Introduction to Monte Carlo Techniques in High Energy Physics

Torbjörn Sjöstrand

How are complicated multiparticle events created? How can such events be simulated with a computer?
Lectures Overview

yesterday:  

Introduction  
the Standard Model  
Quantum Mechanics  
the role of Event Generators

Monte Carlo  
random numbers  
integration  
simulation

today:  

Physics  
hard interactions  
parton showers  
multiparton interactions  
hadronization

Generators  
HERWIG, PYTHIA, SHERPA  
MadGraph, AlpGen, ...  
common standards
Warning: schematic only, everything simplified, nothing to scale, . . .

Incoming beams: parton densities
Hard subprocess: described by matrix elements
Resonance decays: correlated with hard subprocess
Initial-state radiation: spacelike parton showers
Final-state radiation: timelike parton showers
Multiple parton–parton interactions . . .
...with its initial- and final-state radiation
Beam remnants and other outgoing partons
Everything is connected by colour confinement strings
Recall! Not to scale: strings are of hadronic widths
The strings fragment to produce primary hadrons
Many hadrons are unstable and decay further
These are the particles that hit the detector
Hard Processes

Defines the character of the event: QCD, top, $Z^0$, $W^\pm$, $H^0$, SUSY, ED, TC, ...

Obtained by perturbation theory:

$$\hat{\sigma}_{Z^0} = \alpha_{em} M_0 + \alpha_{em} \alpha_s M_1 + \alpha_{em} \alpha_s^2 M_2 + \ldots$$

$$= \text{LO} + \text{NLO} + \text{NNLO} + \ldots$$

Involves subtle cancellations between real and virtual contributions:

I. Lowest order, \( \mathcal{O}(\alpha_{em}) \):

\( q\bar{q} \rightarrow Z^0 \)

II. First-order real, \( \mathcal{O}(\alpha_{em}\alpha_s) \):

\( q\bar{q} \rightarrow Z^0g \text{ etc.} \)

III. First-order virtual, \( \mathcal{O}(\alpha_{em}\alpha_s) \):

\( q\bar{q} \rightarrow Z^0 \text{ with loops} \)

These days

- LO easy
- NLO tough but doable
- NNLO only in very few cases
Accelerated electric charges radiate photons, see e.g. J.D. Jackson, *Classical Electrodynamics.*

A charge $ze$ that changes its velocity vector from $\beta$ to $\beta'$ radiates a spectrum of photons that depends on its trajectory. In the long-wavelength limit it reduces to

$$\lim_{\omega \to 0} \frac{d^2 I}{d\omega d\Omega} = \frac{z^2 e^2}{4\pi^2} \left| \epsilon^* \left( \frac{\beta'}{1 - n\beta'} - \frac{\beta}{1 - n\beta} \right) \right|^2$$

where $n$ is a vector on the unit sphere $\Omega$, $\omega$ is the energy of the radiated photon, and $\epsilon$ its polarization.

1. For fast particles radiation collinear with the initial ($\beta$) and final ($\beta'$) directions is strongly enhanced.
2. $dN/d\omega = (1/\omega)dI/d\omega \propto 1/\omega$ so infinitely many infinitely soft photons are emitted, but the net energy taken away is finite.
3. For $\omega \to 0$ the radiation pattern is independent of the spin of the radiator $\Rightarrow$ universality.
QCD \approx QED with \quad e \rightarrow e\gamma \Rightarrow q \rightarrow qg
\quad \alpha_{em} \Rightarrow (4/3)\alpha_s

More precisely:

\[
d\mathcal{P}_{q \rightarrow qg} \approx \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} \frac{4}{3} \frac{1 + z^2}{1 - z} \, dz
\]

where

- mass (or collinear) singularity:

\[
\frac{dQ^2}{Q^2} \approx \frac{d\theta^2}{\theta^2} \approx \frac{dm^2}{m^2} \approx \frac{dp^2_{\perp}}{p^2_{\perp}}
\]

- soft singularity:

\[
z \approx \frac{E_{q, after}}{E_{q, before}} = 1 - \frac{\omega}{E_{q, before}} \quad \text{so} \quad \frac{dz}{1 - z} = \frac{d\omega}{\omega}
\]
But QCD is non-Abelian so additionally

- \( g \to gg \) similarly divergent
- \( \alpha_s(Q^2) \) diverges for \( Q^2 \to 0 \)

DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi)

\[
\frac{dP_{a\to bc}}{dQ^2} = \frac{\alpha_s(Q^2)}{2\pi} \frac{dQ^2}{Q^2} P_{a\to bc}(z) \, dz
\]

\[
P_{q\to qg} = \frac{4}{3} \frac{1 + z^2}{1 - z}
\]

\[
P_{g\to gg} = \frac{3}{z(1 - z)} \frac{(1 - z(1 - z))^2}{(1 - z(1 - z))^2}
\]

\[
P_{g\to q\bar{q}} = \frac{n_f}{2} \left( z^2 + (1 - z)^2 \right) \quad (n_f = \text{no. of quark flavours})
\]
The iterative structure

Generalizes to many consecutive emissions if strongly ordered,
\[ Q_1^2 \gg Q_2^2 \gg Q_3^2 \ldots \approx \text{time-ordered} \].
To cover “all” of phase space use DGLAP in whole region
\[ Q_1^2 > Q_2^2 > Q_3^2 \ldots, \text{ although only approximately valid.} \]

Must be clever when you write a shower algorithm.

Iteration gives
final-state
parton showers:

Need soft/collinear cuts to stay away from nonperturbative physics.
Details model-dependent, but around 1 GeV scale.
The Parton-Shower Approach

Showers: approximation method for higher-order matrix elements.

Universality: any matrix element reduces to DGLAP in collinear limit.

\[ 2 \rightarrow n \approx (2 \rightarrow 2) \oplus \text{ISR} \oplus \text{FSR} \]

ISR = Initial-State Radiation: \( Q_i^2 \sim -m^2 > 0 \) increasing

FSR = Final-State Radiation: \( Q_i^2 \sim m^2 > 0 \) decreasing
Recall discussion on radioactive decays from Lecture 1:

Naively $P(t) = c \implies N(t) = 1 - ct$. Wrong! Conservation of probability driven by depletion:

A given nucleus can only decay once

Correctly

$P(t) = cN(t) \implies N(t) = \exp(-ct)$

i.e. exponential dampening

$P(t) = c \exp(-ct)$

For $P(t) = f(t)N(t)$, $f(t) \geq 0$, this generalizes to

$$P(t) = f(t) \exp \left( - \int_0^t f(t') \, dt' \right)$$

which can be Monte Carlo-simulated using the veto algorithm.
Correspondingly, with $Q \sim 1/t$ (Heisenberg)

$$d\mathcal{P}_{a \rightarrow bc} = \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} P_{a \rightarrow bc}(z) \, dz$$

$$\times \exp \left( -\sum_{b,c} \int_{Q^2}^{Q_{\max}^2} \frac{dQ'^2}{Q'^2} \int \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z') \, dz' \right)$$

where the exponent is (one definition of) the Sudakov form factor.

A given parton can only branch once, i.e. if it did not already do so

Note that $\sum_{b,c} \int \int d\mathcal{P}_{a \rightarrow bc} \equiv 1 \Rightarrow convenient\ for\ Monte\ Carlo$

($\equiv 1$ if extended over whole phase space, else possibly nothing happens before you reach $Q_0 \approx 1$ GeV).
Showers: approximation to matrix elements (MEs).
Shower emissions \( \approx \) real part of higher orders.
Shower Sudakovs \( \approx \) virtual (loop) part of higher orders.

Real MEs manageable, virtual MEs tough, so want to combine.
Key issue is to avoid doublecounting.

Active and rich field of study, with many methods, e.g.:
- Merging: correct first shower emission to MEs, using veto algorithm.
- CKKW(-L): generate several multiplicities with real MEs, use shower Sudakovs to include virtual corrections.
- MLM: real MEs as above, but veto events that showers migrate to another jet multiplicity.
- MC@NLO: split \( \sigma_{NLO} \) into real LO+1 event and real+virtual LO ones.
- POWHEG: like merging, with overall rate normalized to \( \sigma_{NLO} \).
Parton Distribution Functions (PDFs)

Hadrons are composite: $p = uu d + \text{gluons} + \text{sea } q\bar{q}$.
Higher virtuality scale $Q \Rightarrow$ more partons resolved.

$f_i(x, Q^2) =$ number density of partons $i$ at momentum fraction $x$ and probing scale $Q^2$.

Initial conditions at small $Q^2_0$ unknown: nonperturbative.
Resolution dependence perturbative, by DGLAP:

DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi)

$$\frac{df_b(x, Q^2)}{d(ln Q^2)} = \sum_a \int_x^1 \frac{dz}{z} f_a(y, Q^2) \frac{\alpha_s}{2\pi} P_{a \rightarrow bc} \left( z = \frac{x}{y} \right)$$

$$\sigma = \int_0^1 dx_1 \int_0^1 dx_2 f_i(x_1, Q^2) f_j(x_2, Q^2) \hat{\sigma}(x_1 x_2 E_{cm}^2, Q^2)$$

Continuous ongoing activity to provide best overall fit to data, consistent with theoretical evolution: CTEQ, MSTW, NNPDF, . . .
PDF examples

Convenient plotting interface:
http://hepdata.cedar.ac.uk/pdf/pdf3.html
What is Pileup?

Protons in LHC collected in bunches.
1 bunch \( \approx 1.5 \times 10^{11} \) protons.
Two bunches passing through each other \( \Rightarrow \) several pp collisions.
Current LHC machine conditions \( \Rightarrow \mu = \langle n \rangle \approx 20 \).
Pileup introduces no new physics, so is uninteresting here.

But analogy: a proton is a bunch of partons (quarks and gluons) so several parton–parton collisions when two protons cross.
Many parton-parton interactions per pp event: MPI.

Most have small $p_\perp$, $\sim 2$ GeV
\[ \Rightarrow \] not visible as separate jets, but contribute to event activity.

Solid evidence that MPIs play central role for event structure.

Problem:

$$\sigma_{\text{int}} = \int \int \int dx_1 \, dx_2 \, dp_\perp^2 \, f_1(x_1, p_\perp^2) \, f_2(x_2, p_\perp^2) \, \frac{d\hat{\sigma}}{dp_\perp^2} = \infty$$

since $\int dx \, f(x, p_\perp^2) = \infty$ and $d\hat{\sigma}/dp_\perp^2 \approx 1/p_\perp^4 \to \infty$ for $p_\perp \to 0$.

Requires empirical dampening at small $p_\perp$, owing to colour screening (proton finite size).

Many aspects beyond pure theory $\Rightarrow$ model building.
Nonperturbative $\Rightarrow$ not calculable from first principles!

Model building = ideology + “cookbook”

Two common approaches:

- **String fragmentation**: most ideological
- **Cluster fragmentation**: simplest

Both contain many parameters to be determined from data, preferably LEP $e^+e^- \rightarrow \gamma^*/Z^0 \rightarrow q\bar{q}/q\bar{q}g/\ldots$
In QCD, for large charge separation, field lines seem to be compressed to tubelike region(s) ⇒ string(s)

by self-interactions among soft gluons in the “vacuum”.

Gives linear confinement with string tension:
\[ F(r) \approx \text{const} = \kappa \approx 1 \text{ GeV/fm} \quad \iff \quad V(r) \approx \kappa r \]

String breaks into hadrons along its length, with roughly uniform probability in rapidity, by formation of new $q\bar{q}$ pairs that screen endpoint colours.
The Lund Gluon Picture

Gluon = kink on string

Force ratio gluon/ quark = 2,
cf. QCD $N_C/C_F = 9/4$, $\rightarrow 2$ for $N_C \rightarrow \infty$

No new parameters introduced for gluon jets!
The HERWIG Cluster Model

“Preconfinement”: colour flow is local in coherent shower evolution

1. Introduce forced $g \rightarrow q\bar{q}$ branchings
2. Form colour singlet clusters
3. Clusters decay isotropically to 2 hadrons according to phase space weight
Event Generators
# The Generator Landscape

<table>
<thead>
<tr>
<th>Hard Processes</th>
<th>General-Purpose</th>
<th>Specialized</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resonance Decays</td>
<td>HERWIG</td>
<td>MadGraph, AlpGen, …</td>
</tr>
<tr>
<td>Parton Showers</td>
<td>PYTHIA</td>
<td>HDECAY, …</td>
</tr>
<tr>
<td>Underlying Event</td>
<td>SHERPA</td>
<td>Ariadne/LDC, VINCIA, …</td>
</tr>
<tr>
<td>Hadronization</td>
<td>……</td>
<td>PHOJET/DPMJET</td>
</tr>
<tr>
<td>Ordinary Decays</td>
<td></td>
<td>none (?)</td>
</tr>
</tbody>
</table>

Specialized often best at given task, but need General-Purpose core
HERWIG, PYTHIA and SHERPA offer convenient frameworks for LHC physics studies, but with slightly different emphasis:

**PYTHIA (successor to JETSET, begun in 1978):**
- originated in hadronization studies: the Lund string
- leading in development of MPI for MB/UE
- pragmatic attitude to showers & matching

**HERWIG (successor to EARWIG, begun in 1984):**
- originated in coherent-shower studies (angular ordering)
- cluster hadronization & underlying event pragmatic add-on
- large process library with spin correlations in decays

**SHERPA (APACIC++/AMEGIC++, begun in 2000):**
- own matrix-element calculator/generator
- extensive machinery for CKKW ME/PS matching
- hadronization & min-bias physics under development

**PYTHIA & HERWIG originally in Fortran, now all C++**
A Commercial: MCnet

★ “Trade Union” of (QCD) Event Generator developers ★
Collects HERWIG, SHERPA and PYTHIA.
Also ThePEG, ARIADNE, VINCIA, . . . ,
generator validation (RIVET) and tuning (PROFESSOR)
(CERN, Durham, Lund, Karlsruhe, UC London, + associated).

★ Funded by EU Marie Curie training network 2007–2010 ★
New applications for continued activities: no luck so far.
★ Annual Monte Carlo school: ★
Next: MCnet-LPCC, 23–27 July 2012, CERN
+ Lectures on QCD & Generators at many other schools +
Much relevant material at http://www.montecarlonet.org/

MCPLOTS: repository of comparisons between generators and
data, based on RIVET, see http://mcplots.cern.ch/
The Bigger Picture

ME Generator

ME Expression

SUSY/... spectrum calculation

Process Selection

Resonance Decays

Phase Space Generation

PDF Library

Parton Showers

Multiple Interactions

Beam Remnants

Hadronization

Ordinary Decays

Detector Simulation

τ Decays

B Decays

Need standardized interfaces: see next!
A. Fundamental objects

| 1  | d   | 11  | e⁻   | 21  | g   | 32  | Z⁰   |
| 2  | u   | 12  | νₑ   | 22  | γ   | 33  | Z''⁰ |
| 3  | s   | 13  | μ⁻   | 23  | Z⁰  | 34  | W⁺   |
| 4  | c   | 14  | ν₁μ  | 24  | W⁺  | 35  | H⁰   |
| 5  | b   | 15  | τ⁻   | 25  | h⁰  | 36  | A⁰   |
| 6  | t   | 16  | ν₉τ  | 37  | H⁺  |

Add — sign for antiparticle, where appropriate

B. Mesons

100 | q₁ | + 10 | q₂ | + (2s + 1) with | q₁ | ≥ | q₂ |

C. Baryons

1000 q₁ + 100 q₂ + 10 q₃ + (2s + 1)

with q₁ ≥ q₂ ≥ q₃, or Λ-like q₁ ≥ q₃ ≥ q₂

...and many more
Interfaces

- **LHA**: Les Houches Accord, transfers info on processes, cross sections, parton-level events, ..., via two Fortran commonblocks.
- **LHEF**: Les Houches Event Files, same information, but stored as a single plaintext file.
- **HepMC**: output of complete generated events (intermediate stages and final state with hundreds of particles) for subsequent detector simulation or analysis.
- **SLHA**: SUSY Les Houches Accord, file with info on SUSY (or other BSM) model: parameters, masses, mixing matrices, branching ratios, ... .
- **LHAPDF**: uniform interface to PDF parametrizations.
- **Binoth LHA**: one-loop virtual corrections.
- **FeynRules**: nascent standard for input of Lagrangian and output of Feynman rules.
- ...
The Road Ahead

- Event generators crucial since the start of LHC studies.
- Qualitatively predictive already 25 years ago.
- Quantitatively steady progress, continuing today:
  ★ continuous dialogue with experimental community,
  ★ more powerful computational techniques and computers,
  ★ new ideas.
- As LHC needs to study more rare phenomena and more subtle effects, generators must keep up by increased precision.

But it often happens that the physics simulations provided by the Monte Carlo generators carry the authority of data itself. They look like data and feel like data, and if one is not careful they are accepted as if they were data.

J.D. Bjorken

from a talk given at the 75th anniversary celebration of the Max-Planck Institute of Physics, Munich, Germany, December 10th, 1992. As quoted in: Beam Line, Winter 1992, Vol. 22, No. 4
Thank You —
— and Good Hunting!