Side injection of electrons

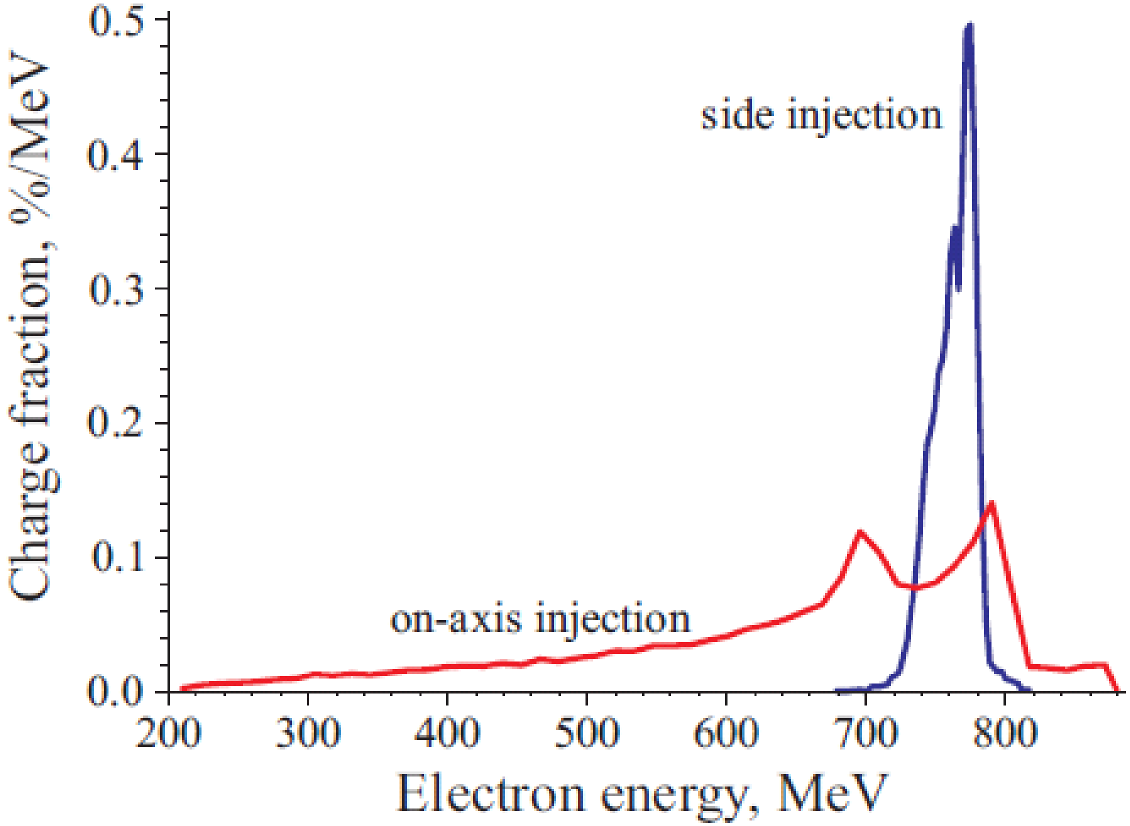
Konstantin Lotov

Budker Institute of Nuclear Physics SB RAS & Novosibirsk State University, Novosibirsk, Russia

Motivation for in-depth study:

Halfway electron injection is a must because of phase velocity considerations

Side injection provides much better quality of the accelerated electron beam



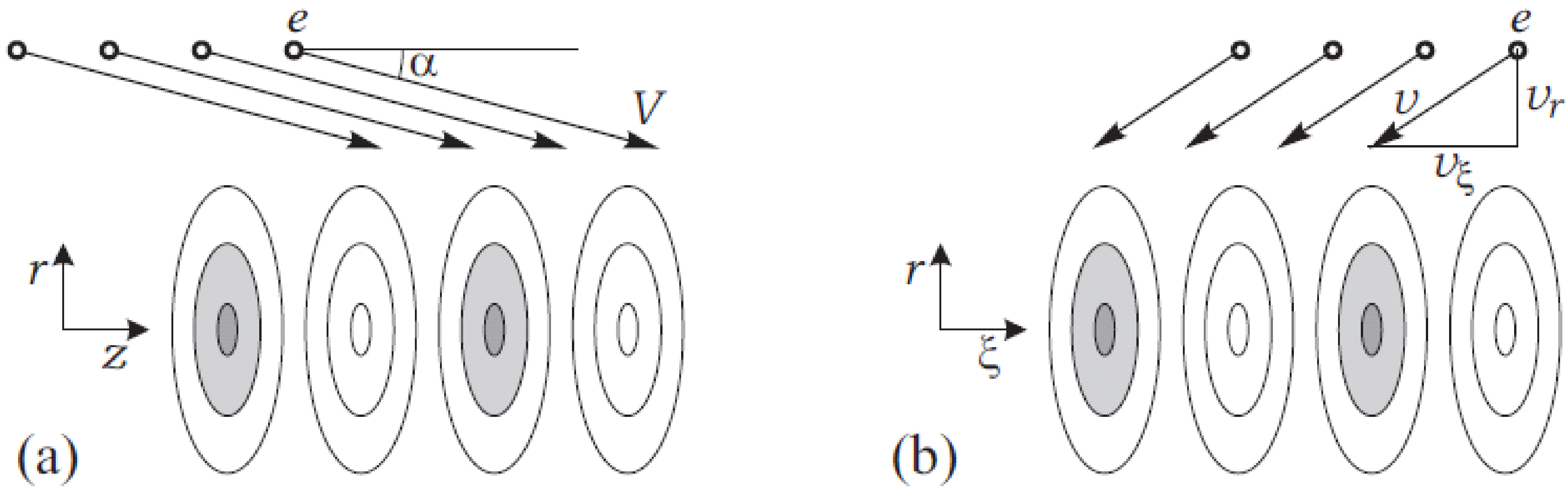
Spectra of test electrons after (last) 4 meters of proton-driven wakefield acceleration for side injection and on-axis injection.

The process has 3 stages:

- penetration
- trapping
- acceleration

Penetration of electrons into the wakefield - 1

If electrons are directed exactly to the axis, then the problem can be studied in a unified way. The geometry in laboratory (a) and co-moving (b) coordinates:



$$\xi = z - V_w t \qquad v_\xi = V\cos\alpha - V_w, \qquad v_r = -V\sin\alpha \approx -\alpha c$$

$$(\text{usually } \alpha\Gamma \ll 1 \text{ and } v_\xi \approx V - V_w)$$

Usually the wakefield potential energy Φ at large radii is

$$\Phi(r,\xi) = \Phi_0 \cos(k_p\xi) K_0(k_p r) \approx \Phi_0 \cos(k_p\xi)\sqrt{\frac{\pi}{2}}\frac{e^{-k_p r}}{\sqrt{k_p r}} \approx \Phi_1 \cos(k_p\xi)\, e^{-k_p (r-r_0)}.$$

Penetration of electrons into the wakefield - 2

Universal law of motion

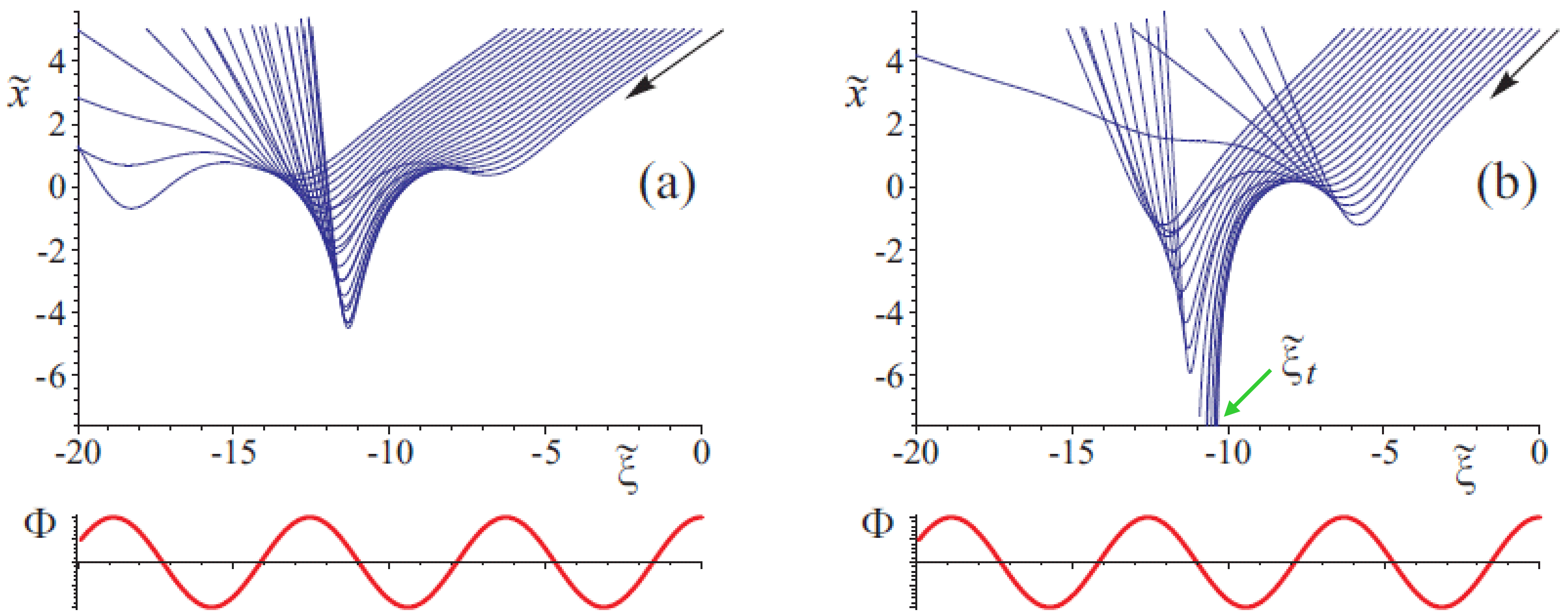
$$\frac{d^2\tilde{x}}{d\tilde{t}^2} = \cos(\tilde{t} - \tilde{\xi}_0)\, e^{-\tilde{x}}$$

with initial condition

$$\tilde{v} = \frac{d\tilde{x}}{d\tilde{t}} = -\frac{v_r}{v_\xi} = \frac{\alpha c}{V - V_w}$$

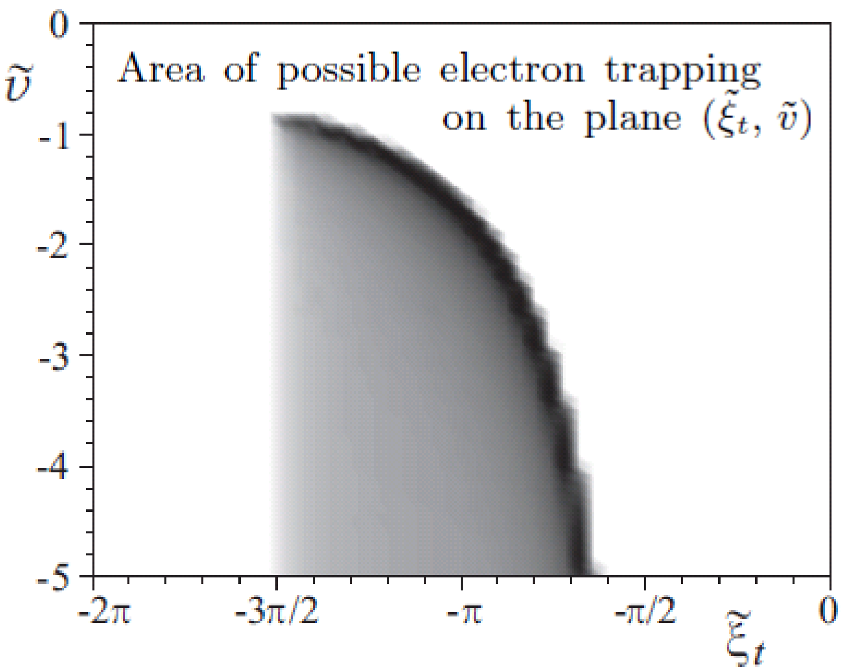
in dimensionless variables

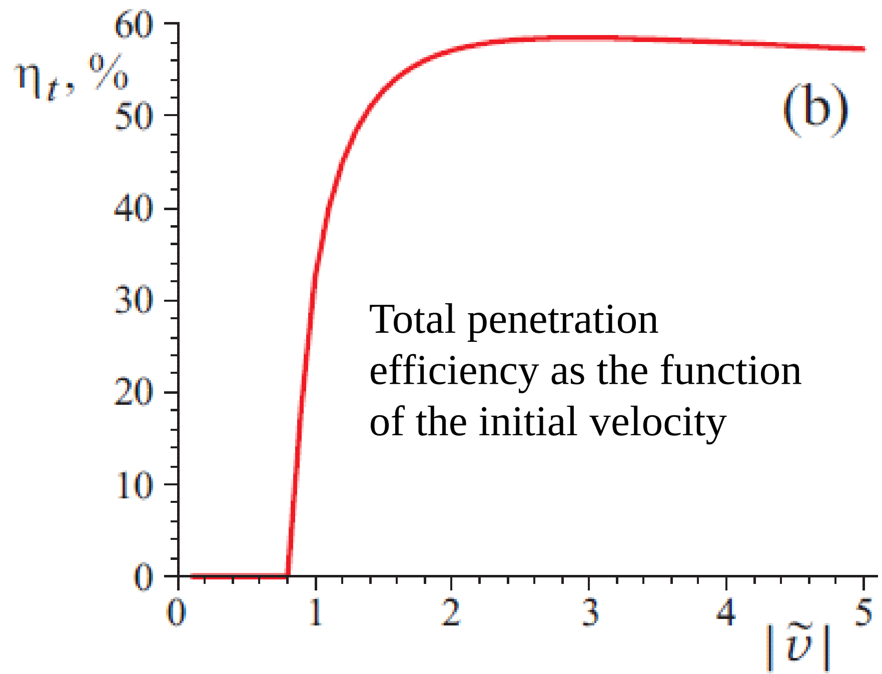
$$\tilde{t} = -k_p v_\xi t, \qquad \tilde{\xi}_0 = k_p \xi_0, \qquad \tilde{\xi} = \tilde{\xi}_0 - \tilde{t}, \qquad \tilde{x} = k_p(r - r_0) - \ln\left(\frac{\Phi_1}{\Gamma m v_\xi^2}\right)$$

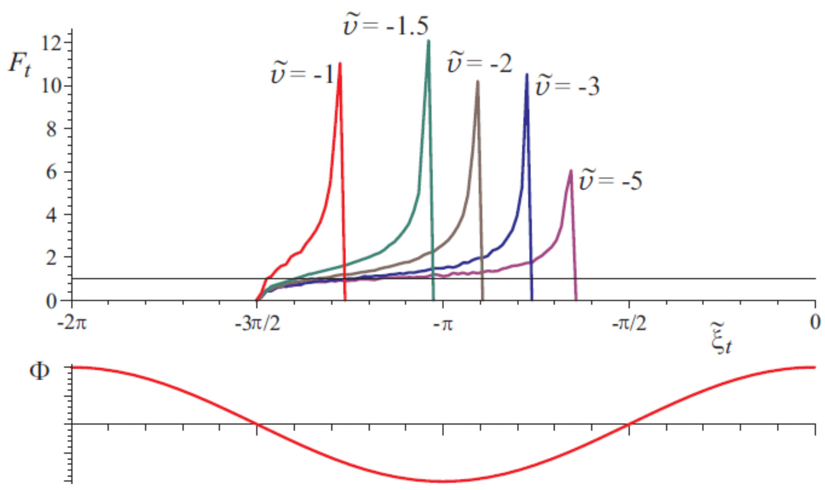


Family of electron trajectories for $\tilde{v} = -0.7$ (a) and $\tilde{v} = -1$ (b). Lower graphs show the location of potential wells and humps.

Penetration of electrons into the wakefield - 3







Flux density F_t of trapped electrons as the function of final phase $\tilde{\xi}_t$ for different values of the initial velocity $\tilde{v}$. Thin line is the flux density of incoming electrons. The lower graph shows the location of potential wells and humps.

Penetration of electrons into the wakefield - summary

There is an optimum angle for side injection of electrons into the plasma wakefield:

$$|\tilde{v}| \sim 1 \quad \text{or} \quad \alpha_{\text{opt}} \sim \frac{V_w - V}{c} \quad \text{or} \quad \alpha_{\text{opt}} \sim \frac{1}{2\Gamma_w^2}$$

At smaller angles, all electrons are reflected back.

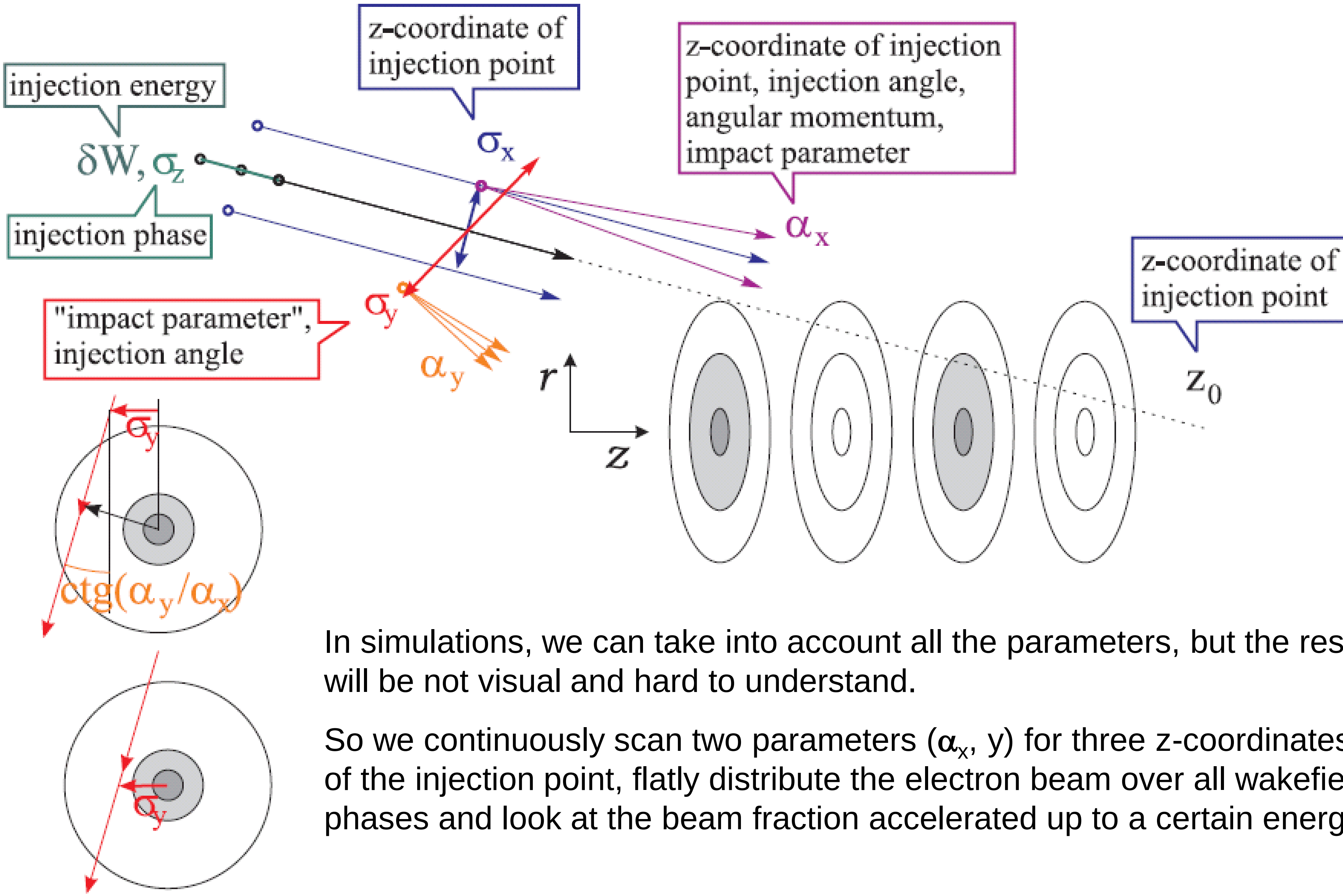
At larger angles, electrons enter the wakefield with superfluous transverse momentum that is unfavorable for trapping.

Separation of electrons occur at large radius: $r_0 \approx k_p^{-1} \ln\left(\frac{\Phi_0 \Gamma^3}{mc^2}\right)$

For wide electron bunches, the angular momentum of electrons complicates the treatment. Computer simulations are necessary.

More details:K.V.Lotov, Optimum angle for side injection of electrons into linear plasma wakefields. Journal of Plasma Physics, 2012

Trapping of a wide electron bunch



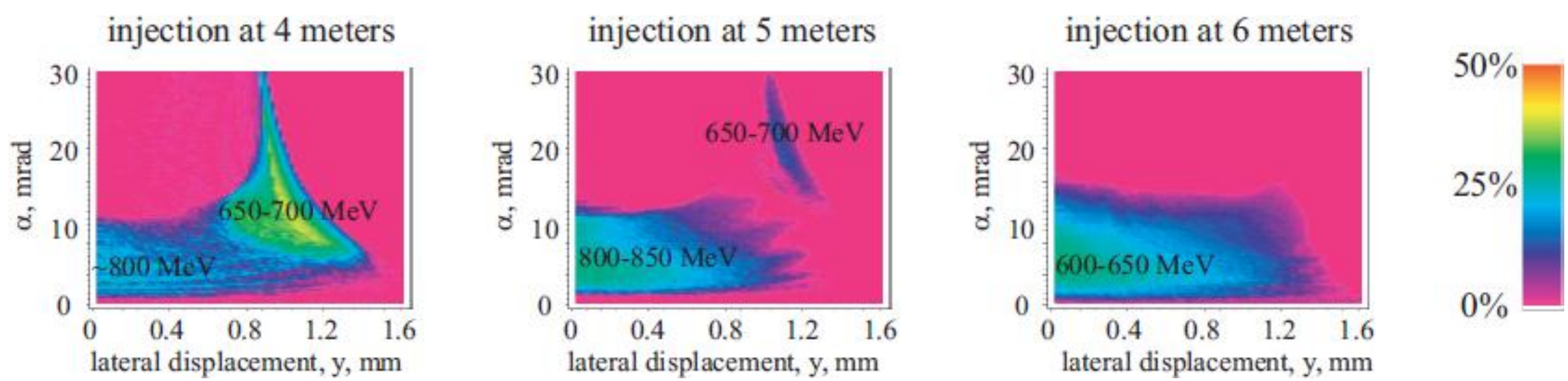
In simulations, we can take into account all the parameters, but the result will be not visual and hard to understand.

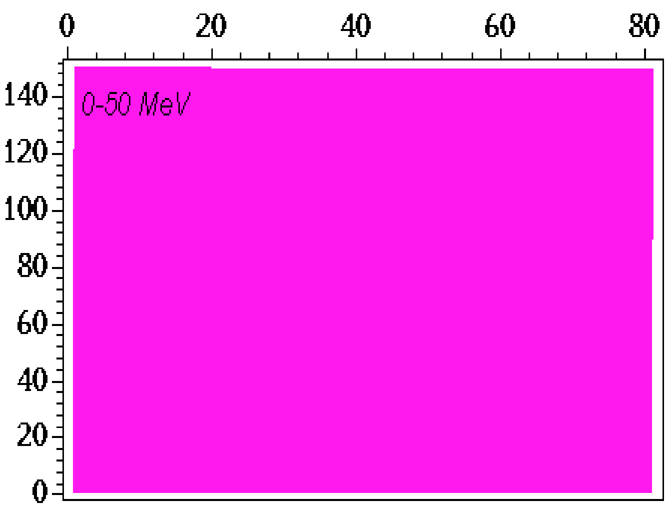
So we continuously scan two parameters (α_x, y) for three z-coordinates of the injection point, flatly distribute the electron beam over all wakefield phases and look at the beam fraction accelerated up to a certain energy.

Trapping of a wide electron bunch (results for 10 MeV beam)

We continuously scan two parameters (α_x, y) for three z-coordinates of the injection point, flatly distribute the electron beam over all wakefield phases and look at the beam fraction accelerated up to a certain energy.

W=10 MeV



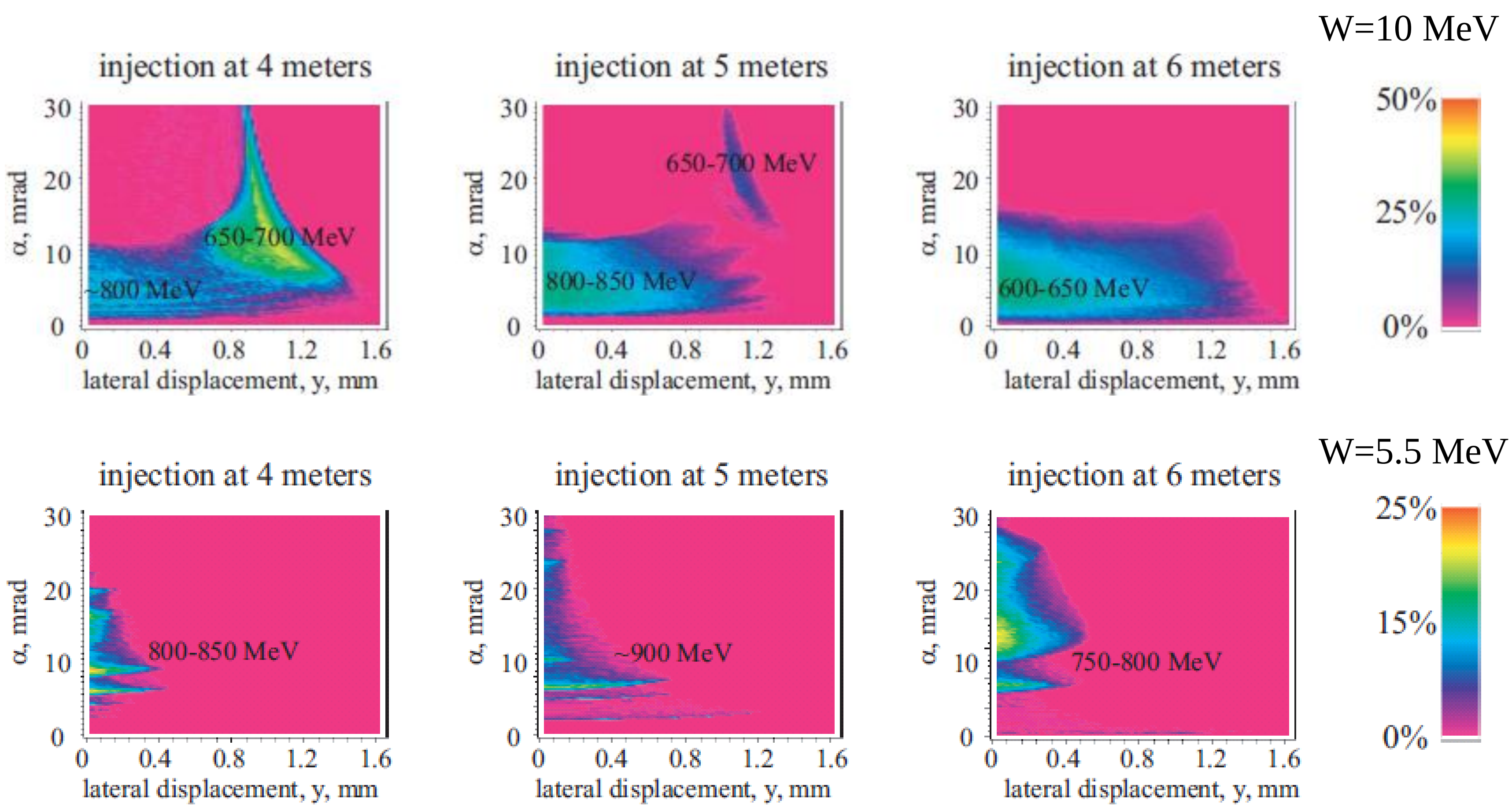


We trap 30% of electrons hitting the spot of the size 1.5 mm (*y*) x 10 mm (*x*) (full width) in real space and 10 x 10 mrad^2 in angle space.

10 mm (*x*) is 1 meter tolerance in z-coordinate of the injection point times 5 mrad (optimum angle) times 2 (full width)

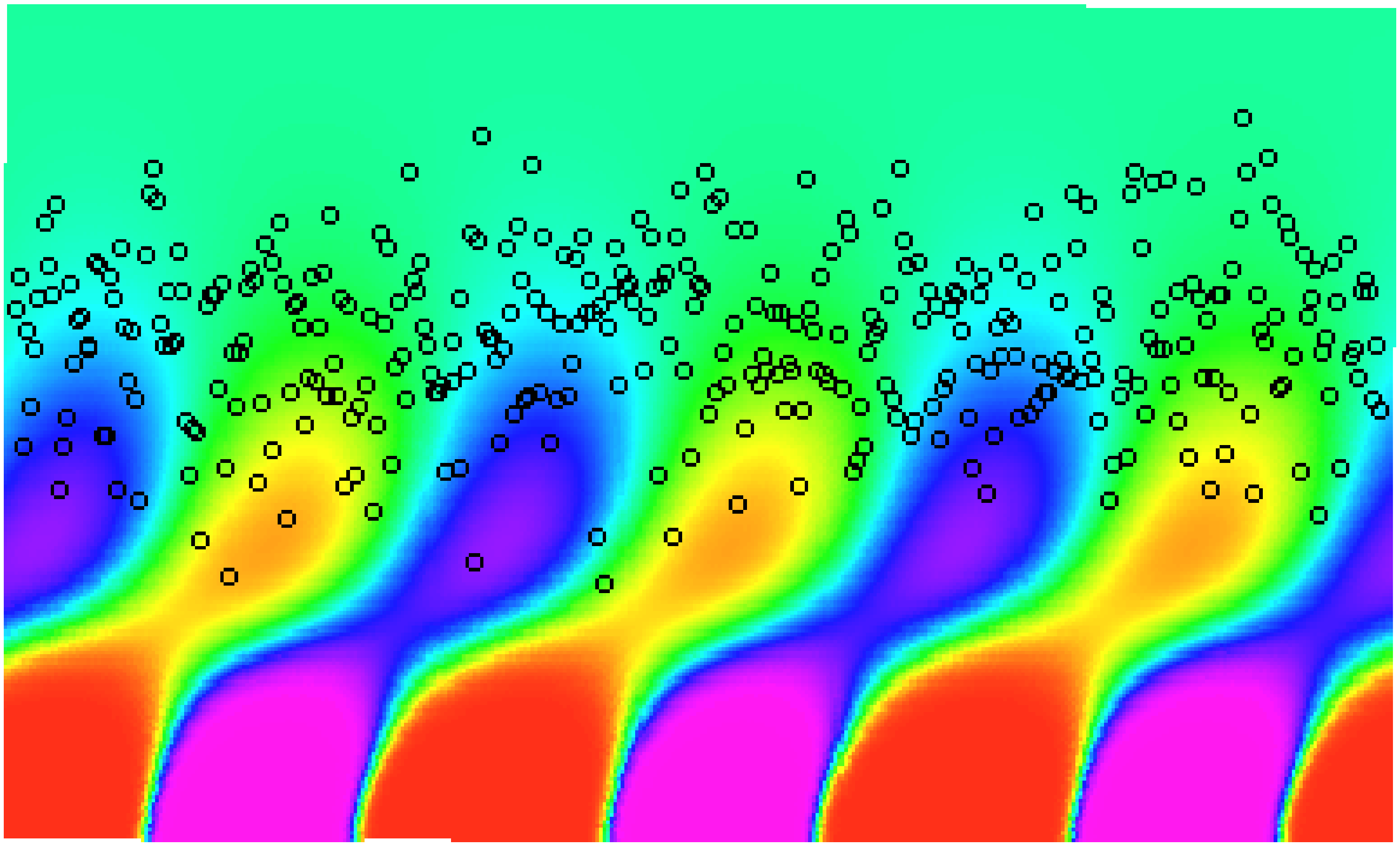
10 mrad in y: if $\alpha_y > \alpha_x$, then α_y becomes the injection angle, otherwise α_y has little effect on trapping

Trapping of a wide electron bunch (results for 5.5 MeV beam)



Trapping is much worse, but this is SPS-LHC wakefield. (Maybe) for optimized regimes the trapping threshold energy is lower.

Acceleration of electrons



Thank you