# Peccei-Quinn Symmetry as the origin of Dirac Neutrino Masses

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# Outline

☆ Strong CP problem and PQ symmetry

☆ Neutrino mass and related models

☆ Connection of strong CP and Dirac neutrino mass

☆ Conclusion and discussion

# Strong CP Problem and PQ symmetry

The CP odd term in SM

$$L = \bar{\theta}\frac{\alpha_s}{8\pi}G^{\mu\nu a}\tilde{G}^{a}_{\mu\nu} \quad , \quad \text{with } \bar{\theta} = \theta - \arg(\det(M_u)) - \arg(\det(M_d))$$

The QCD vacuum state can be parametrized as $\left|\theta\right\rangle = \sum_{n=-\infty}^{n=\infty} e^{in\theta}\left|n\right\rangle$

$\Theta$ is periodic with period $2\pi$

The current experimental upper bound set the constraint from the neutron electric dipole moment $\bar{\theta} < 0.6\times 10^{-10}$

The extremely suppressed quantity is called the strong CP problem

Peccei Quinn solution

Introduce a global chiral $U(1)_{PQ}$ symmetry.

Make $\bar{\theta}$ a dynamical variable, and the shift symmetry of the Nambu-Goldstone boson axion, corresponding to $U(1)_{PQ}$ will set $\bar{\theta}$ zero at classical potential.

$$L = \left( \bar{\theta} - \frac{a}{f_a} \right) \frac{\alpha_s}{8\pi} G^{\mu\nu a} \tilde{G}^a_{\mu\nu}$$

Two Higgs doublet model can realize the idea

$$L_y = \bar{Q}_L \tilde{H}_1 u_R + \bar{Q}_L H_2 d_R + \bar{L}_L H_2 l_R + \text{h.c.}$$

The action is invariant classically under $U(1)_{PQ}$ transformation

The chiral anomaly will break the shift symmetry

$$m_a \simeq \frac{\sqrt{m_u m_d}}{(m_u + m_d)} \frac{f_\pi m_\pi}{f_a} \sim \frac{\Lambda_{\text{QCD}}{}^2}{f_a}$$

The laboratory and outer space search provides the bounds

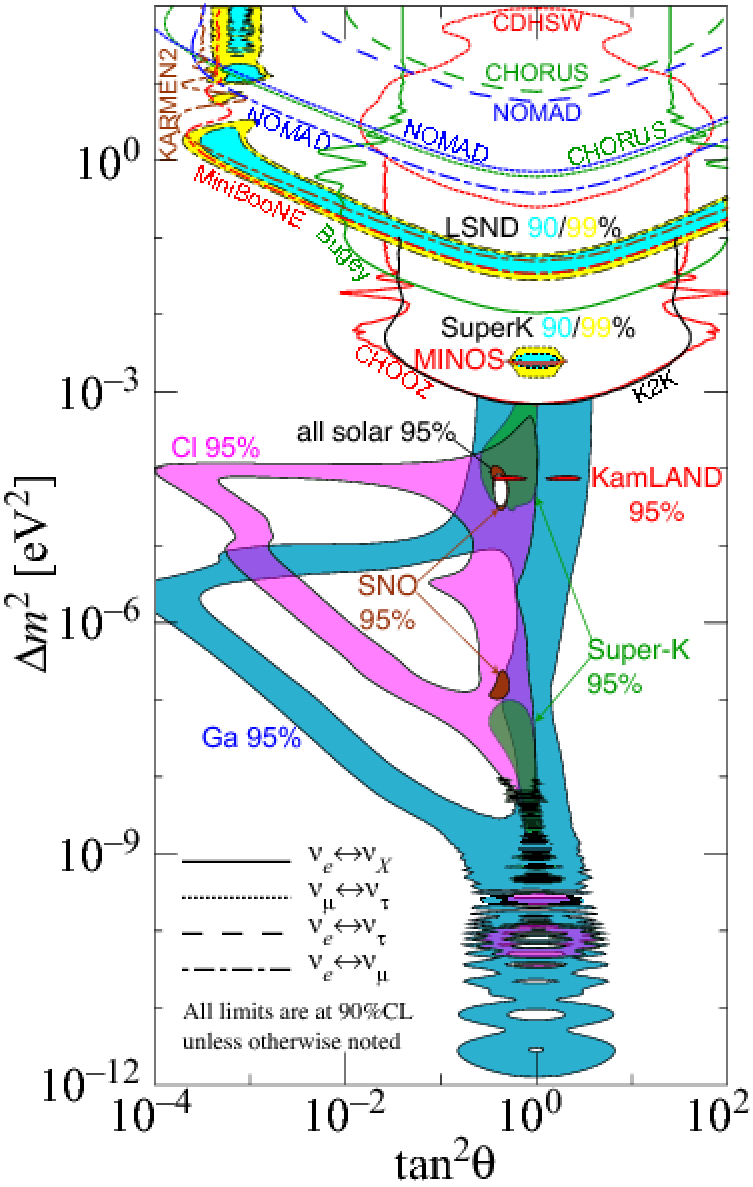
$$10^9\,\text{GeV} < f_a < 3\times 10^{11}\,\text{GeV} \qquad 3\times 10^{-3}\,\text{eV} > m_a > 10^{-6}\,\text{eV}$$

The ordinary PQ model with $f_a$ around electroweak scale has been ruled out.

Invisible axion: large VEV for the scalar singlet

Kim-Shifman-Vainstein-Zakharov (KSVZ) :The PQ charges is only carried by the heavy vector quarks and the Higgs singlet

Dine-Fischler-Srednicki-Zhitnitsky (DFSZ) :Two Higgs doublet and one Higgs singlet are required




H.Murayama(PDG2010)

# Neutrino oscillation experiments

$\Delta m_{12}^2 = (7.59^{+0.20}_{-0.21}) \times 10^{-5}\,\mathrm{eV}^2$ SNO (2008)

$|\Delta m_{23}^2| = (2.43 \pm 0.13) \times 10^{-3}\,\mathrm{eV}^2$ MINOS (2008)

$\sin^2 2\theta_{12}^{\ell} = 0.861^{+0.026}_{-0.022}$ SNO (2010)

$\sin^2 2\theta_{23}^{\ell} > 0.92$ Super Kamiokande(2005)

$\sin^2 2\theta_{13}^{\ell} = 0.089 \pm 0.011$ DAYA BAY (2012)

Best-fit($\pm 1\,\sigma$) PDG (2012)

$$\Delta m_{\odot}^2 = (7.58^{+0.22}_{-0.26}) \times 10^{-5}\,\mathrm{eV}^2$$

$$|\Delta m_{A}^2| = (2.35^{+0.12}_{-0.09}) \times 10^{-3}\,\mathrm{eV}^2$$

$$\sin^2 \theta_{12}^{\ell} = 0.312^{+0.018}_{-0.015}$$

$$\sin^2 \theta_{23}^{\ell} = 0.42^{+0.08}_{-0.03}$$

$$\sin^2 \theta_{13}^{\ell} = 0.0251 \pm 0.0034$$

## Neutrino mass and related models

The masses of neutrinos are observed from neutrino oscillation experiments

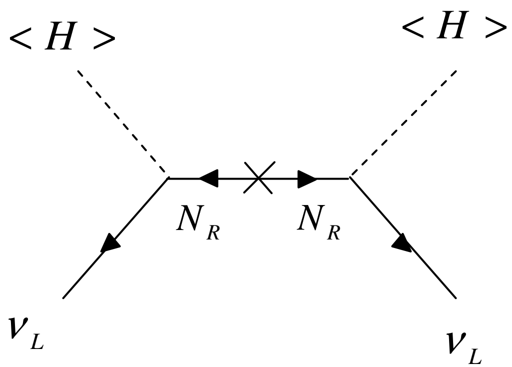
The key point to understand neutrino physics lies on whether the neutrinos are Dirac fermions or Majorana fermions

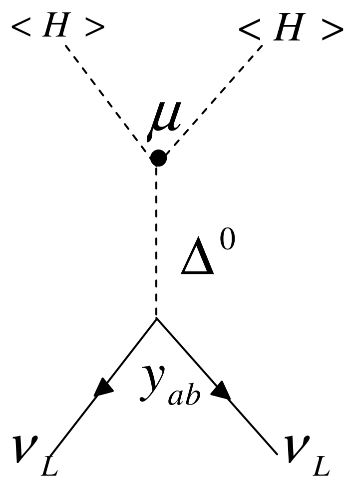
The tiny neutrino masses can be explained in terms of lepton number (L) symmetry which is broken at the scale $\Lambda_L$

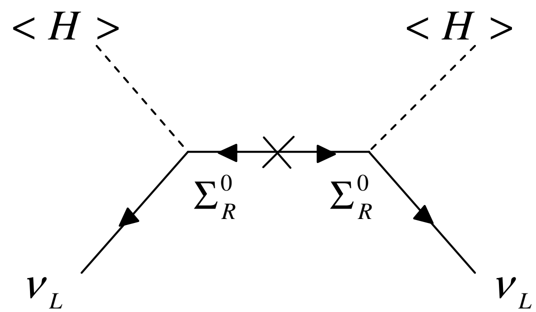
$$m_\nu \propto \frac{HHLL}{\Lambda_L}$$

# Seesaw mechanism

Generate neutrino mass from tree level contribution $\frac{1}{\Lambda} L_L L_L HH$




**type-I**

$N_R : (1,0)$

$$m_\nu \simeq \frac{m_D^{\;2}}{M_R} \;, m_D \ll M_R$$

**type-II**

$\Delta : (3,+1)$

$$m_{\nu ab} \simeq y_{ab} \frac{\mu < v >^2}{M_\Delta^{\;2}}$$

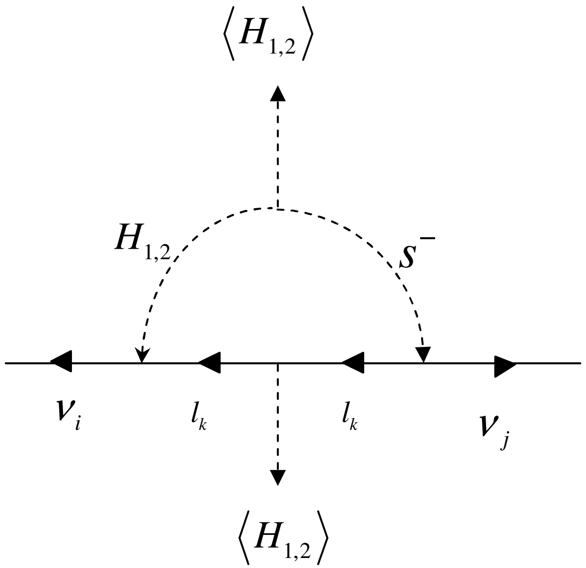
**type-III**

$\Sigma_R : (3,0)$

$$m_\nu \simeq \frac{m_D^{\;2}}{M_\Sigma}, m_D \ll M_\Sigma$$

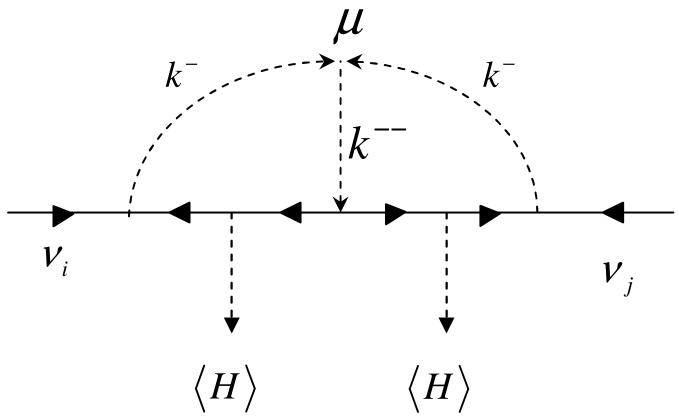
# Loop induced neutrino mass

## Zee model



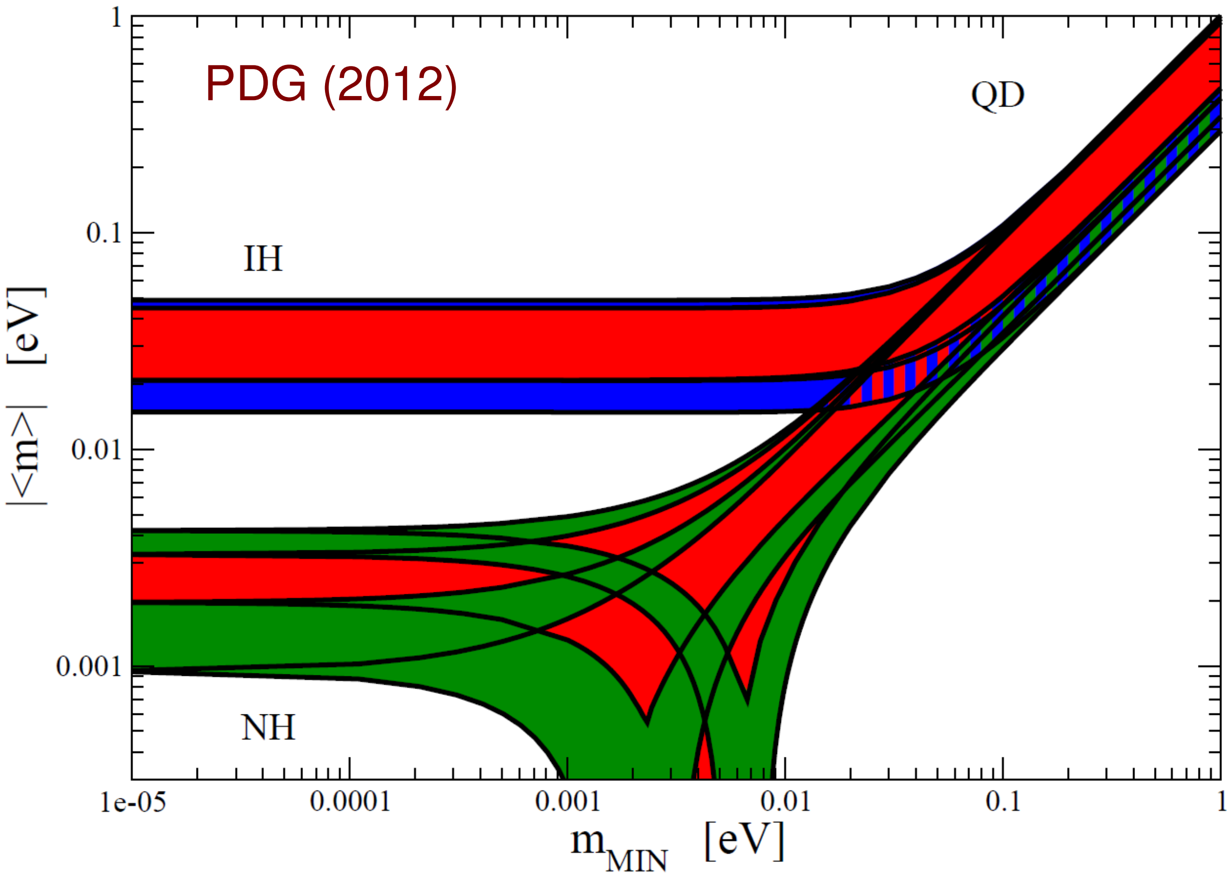

$$L = -\overline{L}^c f(i\sigma_2)Lk^+ - \overline{L}_L(Y_1H_1 + Y_2H_2)l_R + \text{h.c.}$$

## Zee-Babu model




$$L = -\overline{L}^c Y_1 i\sigma_2 Lk^+ - \overline{l_R}^c Y_2 l_R k^{++} - \mu k^- k^- k^{++} + \text{h.c.}$$

Neutrinoless double beta decay ($\nu 0 \beta \beta$) experiment could help to determine the Dirac or Majorana nature of neutrinos




The effective mass <m> depends also on Majorana phases $\alpha_{21}$ , $\alpha_{31}$

# Connection of PQ symmetry and Dirac Neutrino mass

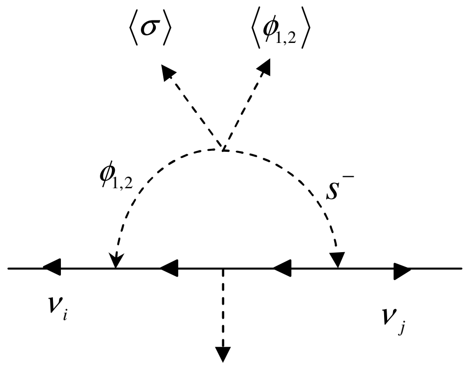
Small quantities arising in physics usually requires the use of new symmetries for explanation

Connect the strong CP problem and light neutrino mass is not a new idea

Davidson and Wali (1982); Mohapatra and Senjanovic (1983); Shafi and Stecker(1984);Langacker, Peccei,Yanagida,(1986);Shin(1987); He and Volkas(1988); Geng and Ng(1988)…….

Most of them work on Majorana mass, especially in type-I seesaw model.

Arason, Ramond,Wright (1990); Bertolini and Santamaria (1990)



# Connect the Dirac mass and PQ symmetry

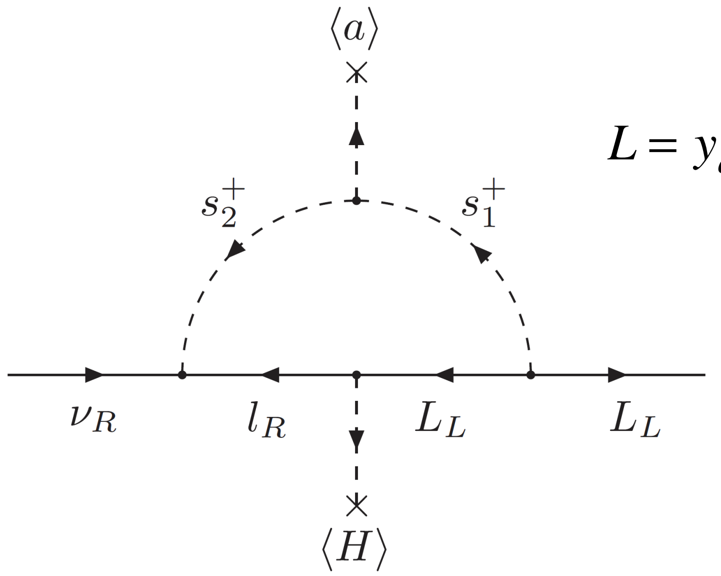
Adding SM gauge singlet neutrinos is the simplest way to acquiring the Dirac mass. Loop induced Dirac mass is the way to explain the nature of the tiny neutrino mass

| | $L_L$ | $l_R$ | $H_1$ | $H_2$ | $\nu_{R_i}$ | $s_1^+$ | $s_2^+$ | $a$ |
|---|---|---|---|---|---|---|---|---|
| $Y$ | $-\frac{1}{2}$ | -1 | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 | 1 | 1 | 0 |
| $L$ | 1 | 1 | 0 | 0 | 1 | -2 | -2 | 0 |
| PQ | 0 | -2 | 2 | -2 | 0 | 0 | 2 | -2 |

Assign the three right-handed neutrinos $\nu_R$ with Dirac nature.

To ensure the existence of PQ symmetry, one way is to introduce two Higgs doublets in a model.

Add two charge singlets $s_1$ ,$s_2$ and the neutral singlet $a$




$$L = y_{\alpha\beta}\overline{L}_{L\alpha} l_{R\beta} H_1 + f_{\alpha\beta}\overline{L}^c_{L\alpha} i\sigma_2 (L_{L\beta}) s_1^+ + h_{\alpha i}\overline{l}^c_{R\alpha} \nu_{Ri} s_2^+ + \text{h.c.}$$

without the imposing of axion, the diagram is similar to Nasri *et al. (2002)*; Sugiyama *et al. (2011).*

$$V = -\mu_1^2 H_1^* H_1 - \mu_2^2 H_2^* H_2 - \mu_a^2 |a|^2 + \mu_{s1}^2 |s_1|^2 + \mu_{s2}^2 |s_2|^2$$
$$+\lambda_1 (H_1^* H_1)(H_1^* H_1) + \lambda_2 (H_2^* H_2)(H_2^* H_2) + \lambda_3 (H_1^* H_1)(H_2^* H_2) + \lambda_4 (H_1^* H_2)(H_2^* H_1)$$
$$+(H_1^* H_1)[d_1 |s_1|^2 + d_2 |s_2|^2 + d_a |a|^2] + (H_2^* H_2)[g_1 |s_1|^2 + g_2 |s_2|^2 + g_a |a|^2]$$
$$+h_1 |s_1|^4 + h_2 |s_2|^4 + h_a |a|^4 + h_3 |s_1|^2 |s_2|^2 + |a|^2 (h_{a1} |s_1|^2 + h_{a2} |s_2|^2)$$
$$+[h_5 (H_2^* H_1) a^2 + \mu s_1^- s_2^+ a + \text{h.c.}]$$

The loop induced Dirac neutrino mass matrix is given by

$$(M_\nu)_{\alpha i} = -\frac{1}{8\pi^2} f_{\alpha\beta} h_{\beta i} m_\beta \frac{\mu f_a}{m_{s1}^2 - m_{s2}^2} \log\frac{m_{s1}^2}{m_{s2}^2} \approx -\frac{1}{8\pi^2} f_{\alpha\beta} h_{\beta i} \frac{\mu m_\beta}{m_a} \frac{\Lambda_{QCD}^2}{m_{s1}^2 - m_{s2}^2} \log\frac{m_{s1}^2}{m_{s2}^2}$$

The large value of $f_a$ lifts up the scale of Dirac neutrino masses. In order to keep the smallness of neutrino mass matrix, other coupling in the neutrino formula should be suppressed.

$$M_\nu \approx 0.1\text{eV},\ \mathrm{m}_{s1,2} \sim O(100-1000)\text{GeV}, f_a \approx 10^{10}\,\text{GeV},$$

$$\text{we have } f \sim 10^{-3},\ h \sim 10^{-2}, \mu \sim O(100)\text{keV}$$

The small $\mu$ can be explained by Froggatt-Nielsen mechanism with PQ-symmetry. The impose of heavy Majorana neutrino mass can also release the limit on $\mu$ .

The neutrino mass relation

$$FH = V_{\mathrm{PMNS}} M_{\nu diag}$$

where $H = M_{l\,diag}(-hV_R^{\nu\dagger}),\ F = C\,f,\quad C = \frac{1}{8\pi^2}\frac{\mu f_a}{m_{s1}^2 - m_{s2}^2}\log\frac{m_{s1}^2}{m_{s2}^2}$

## Normal Hierarchy

$$F_{e\mu} = \frac{V_{\tau 1}^*}{V_{e1}^*}F_{\mu\tau}\ ,\ F_{e\tau} = \frac{V_{\mu 1}^*}{V_{e1}^*}F_{\mu\tau},\ H_{\mu i} = \frac{V_{\mu 1}^*}{V_{e1}^*}H_{ei} - \frac{m_{\nu i}}{F_{\mu\tau}}V_{\tau i}, H_{\tau i} = \frac{V_{\tau 1}^*}{V_{e1}^*}H_{ei} + \frac{m_{\nu i}}{F_{\mu\tau}}V_{\mu i}$$

## Inverted Hierarchy

$$F_{e\mu} = \frac{V_{\tau 3}^*}{V_{e3}^*}F_{\mu\tau}\ ,\ F_{e\tau} = -\frac{V_{\mu 3}^*}{V_{e3}^*}F_{\mu\tau},\ H_{\mu i} = \frac{V_{\mu 3}^*}{V_{e3}^*}H_{ei} - \frac{m_{\nu i}}{F_{\mu\tau}}V_{\tau i}, H_{\tau i} = \frac{V_{\tau 3}^*}{V_{e3}^*}H_{ei} + \frac{m_{\nu i}}{F_{\mu\tau}}V_{\mu i}$$

Constraint from $\mu \rightarrow e\gamma$

Current bounds $B(\mu \rightarrow e\gamma) < 2.4 \times 10^{-12}$

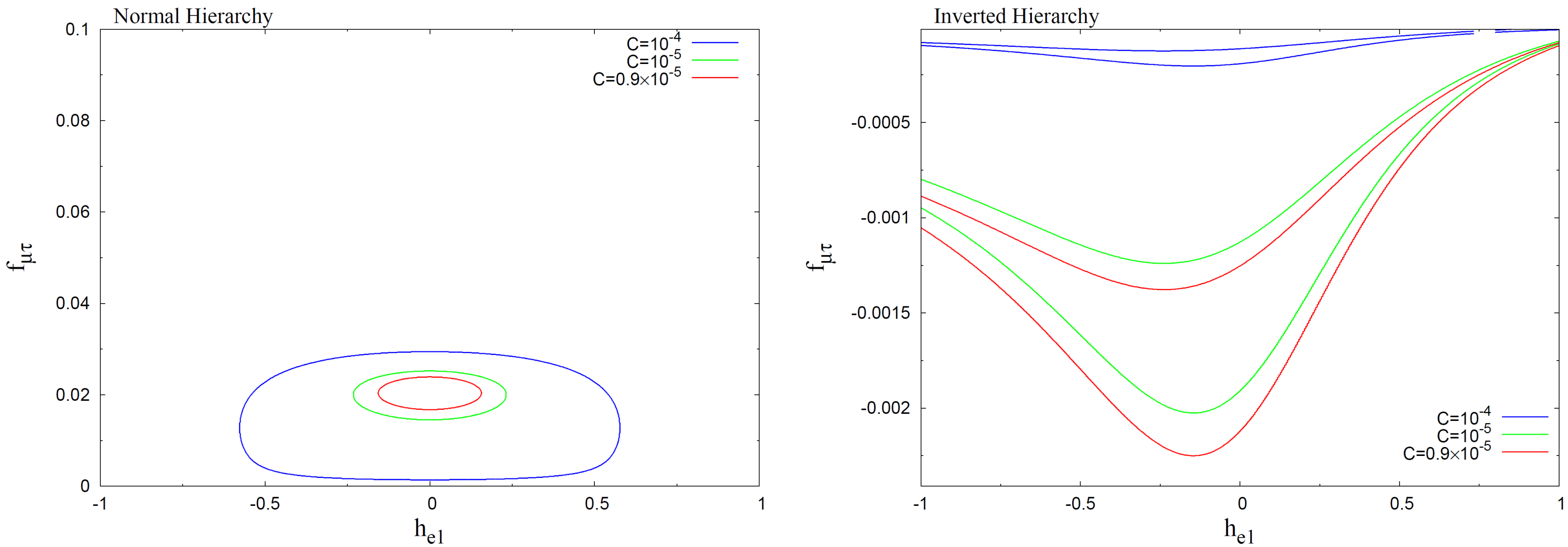
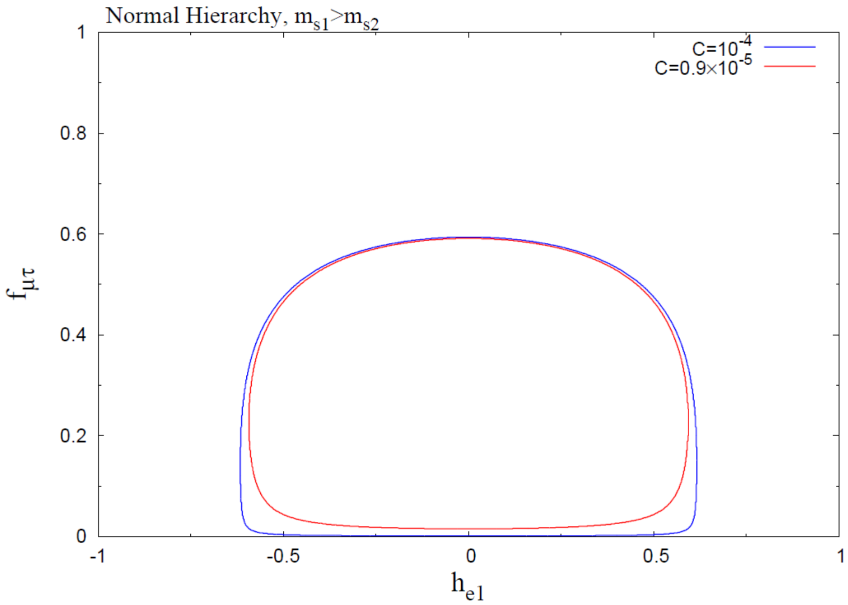
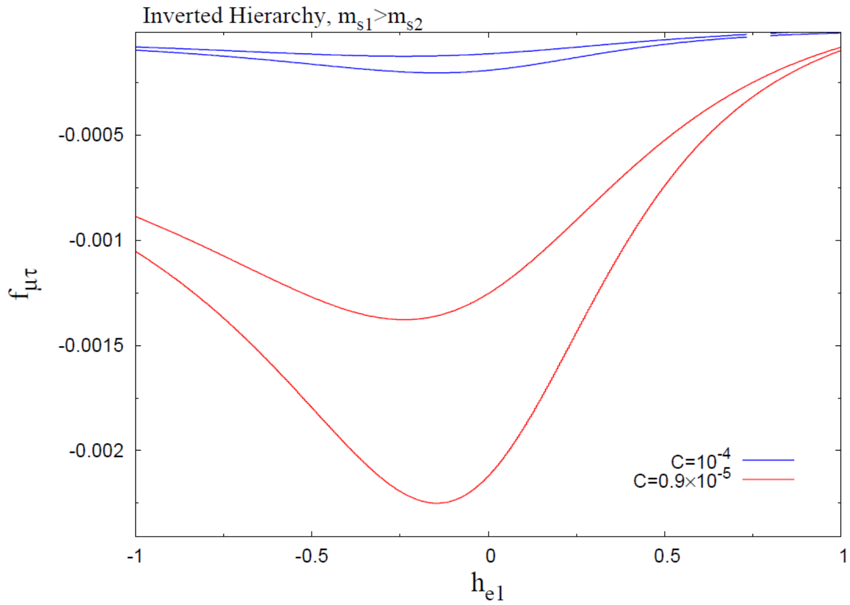



FIG. 2: The allowed region in $h_{e1} - f_{\mu\tau}$ plane in (a) the normal hierarchy, and (b) the inverted hierarchy. Different curves are generated with different $C$ (blue: $C = 10^{-4}$, green: $C = 10^{-5}$, red: $C = 0.9 \times 10^{-5}$). The other parameters are taken as $m_{s1} = m_{s2} = 500\text{GeV}$, $h_{e2} = h_{e3} = 0.5$.

The mass hierarchy between charged scalars is allowed, but the relation $m_{s1} m_{s2} > \mu f_a$ can not be violated.

For the case $m_{s1} = 20 m_{s2}$



Muon g-2

$$\Delta a_\mu = -\frac{m_\mu{}^2}{96\pi^2}[4\sum_\alpha |f_{\mu\alpha}|^2 \frac{1}{m_{s1}{}^2} + \sum_\alpha |h_{\mu i}|^2 \frac{1}{m_{s2}{}^2}$$
$$+\frac{m_\mu{}^2}{v^2}(\frac{14c_{\beta-\alpha}{}^2}{m_h{}^2\cos^2\beta} + \frac{14s_{\beta-\alpha}{}^2}{m_H{}^2\cos^2\beta} - \frac{22\tan^2\beta}{m_A{}^2} + \frac{\tan^2\beta}{m_{H^+}^2})] \approx -10^{-14}$$

less than current results $a_\mu(\exp) - a_\mu(\mathrm{SM}) = (287 \pm 63 \pm 49)\times 10^{-11}$

The exchange of singly charged scalars gives the EDM to the charged leptons at one-loop level

$$d_l = -\frac{e}{(4\pi)^2}\sum_i \mathrm{Im}(2h_{li}^*(fV)_{il})\frac{\mu f_a}{m_{s1}^2 - m_{s2}^2}[\frac{1}{m_{s1}}B(\frac{m_{\nu i}^2}{m_{s1}^2}) - \frac{1}{m_{s2}}B(\frac{m_{\nu i}^2}{m_{s2}^2})]\ ,\ B = \frac{\sqrt{x}}{2(1-x)^2}(1+x+\frac{2x\ln(x)}{1-x})$$

The extra contribution is smaller than $10^{-35}$ e cm, smaller than current experimental sensitivity $10^{-27}$ e cm

The present upper bound of neutrino magnetic moment is

$$\mu_\nu < 3.2\times10^{-11}\mu_B \quad \text{with } \mu_B \equiv e/2m_e$$

SM + Dirac right-handed neutrino gives

$$\mu_\nu = 3.2\times10^{-19}\mu_B(m_{\nu i}/\mathrm{eV})$$

The main new contribution comes from the mixing of $s_1$ and $s_2$ in the loop

$$|\mu_\nu^s| = 2m_i e(\log\frac{m_{s2}^2}{m_{s1}^2})^{-1}(\frac{1}{m_{s1}{}^2} - \frac{1}{m_{s2}^2}) \approx 10^{-19}\mu_B$$

The contributions from single charged scalar exchange are less than $10^{-25}\mu_B$

## Axion Dark Matter

Axion can be regarded as a cold dark matter which is produced at the epoch with the Hubble constant H~3×10$^{-9}$ eV

$$\Omega_a \simeq \frac{1}{2}\left(\frac{0.6\times 10^{-5}\,\mathrm{eV}}{m_a}\right)^{\frac{7}{6}}\left(\frac{0.7}{h}\right)^2$$

The CDM axions would form a Bose-Einstein condensate due to their gravitational interactions.

Sikivie and Yang(2009)

The axion coupling to neutrinos could be different from ordinary DFSZ model due to the different assignment to right-handed neutrinos

# Signal from H→ $\gamma$ $\gamma$ and H→Z $\gamma$

The branching ratio of Higgs decay into two photons excess about $R_{\gamma\gamma}$ =1.56±0.43 and $R_{\gamma\gamma}$ = 1.8±0.4 times than the SM prediction in both CMS and ATLAS collaborations data

$$R_{\gamma\gamma(Z\gamma)} \equiv \frac{\sigma(pp \to h)\Gamma(h \to \gamma\gamma(Z\gamma))}{\sigma_{SM}(pp \to h)\Gamma_{SM}(h \to \gamma\gamma(Z\gamma))}$$

The two Higgs doublet model affects the Higgs production cross section by

$$\sigma_0 = \frac{G_F \alpha_s^{\ 2}}{128\sqrt{2}\pi} \left| \frac{1}{2}\left(\frac{c_\alpha}{t_\beta} + s_\alpha\right) A_{1/2}(\tau_t) - \frac{1}{2}(c_\alpha t_\beta - s_\alpha) A_{1/2}(\tau_b) \right|^2$$

Charged scalars from the $SU(2)_L$ doublet and the singlets affects the final states production rates

# Isolate the $SU(2)_L$ singlet scalar contribution from $R_{\gamma\gamma}$ and $R_{Z\gamma}$

$$\Gamma_{\gamma\gamma} = \frac{G_F\alpha^2 m_h^3}{128\sqrt{2}\pi^3}\Big|(\frac{c_\alpha}{\tan\beta}+s_\alpha)\frac{4}{3}A_{1/2}(\tau_t)-(c_\alpha\tan\beta-s_\alpha)\frac{1}{3}A_{1/2}(\tau_b)+s_\alpha A_1(\tau_W) + \frac{v}{2}\frac{\mu_{H^+}}{m_H^2}A_0(\tau_{H_i})+\frac{v}{2}\frac{\mu_{s^+}}{m_s^2}A_0(\tau_s)\Big|^2$$

$$\Gamma_{Z\gamma} = \frac{G_F^2 m_W^2\alpha m_h^3}{64\pi^4}(1-m_Z^2/m_H^2)^3\Big|(\frac{c_\alpha}{\tan\beta}+s_\alpha)\frac{2(1-8s_W^2/3)}{c_W}A_{1/2}(\tau_t,\lambda_t) + (c_\alpha\tan\beta-s_\alpha)\frac{1-4s_W^2/3}{c_W}A_{1/2}(\tau_b,\lambda_b) + s_\alpha A_1(\tau_W,\lambda_W)+(\frac{1}{2}-s_w^2)\frac{v\mu_{H^+}}{m_{H^+}^2}A_0(\tau_{H^+},\lambda_{H^+})+(-s_w^2)\frac{v\mu_{s^+}}{m_{s^+}^2}A_0(\tau_{s^+},\lambda_{s^+})\Big|^2$$

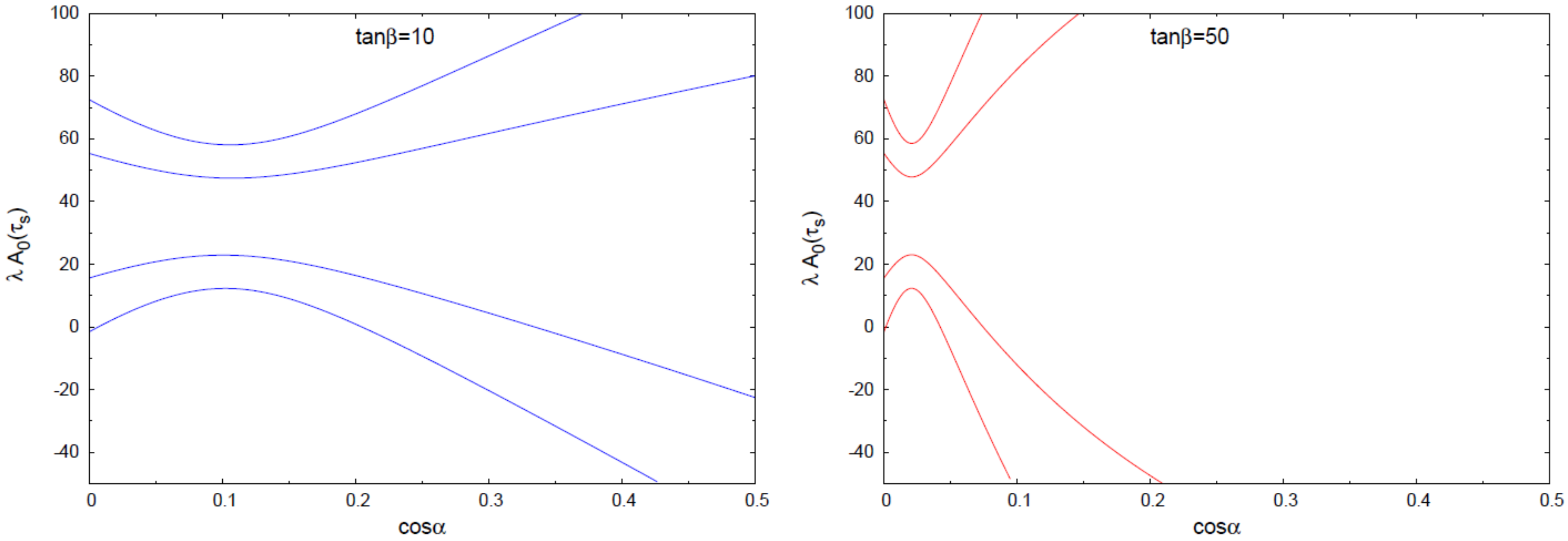
$$\Gamma_{\gamma\gamma} = F_{\gamma\gamma}|A_W(c_\alpha)+A_t(c_\alpha,t_\beta)+A_b(c_\alpha,t_\beta)+\frac{1}{2}B_H+\frac{1}{2}B_s|^2\,,$$

$$\Gamma_{Z\gamma} = F_{Z\gamma}|A'_W(c_\alpha)+A'_t(c_\alpha,t_\beta)+A'_b(c_\alpha,t_\beta)+(\frac{1}{2}-s_w^2)k_H B_H+(-s_w^2)k_s B_s|^2,$$

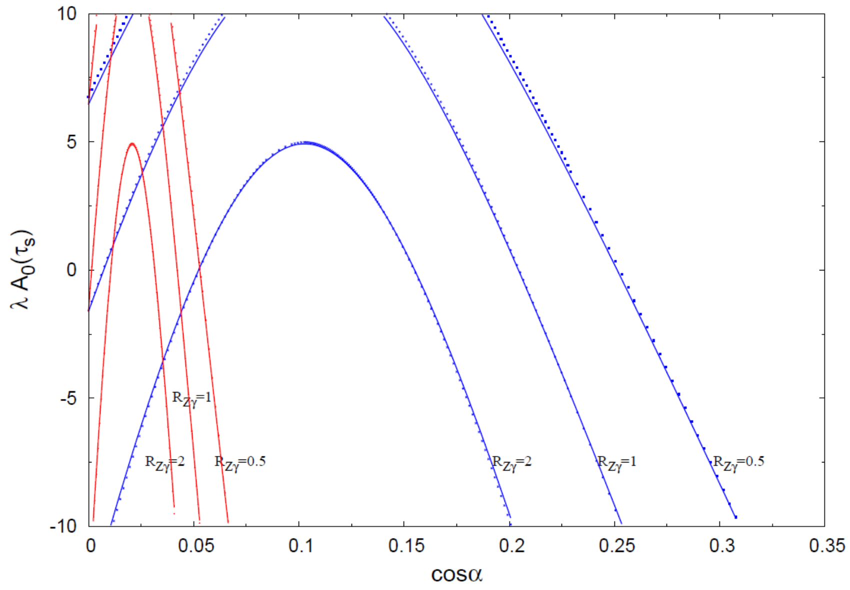
$$B_s=\lambda A_0(\tau_s), B_H=\lambda' A_0(\tau_H),\ \lambda=\frac{v\mu_{hs^+s^-}}{m_s^2},\ \ \lambda'=\frac{v\mu_{hH^+H^-}}{m_H^2},\ k_s=\frac{A_0(\tau_s,\lambda_s)}{A_0(\tau_s)}, k_H=\frac{A_0(\tau_H,\lambda_H)}{A_0(\tau_H)}$$

$k_H$ and $k_s$ move slowly from 0.53~0.5 for scalar mass >100GeV

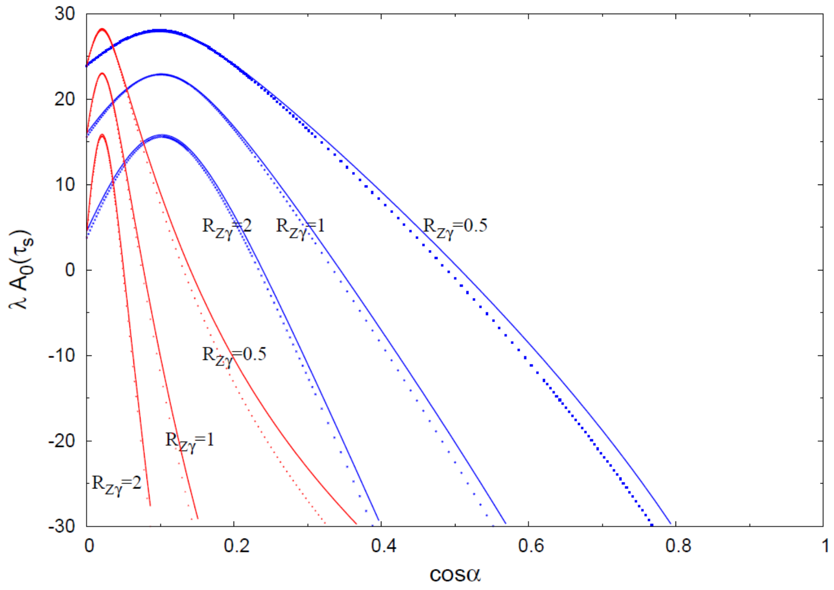
By taking $R_{\gamma\gamma}$ =1.5, $R_{Z\gamma}$ =1, express the charged singlet amplitudes $\lambda A_0(\tau_s)$ as function of cos $\alpha$



By taking $R_{\gamma\gamma}$ =1.5, $R_{Z\gamma}$ =0.5, 1, 2




$$\frac{\mathrm{Re}(A_{\gamma\gamma})}{\mathrm{Re}(A_{\gamma\gamma}^{\mathrm{SM}})}>0,\quad \frac{\mathrm{Re}(A_{Z\gamma})}{\mathrm{Re}(A_{Z\gamma}^{\mathrm{SM}})}>0$$




$$\frac{\mathrm{Re}(A_{\gamma\gamma})}{\mathrm{Re}(A_{\gamma\gamma}^{\mathrm{SM}})}<0,\quad \frac{\mathrm{Re}(A_{Z\gamma})}{\mathrm{Re}(A_{Z\gamma}^{\mathrm{SM}})}>0$$

# Discussion and Conclusion

We investigate a model that neutrinos are Dirac fermions and their masses are generated from the Peccei-Quinn symmetry breaking at one-loop level

Leptonic rare decays constrain the model parameters severely, and the model can be tested in the near future.

Recent and future Higgs signals for $H \to \gamma\ \gamma$ and $H \to Z\gamma$ could help to extract the signal from scalars in this model.