Multiple Scattering (MSC) in Geant4

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Intro.

- MICE performance predicted using the cooling formula (CF):
- G4MICE ≠ CF

  (see prev CMs)

$$\frac{d\varepsilon_n}{dz} = \frac{-\varepsilon_n}{\beta^2 E}\left\langle \frac{dE}{dX} \right\rangle + \frac{\beta_t \left(0.014\ \mathrm{GeV}\right)^2}{2\beta^3 E m_\mu X_0}$$

- MSC typically approx.:
  - CF uses Rossi-Greisen (1961)
  - Somewhat crude

$$\theta^{rms}_{plane} = 14\mathrm{MeV}\,\frac{\sqrt{X/X_0}}{p\beta}$$

- Two routes to MSC Monte Carlo
  - *detailed* – all collisions/interactions simulated (e.g. ELMS)
  - *condensed* – use angular / probability distributions (most MC codes)

Multiple Scattering Approximations

1961 - Rossi-Greisen

- 21 MeV in orig. paper
- Strong path length & Z dependence

$$\theta^{rms}_{plane} = 14\text{MeV} \frac{\sqrt{X / X_0}}{p\beta}$$

1974 - Highland (PDG) correction

$$\theta^{rms}_{plane} = \frac{13.6\text{MeV}}{p\beta} z \sqrt{x/X_0}\left[1 + 0.038 \ln(x/X_0)\right]$$

- removes path length dep.
- ***Z dep. remains however***

1990 - Lynch & Dahl expression

$$\theta^2_{rms,plane} = \frac{\chi_c^2}{1+F^2}\left[\frac{1+\nu}{\nu}\ln(1+\nu) - 1\right]$$

- *"much better approximation...agrees with Moliere scattering to 2% for all Z"*
- ...**Derivation / comparison with Highland / Moliere not supplied!**
- doesn't seem to have replaced Highland...

Moliere Theory (1949, Bethe 1952)

- Connects small angle Gaussian region with large angle single scattering.
- Described using two angles:

χ_a^2 **Screening angle**, below which scattering suppressed due to atomic screening

$$\chi_c^2 = 4\pi N t e^4 \, Z(Z+1) / (pv)^2$$

Critical angle, on average only 1 collision with $\theta > \chi_c^2$ through a scatterer.

- Bethe : $Z^2 \rightarrow Z(Z+1)$ to incl. inelastic scattering from atomic e^-s
  - Assumes e^- scattering shape same as scattering from the nucleus
  - *Fano* (1954) and others disputed this, but experiments by *Shen et al* (1979) supported Bethe.
- Thomas-Fermi model to describe e^- screening of the nucleus
  - Inaccurate in low Z materials!
- **MuScat** comparison
- Recent work by Tollestrup et al (2000), Fernow (1998, 2006)
  - Use their own form factors, avoiding T-F.

MuScat results

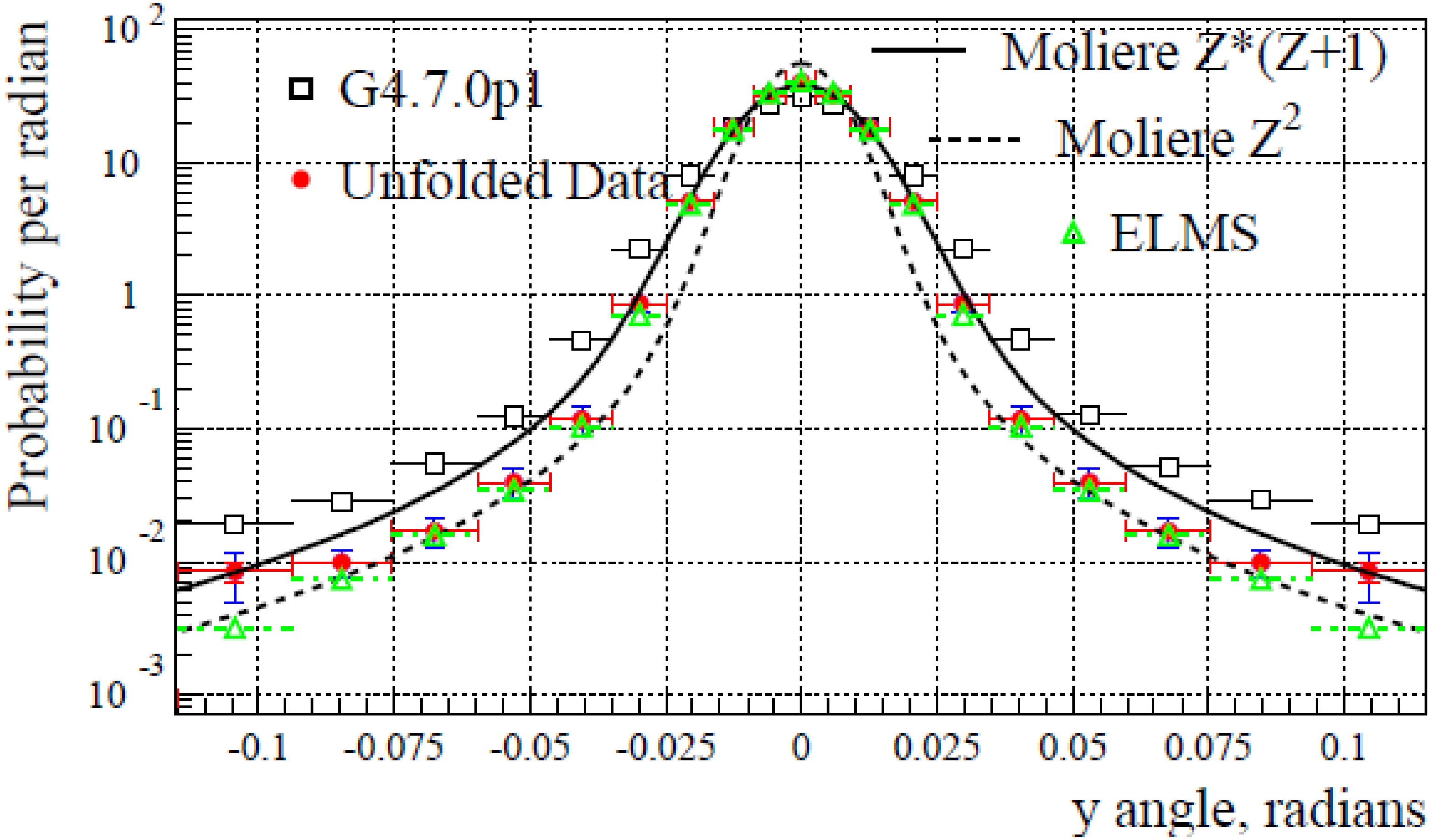


Fig. 22. The projected scattering angle distribution in data and simulation for 159 mm of liquid H_2.

Fernow (2006) using ICOOL

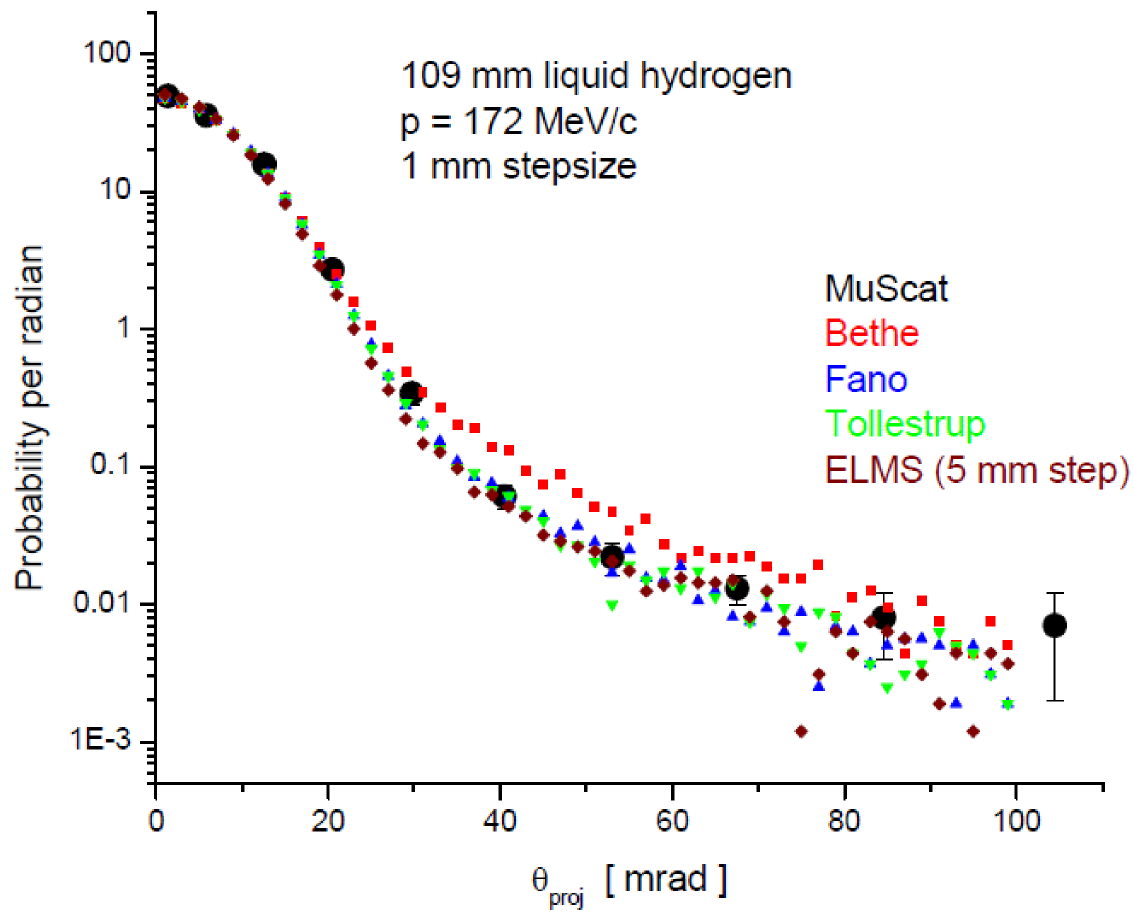


Figure 1. Comparison of models and data for liquid hydrogen.

Scattering in Geant4

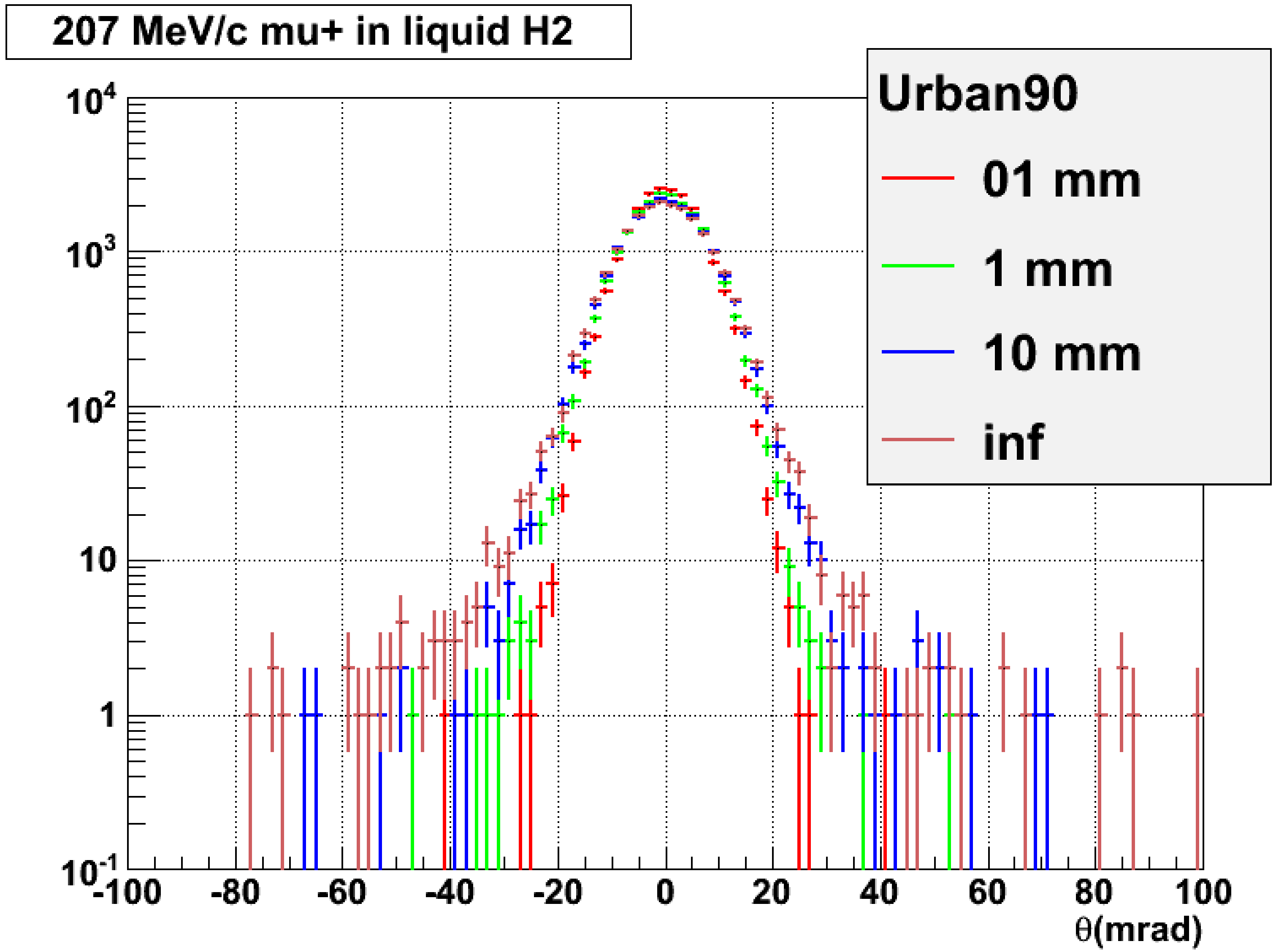
- Urban Model, based on **Lewis Theory**.
  - Uses model functions to determine angular & spatial distributions
  - Sep. parameterisations of the central part of the scattering angle and of its tail.

- Uses Highland formula as a parameter
  - a shortcut to achieve a compromise between performance & accuracy

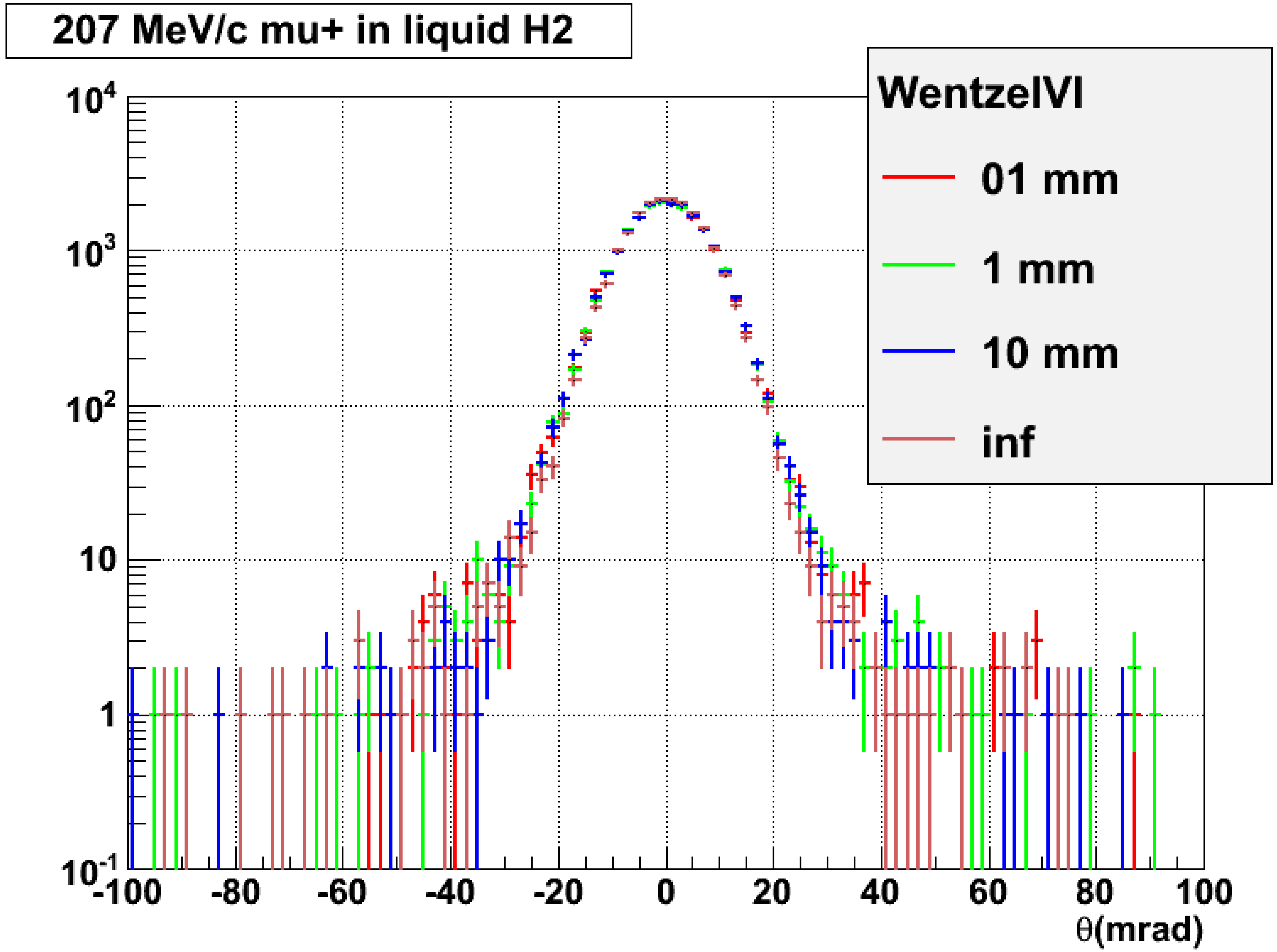
"Lewis theory is the base for many multiple scattering algorithms. Moliere theory is formulated in term of theta - it is initially a small angle model not assuming backscattering. However, ***both use a common formalism****."*

- Step length dependency corrected in g4.9.5.p01 release (3/12)

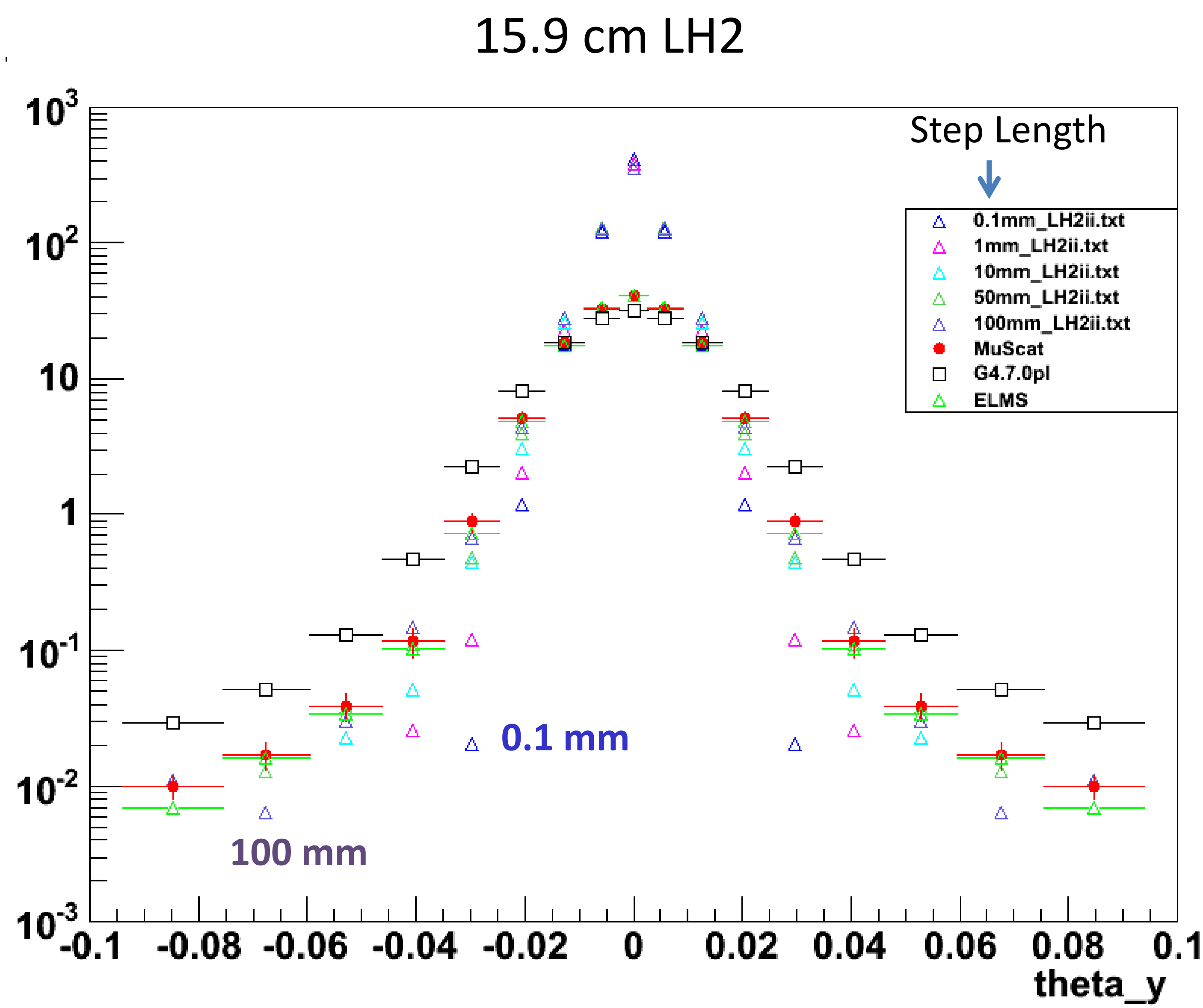
Geant4.9.5 - old G4UrbanMscModel90



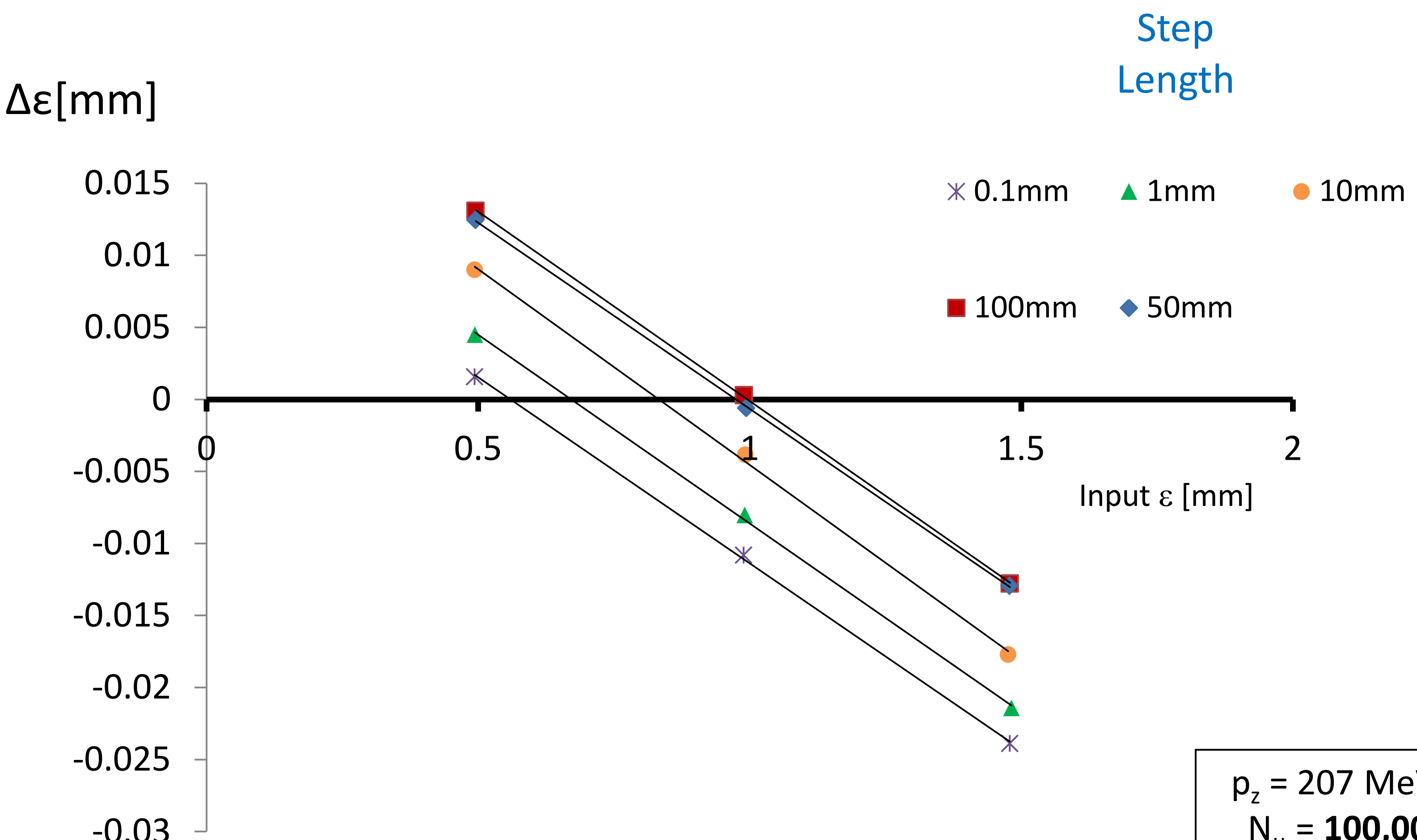
Geant4.9.5.p01



G4MICE (G4.9.5.p01) – MuScat comparison



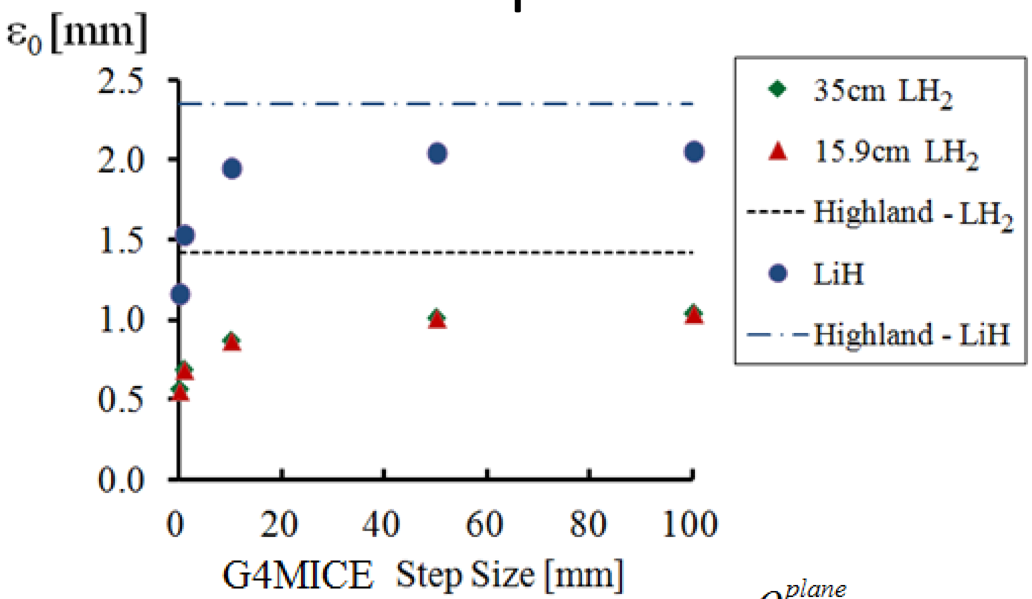
Cooling in 15.9cm LH_2



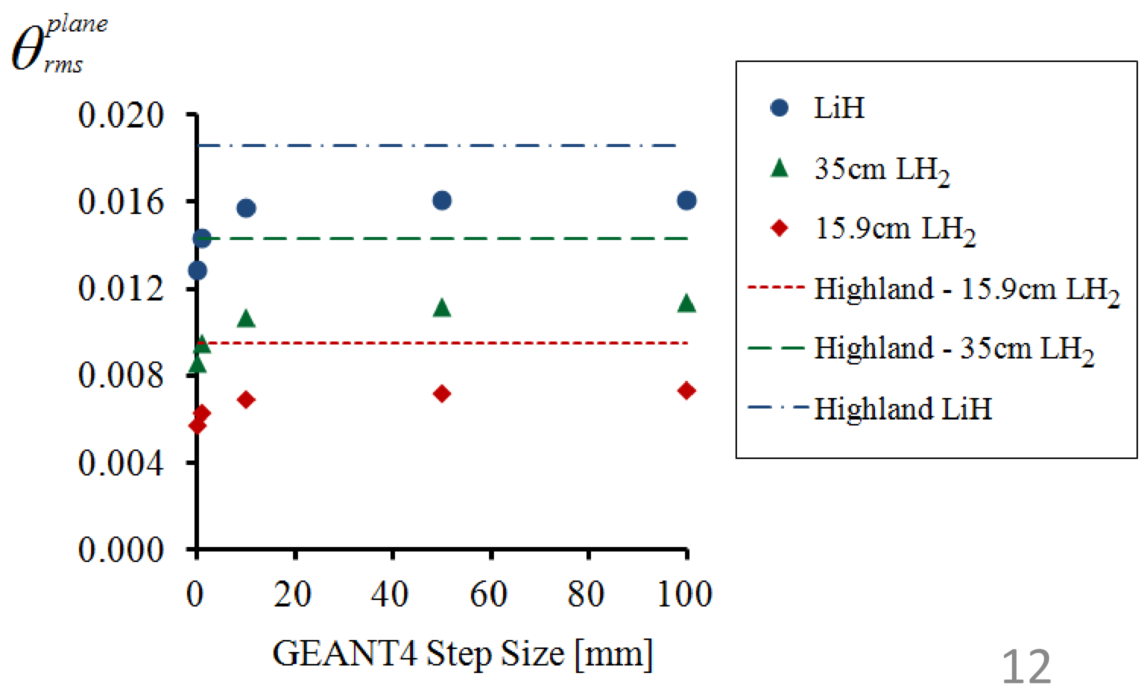
p_z = 207 MeV/c
N_μ = **100,000**
Step IV fields

No AFC (i.e. no Al windows)

Eq. Emittance & Scattering Angle



(Extrapolated from prev. slide)



On axis, mono-energetic beams sent through a block of absorber

Summary

- G4MICE/MAUS use Geant4 physics libraries.
  - Change max. step length using *G4StepMax* parameter
- MSC & ε_0 predictions dep. on Step Length**!**

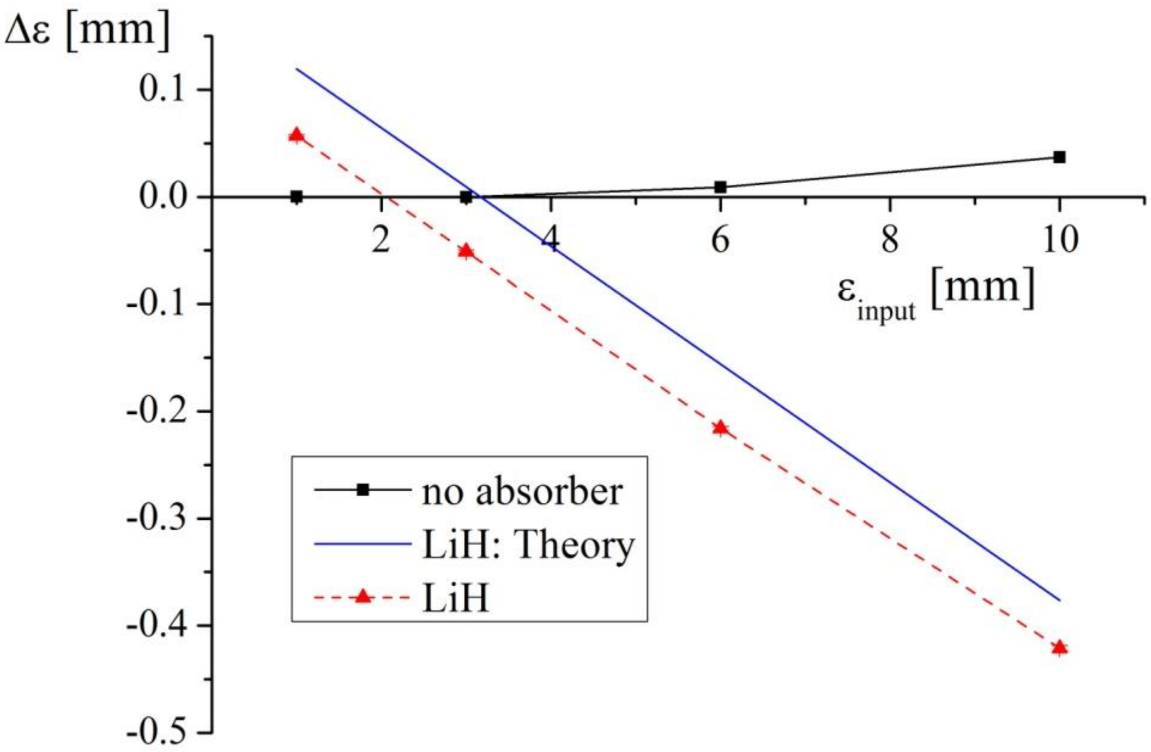
- Past Step Length dependencies in Geant4
  - Now corrected in Geant4.9.5.p01 (March 2012).

- New version tested with G4MICE – **no change**.
  - Issue with *G4StepMax* or in G4MICE-Geant4 interaction...?
- In contact with the Geant4 developers
  - Likely I'm still using the old Urban Model…**i.e. Geant4.9.5**

➢ Fix MSC & compare with predictions
  - CF, Lynch & Dahl, Highland etc

EXTRAS

G4MICE: Step IV 63mm LiH



Sample G4MICE geometry

```
Configuration Step IV
{

Dimensions 6. 6. 31. m
PropertyString Material Galactic
PropertyDouble G4StepMax 1 mm

Module MuScatTargets/LH2ii_15.9cm.dat
{
  Position 0. 0. -2.75   m
  Rotation 0. 0. 0. degree
}

Module Tests/VirtualPlane.dat
{
   Position 0. 0. -3 m
   Rotation 0. 0. 0. degree
   PropertyString IndependentVariable Z
   PropertyBool   RepeatModule 1
   PropertyInt    NumberOfRepeats 21
   PropertyHep3Vector RepeatTranslation 0 0 5 cm
   PropertyHep3Vector RepeatRotation 0 0 0
   PropertyDouble RepeatScaleFactor 1
}
}
```

Geant4 vs MuScat

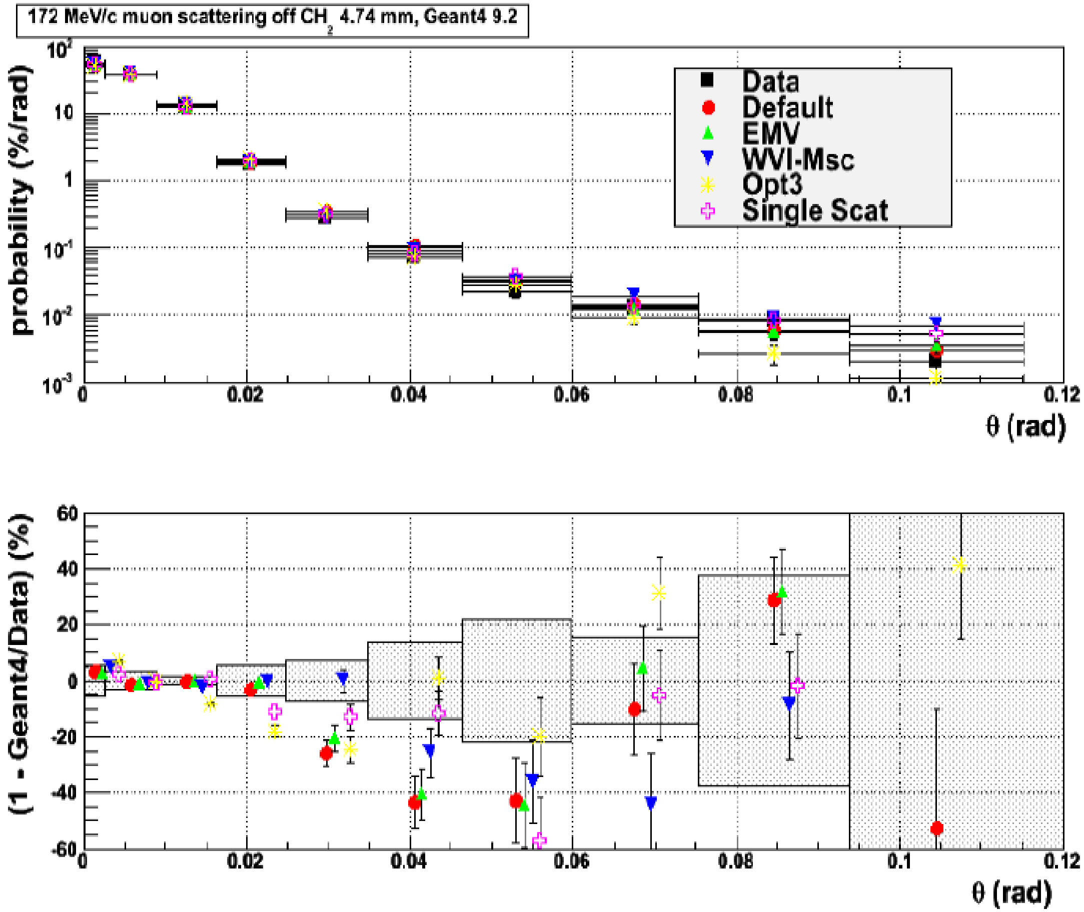
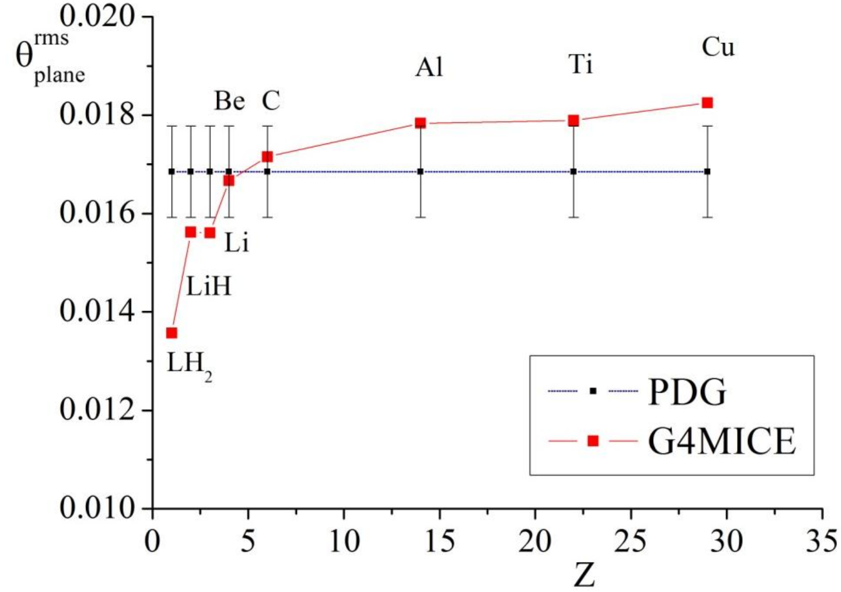


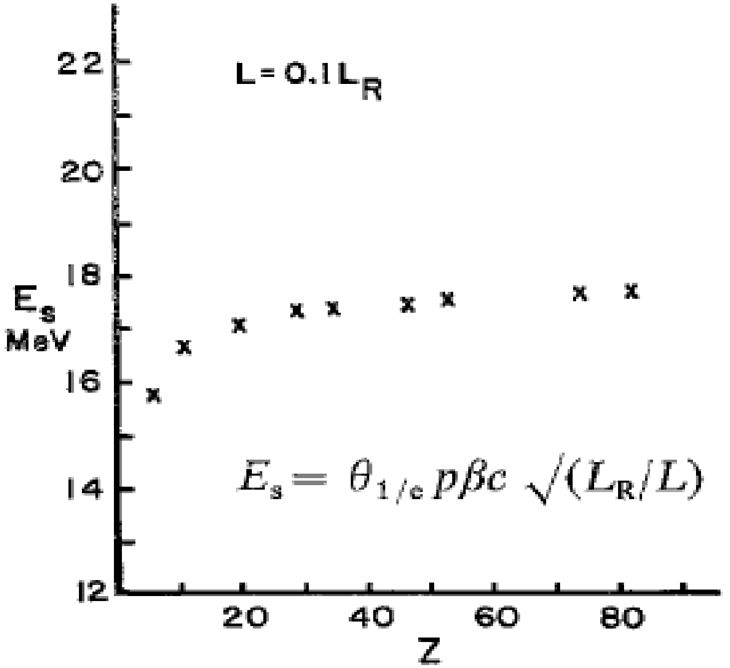
Figure 4. Geant4 simulation of μ+ scattering off thin polyethylene target versus MuScat data [22]: top – scattering angle, bottom – relative difference of Monte Carlo and the data, hashed area shows one standard deviation. The *WentzelVI* MSC model ($\chi^2/n = 1.69$) and the single scattering model ($\chi^2/n = 1.40$) describe better the tail of the distribution than the *Urban* MSC model ($\chi^2/n = 2.14$).

From: Geant4 models for simulation of multiple scattering
V N Ivanchenko *et al* 2010 *J. Phys.: Conf. Ser.* **219** 032045

Intro (2) - from CM32 talk

- Simple PDG approx. unsuitable
- x/X_0 scaling poor at low Z
- e^- screening calc. increasingly inaccurate





$$E_s = \theta_{1/e}\, p\beta c\, \sqrt{(L_R/L)}$$

V.L. Highland, Nucl. Instrum. Methods 129, 497 (1975)

Step IV – from CM32 talk

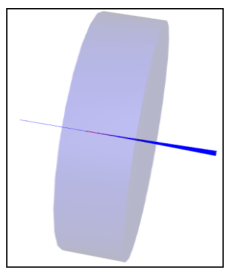
- Measure ε_0 in different materials
- Check results are consistent with theory – **how?**

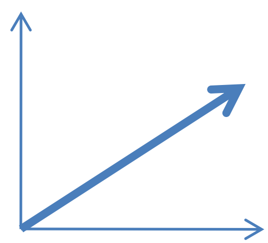
→ Compare with G4MICE

- GEANT4.9.2

Z	X_o		*x* [cm]
1	63.04	LH_2	57.61
3	82.78	Li	10.06
4	65.19	Be	2.29
6	42.7	C	1.39
2.00	79.62	LiH	6.30

G4MICE
N_μ=100,000
pencil beam on axis
no fields





$$\theta_x = a\tan(P_x / P_z)$$

$$\theta_{plane}^{rms} = \sigma_x$$