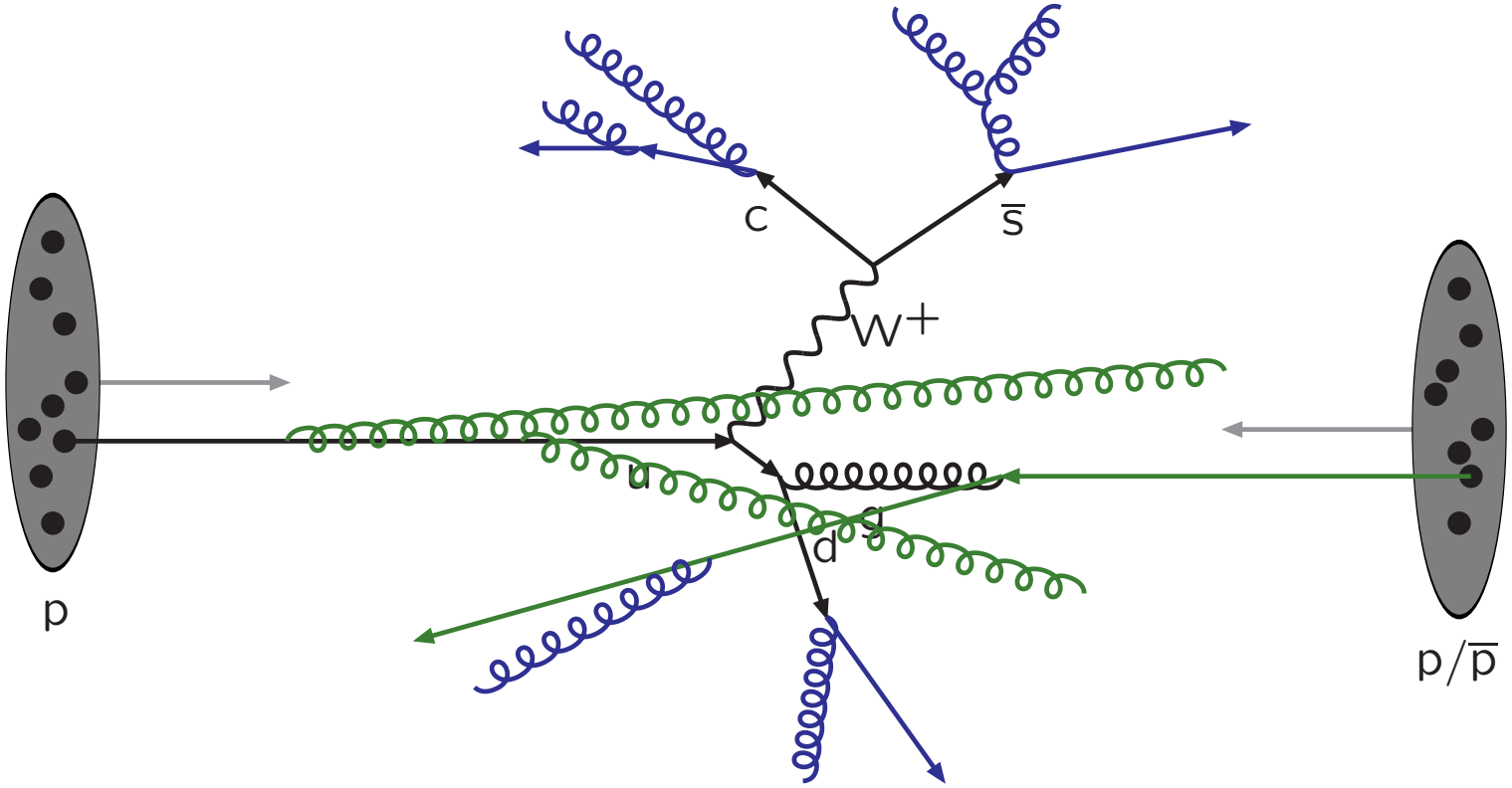
Resonance decays: correlated with hard subprocess

The Structure of an Event – 4



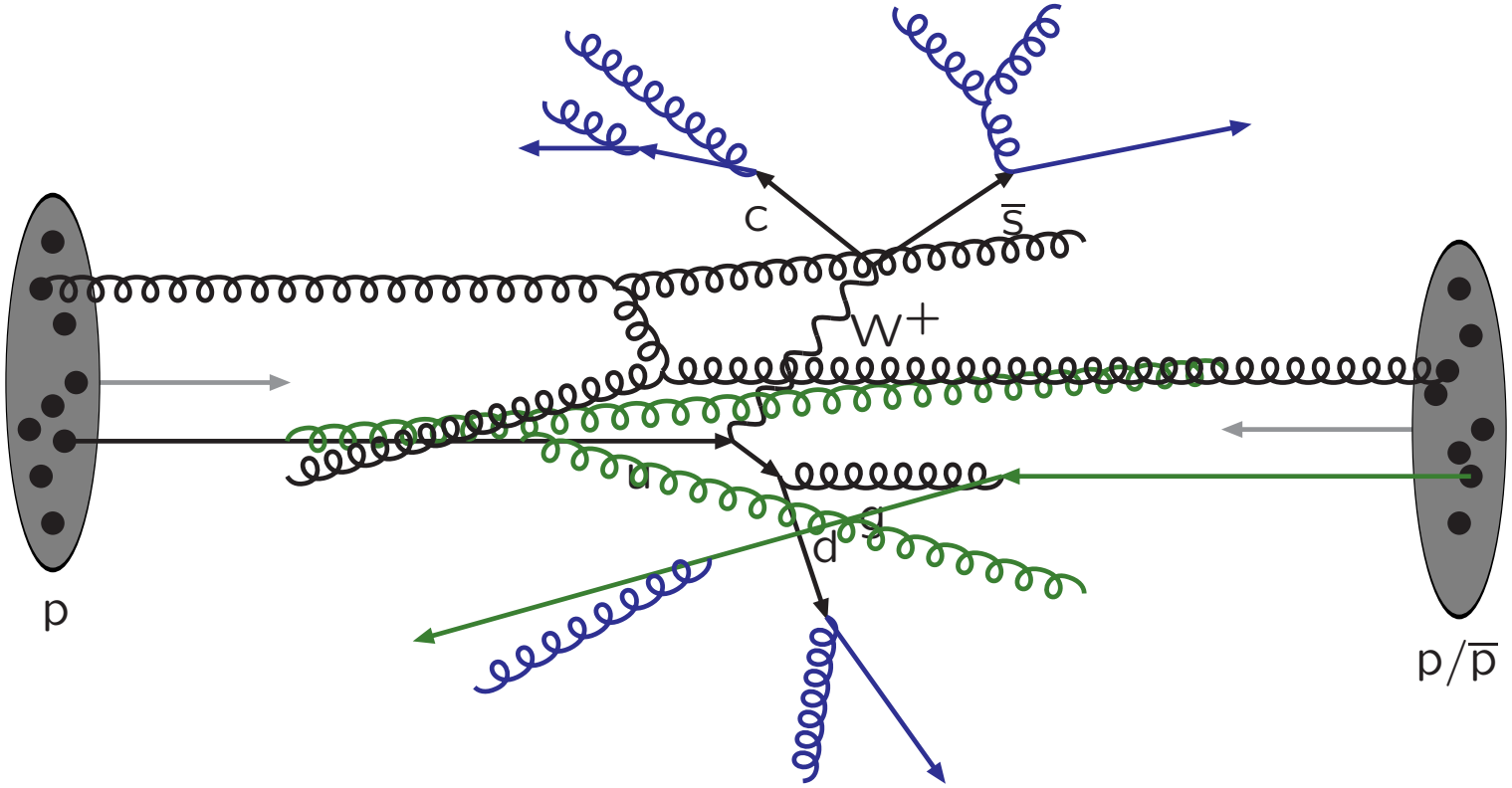
Initial-state radiation: spacelike parton showers

The Structure of an Event – 5



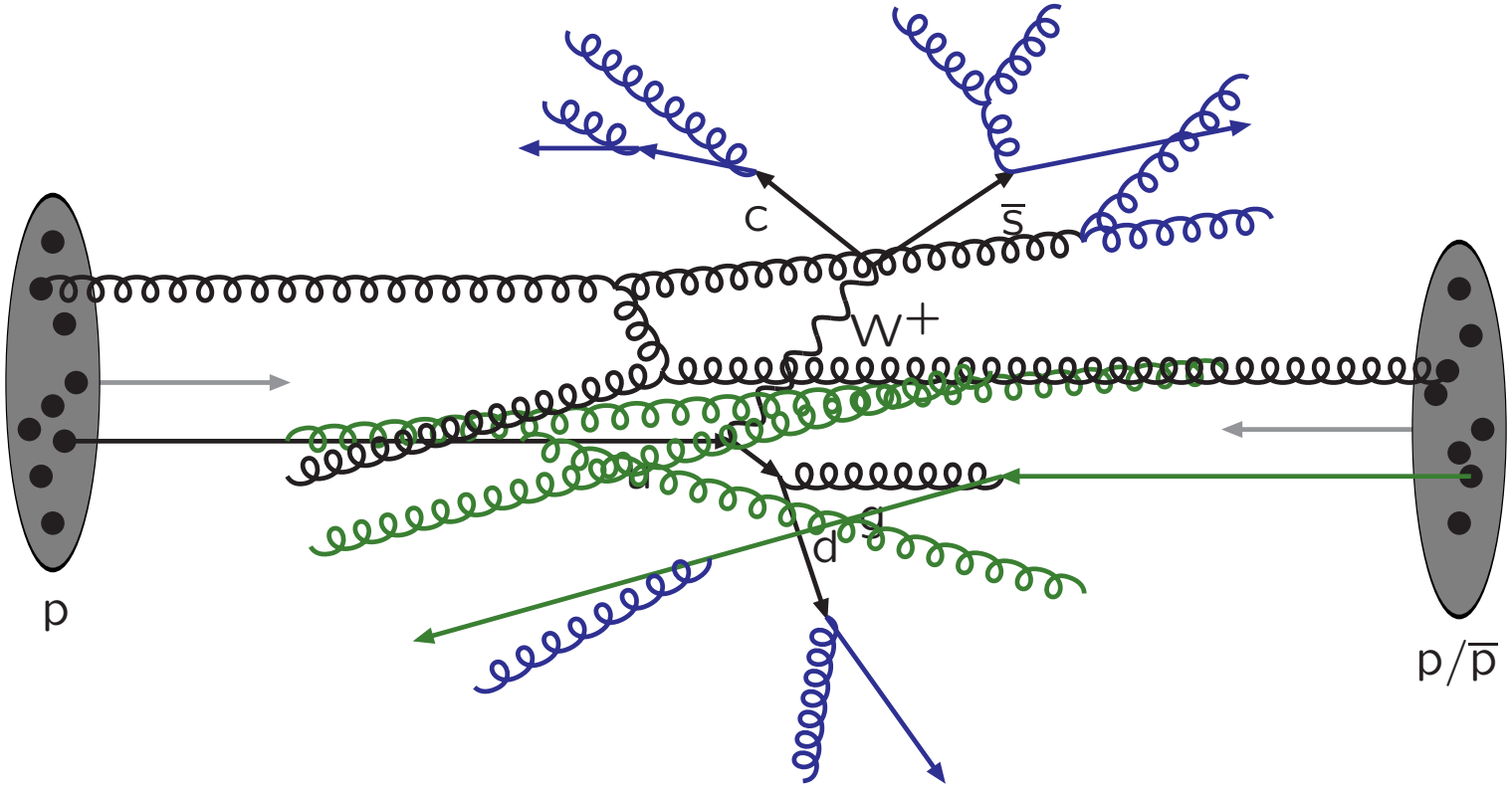
Final-state radiation: timelike parton showers

The Structure of an Event – 6



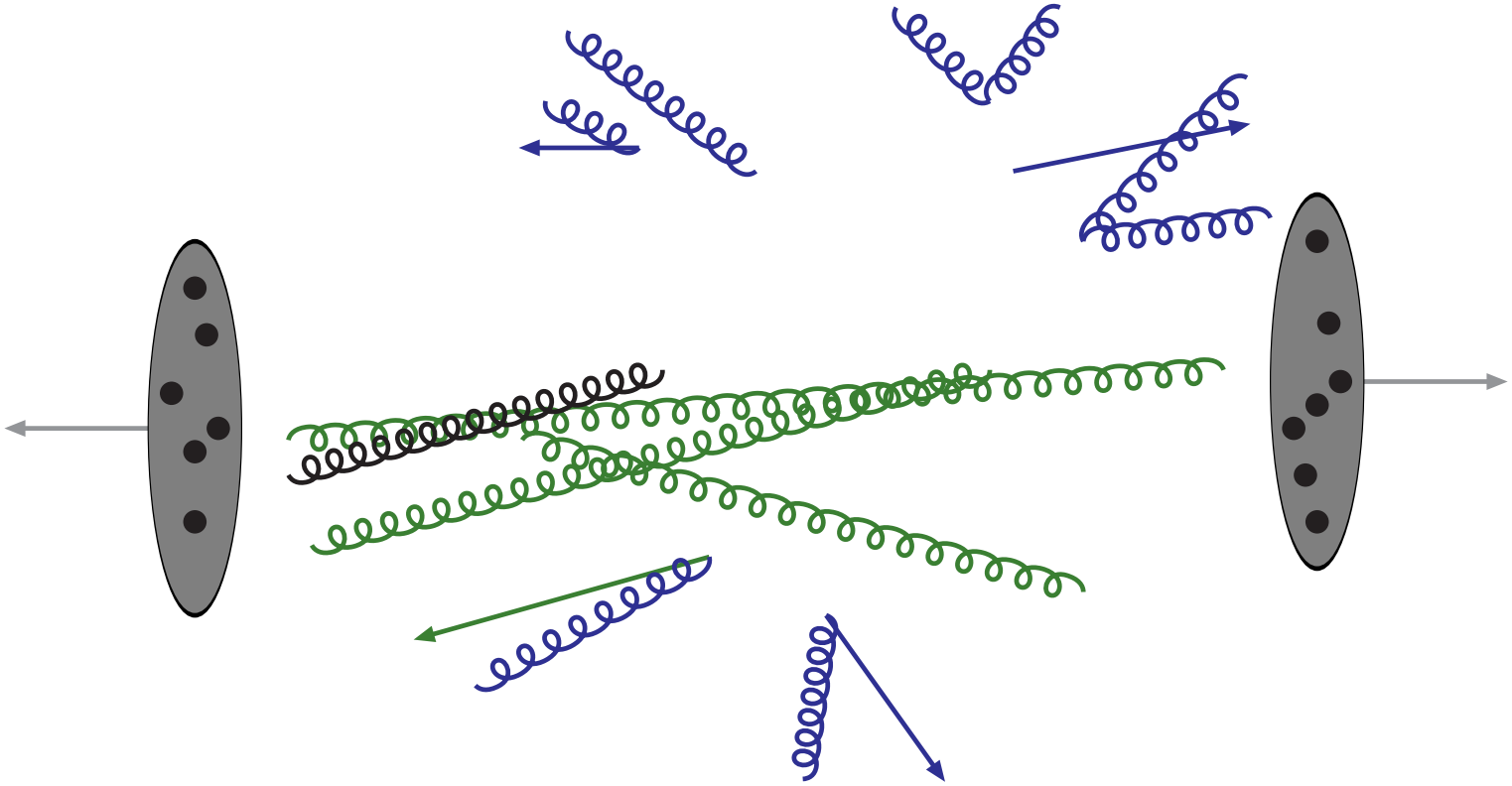
Multiple parton–parton interactions . . .

The Structure of an Event – 7



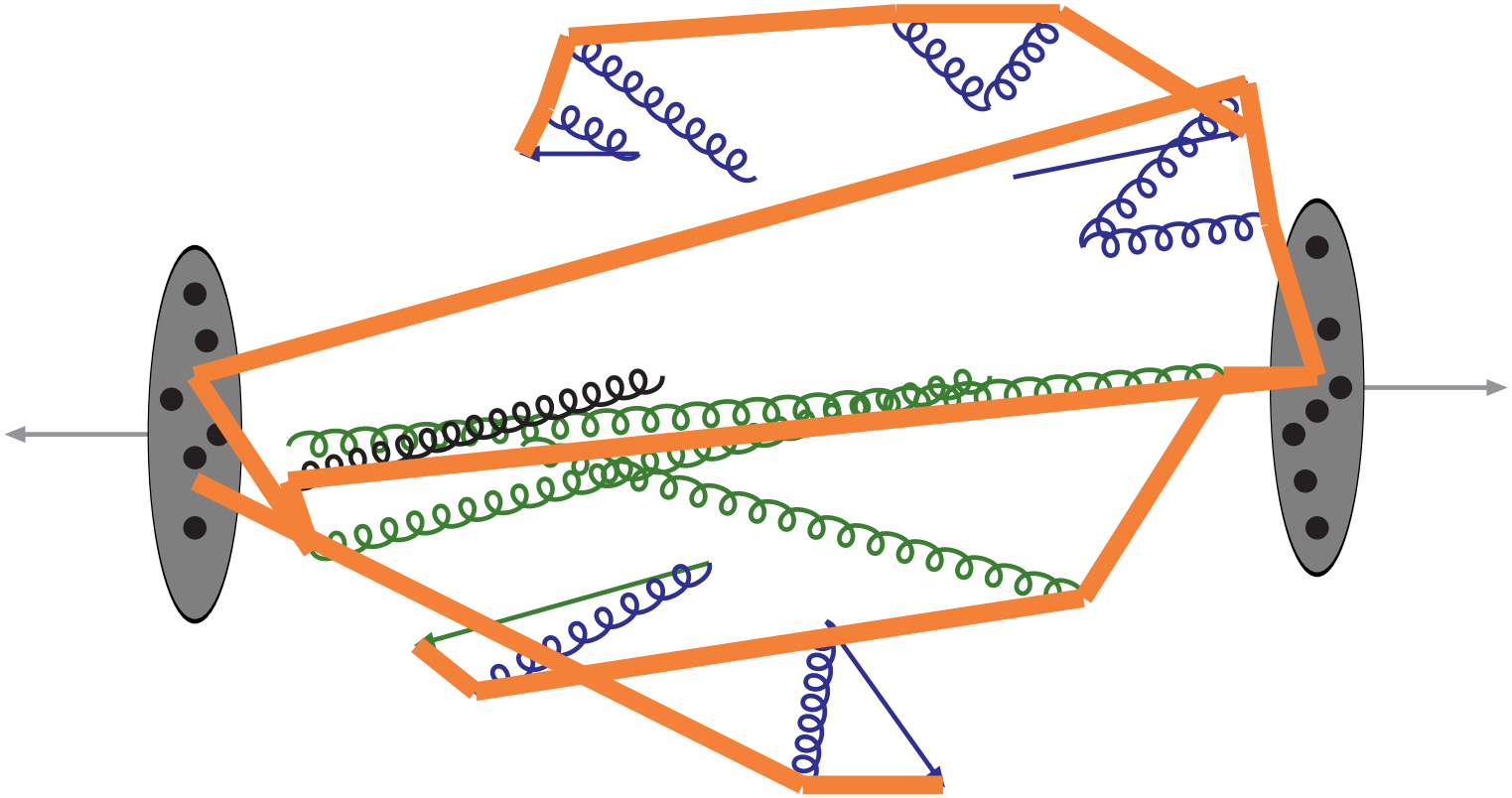
. . . with its initial- and final-state radiation

The Structure of an Event – 8



Beam remnants and other outgoing partons

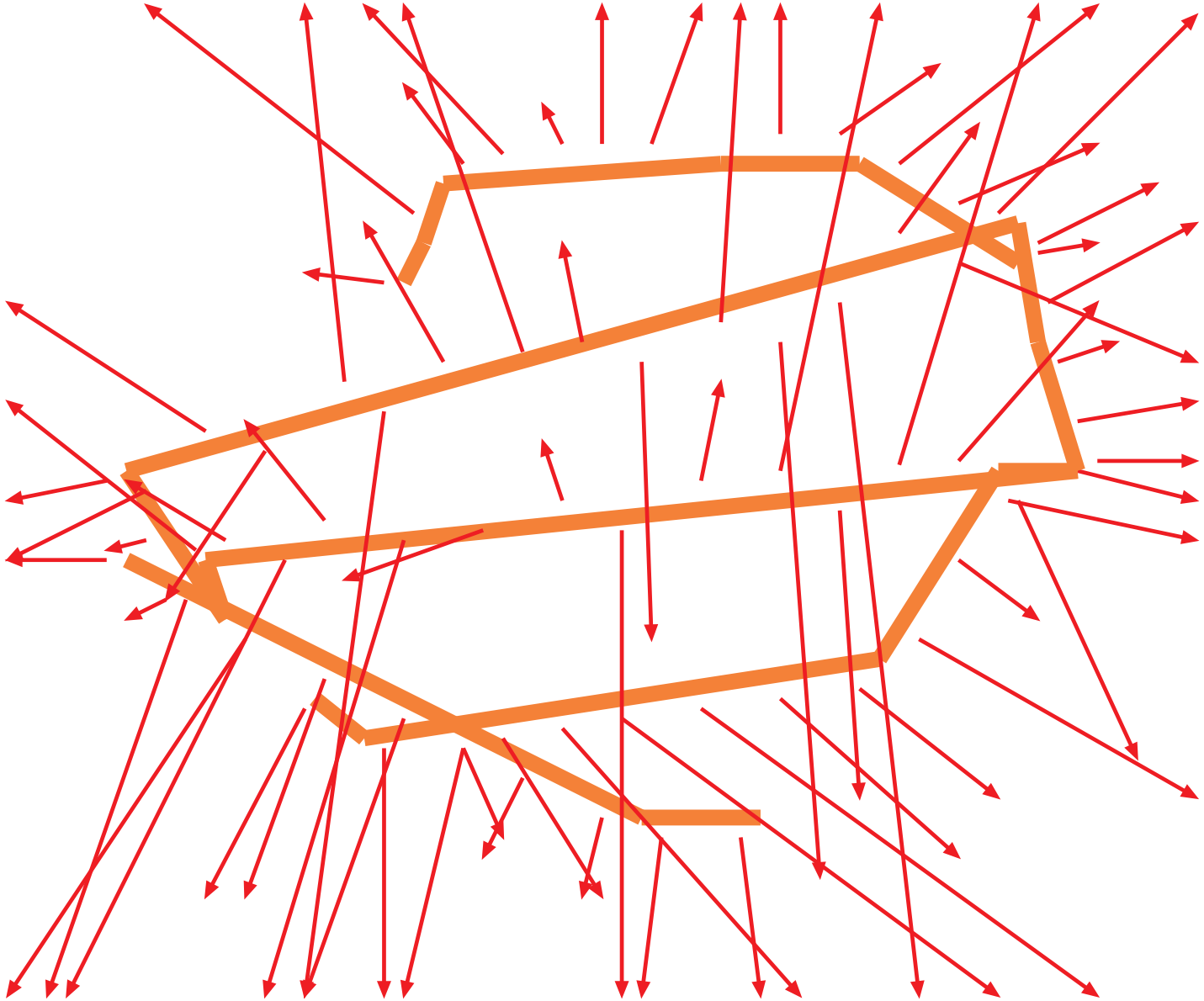
The Structure of an Event – 9



Everything is connected by colour confinement strings

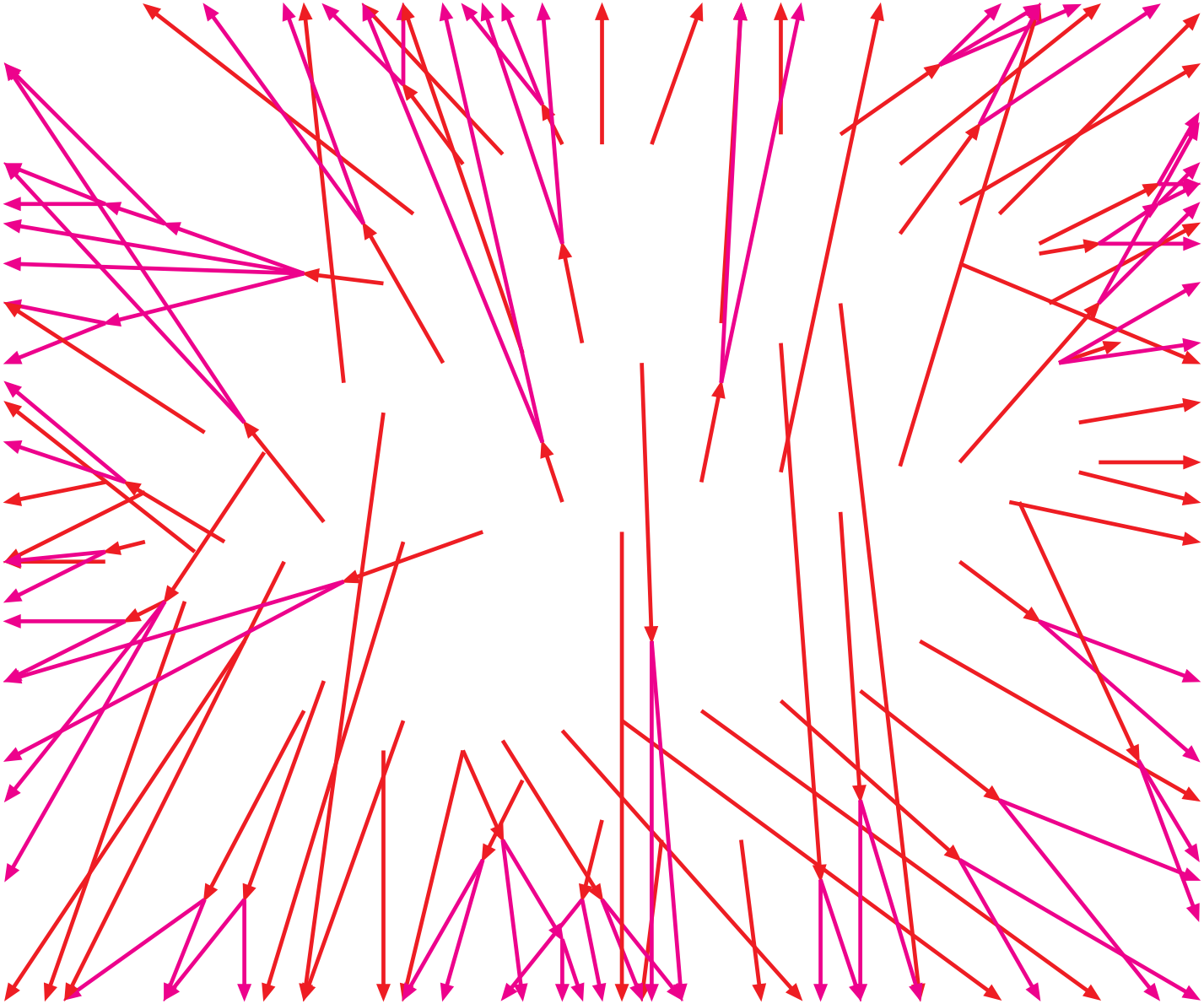
Recall! Not to scale: strings are of hadronic widths

The Structure of an Event – 10



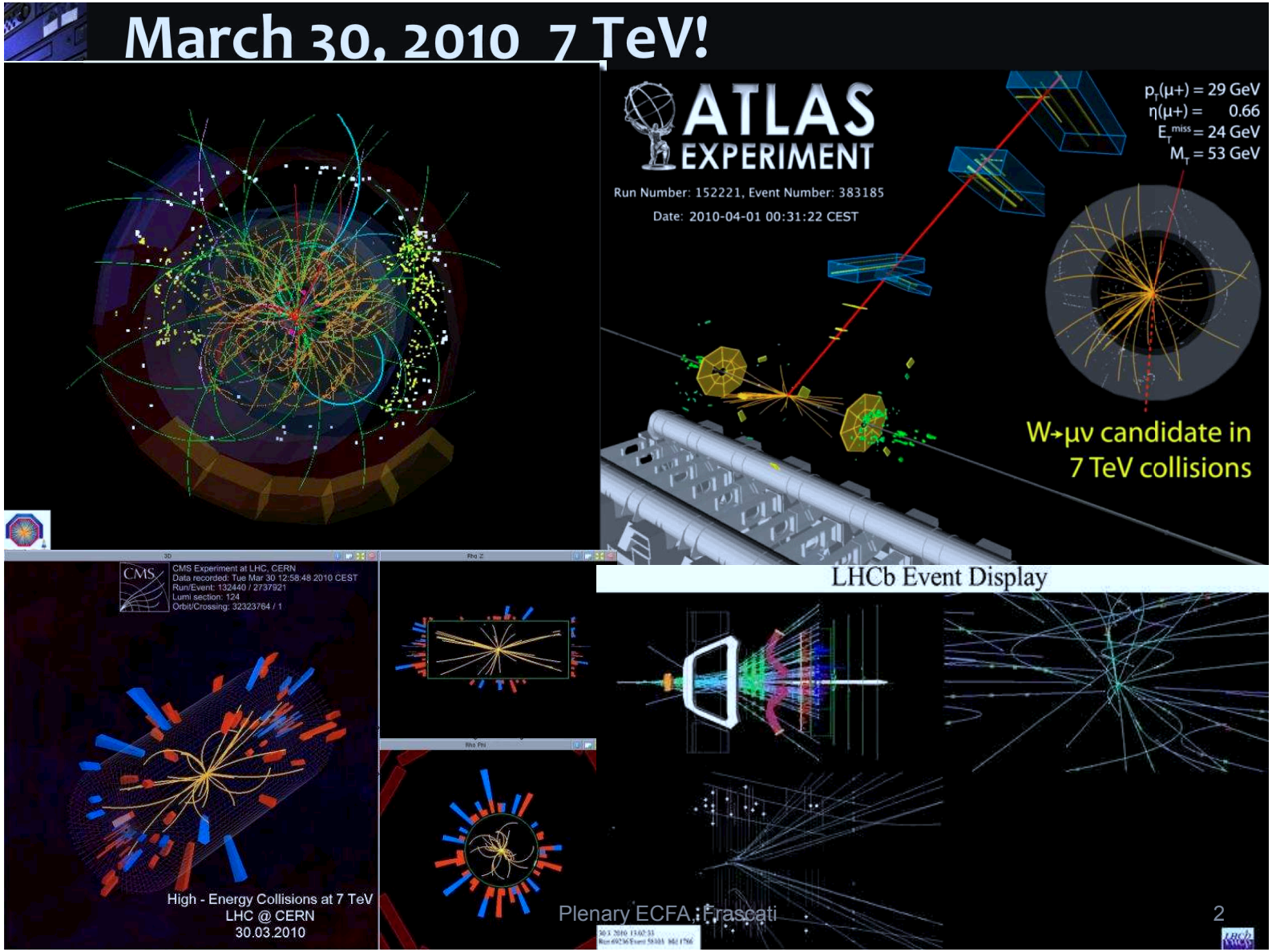
The strings fragment to produce primary hadrons

The Structure of an Event – 11



Many hadrons are unstable and decay further

The Structure of an Event – 12



These are the particles that hit the detector

Hard Processes

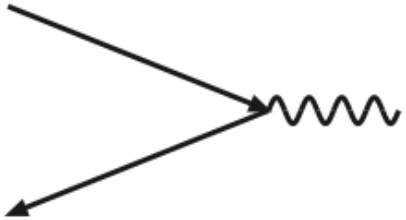
Defines the character of the event:
QCD, top, Z^0, $W^\pm$, H^0, SUSY, ED, TC, ...

Obtained by perturbation theory:

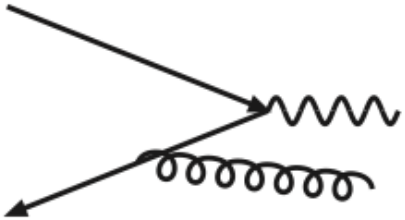
$$\begin{aligned}\hat{\sigma}_{Z^0} &= \alpha_{\rm em} M_0 + \alpha_{\rm em}\alpha_{\rm s} M_1 + \alpha_{\rm em}\alpha_{\rm s}^2 M_2 + \ldots \\ &= \text{LO} + \text{NLO} + \text{NNLO} + \ldots\end{aligned}$$

Involves subtle cancellations between real and virtual contributions:

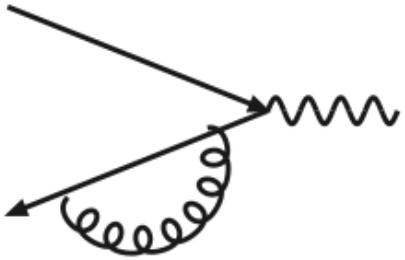
I. Lowest order, $\mathcal{O}(\alpha_{\rm em})$: $q\bar{q} \to Z^0$



II. First-order real, $\mathcal{O}(\alpha_{\rm em}\alpha_{\rm s})$: $q\bar{q} \to Z^0 g$ etc.



III. First-order virtual, $\mathcal{O}(\alpha_{\rm em}\alpha_{\rm s})$: $q\bar{q} \to Z^0$ with loops



These days
- LO easy
- NLO tough but doable
- NNLO only in very few cases

QED Bremsstrahlung

Accelerated electric charges radiate photons,

see e.g. J.D. Jackson, *Classical Electrodynamics*.

A charge ze that changes its velocity vector from $\boldsymbol{\beta}$ to $\boldsymbol{\beta}'$ radiates a spectrum of photons that depends on its trajectory. In the long-wavelength limit it reduces to

$$\lim_{\omega \to 0} \frac{\mathrm{d}^2 I}{\mathrm{d}\omega \mathrm{d}\Omega} = \frac{z^2 e^2}{4\pi^2} \left| \boldsymbol{\epsilon}^* \left(\frac{\boldsymbol{\beta}'}{1 - \mathbf{n}\boldsymbol{\beta}'} - \frac{\boldsymbol{\beta}}{1 - \mathbf{n}\boldsymbol{\beta}} \right) \right|^2$$

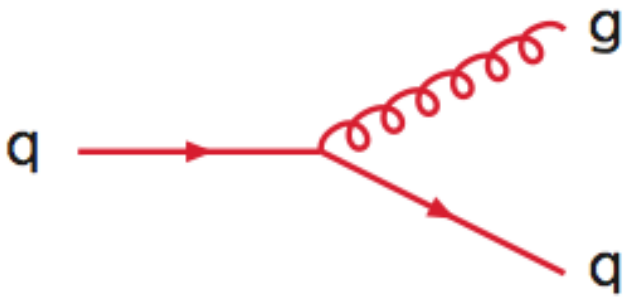
where $\mathbf{n}$ is a vector on the unit sphere Ω, ω is the energy of the radiated photon, and $\boldsymbol{\epsilon}$ its polarization.

1. For fast particles radiation collinear with the initial ($\boldsymbol{\beta}$) and final ($\boldsymbol{\beta}'$) directions is strongly enhanced.
2. $\mathrm{d}N/\mathrm{d}\omega = (1/\omega)\mathrm{d}I/\mathrm{d}\omega \propto 1/\omega$ so infinitely many infinitely soft photons are emitted, but the net energy taken away is finite.
3. For $\omega \to 0$ the radiation pattern is independent of the spin of the radiator $\Rightarrow$ universality.

QCD Bremsstrahlung – 1

QCD $\approx$ QED with $\quad \mathrm{e} \to \mathrm{e}\gamma \Rightarrow \mathrm{q} \to \mathrm{qg}$

$\alpha_{\mathrm{em}} \Rightarrow (4/3)\alpha_{\mathrm{s}}$



More precisely:

$$\mathrm{d}\mathcal{P}_{\mathrm{q}\to\mathrm{qg}} \approx \frac{\alpha_{\mathrm{s}}}{2\pi}\,\frac{\mathrm{d}Q^2}{Q^2}\,\frac{4}{3}\,\frac{1+z^2}{1-z}\,\mathrm{d}z$$

where

- mass (or collinear) singularity:

$$\frac{\mathrm{d}Q^2}{Q^2} \approx \frac{\mathrm{d}\theta^2}{\theta^2} \approx \frac{\mathrm{d}m^2}{m^2} \approx \frac{\mathrm{d}p_\perp^2}{p_\perp^2}$$

- soft singularity:

$$z \approx \frac{E_{\mathrm{q,after}}}{E_{\mathrm{q,before}}} = 1 - \frac{\omega}{E_{\mathrm{q,before}}} \quad \text{so} \quad \frac{\mathrm{d}z}{1-z} = \frac{\mathrm{d}\omega}{\omega}$$

QCD Bremsstrahlung – 2

But QCD is non-Abelian so additionally

- $\mathrm{g} \to \mathrm{gg}$ similarly divergent
- $\alpha_s(Q^2)$ diverges for $Q^2 \to 0$

DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi)

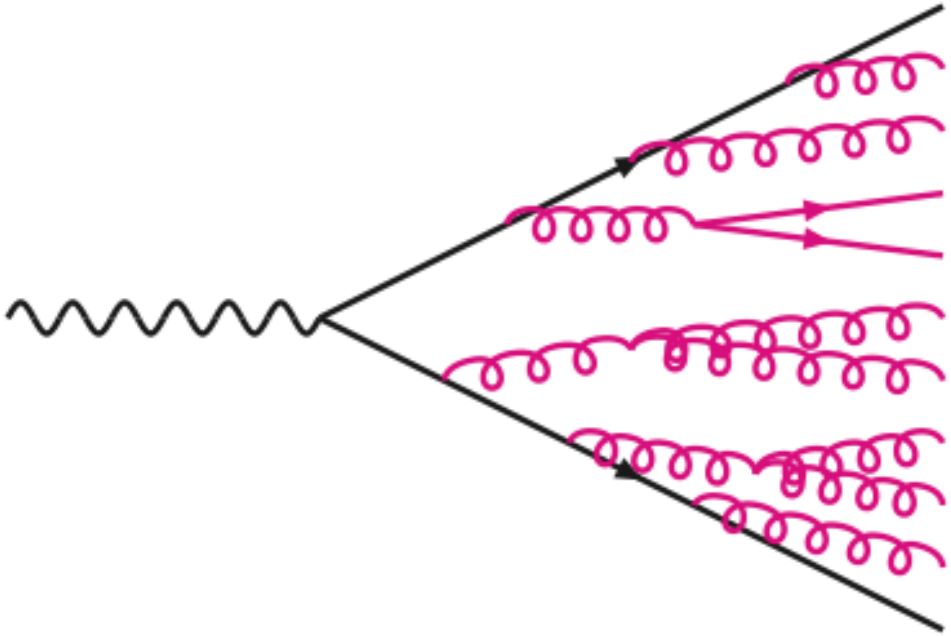
$$
\begin{aligned}
\mathrm{d}\mathcal{P}_{a\to bc} &= \frac{\alpha_\mathrm{s}(Q^2)}{2\pi}\,\frac{\mathrm{d}Q^2}{Q^2}\,P_{a\to bc}(z)\,\mathrm{d}z \\
P_{\mathrm{q}\to\mathrm{qg}} &= \frac{4}{3}\,\frac{1+z^2}{1-z} \\
P_{\mathrm{g}\to\mathrm{gg}} &= 3\,\frac{(1-z(1-z))^2}{z(1-z)} \\
P_{\mathrm{g}\to\mathrm{q\bar{q}}} &= \frac{n_f}{2}\,(z^2+(1-z)^2) \quad (n_f = \text{no. of quark flavours})
\end{aligned}
$$

The iterative structure

Generalizes to many consecutive emissions if strongly ordered, $Q_1^2 \gg Q_2^2 \gg Q_3^2 \ldots$ ($\approx$ time-ordered).
To cover "all" of phase space use DGLAP in whole region $Q_1^2 > Q_2^2 > Q_3^2 \ldots$, although only approximately valid.

Must be clever when you write a shower algorithm.

Iteration gives final-state parton showers:



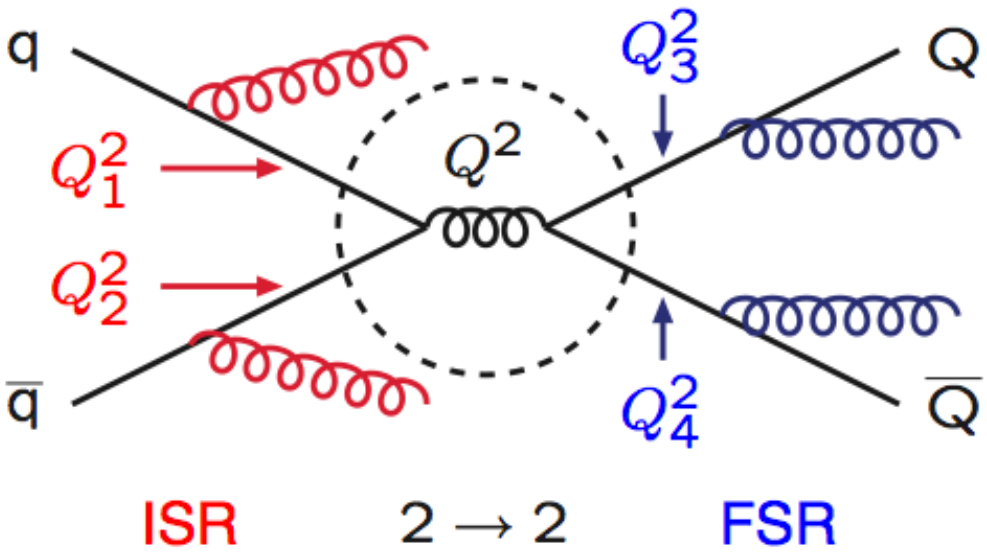
Need soft/collinear cuts to stay away from nonperturbative physics.
Details model-dependent, but around 1 GeV scale.

The Parton-Shower Approach

Showers: approximation method for higher-order matrix elements.

Universality: any matrix element reduces to DGLAP in collinear limit.

$$2 \to n \quad \approx \quad (2 \to 2) \oplus \text{ISR} \oplus \text{FSR}$$

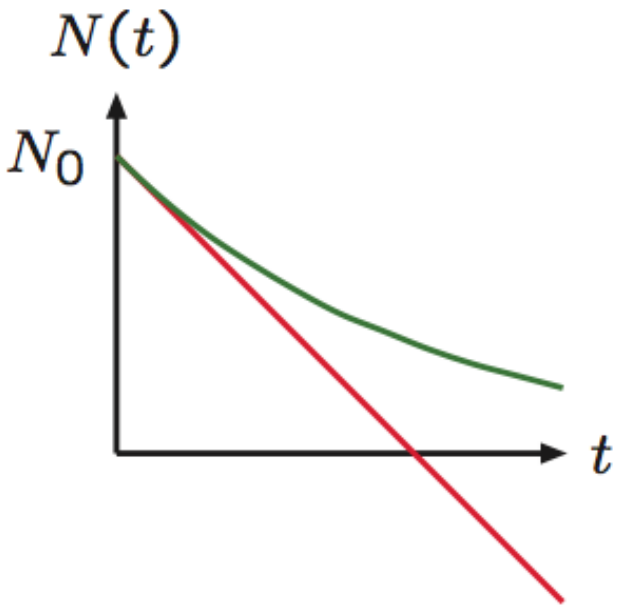


ISR = Initial-State Radiation: $Q_i^2 \sim -m^2 > 0$ increasing

FSR = Final-State Radiation: $Q_i^2 \sim m^2 > 0$ decreasing

Reminder: Radioactive Decay and the Veto Algorithm

Recall discussion on radioactive decays from Lecture 1:



Naively $P(t) = c \Longrightarrow N(t) = 1 - ct$.
Wrong! Conservation of probability driven by depletion:
a given nucleus can only decay once

Correctly
$P(t) = cN(t) \Longrightarrow N(t) = \exp(-ct)$
i.e. exponential dampening
$P(t) = c\exp(-ct)$

For $P(t) = f(t)N(t)$, $f(t) \geq 0$, this generalizes to

$$P(t) = f(t)\ \exp\left(-\int_0^t f(t')\,\mathrm{d}t'\right)$$

which can be Monte Carlo-simulated using the veto algorithm.

The Sudakov form factor

Correspondingly, with $Q \sim 1/t$ (Heisenberg)

$$\begin{aligned} \mathrm{d}\mathcal{P}_{a\to bc} &= \frac{\alpha_\mathrm{s}}{2\pi}\frac{\mathrm{d}Q^2}{Q^2}\,P_{a\to bc}(z)\,\mathrm{d}z \\ &\times \exp\left(-\sum_{b,c}\int_{Q^2}^{Q^2_\mathrm{max}}\frac{\mathrm{d}Q'^2}{Q'^2}\int\frac{\alpha_\mathrm{s}}{2\pi}\,P_{a\to bc}(z')\,\mathrm{d}z'\right) \end{aligned}$$

where the exponent is (one definition of) the Sudakov form factor.

A given parton can only branch once, i.e. if it did not already do so

Note that $\sum_{b,c}\int\int\ \mathrm{d}\mathcal{P}_{a\to bc} \equiv 1 \Rightarrow$ *convenient for Monte Carlo* ($\equiv 1$ if extended over whole phase space, else possibly nothing happens before you reach $Q_0 \approx 1$ GeV).

Matching Showers and Matrix Elements

Showers: approximation to matrix elements (MEs).
Shower emissions $\approx$ real part of higher orders.
Shower Sudakovs $\approx$ virtual (loop) part of higher orders.

Real MEs manageable, virtual MEs tough, so want to combine.
Key issue is to avoid doublecounting.

Active and rich field of study, with many methods, e.g.:

- Merging: correct first shower emission to MEs, using veto algorithm.
- CKKW(-L): generate several multiplicities with real MEs, use shower Sudakovs to include virtual corrections.
- MLM: real MEs as above, but veto events that showers migrate to another jet multiplicity.
- MC@NLO: split σ_{NLO} into real LO+1 event and real+virtual LO ones.
- POWHEG: like merging, with overall rate normalized to σ_{NLO}.

Parton Distribution Functions (PDFs)

Hadrons are composite: $\mathrm{p} = \mathrm{uud}$ + gluons + sea $\mathrm{q\overline{q}}$.
Higher virtuality scale $Q \Rightarrow$ more partons resolved.
$f_i(x, Q^2)$ = number density of partons i at momentum fraction x and probing scale Q^2.

Initial conditions at small Q_0^2 unknown: nonperturbative.

Resolution dependence perturbative, by DGLAP:

DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi)

$$\frac{\mathrm{d}f_b(x, Q^2)}{\mathrm{d}(\ln Q^2)} = \sum_a \int_x^1 \frac{\mathrm{d}z}{z} f_a(y, Q^2) \frac{\alpha_\mathrm{s}}{2\pi} P_{a\to bc}\left(z = \frac{x}{y}\right)$$

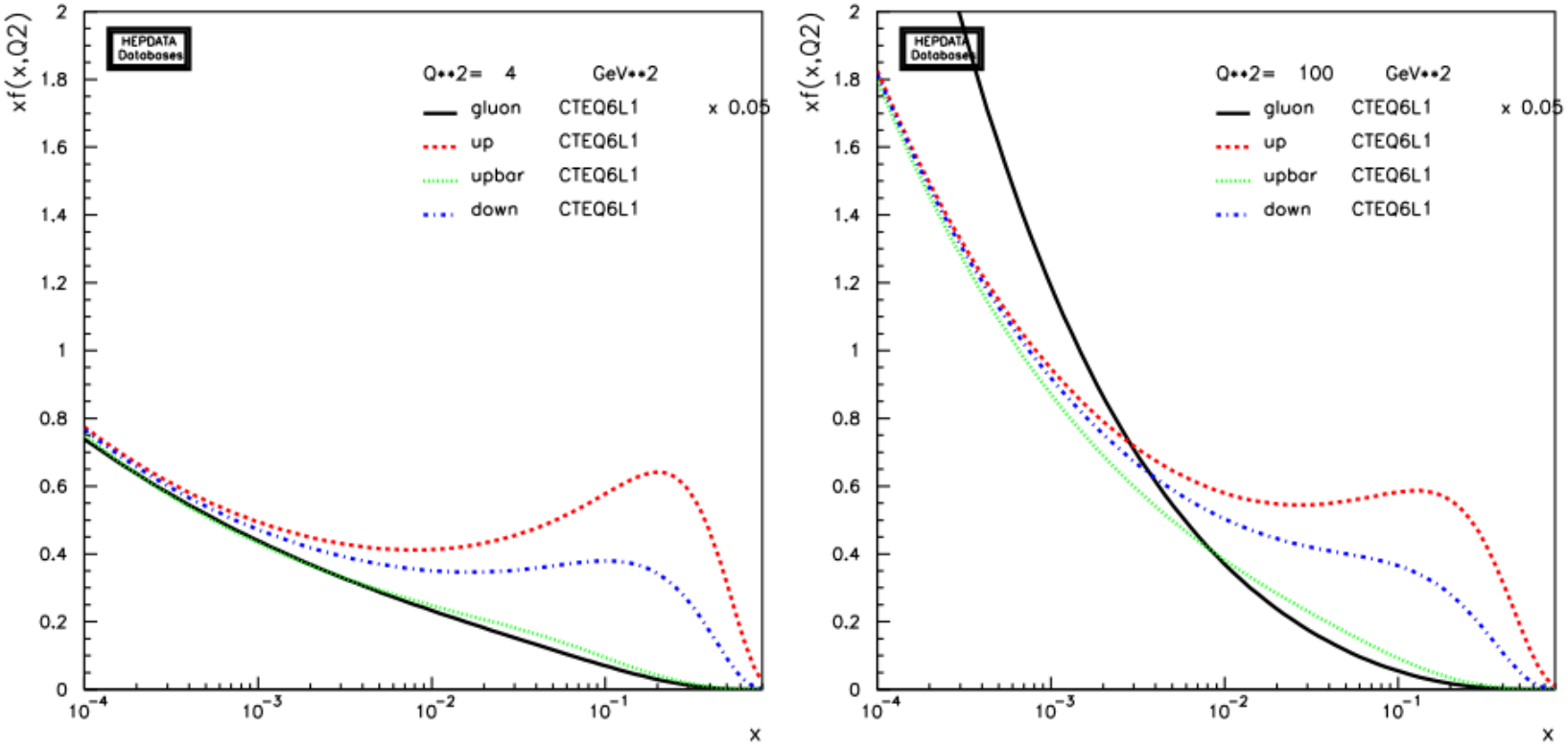
$$\sigma = \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2\, f_i(x_1, Q^2)\, f_j(x_2, Q^2)\, \hat{\sigma}(x_1 x_2 E_\mathrm{cm}^2, Q^2)$$

Continuous ongoing activity to provide best overall fit to data, consistent with theoretical evolution: CTEQ, MSTW, NNPDF, ...

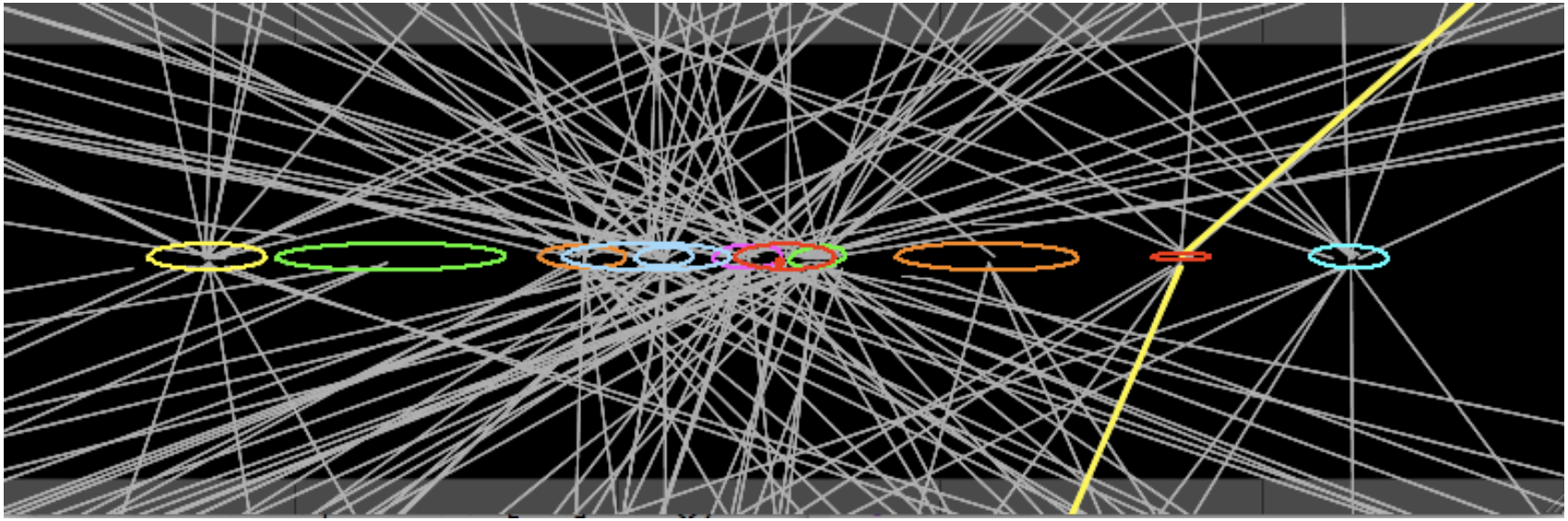
PDF examples

Convenient plotting interface:

http://hepdata.cedar.ac.uk/pdf/pdf3.html



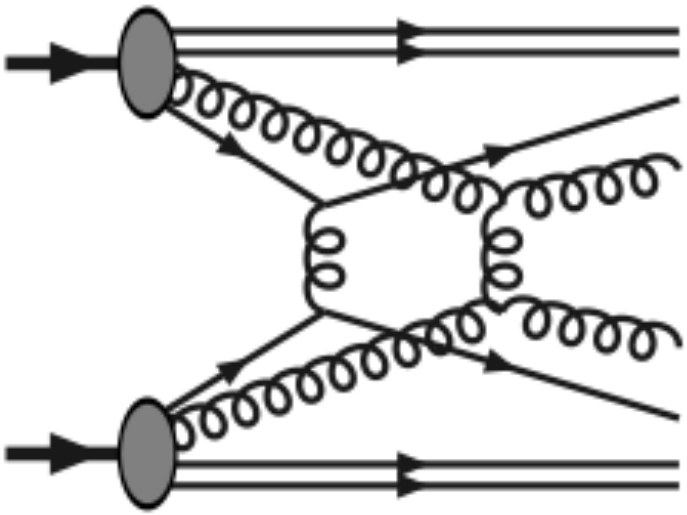
What is Pileup?



Protons in LHC collected in bunches.
1 bunch $\approx 1.5 \times 10^{11}$ protons.
Two bunches passing through each other $\Rightarrow$ several pp collisions.
Current LHC machine conditions $\Rightarrow \mu = \langle n \rangle \approx 20$.
Pileup introduces no new physics, so is uninteresting here.

But analogy: a proton is a bunch of partons (quarks and gluons)
so several parton–parton collisions when two protons cross.

MultiParton Interactions (MPIs)



Many parton-parton interactions per pp event: MPI.

Most have small $p_\perp, \sim 2$ GeV $\Rightarrow$ not visible as separate jets, but contribute to event activity.

Solid evidence that MPIs play central role for event structure.

Problem:

$$\sigma_{\rm int} = \iiint \mathrm{d}x_1\, \mathrm{d}x_2\, \mathrm{d}p_\perp^2\, f_1(x_1, p_\perp^2)\, f_2(x_2, p_\perp^2)\, \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}p_\perp^2} = \infty$$

since $\int \mathrm{d}x\, f(x, p_\perp^2) = \infty$ and $\mathrm{d}\hat{\sigma}/\mathrm{d}p_\perp^2 \approx 1/p_\perp^4 \to \infty$ for $p_\perp \to 0$.

Requires empirical dampening at small $p_\perp$, owing to colour screening (proton finite size).

Many aspects beyond pure theory $\Rightarrow$ model building.

Hadronization

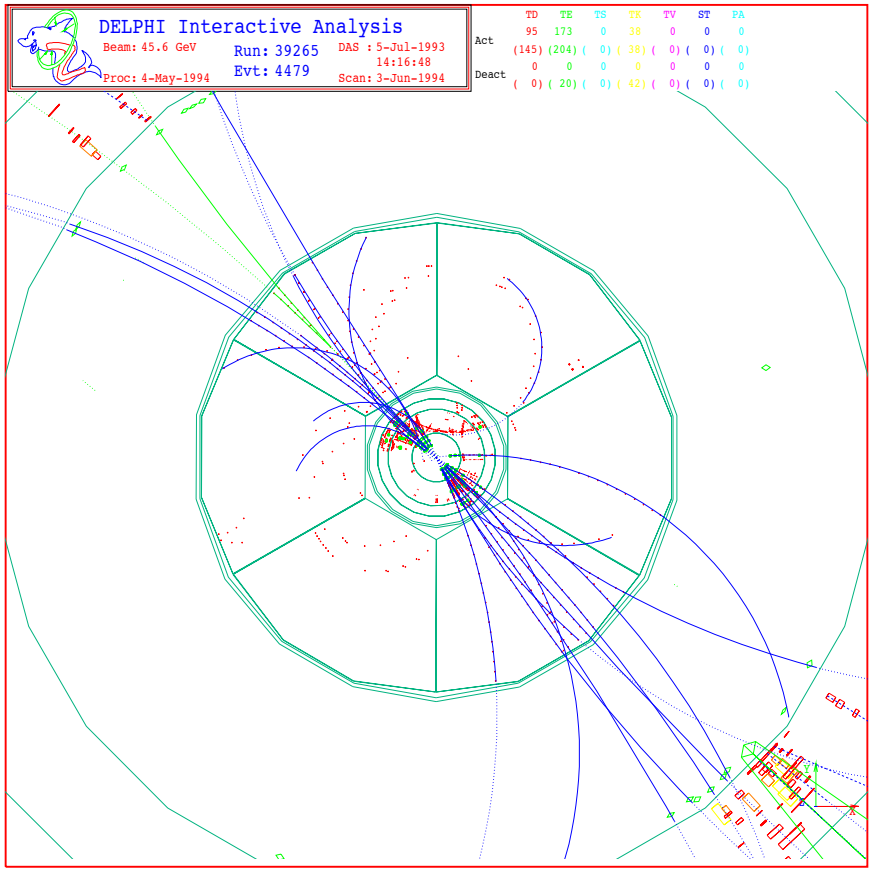
Nonperturbative $\Longrightarrow$ not calculable from first principles!

Model building = ideology + "cookbook"

Two common approaches:

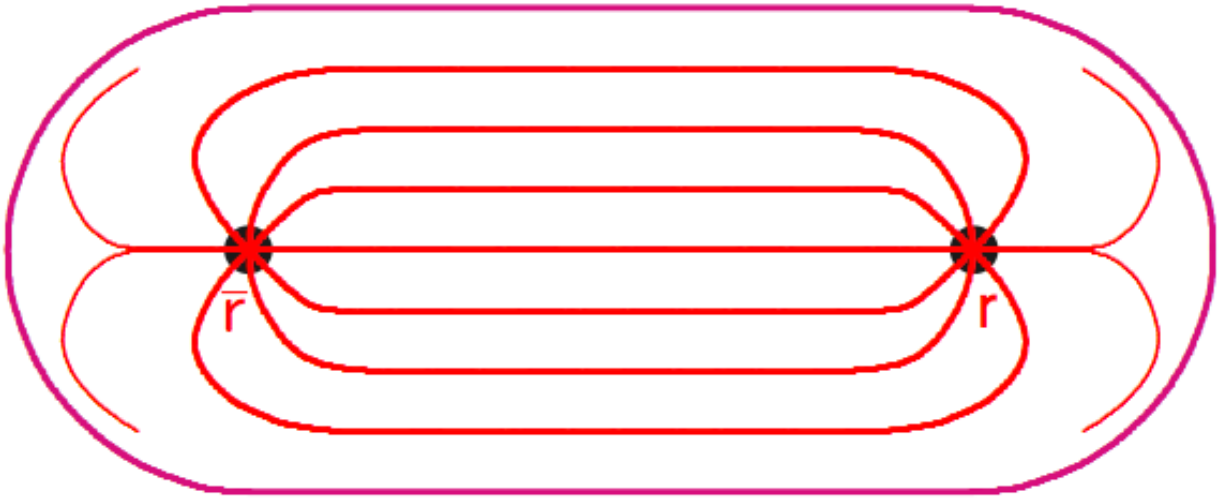
- String fragmentation: most ideological
- Cluster fragmentation: simplest

Both contain many parameters to be determined from data, preferably LEP $e^+e^- \to \gamma^*/Z^0 \to q\bar{q}/q\bar{q}g/\ldots$



The Lund String Model

In QCD, for large charge separation, field lines seem to be compressed to tubelike region(s) $\Rightarrow$ **string(s)**



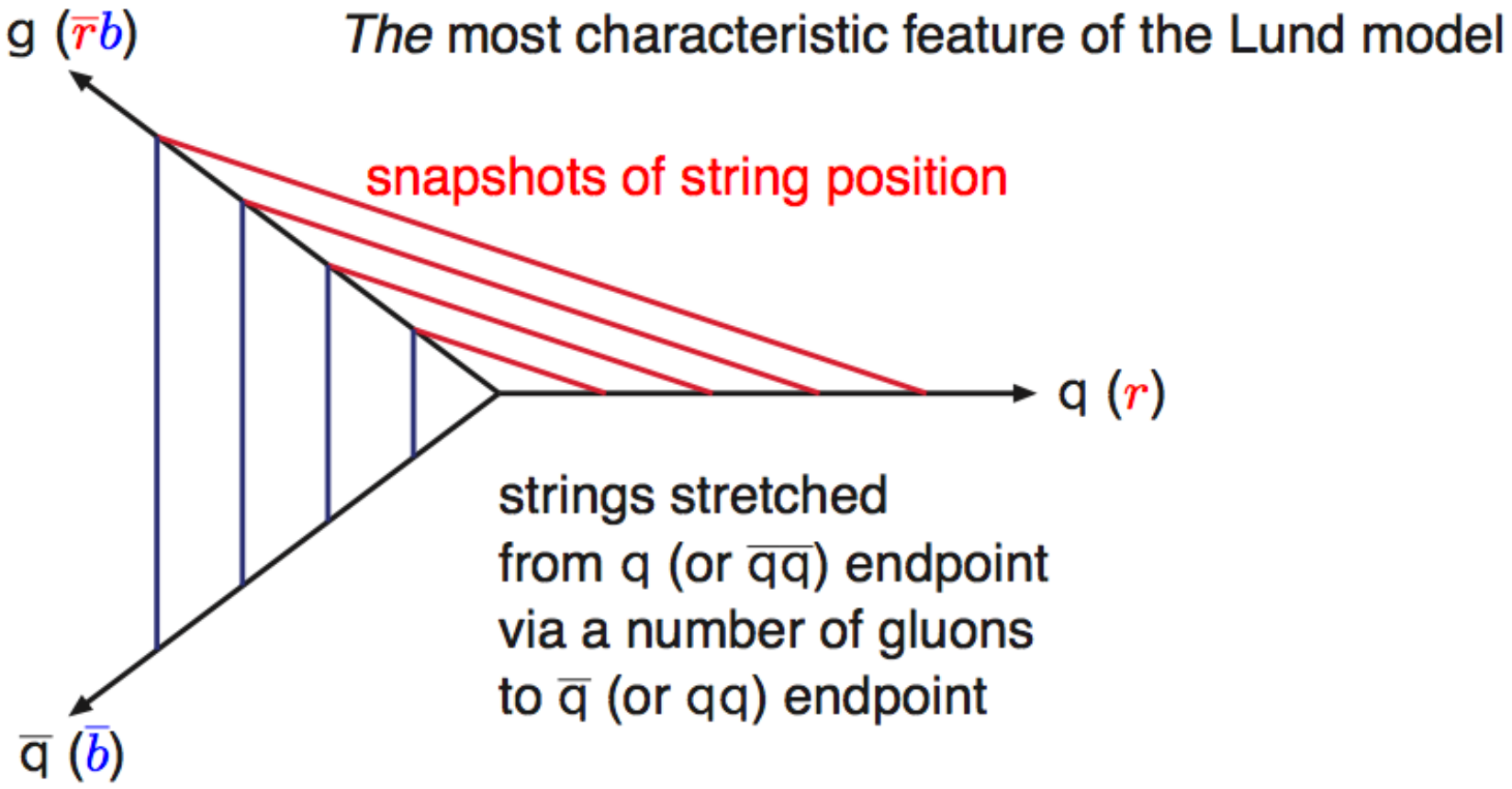
by self-interactions among soft gluons in the "vacuum".

Gives linear confinement with string tension:

$F(r) \approx \mathrm{const} = \kappa \approx 1\ \mathrm{GeV/fm} \quad \Longleftrightarrow \quad V(r) \approx \kappa r$

String breaks into hadrons along its length,
with roughly uniform probability in rapidity,
by formation of new $\mathrm{q\bar{q}}$ pairs that screen endpoint colours.

The Lund Gluon Picture

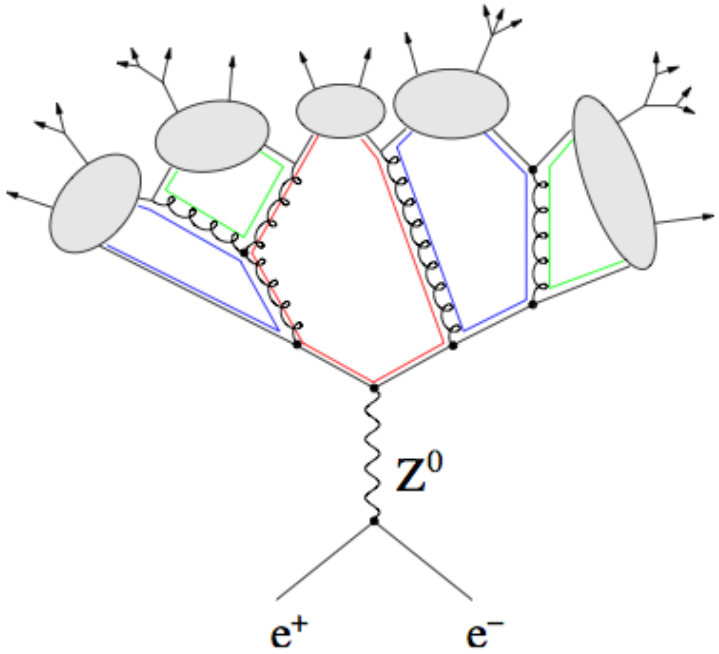


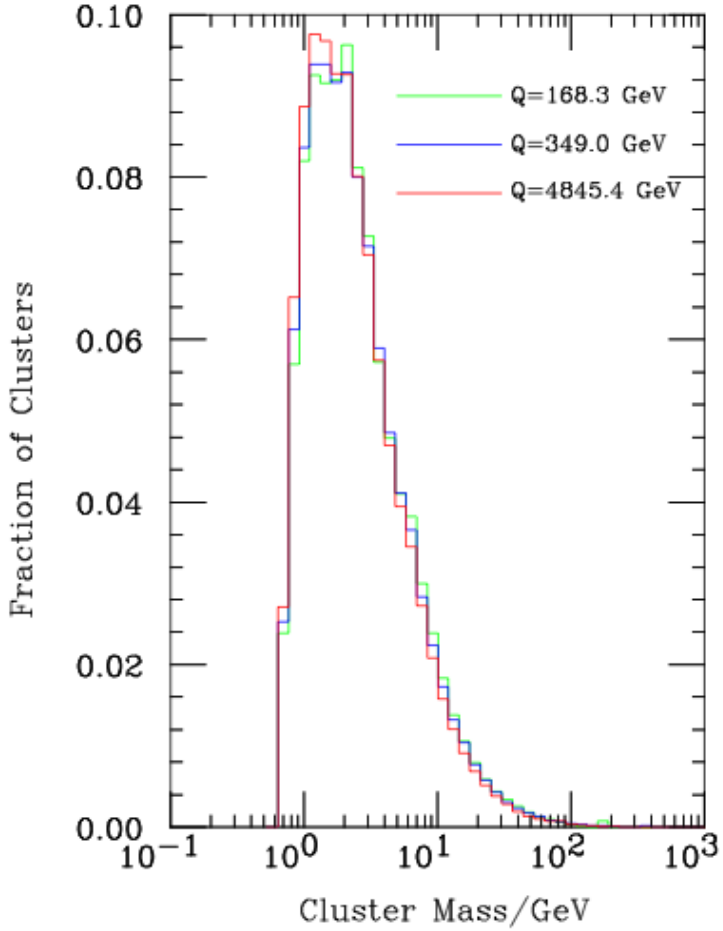
Gluon = kink on string

Force ratio gluon/ quark = 2,
cf. QCD $N_C/C_F = 9/4$, $\rightarrow 2$ for $N_C \rightarrow \infty$
No new parameters introduced for gluon jets!

The HERWIG Cluster Model

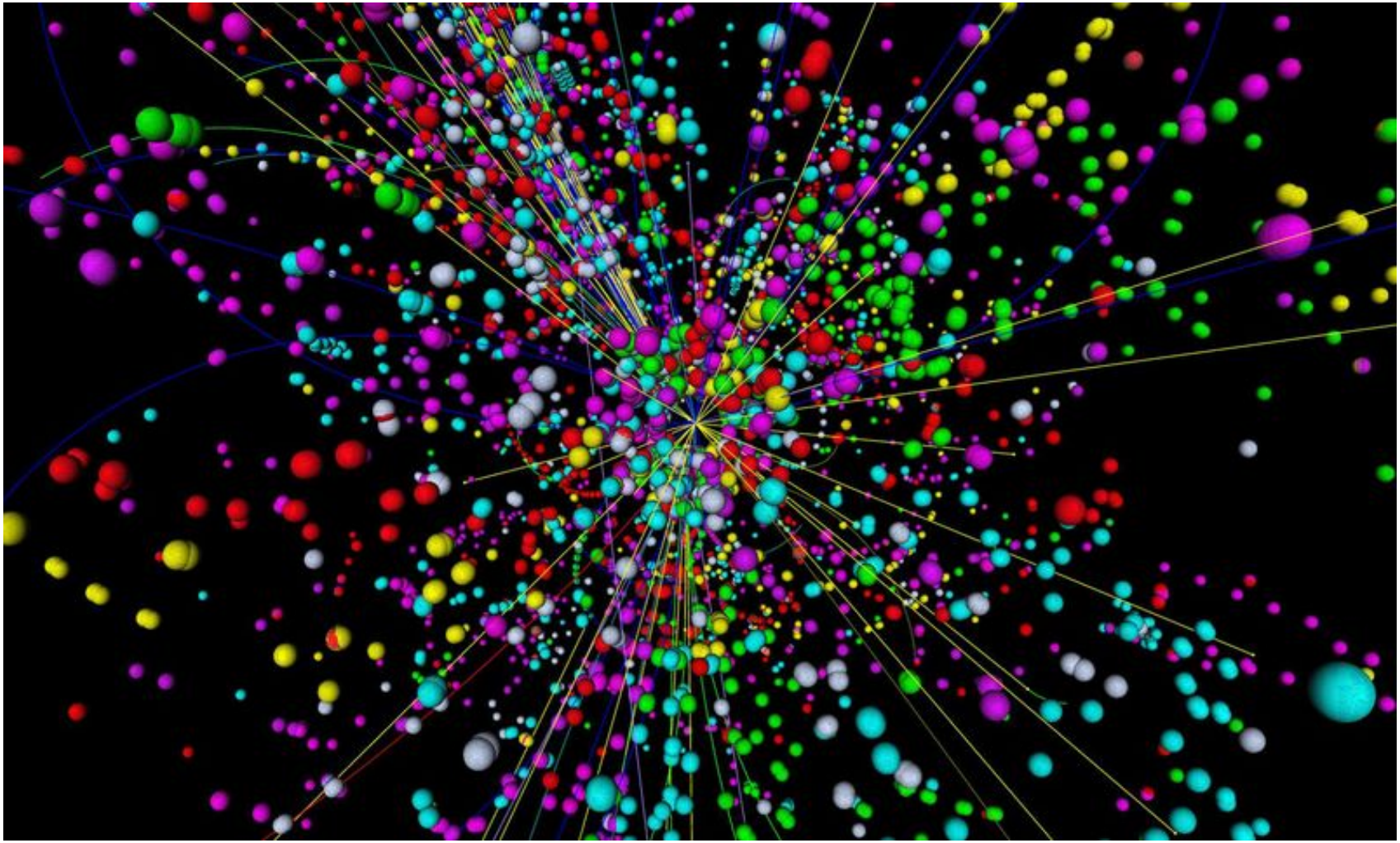
"Preconfinement":
colour flow is local
in coherent shower evolution





1. Introduce forced $g \to q\bar{q}$ branchings
2. Form colour singlet clusters
3. Clusters decay isotropically to 2 hadrons according to phase space weight

Event Generators



The Generator Landscape

	General-Purpose	Specialized
Hard Processes	HERWIG	MadGraph, AlpGen, . . .
Resonance Decays		HDECAY, . . .
Parton Showers	PYTHIA	Ariadne/LDC, VINCIA, . . .
Underlying Event	SHERPA	PHOJET/DPMJET
Hadronization	.....	none (?)
Ordinary Decays		TAUOLA, EvtGen

Specialized often best at given task, but need General-Purpose core

The Workhorses: What are the Differences?

HERWIG, PYTHIA and SHERPA offer convenient frameworks for LHC physics studies, but with slightly different emphasis:

PYTHIA (successor to JETSET, begun in 1978):

- originated in hadronization studies: the Lund string
- leading in development of MPI for MB/UE
- pragmatic attitude to showers & matching

HERWIG (successor to EARWIG, begun in 1984):

- originated in coherent-shower studies (angular ordering)
- cluster hadronization & underlying event pragmatic add-on
- large process library with spin correlations in decays

SHERPA (APACIC++/AMEGIC++, begun in 2000):

- own matrix-element calculator/generator
- extensive machinery for CKKW ME/PS matching
- hadronization & min-bias physics under development

PYTHIA & HERWIG originally in Fortran, now all C++

A Commercial: MCnet

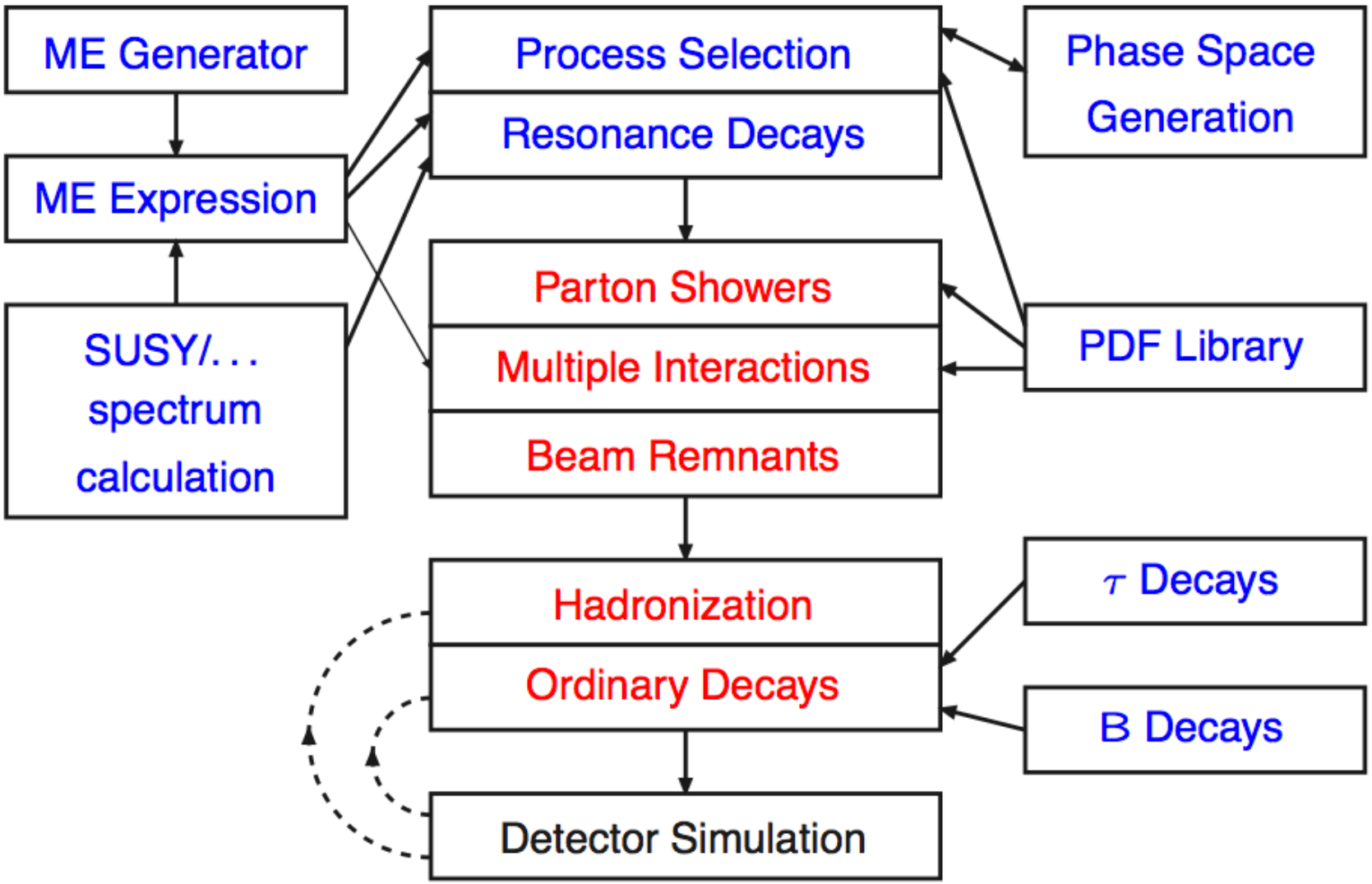
⋆ "Trade Union" of (QCD) Event Generator developers ⋆
Collects HERWIG, SHERPA and PYTHIA.
Also ThePEG, ARIADNE, VINCIA, . . . ,
generator validation (RIVET) and tuning (PROFESSOR)
(CERN, Durham, Lund, Karlsruhe, UC London, + associated).

⋆ Funded by EU Marie Curie training network 2007–2010 ⋆
New applications for continued activities: no luck so far.

⋆ Annual Monte Carlo school: ⋆
Next: MCnet-LPCC, 23–27 July 2012, CERN
+ Lectures on QCD & Generators at many other schools +
Much relevant material at `http://www.montecarlonet.org/`

MCPLOTS: repository of comparisons between generators and data, based on RIVET, see `http://mcplots.cern.ch/`

The Bigger Picture



Need standardized interfaces: see next!

PDG Particle Codes

A. Fundamental objects

1	d	11	e^-	21	g				
2	u	12	ν_e	22	γ	32	Z'^0		
3	s	13	μ^-	23	Z^0	33	Z''^0		
4	c	14	ν_μ	24	W^+	34	W'^+		
5	b	15	τ^-	25	h^0	35	H^0	37	H^+
6	t	16	ν_τ			36	A^0	39	Graviton

add $-$ sign for antiparticle, where appropriate

B. Mesons

$100\,|q_1| + 10\,|q_2| + (2s+1)$ with $|q_1| \geq |q_2|$

C. Baryons

$1000\,q_1 + 100\,q_2 + 10\,q_3 + (2s+1)$

with $q_1 \geq q_2 \geq q_3$, or Λ-like $q_1 \geq q_3 \geq q_2$

. . . and many more

Interfaces

- LHA: Les Houches Accord,
  transfers info on processes, cross sections, parton-level events, . . . , via two Fortran commonblocks
- LHEF: Les Houches Event Files,
  same information, but stored as a single plaintext file
- HepMC: output of complete generated events
  (intermediate stages and final state with hundreds of particles)
  for subsequent detector simulation or analysis
- SLHA: SUSY Les Houches Accord,
  file with info on SUSY (or other BSM) model:
  parameters, masses, mixing matrices, branching ratios, ...
- LHAPDF: uniform interface to PDF parametrizations
- Binoth LHA: one-loop virtual corrections
- FeynRules: nascent standard for input of Lagrangian and output of Feynman rules
- . . .

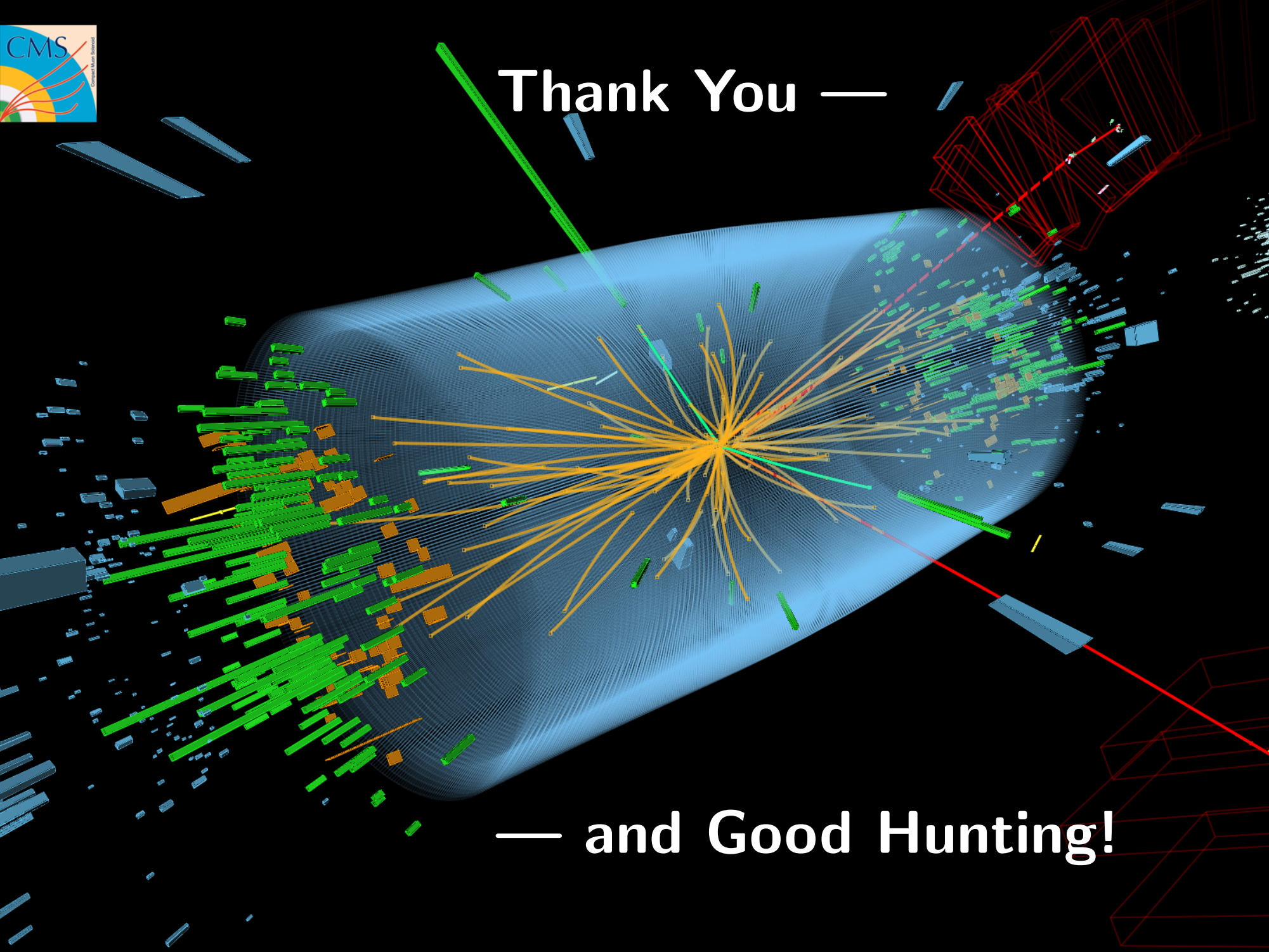
The Road Ahead

- Event generators crucial since the start of LHC studies.
- Qualitatively predictive already 25 years ago.
- Quantitatively steady progress, continuing today:
  - ⋆ continuous dialogue with experimental community,
  - ⋆ more powerful computational techniques and computers,
  - ⋆ new ideas.
- As LHC needs to study more rare phenomena and more subtle effects, generators must keep up by increased precision.

But it often happens that the physics simulations provided by the Monte Carlo generators carry the authority of data itself. They look like data and feel like data, and if one is not careful they are accepted as if they were data.

J.D. Bjorken

from a talk given at the 75th anniversary celebration of the Max-Planck Institute of Physics, Munich, Germany, December 10th, 1992. As quoted in: Beam Line, Winter 1992, Vol. 22, No. 4


Thank You —
— and Good Hunting!