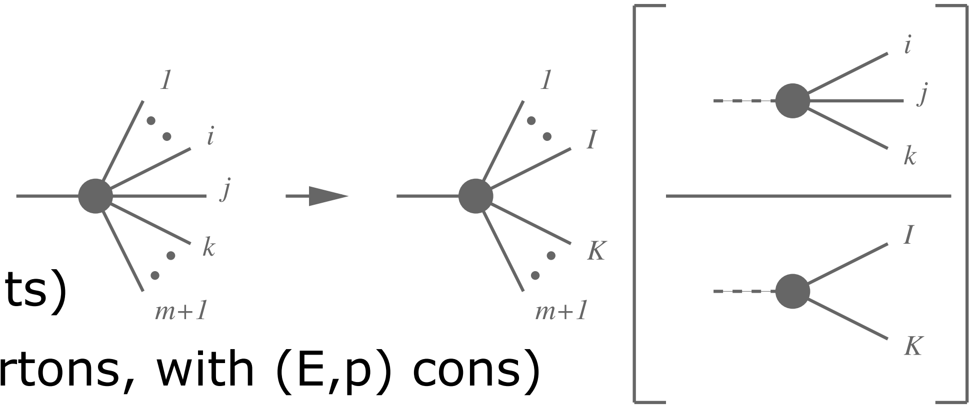
Recap: VINCIA

Plug-in to PYTHIA 8
C++ (~20,000 lines)

Giele, Kosower, Skands, PRD **78** (2008) 014026, PRD **84** (2011) 054003
Gehrmann-de Ridder, Ritzmann, Skands, PRD **85** (2012) 014013
Lopez-Villarejo, Skands, JHEP **11** (2011) 150

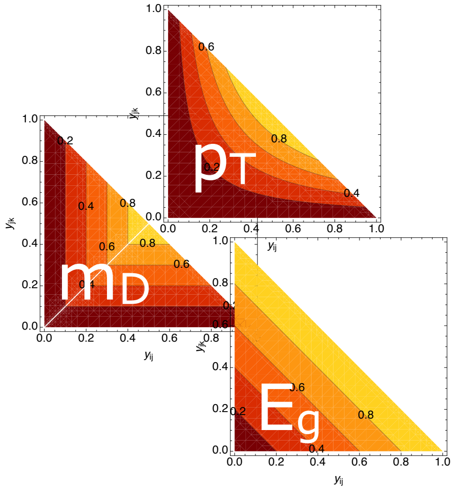


Based on antenna factorization

- of Amplitudes (exact in both soft and collinear limits)
- of Phase Space (LIPS : 2 on-shell → 3 on-shell partons, with (E,p) cons)

Evolution Scale

Infinite family of continuously deformable Q_E
E.g.: transverse momentum, invariant mass, energy
Hard 2→n: "smooth ordering" & LO matching

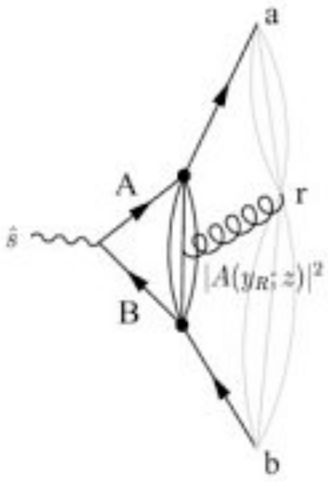


Radiation functions

Laurent-series with arbitrary coefficients, ant_i
E.g.: Gehrmann-Gehrmann-Glover, ARIADNE, MIN, MAX
+ Helicity-dependence & Massive fermions *(c,b,t)*

Kinematics maps

Formalism derived for infinitely deformable $\kappa_{3\to2}$
Special cases: ARIADNE, Kosower, + massive generalizations



vincia.hepforge.org

Changing Paradigm

"Higher-Order Corrections To Timelike Jets"
GeeKS: Giele, Kosower, Skands, PRD 84 (2011) 054003

Ask:

Is it possible to use the all-orders structure that the shower so nicely generates for us, as a substrate, a stratification, on top of which fixed-order amplitudes could be interpreted as corrections, which would be finite everywhere?

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Problems:

Traditional parton showers are *history-dependent* (non-Markovian)

→ Number of generated terms grows like $2^N N!$

\+ Highly complicated expansions

Parton- (or Catani-Seymour) Shower:
After 2 branchings: 8 terms
After 3 branchings: 48 terms
After 4 branchings: 384 terms

Solution: $(MC)^2$: Monte-Carlo Markov Chain

Markovian Antenna Showers (VINCIA)

→ Number of generated terms grows like N

\+ extremely simple expansions

Markovian Antenna Shower:
After 2 branchings: 2 terms
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One-Loop Corrections

Giele, Kosower, Skands, Phys.Rev. D78 (2008) 014026

Trivial Example (for notation): $Z^0 \to q\bar{q}$ First Order (~POWHEG)

Fixed Order: Exclusive 2-jet rate (2 and only 2 jets), at Q = Q$_{\text{had}}$

$$= \underbrace{|M_0^0|^2}_{\text{Born}} \left(1 + \underbrace{\frac{2\,\mathrm{Re}[M_0^0 M_0^{1*}]}{|M_0^0|^2}}_{\text{Virtual}} + \underbrace{\int_0^{Q^2_{\text{had}}} \mathrm{d}\Phi_{\text{ant}}\; g_s^2\; \mathcal{C}\; A_{g/q\bar{q}}}_{\text{Unresolved Real}} \right) \longrightarrow = \frac{|M_1^0|^2}{|M_0^0|^2}$$

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Markov Shower: Exclusive 2-jet rate (2 and only 2 jets), at Q = Q$_{\mathrm{had}}$

$$\underbrace{|M_0^0|^2}_{\text{Born}}\; \underbrace{\Delta(s, Q^2_{\mathrm{had}})}_{\text{Sudakov}} \;=\; |M_0^0|^2 \left(1 - \underbrace{\int_{Q^2_{\mathrm{had}}}^{s} \mathrm{d}\Phi_{\mathrm{ant}}\; g_s^2\, \mathcal{C}\; A_{g/q\bar{q}} + \mathcal{O}(\alpha_s^2)}_{\text{Approximate Virtual + Unresolved Real}} \right)$$

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Born Virtual Unresolved Real

$$= \frac{|M_1^0|^2}{|M_0^0|^2}$$

Markov Shower: Exclusive 2-jet rate (2 and only 2 jets), at Q = Q_{had}

$$|M_0^0|^2\; \Delta(s, Q^2_{\mathrm{had}}) \;=\; |M_0^0|^2 \left(1 - \int_{Q^2_{\mathrm{had}}}^{s} \mathrm{d}\Phi_{\mathrm{ant}}\; g_s^2\; \mathcal{C}\; A_{g/q\bar{q}} + \mathcal{O}(\alpha_s^2) \right)$$

Born Sudakov Approximate Virtual + Unresolved Real

NLO Correction: Subtract and correct by difference

$$\left.\begin{aligned} \frac{2\,\mathrm{Re}[M_0^0 M_0^{1*}]}{|M_0^0|^2} &= \frac{\alpha_s}{2\pi}\, 2C_F\; \left(2I_{q\bar{q}}(\epsilon, \mu^2/m_Z^2) - 4\right) \\ \int_0^s \mathrm{d}\Phi_{\mathrm{ant}}\; 2C_F\; g_s^2\; A_{g/q\bar{q}} &= \frac{\alpha_s}{2\pi}\, 2C_F\; \left(-2I_{q\bar{q}}(\epsilon, \mu^2/m_Z^2) + \frac{19}{4}\right) \end{aligned}\right\} \quad |M_0^0|^2 \to \left(1 + \frac{\alpha_s}{\pi}\right) |M_0^0|^2$$

IR Singularity Operator

$|M|^2_{\mathrm{NLO}} = (1+V)\; |M|^2_{\mathrm{LO}}$

One-Loop Corrections

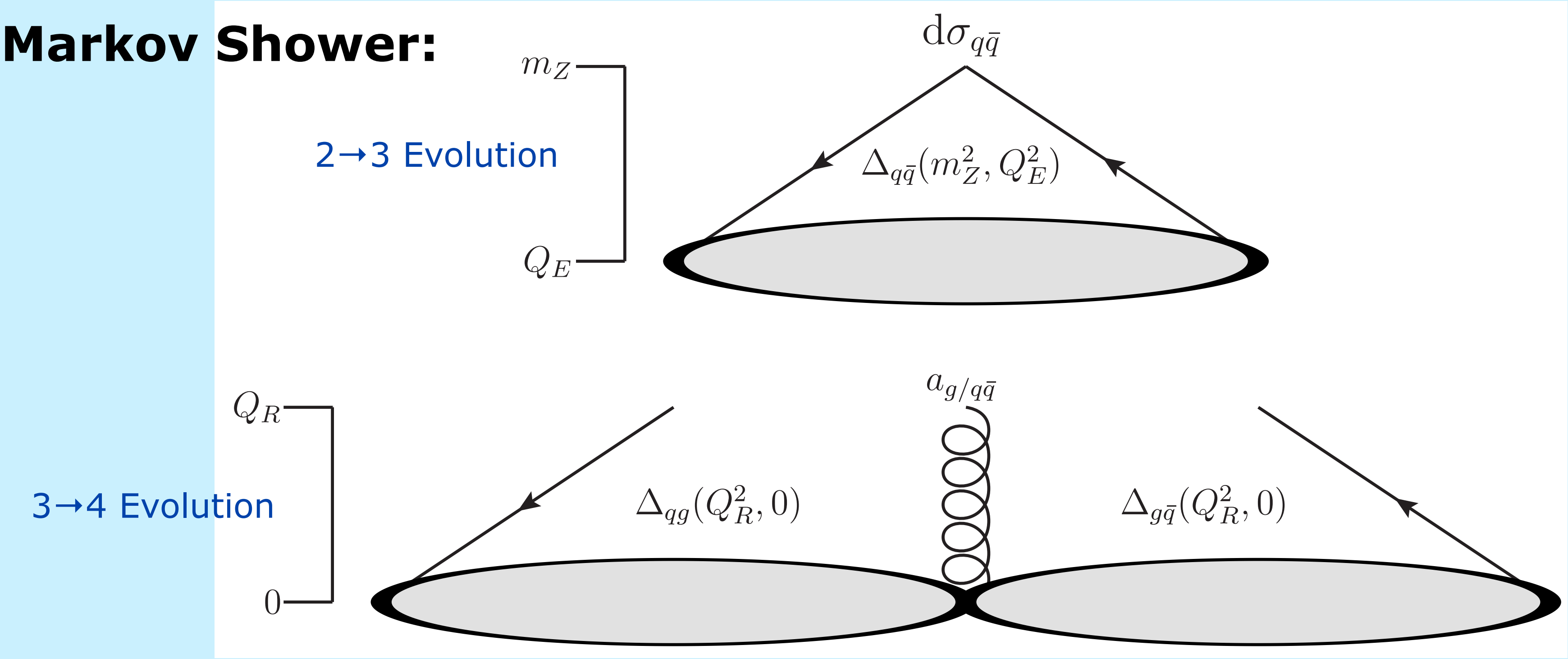
Ongoing work, with E. Laenen & L. Hartgring (NIKHEF)

Getting Serious: second order

Fixed Order: Exclusive 3-jet rate (3 and only 3 jets), at Q = Q$_{\text{had}}$

$$\text{Exact} \;\rightarrow\; \underbrace{|M_1^0|^2}_{\text{Born}} \;+\; \underbrace{2\,\mathrm{Re}[M_1^0 M_1^{1*}]}_{\text{Virtual}} \;+\; \underbrace{\int_0^{Q^2_{\text{had}}} \frac{\mathrm{d}\Phi_2}{\mathrm{d}\Phi_1}|M_2^0|^2}_{\text{Unresolved Real}}$$

Markov Shower:



$$\text{Approximate} \;\rightarrow\; \underbrace{(1+V_0)}_{V_0 = \alpha_s/\pi}\; \underbrace{|M_1^0|^2}_{\mu_R}\; \underbrace{\Delta_2(m_Z^2, Q_1^2)}_{\text{2→3 Evolution}}\; \underbrace{\Delta_3(Q_{R1}^2, Q_{\text{had}}^2)}_{\text{3→4 Evolution}}\;,$$

Master Equation

Ongoing work, with E. Laenen & L. Hartgring (NIKHEF)

NLO Correction: Subtract and correct by difference

$A_{NLO} = A_{LO}\,(1+V_1)$

V0

μR

$$
\begin{aligned}
V_{1Z}(q,g,\bar{q}) \;=\;& \left[\frac{2\,\mathrm{Re}[M_1^0 M_1^{1*}]}{|M_1^0|^2}\right]^{\mathrm{LC}} - \frac{\alpha_s}{\pi} - \frac{\alpha_s}{2\pi}\left(\frac{11N_C - 2n_F}{6}\right)\ln\left(\frac{\mu_{\mathrm{ME}}^2}{\mu_{\mathrm{PS}}^2}\right) \\
&+ \frac{\alpha_s C_A}{2\pi}\left[-2I^{(1)}_{qg}(\epsilon,\mu^2/s_{qg}) - 2I^{(1)}_{qg}(\epsilon,\mu^2/s_{g\bar{q}}) + \frac{34}{3}\right] \\
&+ \frac{\alpha_s n_F}{2\pi}\left[-2I^{(1)}_{qg,F}(\epsilon,\mu^2/s_{qg}) - 2I^{(1)}_{g\bar{q},F}(\epsilon,\mu^2/s_{qg}) - 1\right] \\
&+ \frac{\alpha_s C_A}{2\pi}\left[8\pi^2\int_{Q_1^2}^{m_Z^2} \mathrm{d}\Phi_{\mathrm{ant}}\; A^{\mathrm{std}}_{g/q\bar{q}} + 8\pi^2\int_{Q_1^2}^{m_Z^2} \mathrm{d}\Phi_{\mathrm{ant}}\; \delta A_{g/q\bar{q}} \right.\\
&\left. \quad - \sum_{j=1}^{2} 8\pi^2\int_0^{s_j} \mathrm{d}\Phi_{\mathrm{ant}}\;(1-O_{Ej})\; A^{\mathrm{std}}_{g/qg} + \sum_{j=1}^{2} 8\pi^2\int_0^{s_j} \mathrm{d}\Phi_{\mathrm{ant}}\; \delta A_{g/qg}\right] \\
&+ \frac{\alpha_s n_F}{2\pi}\left[-\sum_{j=1}^{2} 8\pi^2\int_0^{s_j} \mathrm{d}\Phi_{\mathrm{ant}}(1-O_{Sj})\; P_{Aj}\; A^{\mathrm{std}}_{\bar{q}/qg} + \sum_{j=1}^{2} 8\pi^2\int_0^{s_j} \mathrm{d}\Phi_{\mathrm{ant}}\; \delta A_{\bar{q}/qg}\right.\\
&\left. \quad -\frac{1}{6}\,\frac{s_{qg}-s_{g\bar{q}}}{s_{qg}+s_{g\bar{q}}}\ln\left(\frac{s_{qg}}{s_{g\bar{q}}}\right)\right],
\end{aligned}
$$

Standard IR Singularities

Standard Finite Terms

δA = LO Matching Terms (finite)

Gluon Emission IR Singularity

Gluon Splitting IR Singularity

2→3 Sudakov Logs

Q_1 = 3-parton Resolution Scale

$\delta A_{2\to3}$

3→4 Emit

O_{Ej} = Gluon-Emission Ordering Function

$\delta A_{3\to4,\ \mathrm{Emit}}$

3→4 Sudakov Logs

3→4 Split

O_{Sj} = Gluon-Splitting Ordering Function

$\delta A_{3\to4,\ \mathrm{Split}}$

*)Note: here only Leading Color

Loop Corrections

Ongoing work, with E. Laenen & L. Hartgring (NIKHEF)

$(\mathrm{MC})^2$: NLO Z → 2 → 3 Jets + Markov Shower

Size of NLO Correction:
over 3-parton Phase Space

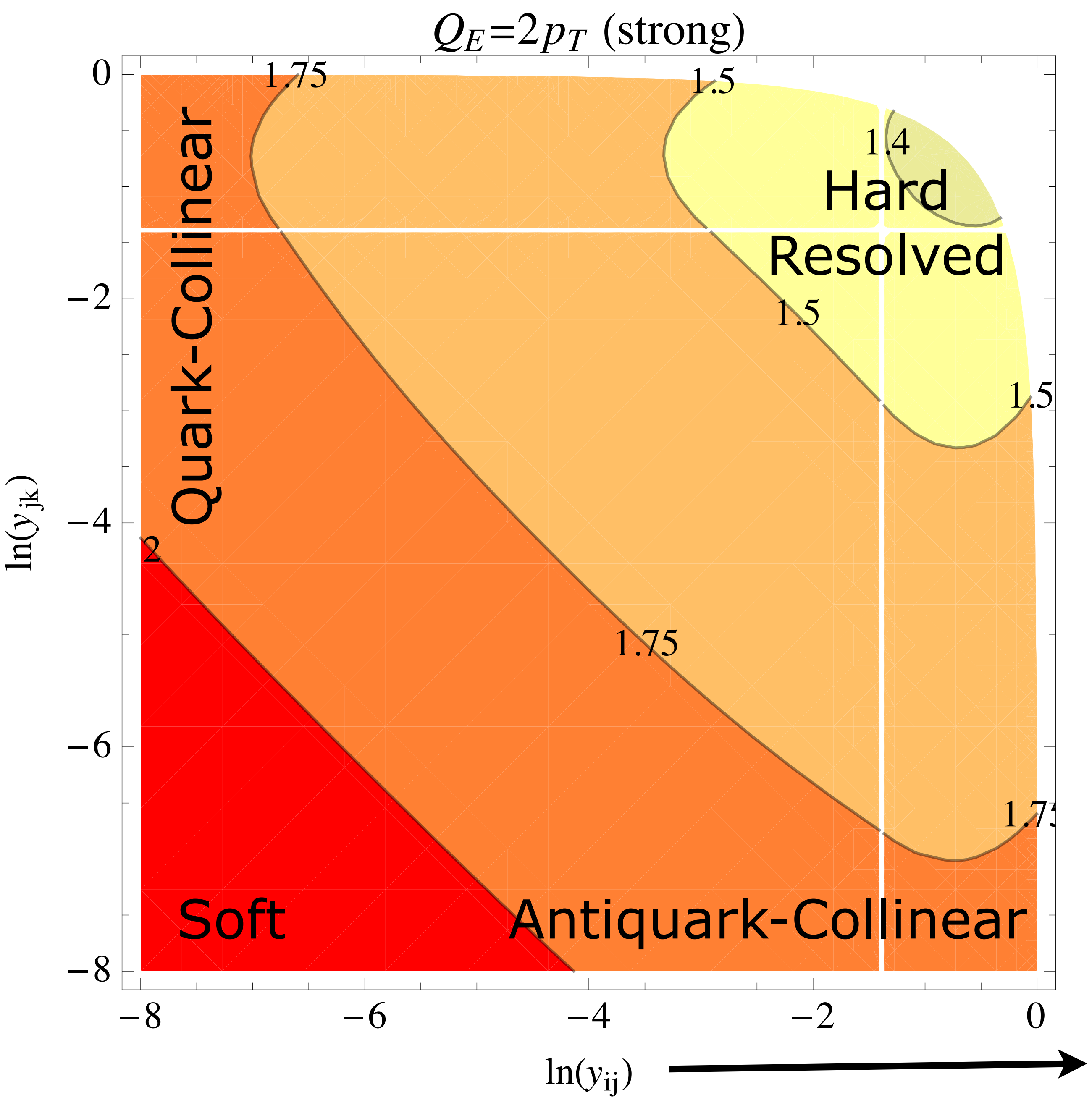
Markov Evolution in:
Transverse Momentum

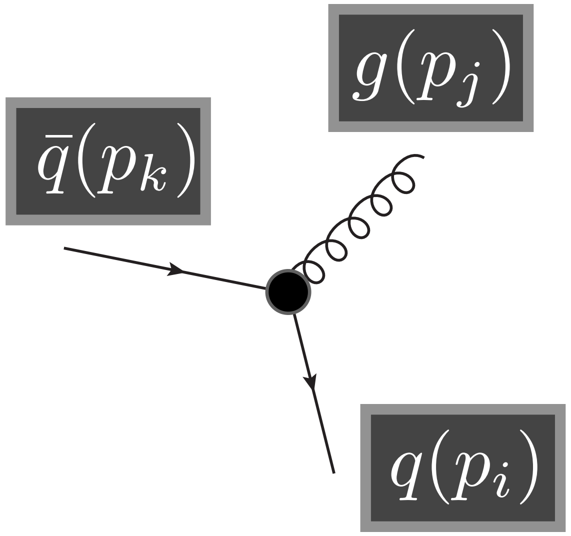
Parameters:

$\alpha_S(M_Z) = 0.12$

$\mu_R = m_Z$

$\Lambda_{QCD} = \Lambda_{MS}$





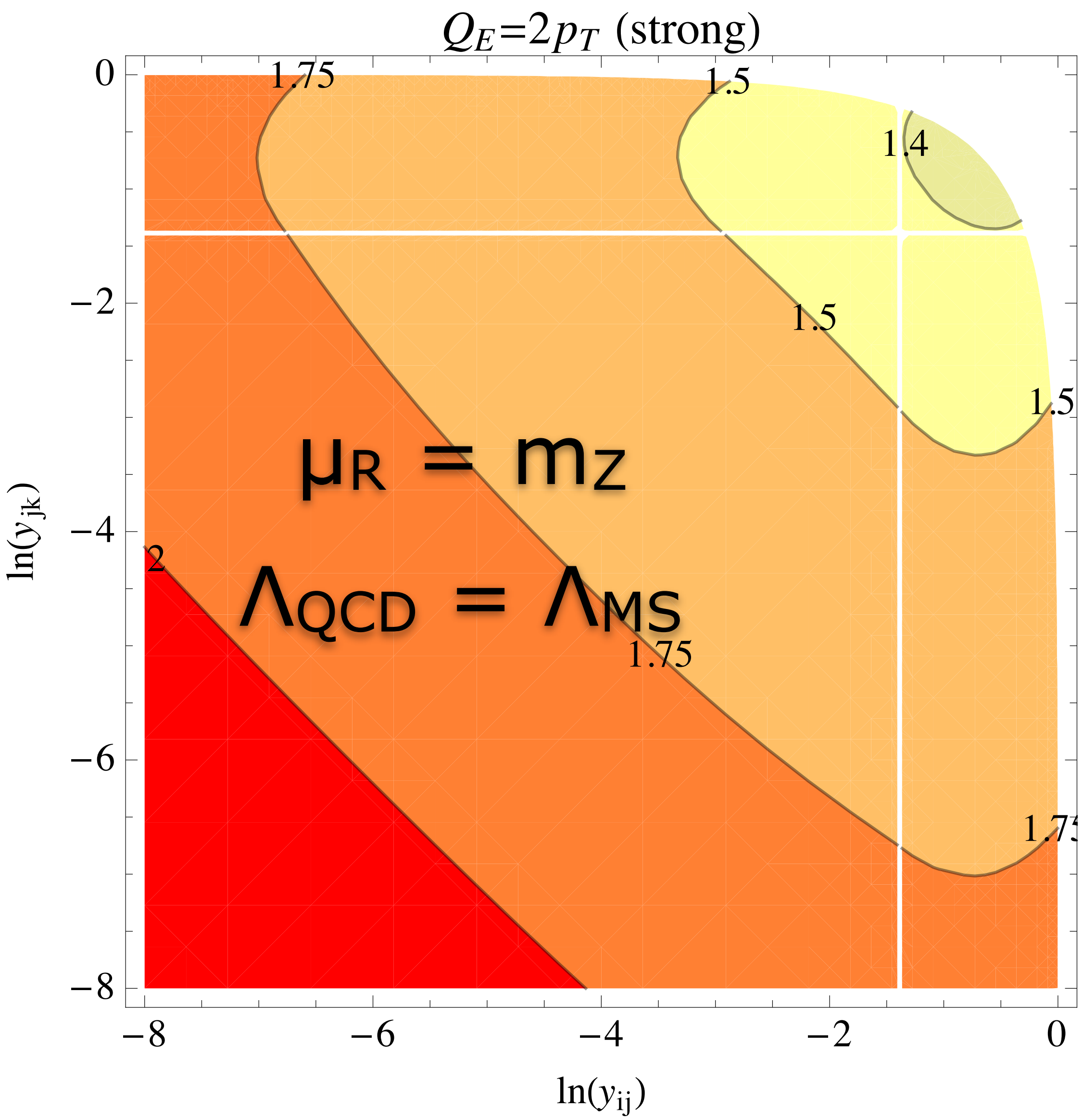
Scaled Invariants

$$y_{ij} = \frac{(p_i \cdot p_j)}{M_Z^2}$$

→ 0 when i||j
& when E_j → 0

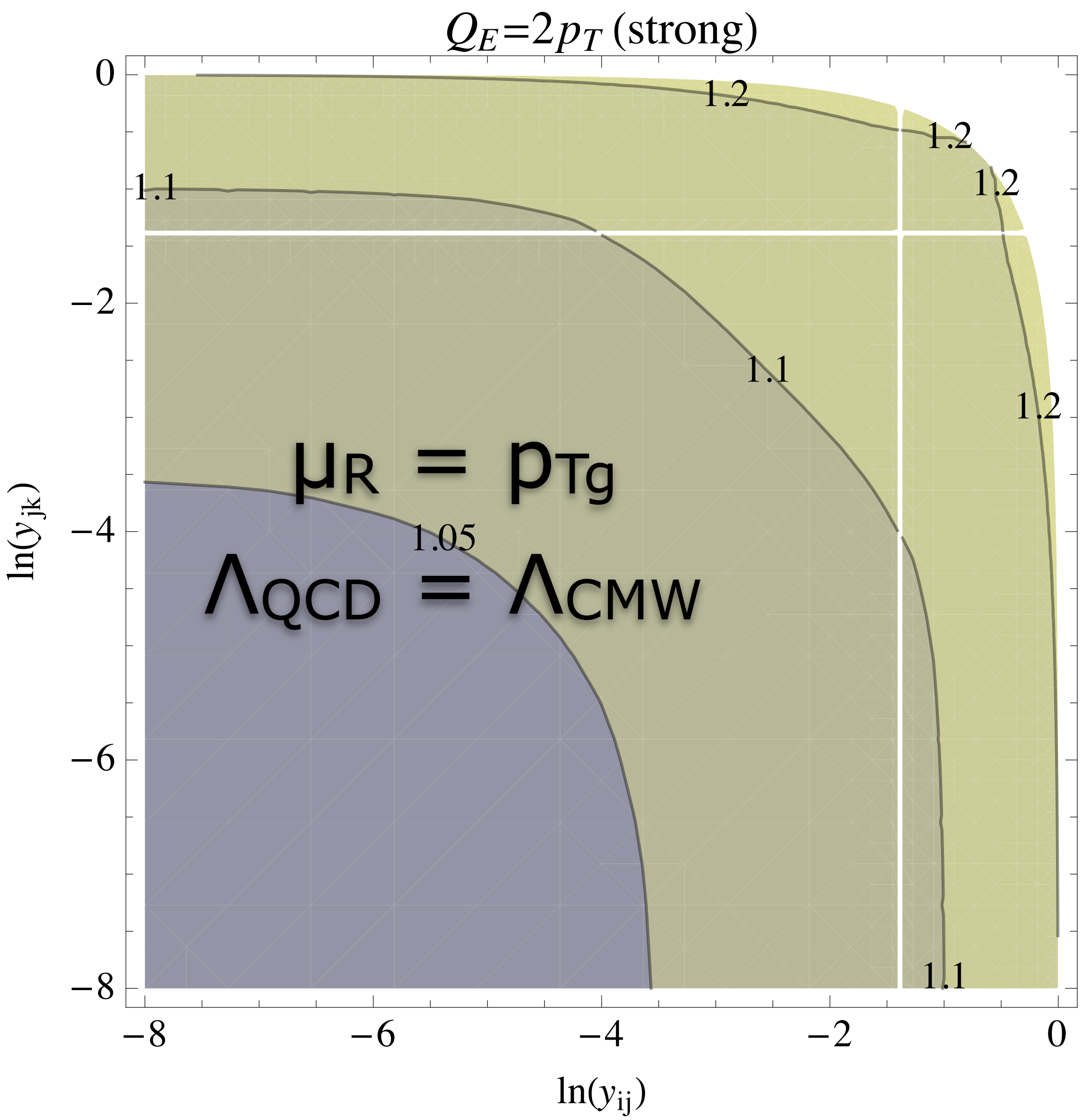
Choice of μ_R

Ongoing work, with E. Laenen & L. Hartgring (NIKHEF)



A) M_Z

"Typical" Fixed-Order Choice



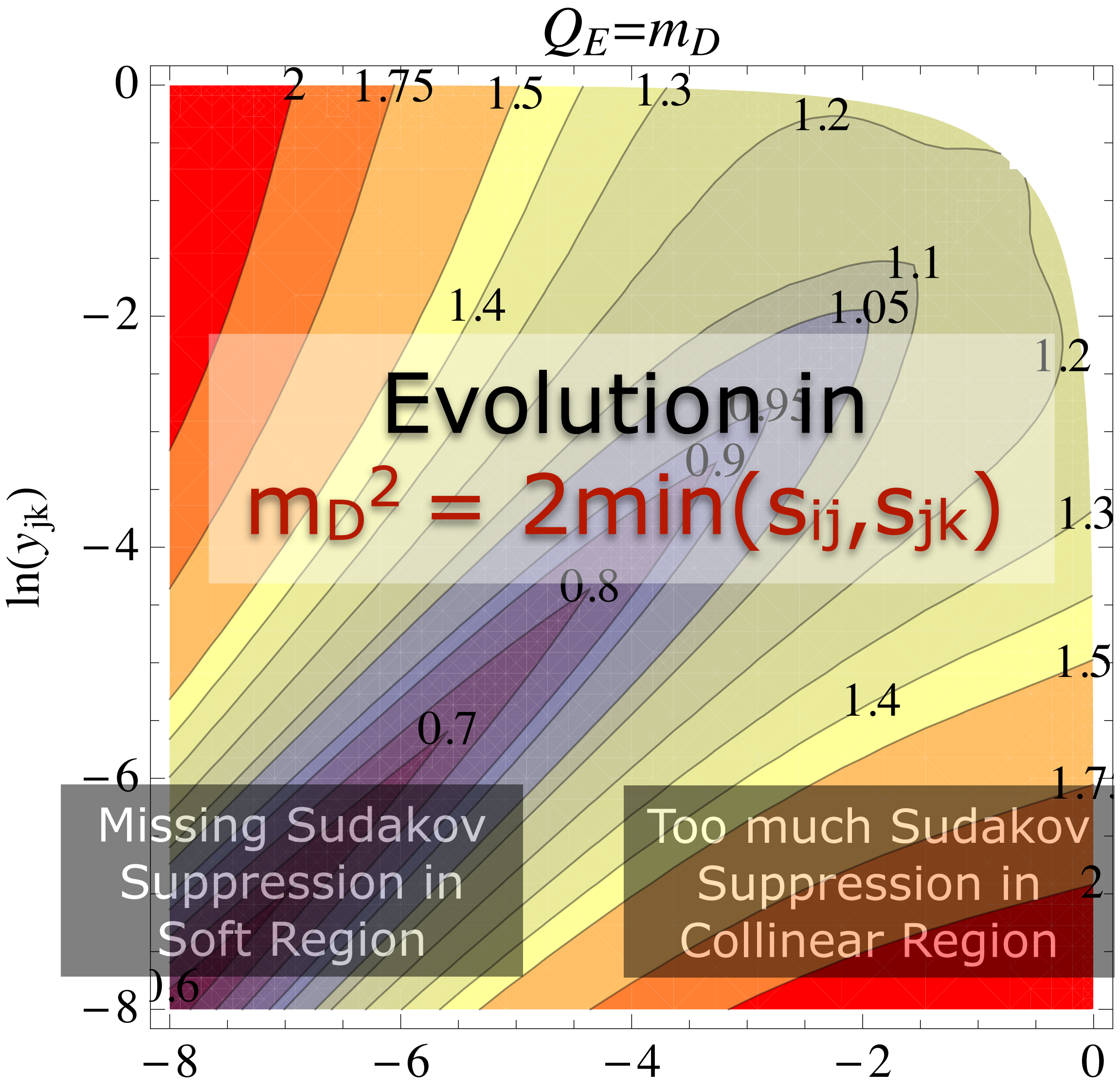
B) p_T

= "Typical" Shower Choice

Markov Evolution in: Transverse Momentum, $\alpha_S(M_Z)$ = 0.12

Choice of Q_{Evol}

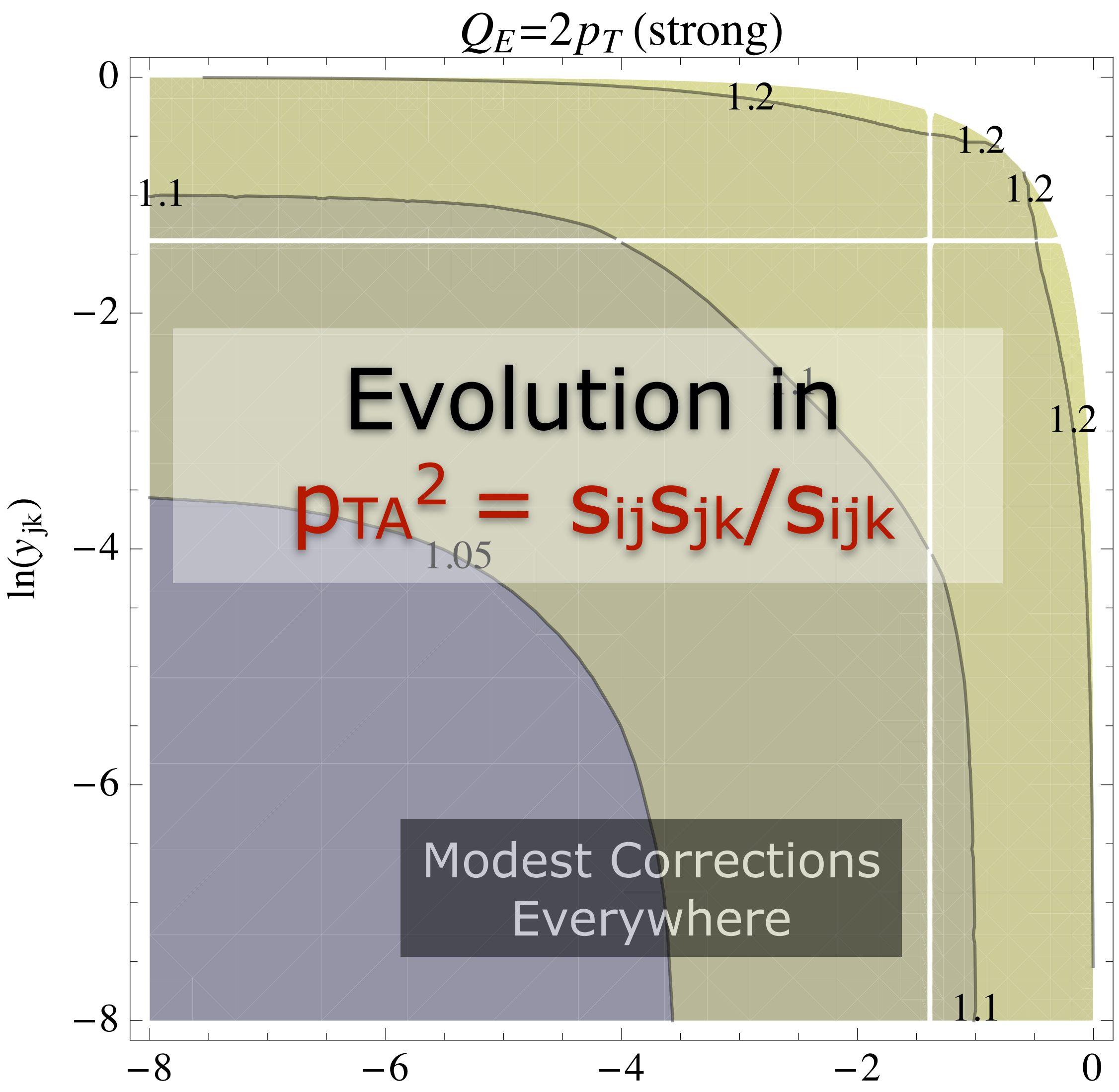
Ongoing work, with E. Laenen & L. Hartgring (NIKHEF)



Evolution in
$m_D^2 = 2\min(s_{ij},s_{jk})$

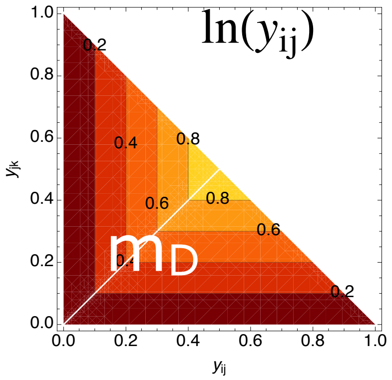
Missing Sudakov Suppression in Soft Region

Too much Sudakov Suppression in Collinear Region



Evolution in
$p_{TA}^2 = s_{ij}s_{jk}/s_{ijk}$

Modest Corrections Everywhere



Parameters:
$\alpha_S(M_Z) = 0.12,$
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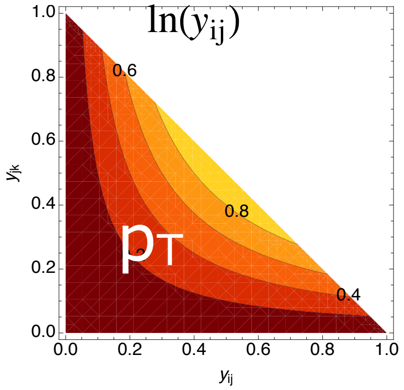
$\Lambda_{QCD} = \Lambda_{CMW}$

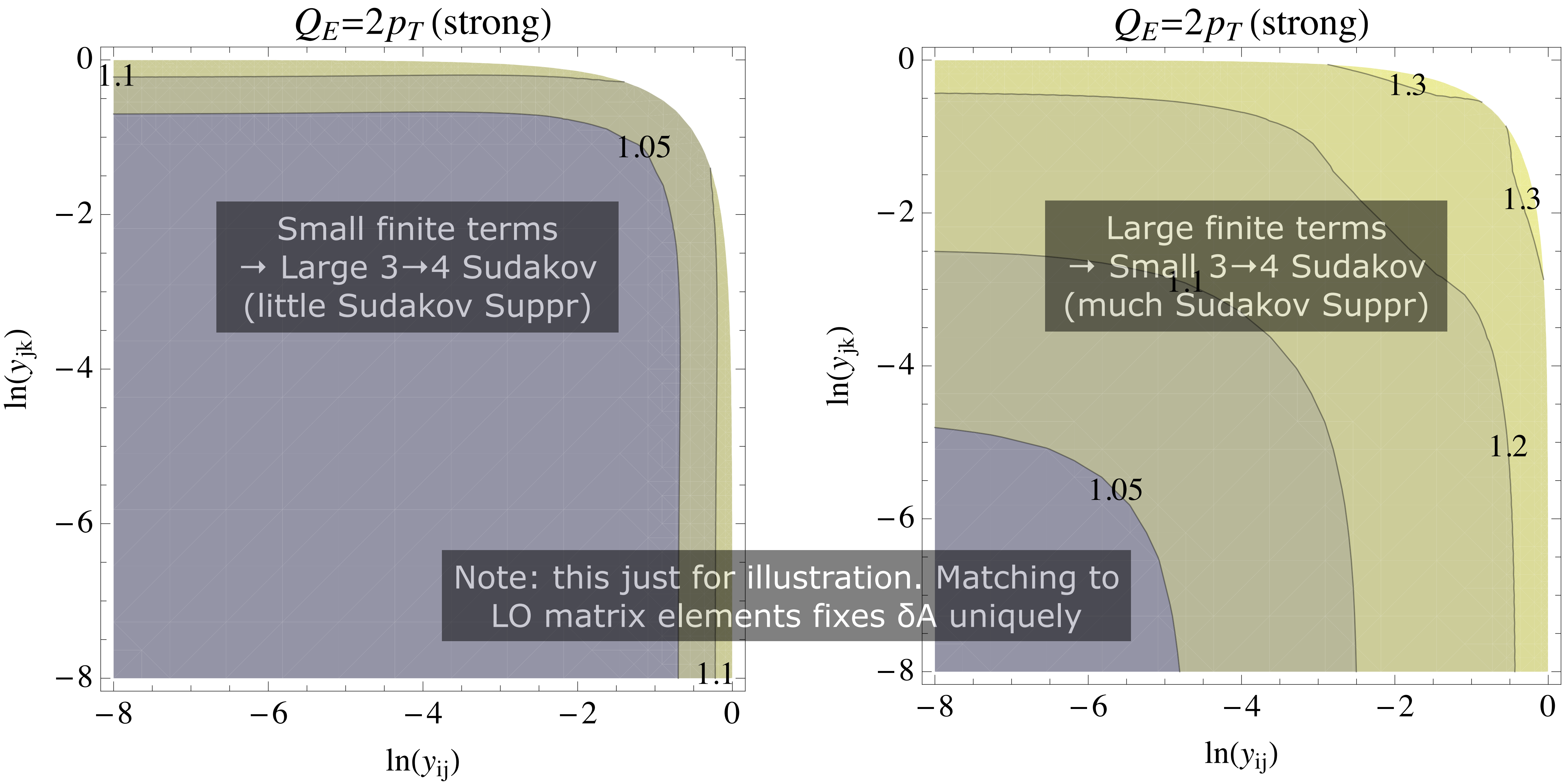


Choice of Finite Terms

Ongoing work, with E. Laenen & L. Hartgring (NIKHEF)

MIN Antennae:
$\delta A_{3\to4} < 0$

MAX Antennae:
$\delta A_{3\to4} > 0$



Small finite terms
→ Large 3→4 Sudakov
(little Sudakov Suppr)

Large finite terms
→ Small 3→4 Sudakov
(much Sudakov Suppr)

Note: this just for illustration. Matching to
LO matrix elements fixes δA uniquely

Parameters: $\alpha_S(M_Z) = 0.12$, $\mu_R = p_{TA}$, $\Lambda_{QCD} = \Lambda_{CMW}$

Outlook

1. **Publish 3 papers** (~ a couple of months: helicities, NLO multileg, ISR)
2. **Apply these corrections to a broader class of processes, including ISR** → LHC phenomenology
3. **Automate correction procedure, via interfaces to one-loop codes ...** (goes slightly beyond Binoth Accord; for LO corrections, we currently use own interface to modified MadGraph ME's)
4. **Variations.** No calculation is more precise than the reliability of its uncertainty estimate → aim for full assessment of TH uncertainties.
5. **Recycle formalism for all-orders shower corrections?**

Phase Space Contours

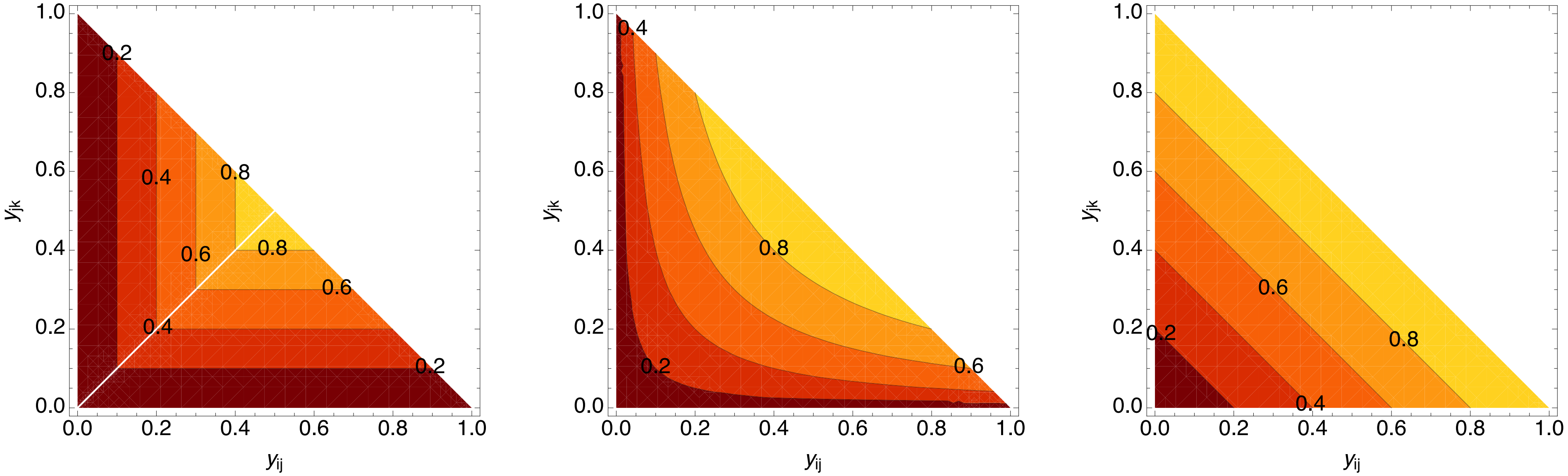
Evolution Variables:

Mass-Ordering ($m^2_{\rm min}$)

$p_\perp$-ordering ($\langle m^2\rangle_{\rm geometric}$)

Energy-Ordering ($\langle m^2\rangle_{\rm arithmetic}$)

Linear in y

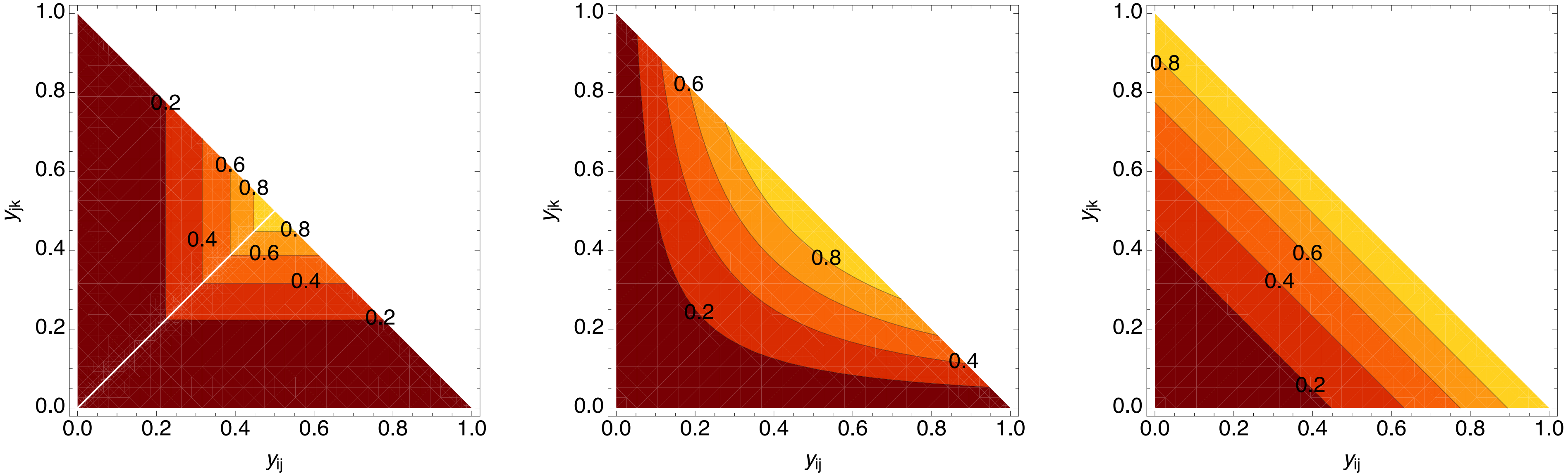


(a) $Q_E^2 = m_D^2 = 2\min(y_{ij},y_{jk})s$

(b) $Q_E^2 = 2p_\perp\sqrt{s} = 2\sqrt{y_{ij}y_{jk}}s$

(c) $Q_E^2 = 2E^*\sqrt{s} = (y_{ij}+y_{jk})s$

Quadratic in y



(d) $Q_E^2 = \frac{m_D^4}{s} = 4\min(y_{ij}^2,y_{jk}^2)s$

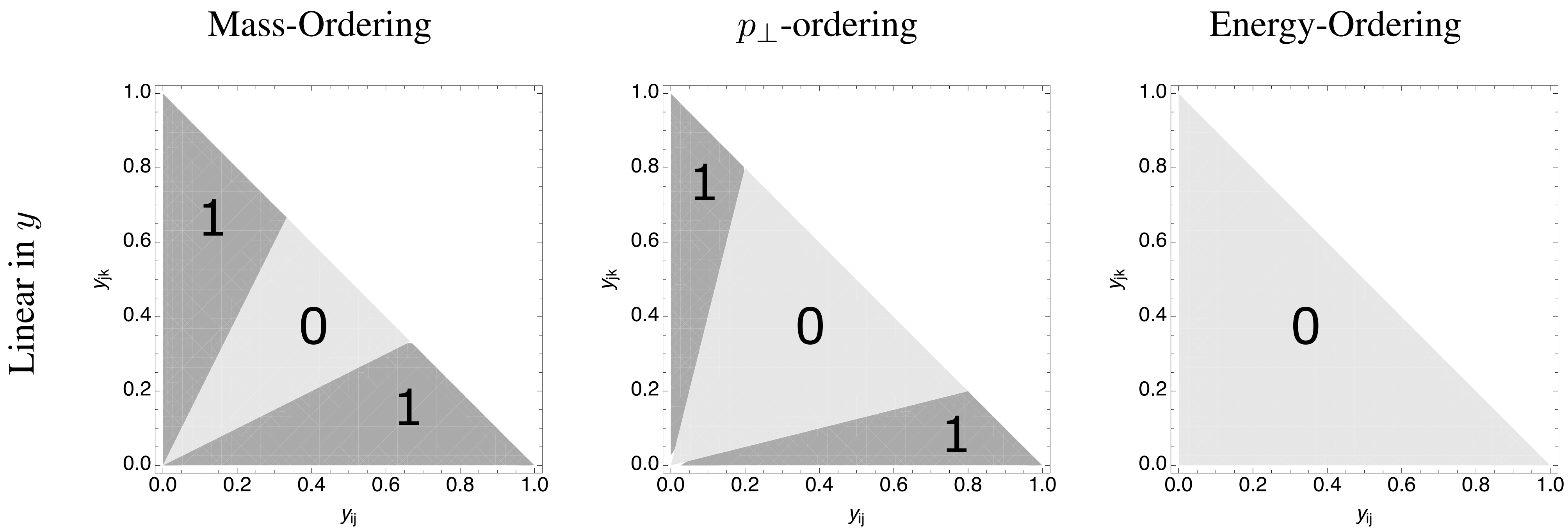
(e) $Q_E^2 = 4p_\perp^2 = 4y_{ij}y_{jk}s$

(f) $Q_E^2 = 4E^{*2} = (y_{ij}+y_{jk})^2 s$

Consequences of Ordering

Number of antennae restricted
by ordering condition

Ongoing work, with E. Laenen & L. Hartgring (NIKHEF)



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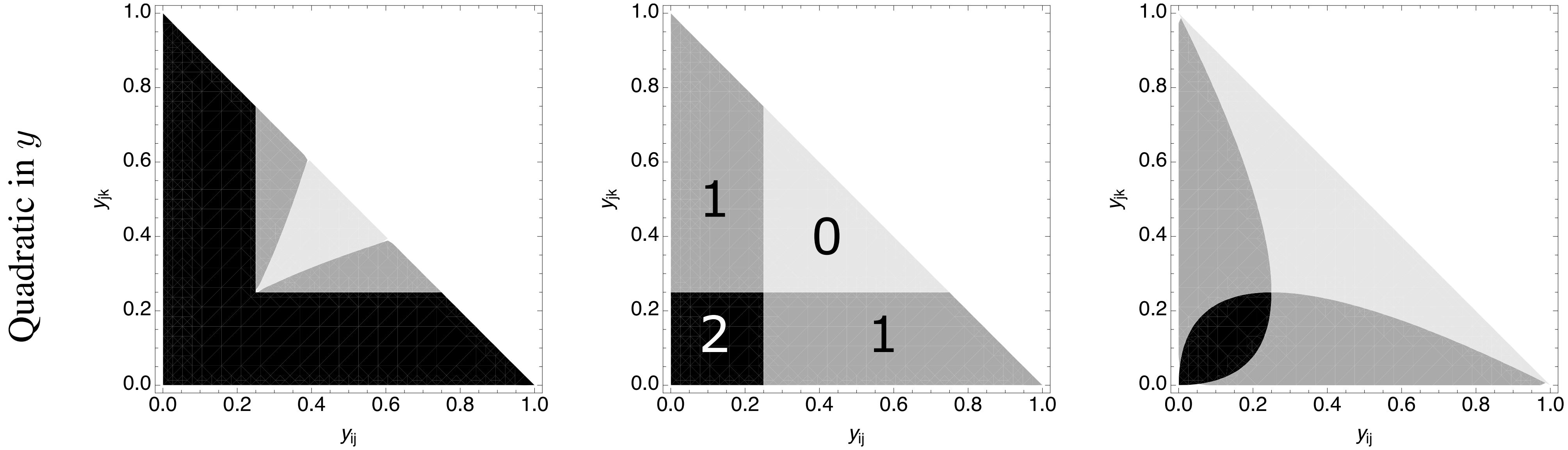

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Solution: $(\mathrm{MC})^2$

"Higher-Order Corrections To Timelike Jets"
GeeKS: Giele, Kosower, Skands, PRD 84 (2011) 054003

Idea:

Start from quasi-conformal all-orders structure (approximate)

Impose exact higher orders as finite corrections

Truncate at fixed **scale** (rather than fixed order)

Bonus: low-scale partonic events → can be hadronized

Problems:

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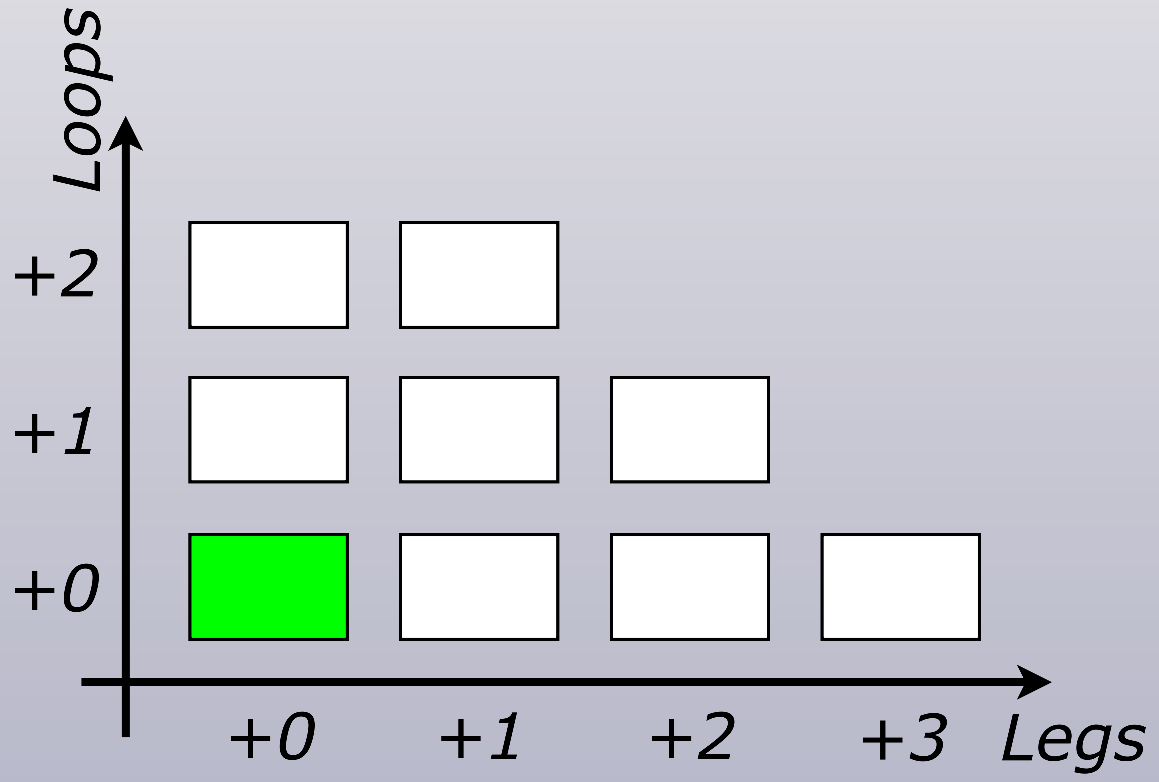
New: Markovian pQCD*

*)pQCD : perturbative QCD

Start at Born level

$$|M_F|^2$$



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Start at Born level

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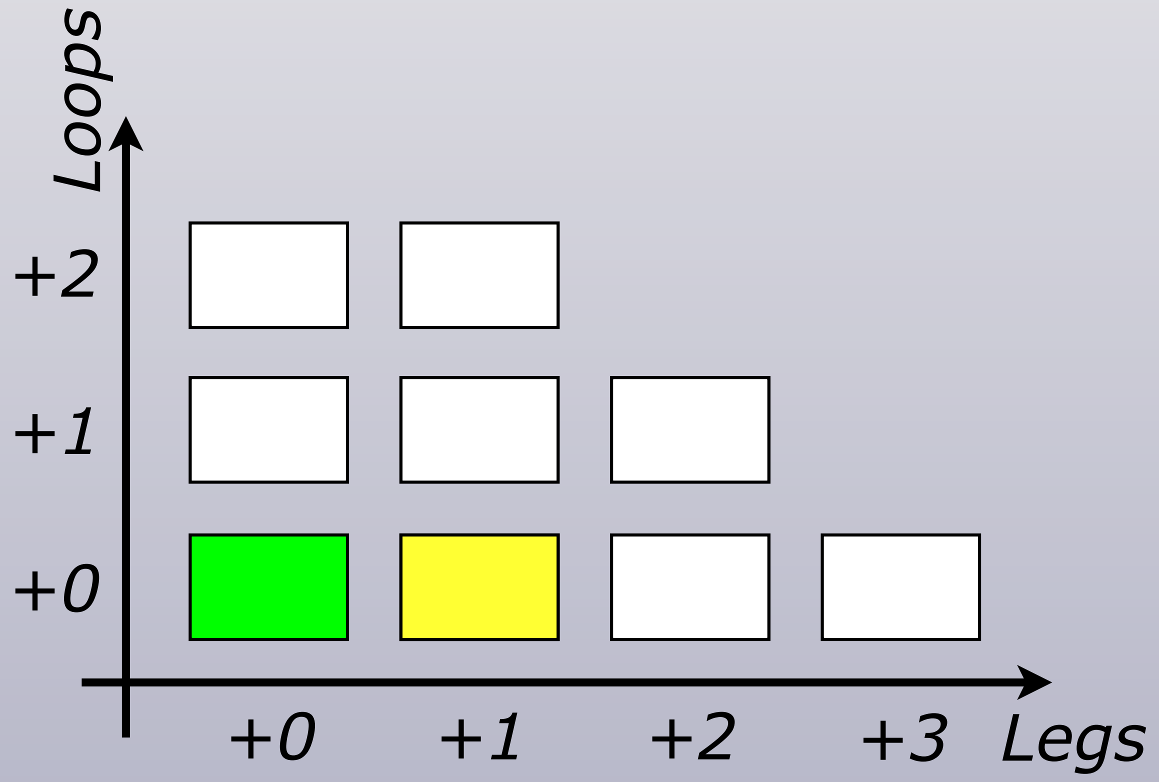
Generate "shower" emission

$$|M_{F+1}|^2 \overset{LL}{\sim} \sum_{i\in\text{ant}} a_i \; |M_F|^2$$



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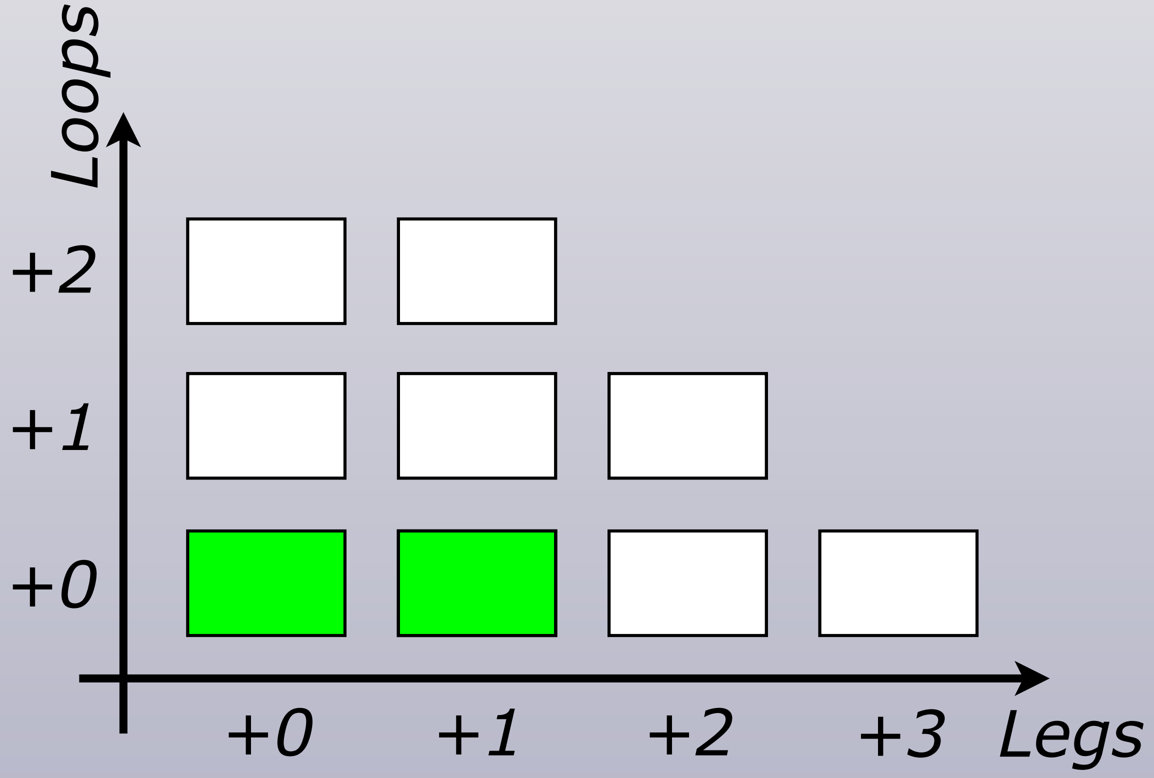
Correct to Matrix Element

$$a_i \to \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2} a_i$$



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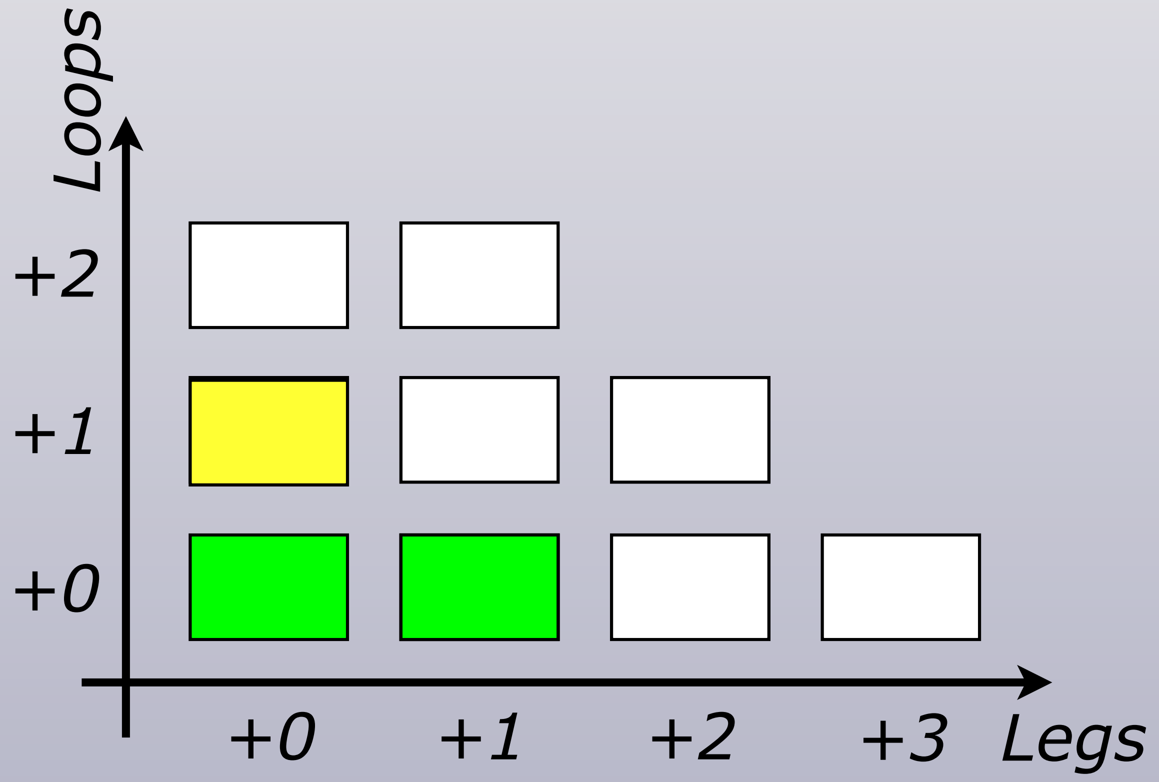
Unitarity of Shower

$$\mathrm{Virtual} = -\int \mathrm{Real}$$



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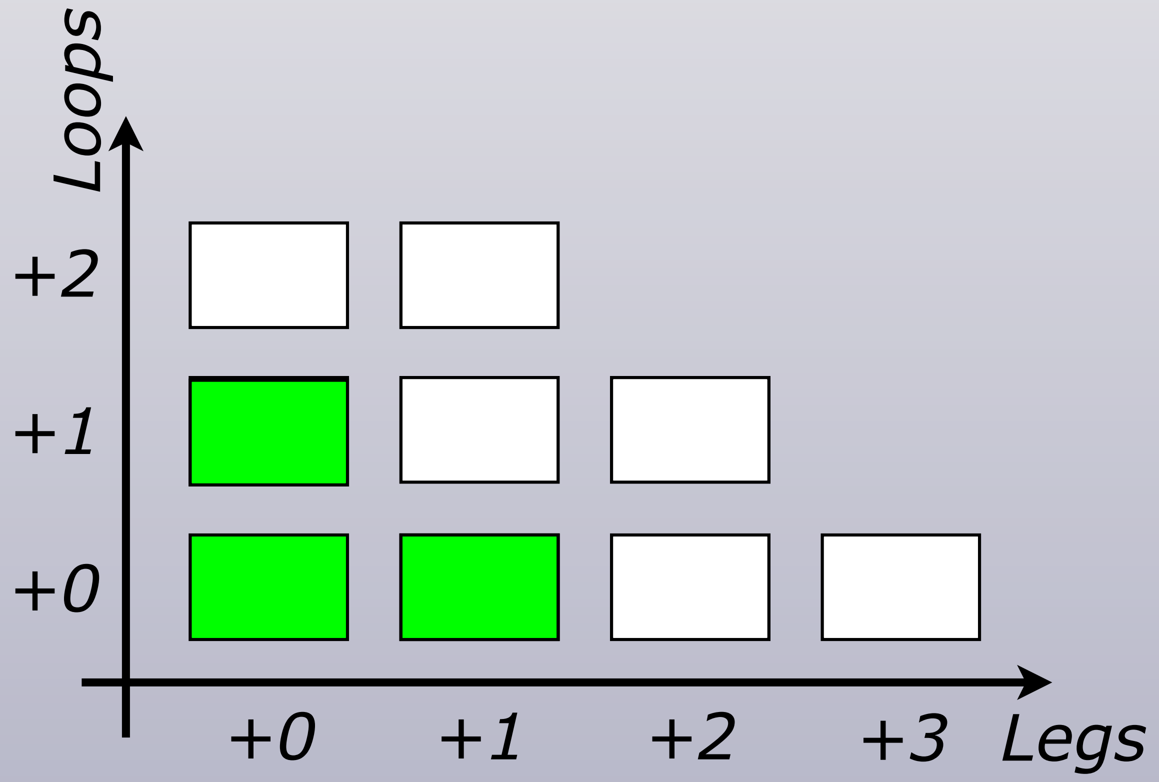
Unitarity of Shower

$$\mathrm{Virtual} = - \int \mathrm{Real}$$

Correct to Matrix Element

$$|M_F|^2 \rightarrow |M_F|^2 + 2\mathrm{Re}[M_F^1 M_F^0] + \int \mathrm{Real}$$



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Generate "shower" emission

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Correct to Matrix Element

$$a_i \to \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2} a_i$$

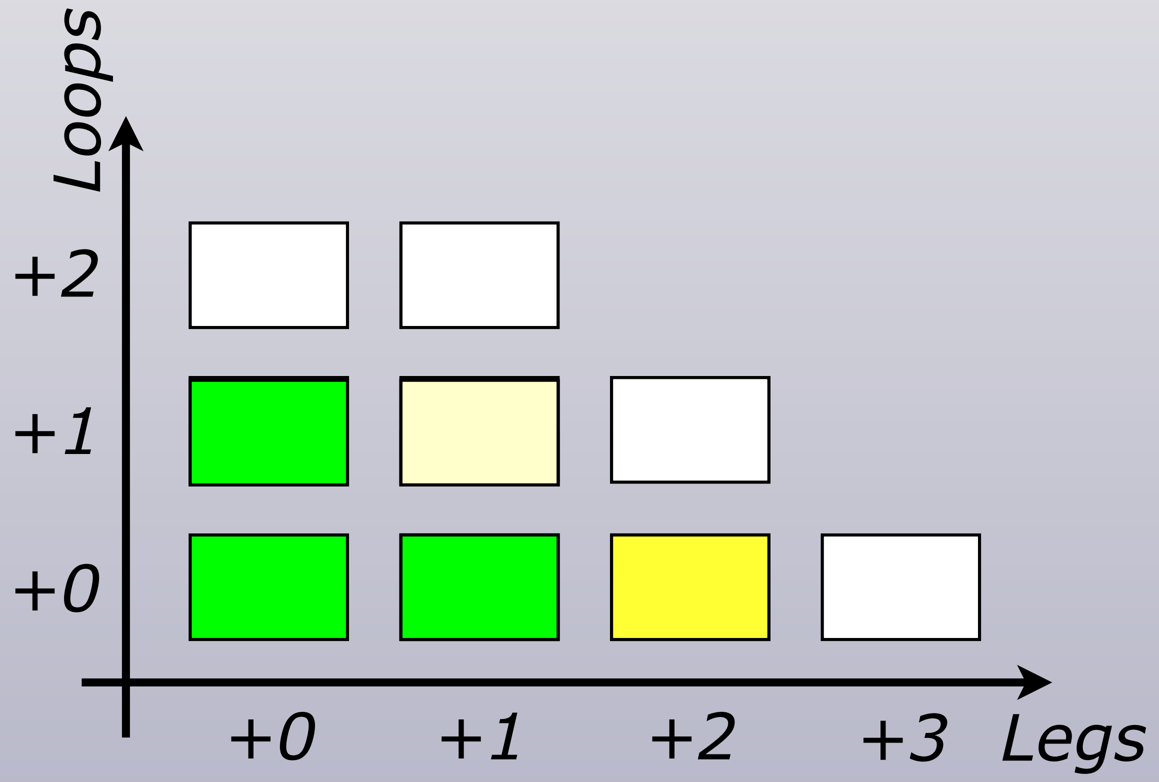
Unitarity of Shower

$$\text{Virtual} = -\int \text{Real}$$

Correct to Matrix Element

$$|M_F|^2 \to |M_F|^2 + 2\text{Re}[M_F^1 M_F^0] + \int \text{Real}$$

Repeat



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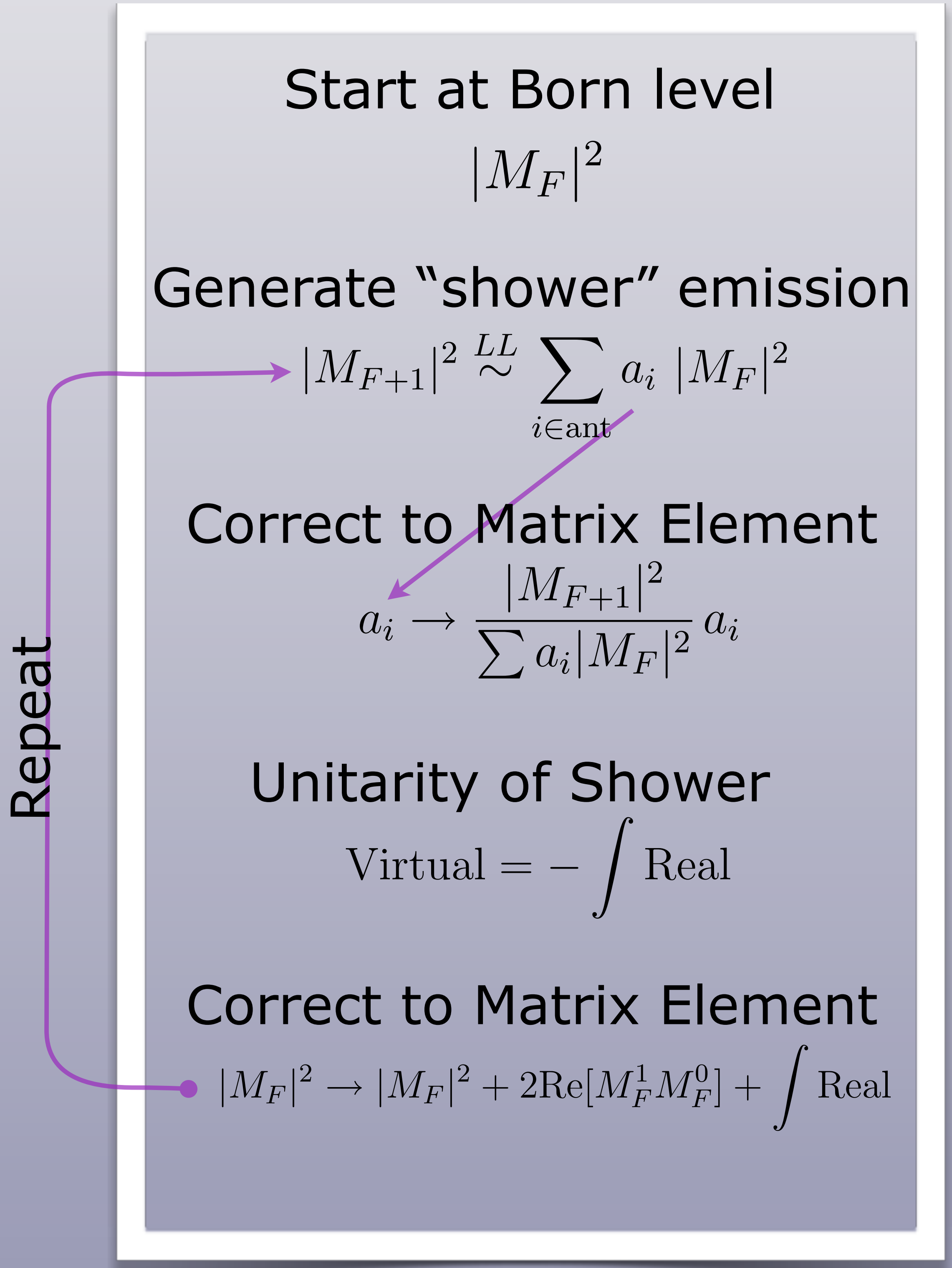
New: Markovian pQCD*

*)pQCD : perturbative QCD



Start at Born level

$$|M_F|^2$$

Generate “shower” emission

$$|M_{F+1}|^2 \overset{LL}{\sim} \sum_{i\in\text{ant}} a_i \; |M_F|^2$$

Correct to Matrix Element

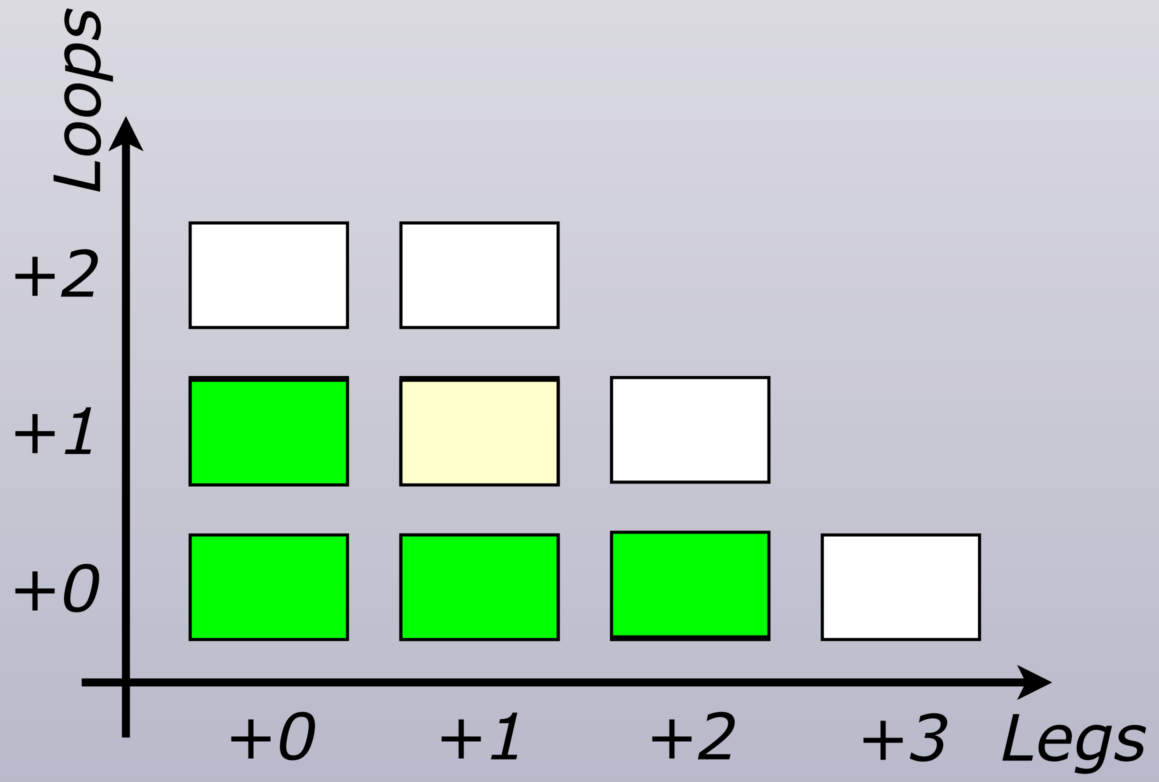
$$a_i \to \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2} a_i$$

Unitarity of Shower

$$\text{Virtual} = -\int \text{Real}$$

Correct to Matrix Element

$$|M_F|^2 \to |M_F|^2 + 2\text{Re}[M_F^1 M_F^0] + \int \text{Real}$$



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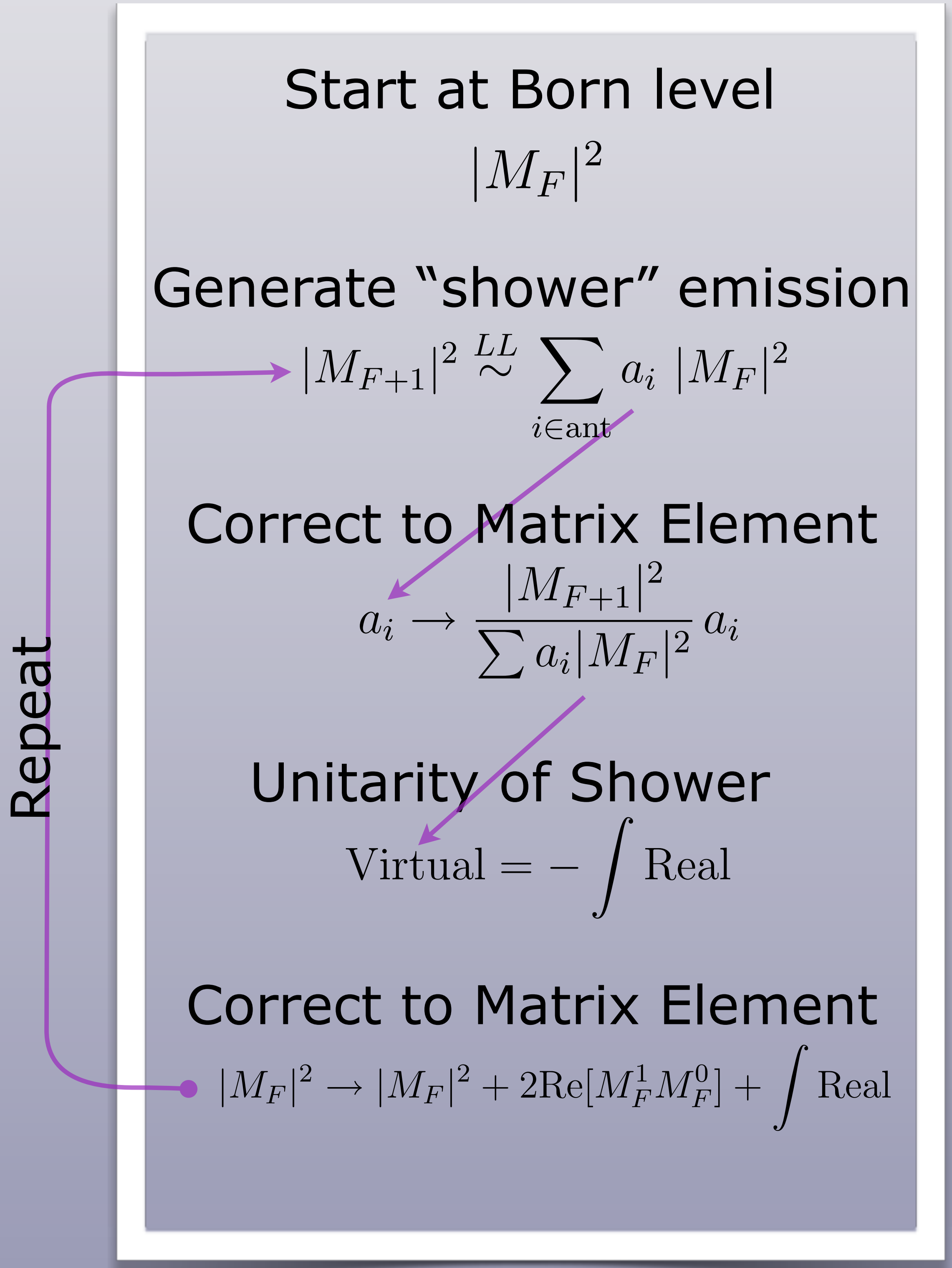

Repeat

Start at Born level

$$|M_F|^2$$

Generate "shower" emission

$$|M_{F+1}|^2 \overset{LL}{\sim} \sum_{i\in\mathrm{ant}} a_i\ |M_F|^2$$

Correct to Matrix Element

$$a_i \to \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2} a_i$$

Unitarity of Shower

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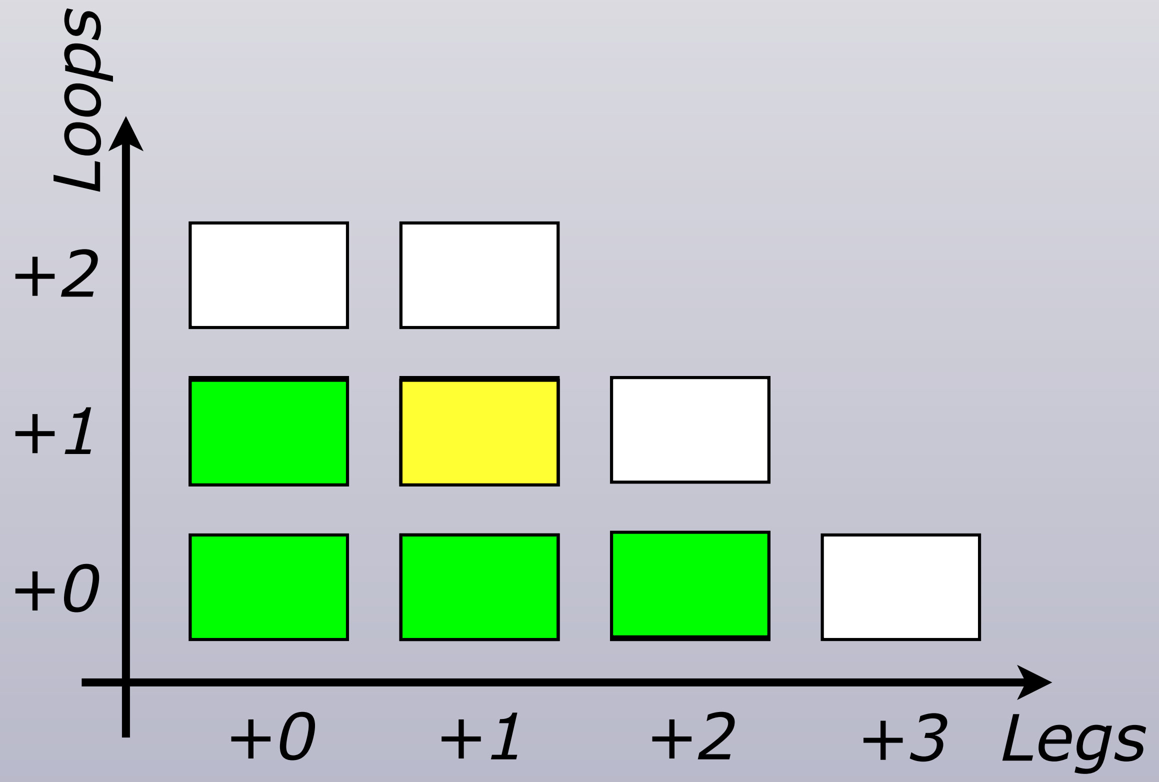

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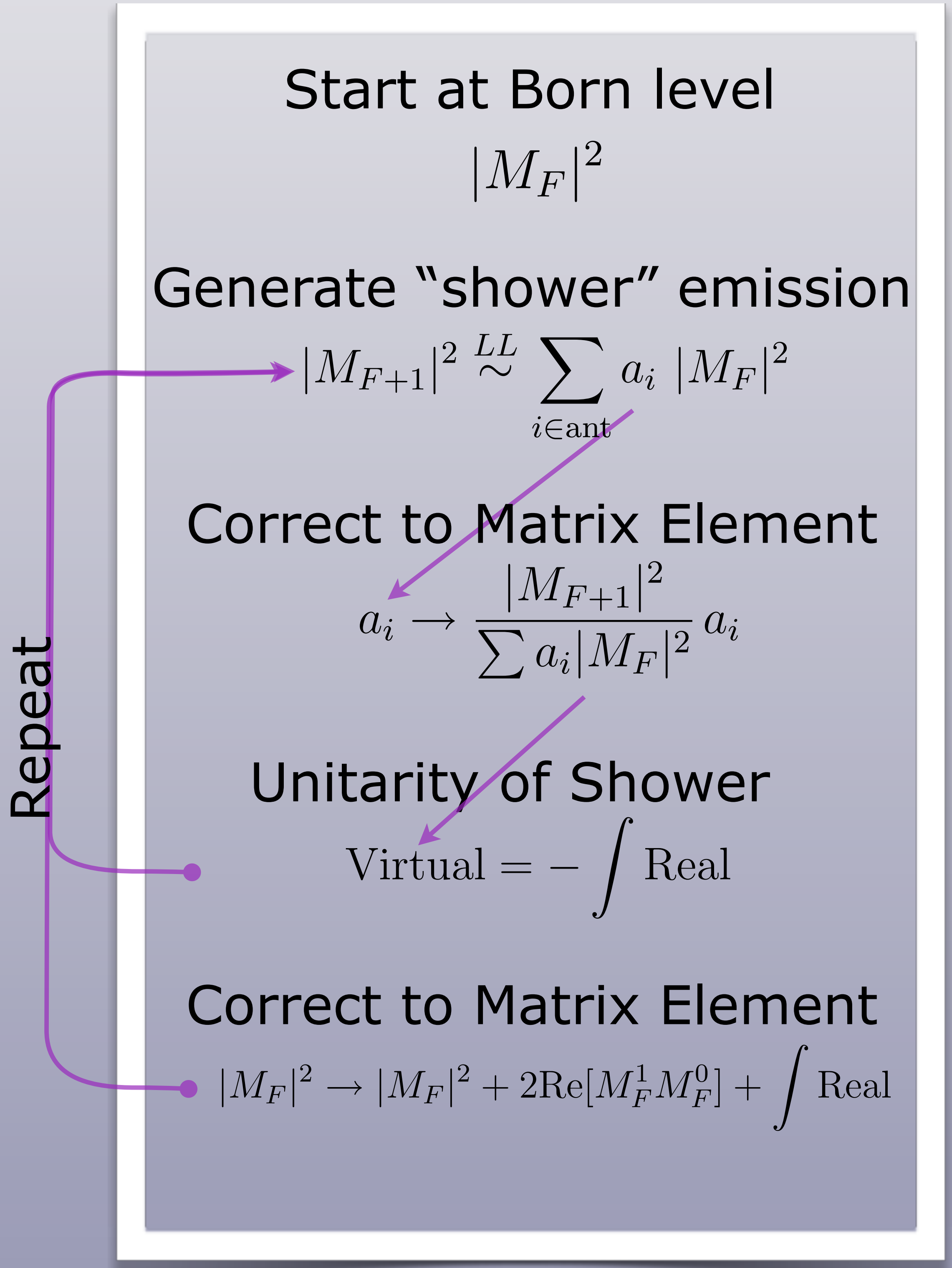

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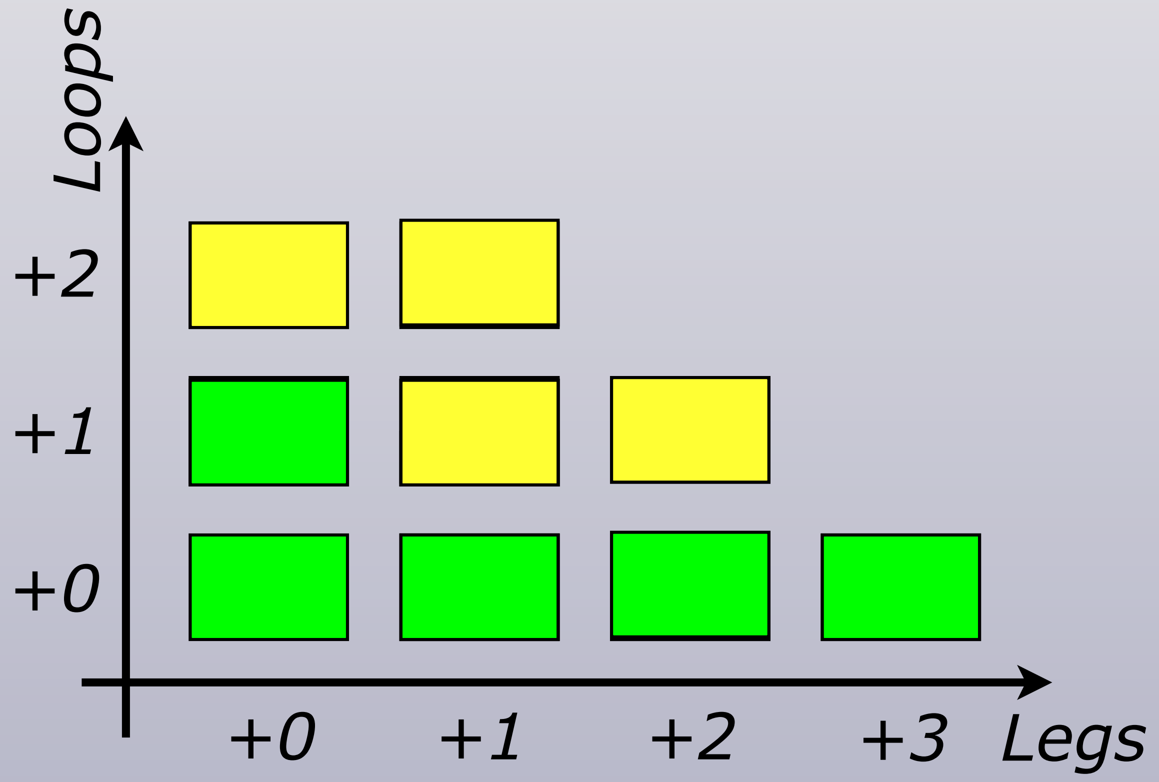

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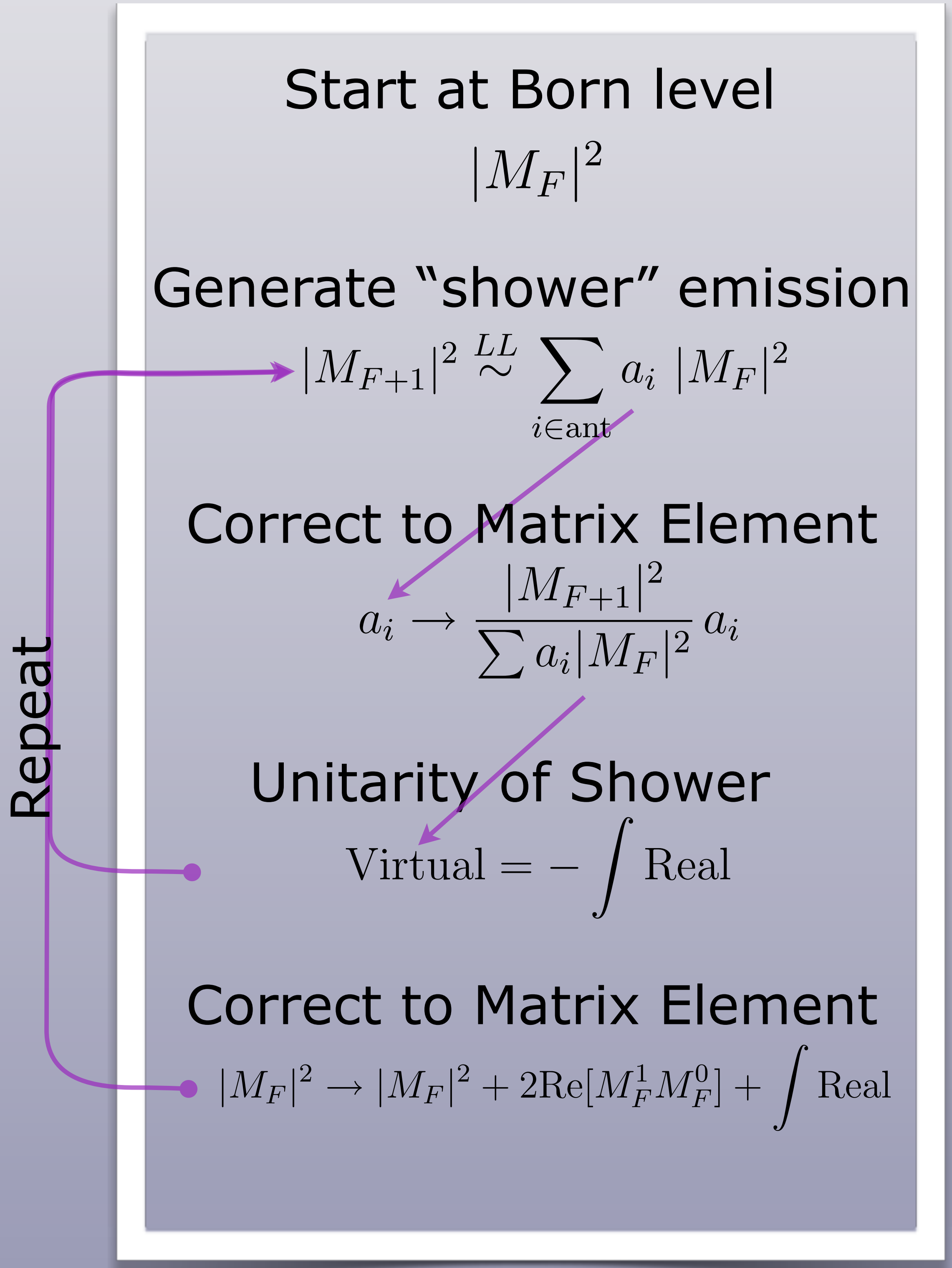

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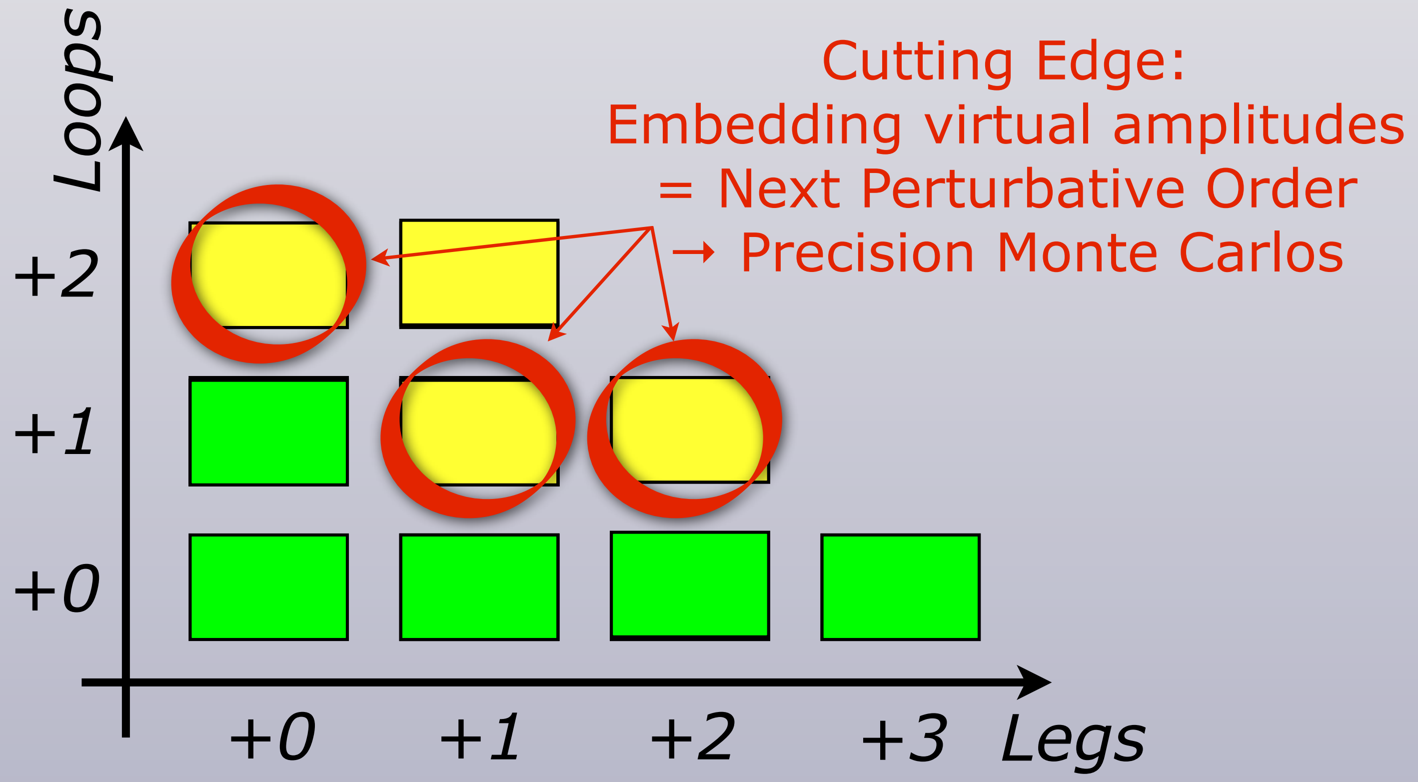


Cutting Edge:
Embedding virtual amplitudes
= Next Perturbative Order
→ Precision Monte Carlos

The VINCIA Code

+

PYTHIA 8

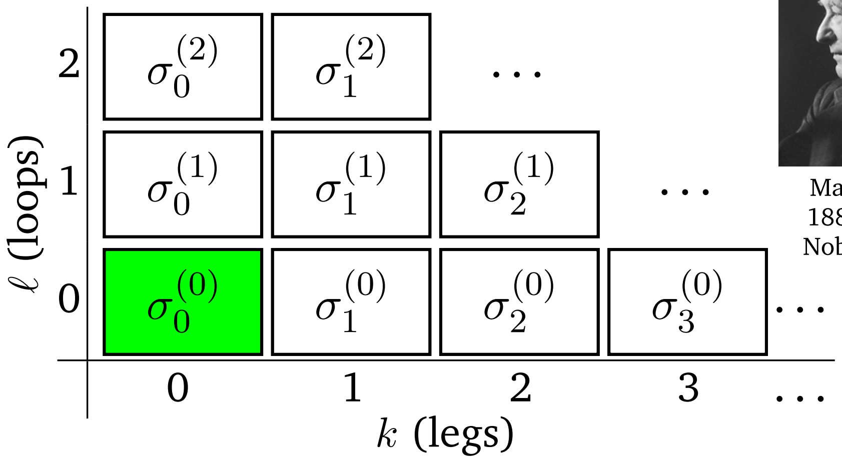
“Higher-Order Corrections To Timelike Jets”
GeeKS: Giele, Kosower, Skands, PRD 84 (2011) 054003

Fixed Order: Recap

Improve by computing quantum corrections, order by order

(from PS, *Introduction to QCD*, TASI 2012, arXiv:1207.2389)

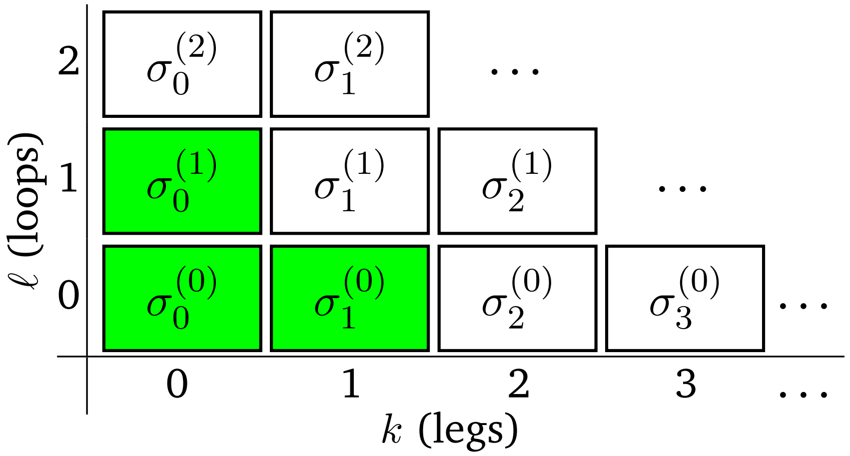
Leading Order



2	$\sigma_0^{(2)}$	$\sigma_1^{(2)}$	$\ldots$		
1	$\sigma_0^{(1)}$	$\sigma_1^{(1)}$	$\sigma_2^{(1)}$	$\ldots$	
0	🟩 $\sigma_0^{(0)}$	$\sigma_1^{(0)}$	$\sigma_2^{(0)}$	$\sigma_3^{(0)}$	$\ldots$
	0	1	2	3	$\ldots$

Max Born,
1882-1970
Nobel 1954

Next-to-Leading Order



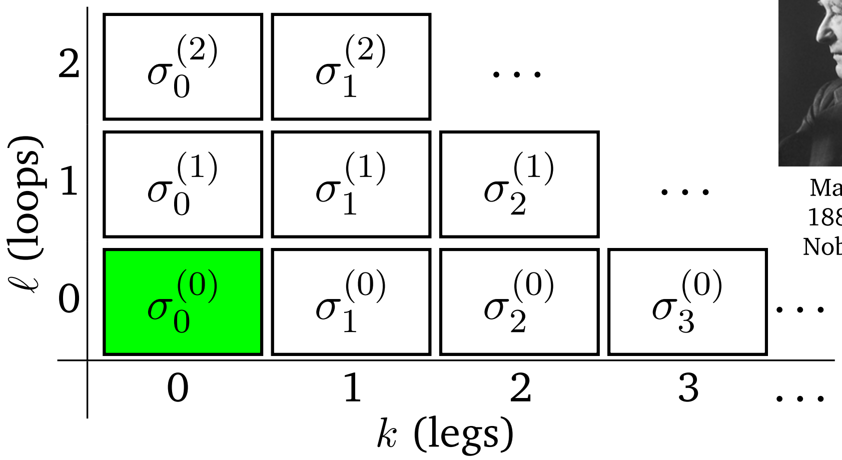
2	$\sigma_0^{(2)}$	$\sigma_1^{(2)}$	$\ldots$		
1	🟩 $\sigma_0^{(1)}$	$\sigma_1^{(1)}$	$\sigma_2^{(1)}$	$\ldots$	
0	🟩 $\sigma_0^{(0)}$	🟩 $\sigma_1^{(0)}$	$\sigma_2^{(0)}$	$\sigma_3^{(0)}$	$\ldots$
	0	1	2	3	$\ldots$

Fixed Order: Recap

Improve by computing quantum corrections, order by order

(from PS, *Introduction to QCD*, TASI 2012, arXiv:1207.2389)

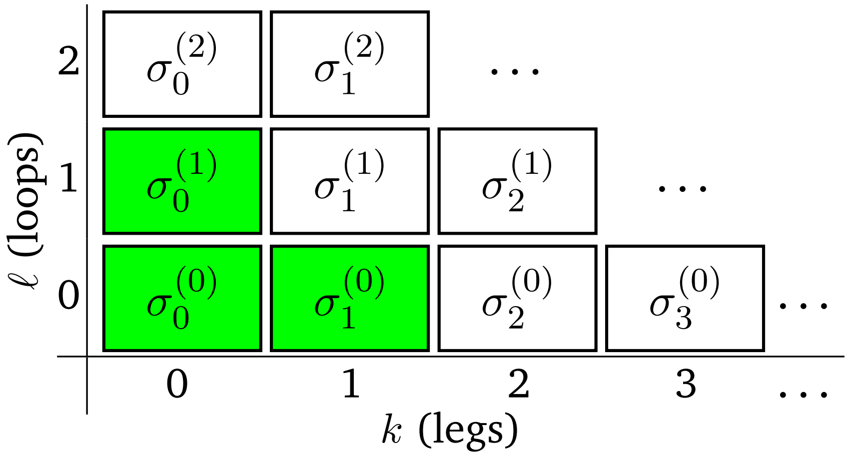
Leading Order



ℓ (loops) 2	$\sigma_0^{(2)}$	$\sigma_1^{(2)}$	$\ldots$		
1	$\sigma_0^{(1)}$	$\sigma_1^{(1)}$	$\sigma_2^{(1)}$	$\ldots$	
0	🟩 $\sigma_0^{(0)}$	$\sigma_1^{(0)}$	$\sigma_2^{(0)}$	$\sigma_3^{(0)}$	$\ldots$
k (legs)	0	1	2	3	$\ldots$

Max Born,
1882-1970
Nobel 1954

Next-to-Leading Order



ℓ (loops) 2	$\sigma_0^{(2)}$	$\sigma_1^{(2)}$	$\ldots$		
1	🟩 $\sigma_0^{(1)}$	$\sigma_1^{(1)}$	$\sigma_2^{(1)}$	$\ldots$	
0	🟩 $\sigma_0^{(0)}$	🟩 $\sigma_1^{(0)}$	$\sigma_2^{(0)}$	$\sigma_3^{(0)}$	$\ldots$
k (legs)	0	1	2	3	$\ldots$

$$\sigma^{\mathrm{NLO}} = \sigma^{\mathrm{Born}} + \int \mathrm{d}\Phi_{F+1} \left|\mathcal{M}_{F+1}^{(0)}\right|^2 + \int \mathrm{d}\Phi_F \, 2\mathrm{Re}\left[\mathcal{M}_F^{(1)} \mathcal{M}_F^{(0)*}\right]$$

→ 1/ϵ² + 1/ϵ + Finite

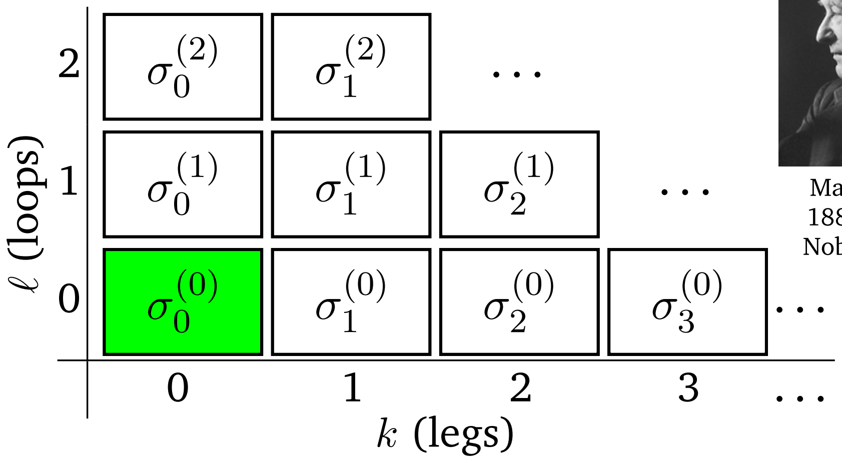
→ -1/ϵ² - 1/ϵ + Finite

Fixed Order: Recap

Improve by computing quantum corrections, order by order

(from PS, *Introduction to QCD*, TASI 2012, arXiv:1207.2389)

Leading Order

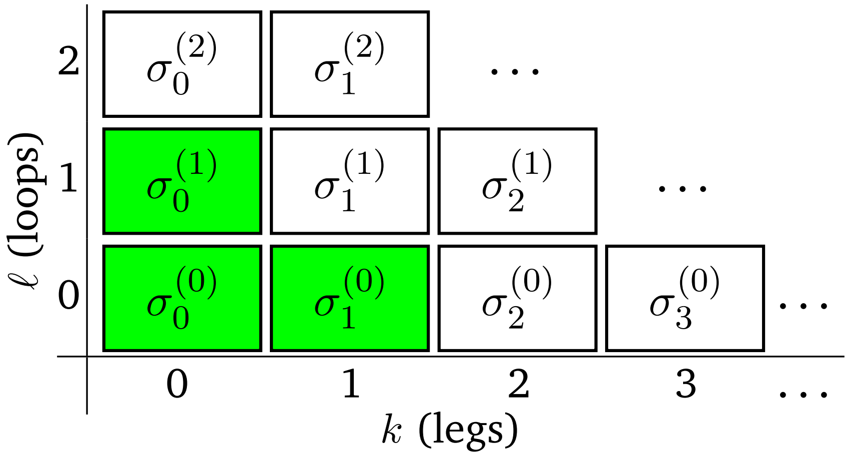


ℓ (loops)					
2	$\sigma_0^{(2)}$	$\sigma_1^{(2)}$	$\dots$		
1	$\sigma_0^{(1)}$	$\sigma_1^{(1)}$	$\sigma_2^{(1)}$	$\dots$	
0	🟩 $\sigma_0^{(0)}$	$\sigma_1^{(0)}$	$\sigma_2^{(0)}$	$\sigma_3^{(0)}$	$\dots$
	0	1	2	3	$\dots$

k (legs)

Max Born,
1882-1970
Nobel 1954

Next-to-Leading Order



ℓ (loops)					
2	$\sigma_0^{(2)}$	$\sigma_1^{(2)}$	$\dots$		
1	🟩 $\sigma_0^{(1)}$	$\sigma_1^{(1)}$	$\sigma_2^{(1)}$	$\dots$	
0	🟩 $\sigma_0^{(0)}$	🟩 $\sigma_1^{(0)}$	$\sigma_2^{(0)}$	$\sigma_3^{(0)}$	$\dots$
	0	1	2	3	$\dots$

k (legs)

$$\sigma^{\rm NLO} = \sigma^{\rm Born} + \underset{\to\ 1/\epsilon^2\ +\ 1/\epsilon\ +\ \text{Finite}}{\int \mathrm{d}\Phi_{F+1} \left|\mathcal{M}^{(0)}_{F+1}\right|^2} + \underset{\to\ -1/\epsilon^2\ -\ 1/\epsilon\ +\ \text{Finite}}{\int \mathrm{d}\Phi_F\, 2\mathrm{Re}\left[\mathcal{M}^{(1)}_F \mathcal{M}^{(0)*}_F\right]}$$

The Subtraction Idea

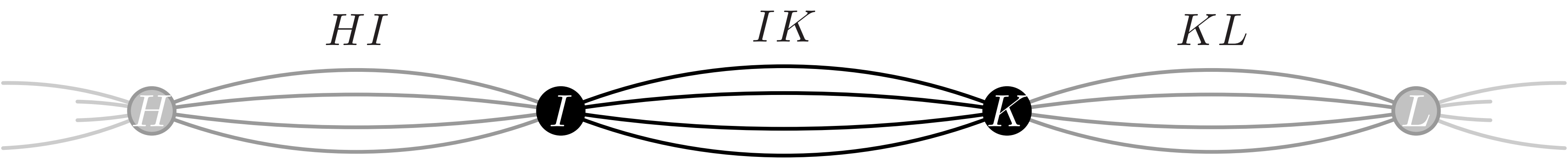
$$= \sigma^{\rm Born} + \int \mathrm{d}\Phi_{F+1} \underbrace{\left(|\mathcal{M}^{(0)}_{F+1}|^2 - \mathrm{d}\sigma^{\rm NLO}_S\right)}_{\text{Finite by Universality}}$$

$$+ \underbrace{\int \mathrm{d}\Phi_F\ 2\mathrm{Re}[\mathcal{M}^{(1)}_F \mathcal{M}^{(0)*}_F] + \int \mathrm{d}\Phi_{F+1}\, \mathrm{d}\sigma^{\rm NLO}_S}_{\text{Finite by KLN}} .$$

Universal
“Subtraction Terms”
(will return to later)

Shower Types

Traditional vs Coherent vs Global vs Sector vs Dipole



	Coll(I)	Soft(IK)
Parton Shower (DGLAP)	a_I	$a_I + a_K$
*Coherent Parton Shower (*HERWIG *[12,40],* PYTHIA6 *[11])*	$\Theta_I a_I$	$\Theta_I a_I + \Theta_K a_K$
*Global Dipole-Antenna (*ARIADNE *[17], GGG [36], WK [32],* VINCIA*)*	$a_{IK} + a_{HI}$	a_{IK}
Sector Dipole-Antenna (LP [41], VINCIA*)*	$\Theta_{IK} a_{IK} + \Theta_{HI} a_{HI}$	a_{IK}
Partitioned-Dipole Shower (SK [23], NS [42], DTW [24], PYTHIA8 *[38],* SHERPA*)*	$a_{I,K} + a_{I,H}$	$a_{I,K} + a_{K,I}$

Figure 2: Schematic overview of how the full collinear singularity of parton I and the soft singularity of the IK pair, respectively, originate in different shower types. (Θ_I and Θ_K represent angular vetos with respect to partons I and K, respectively, and Θ_{IK} represents a sector phase-space veto, see text.)

Global Antennae

$\times$	$\frac{1}{y_{ij}y_{jk}}$	$\frac{1}{y_{ij}}$	$\frac{1}{y_{jk}}$	$\frac{y_{jk}}{y_{ij}}$	$\frac{y_{ij}}{y_{jk}}$	$\frac{y_{jk}^2}{y_{ij}}$	$\frac{y_{ij}^2}{y_{jk}}$	1	y_{ij}	y_{jk}
$q\bar{q} \to qg\bar{q}$										
$++ \to +++$	1	0	0	0	0	0	0	0	0	0
$++ \to +-+$	1	-2	-2	1	1	0	0	2	0	0
$+- \to ++-$	1	0	-2	0	1	0	0	0	0	0
$+- \to +--$	1	-2	0	1	0	0	0	0	0	0
$qg \to qgg$										
$++ \to +++$	1	0	$-\alpha+1$	0	$2\alpha-2$	0	0	0	0	0
$++ \to +-+$	1	-2	-3	1	3	0	-1	3	0	0
$+- \to ++-$	1	0	-3	0	3	0	-1	0	0	0
$+- \to +--$	1	-2	$-\alpha+1$	1	$2\alpha-2$	0	0	0	0	0
$gg \to ggg$										
$++ \to +++$	1	$-\alpha+1$	$-\alpha+1$	$2\alpha-2$	$2\alpha-2$	0	0	0	0	0
$++ \to +-+$	1	-3	-3	3	3	-1	-1	3	1	1
$+- \to ++-$	1	$-\alpha+1$	-3	$2\alpha-2$	3	0	-1	0	0	0
$+- \to +--$	1	-3	$-\alpha+1$	3	$2\alpha-2$	-1	0	0	0	0
$qg \to q\bar{q}'q'$										
$++ \to ++-$	0	0	0	0	0	0	$\frac{1}{2}$	0	0	0
$++ \to +-+$	0	0	$\frac{1}{2}$	0	-1	0	$\frac{1}{2}$	0	0	0
$+- \to ++-$	0	0	$\frac{1}{2}$	0	-1	0	$\frac{1}{2}$	0	0	0
$+- \to +--$	0	0	0	0	0	0	$\frac{1}{2}$	0	0	0
$gg \to g\bar{q}q$										
$++ \to ++-$	0	0	0	0	0	0	$\frac{1}{2}$	0	0	0
$++ \to +-+$	0	0	$\frac{1}{2}$	0	-1	0	$\frac{1}{2}$	0	0	0
$+- \to ++-$	0	0	$\frac{1}{2}$	0	-1	0	$\frac{1}{2}$	0	0	0
$+- \to +-+$	0	0	0	0	0	0	$\frac{1}{2}$	0	0	0

Sector Antennae

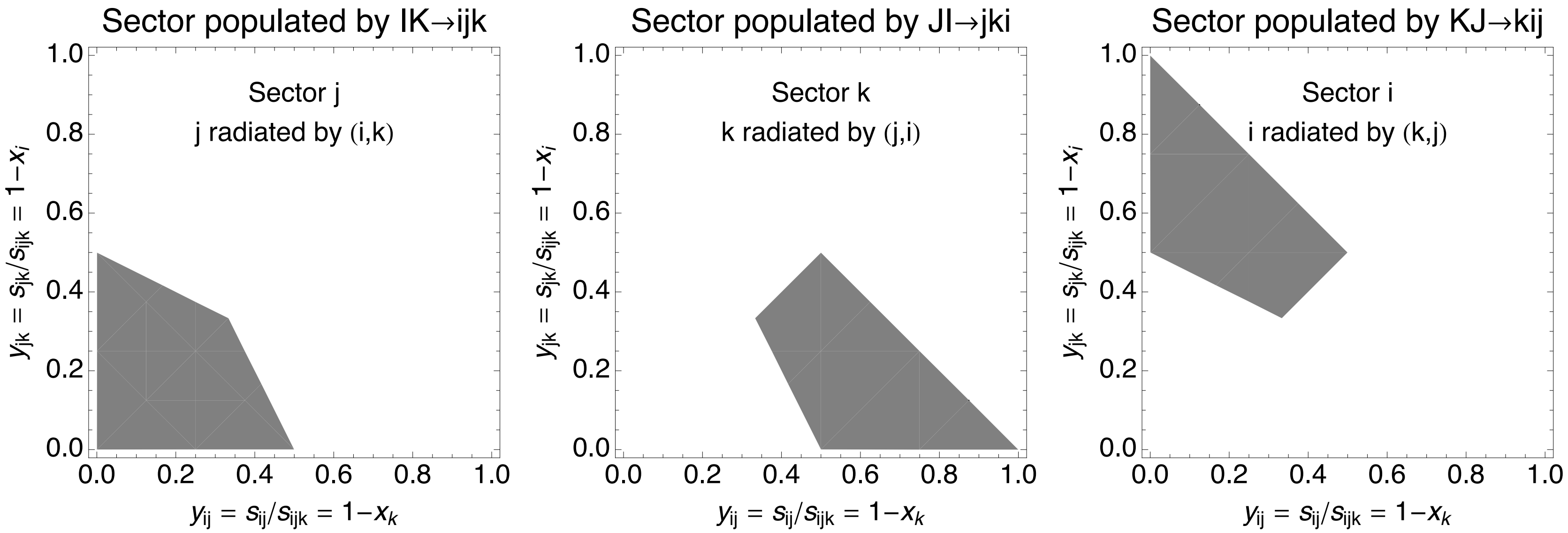
Global

$$\bar{a}^{\rm gl}_{g/qg}(p_i,p_j,p_k) \xrightarrow{s_{jk}\to 0} \frac{1}{s_{jk}}\left(P_{gg\to G}(z) - \frac{2z}{1-z} - z(1-z)\right)$$

→ P(z) = Sum over two neigboring antennae

Sector

Only a single term in each phase space point



→ Full P(z) must be contained in every antenna

$$\begin{aligned}\bar{a}^{\rm sct}_{j/IK}(y_{ij},y_{jk}) = \bar{a}^{\rm gl}_{j/IK}(y_{ij},y_{jk}) \; &+ \; \delta_{Ig}\delta_{H_KH_k}\left\{\delta_{H_IH_i}\delta_{H_IH_j}\left(\frac{1+y_{jk}+y_{jk}^2}{y_{ij}}\right)\right.\\ &+ \; \left.\delta_{H_IH_j}\left(\frac{1}{y_{ij}(1-y_{jk})} - \frac{1+y_{jk}+y_{jk}^2}{y_{ij}}\right)\right\}\\ &+ \; \delta_{Kg}\delta_{H_IH_i}\left\{\delta_{H_IH_j}\delta_{H_KH_k}\left(\frac{1+y_{ij}+y_{ij}^2}{y_{jk}}\right)\right.\\ &+ \; \left.\delta_{H_KH_j}\left(\frac{1}{y_{jk}(1-y_{ij})} - \frac{1+y_{ij}+y_{ij}^2}{y_{jk}}\right)\right\}\end{aligned}$$

Sector = Global + additional collinear terms
(from "neighboring" antenna)

The Denominator

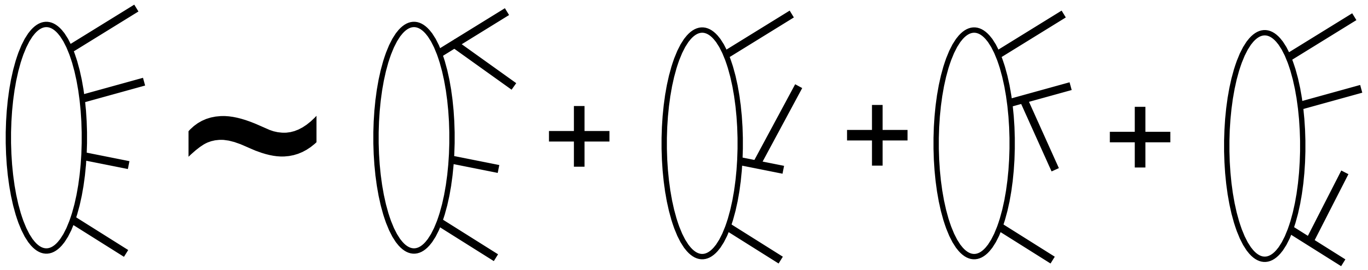
$$a_i \to \frac{|M_{F+1}|^2}{\boxed{\sum a_i |M_F|^2}}$$

In a traditional parton shower, you would face the following problem:

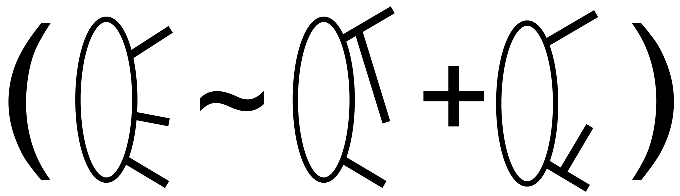
Existing parton showers are *not* really Markov Chains

Further evolution (restart scale) depends on which branching happened last → proliferation of terms

Number of histories contributing to n^{th} branching ∝ **$2^n n!$**



j = 2
→ 4 terms



j = 1
→ 2 terms

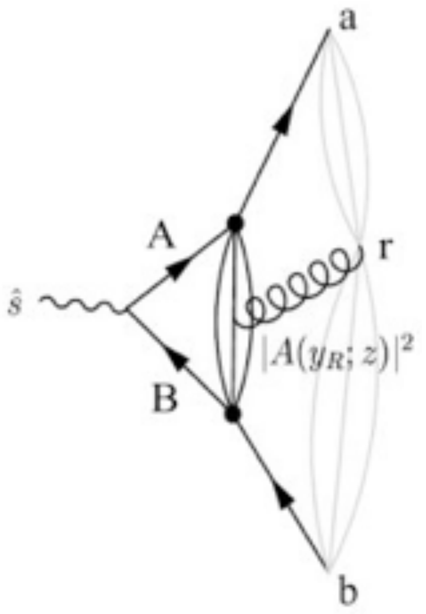
Parton- (or Catani-Seymour) Shower:
After 2 branchings: 8 terms
After 3 branchings: 48 terms
After 4 branchings: 384 terms

(+ parton showers have complicated and/or frame-dependent phase-space mappings, especially at the multi-parton level)

Matched Markovian Antenna Showers

Antenna showers: one term per parton *pair* **$2^n n! \to n!$**

Giele, Kosower, Skands, PRD 84 (2011) 054003



(+ generic Lorentz-invariant and on-shell phase-space factorization)

+ Change "shower restart" to Markov criterion:

Given an *n*-parton configuration, "ordering" scale is

$$Q_{ord} = min(Q_{E1},Q_{E2},...,Q_{En})$$

Unique restart scale, independently of how it was produced

+ Matching: $n! \to n$

Given an *n*-parton configuration, its phase space weight is:

$|M_n|^2$: *Unique weight, independently of how it was produced*

Matched Markovian Antenna Shower:
After 2 branchings: 2 terms
After 3 branchings: 3 terms
After 4 branchings: 4 terms

Parton- (or Catani-Seymour) Shower:
After 2 branchings: 8 terms
After 3 branchings: 48 terms
After 4 branchings: 384 terms

+ **Sector** antennae *Larkosi, Peskin, Phys.Rev. D81 (2010) 054010*
→ 1 term at *any* order *Lopez-Villarejo, Skands, JHEP 1111 (2011) 150*

Approximations

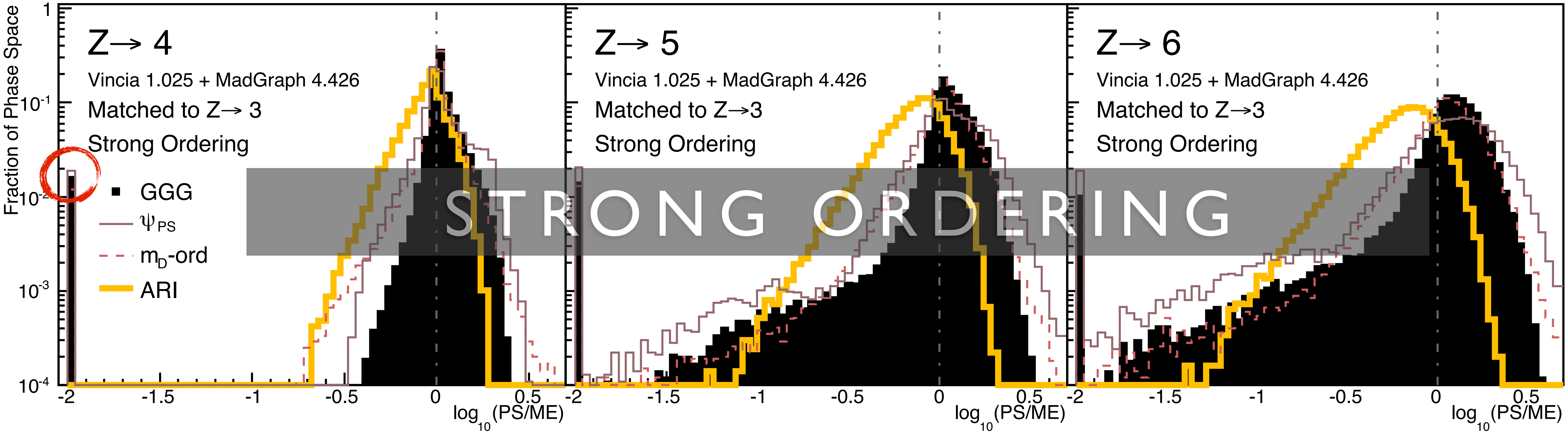
Q: How well do showers do?

Exp: Compare to data. Difficult to interpret; all-orders cocktail including hadronization, tuning, uncertainties, etc

Th: Compare products of splitting functions to full tree-level matrix elements

Plot distribution of **Log$_{10}$(PS/ME)**

(second order) (third order) (fourth order)



Dead Zone: 1-2% of phase space have no strongly ordered paths leading there*

*fine from strict LL point of view: those points correspond to "unordered" non-log-enhanced configurations

2→4

Generate Branchings *without* imposing strong ordering

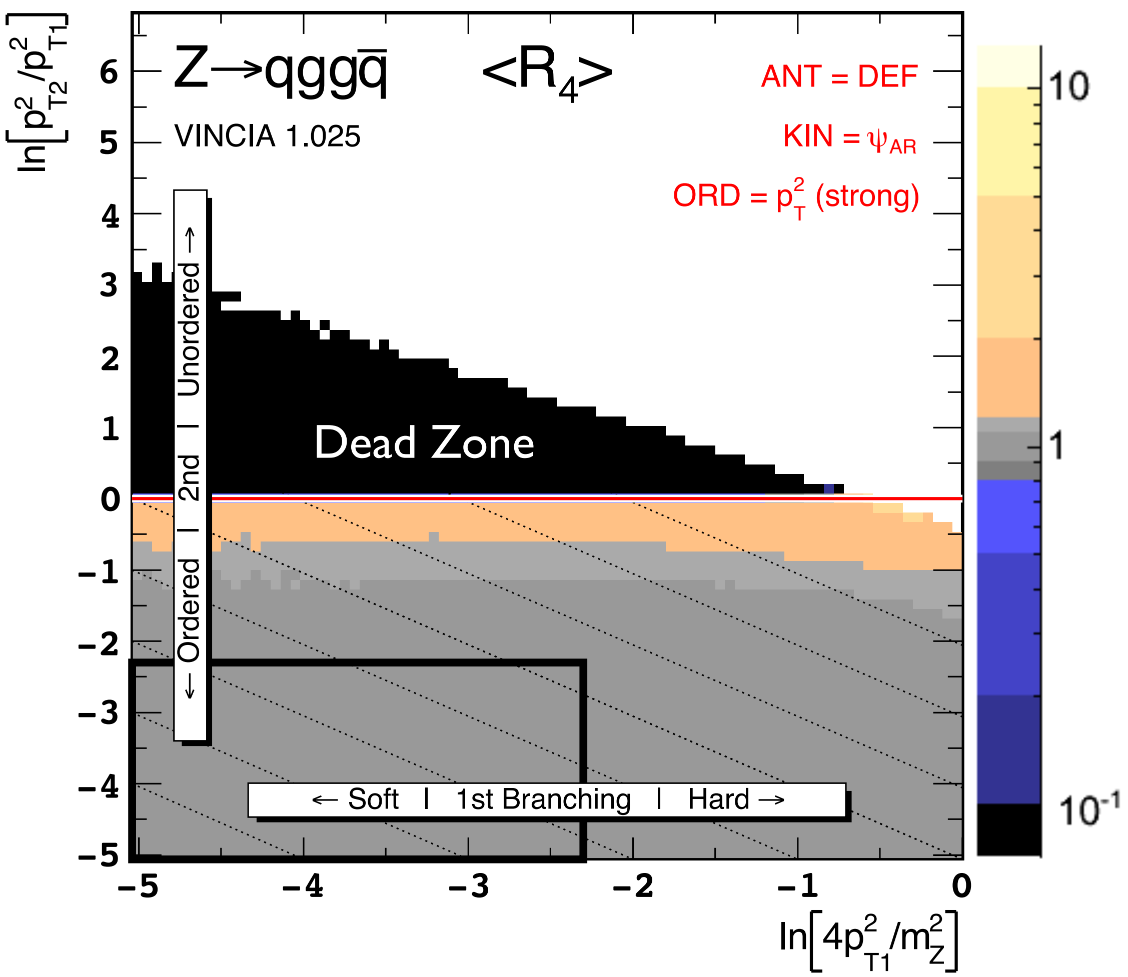
At each step, each dipole allowed to fill its entire phase space

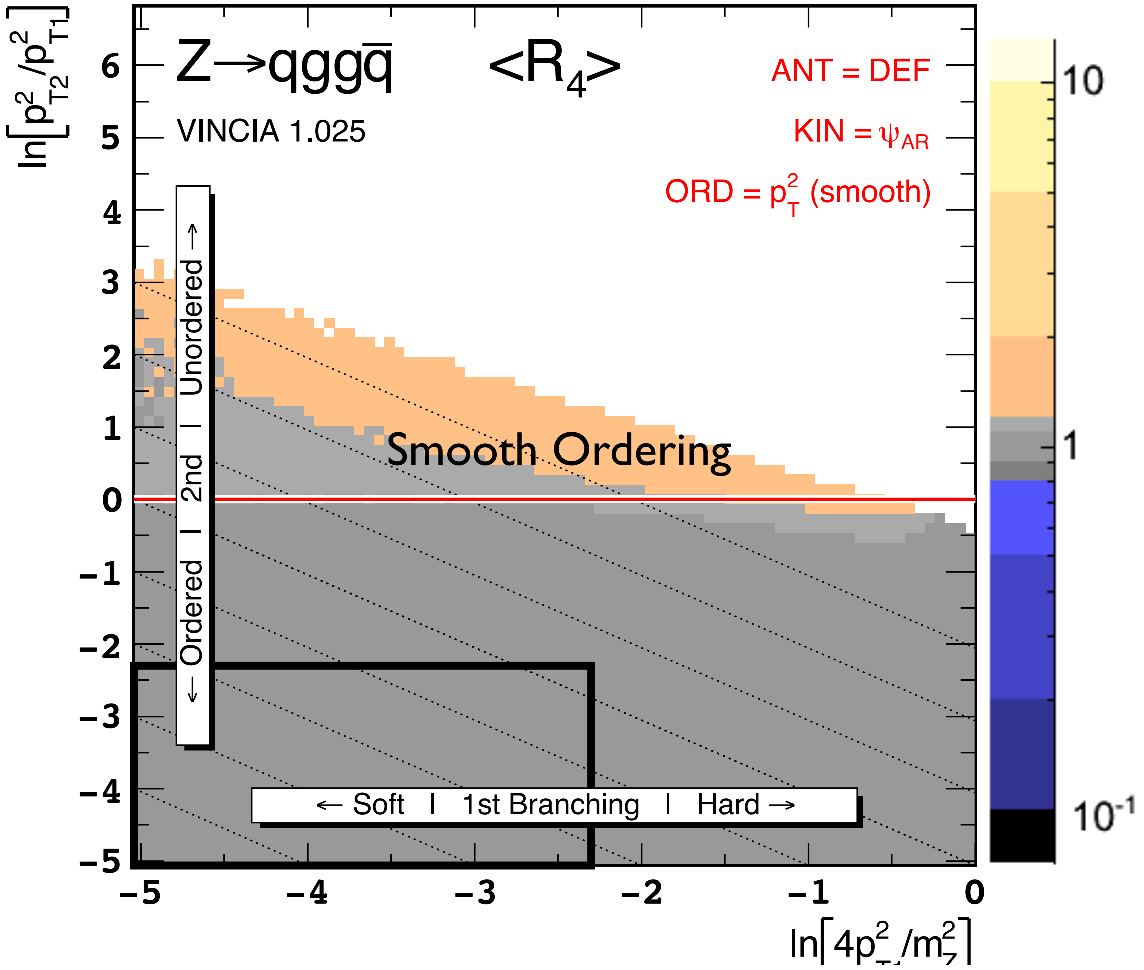
Overcounting removed by matching

+ smooth ordering beyond matched multiplicities

$$\frac{\hat{p}_\perp^2}{\hat{p}_\perp^2 + p_\perp^2} P_{\mathrm{LL}}$$

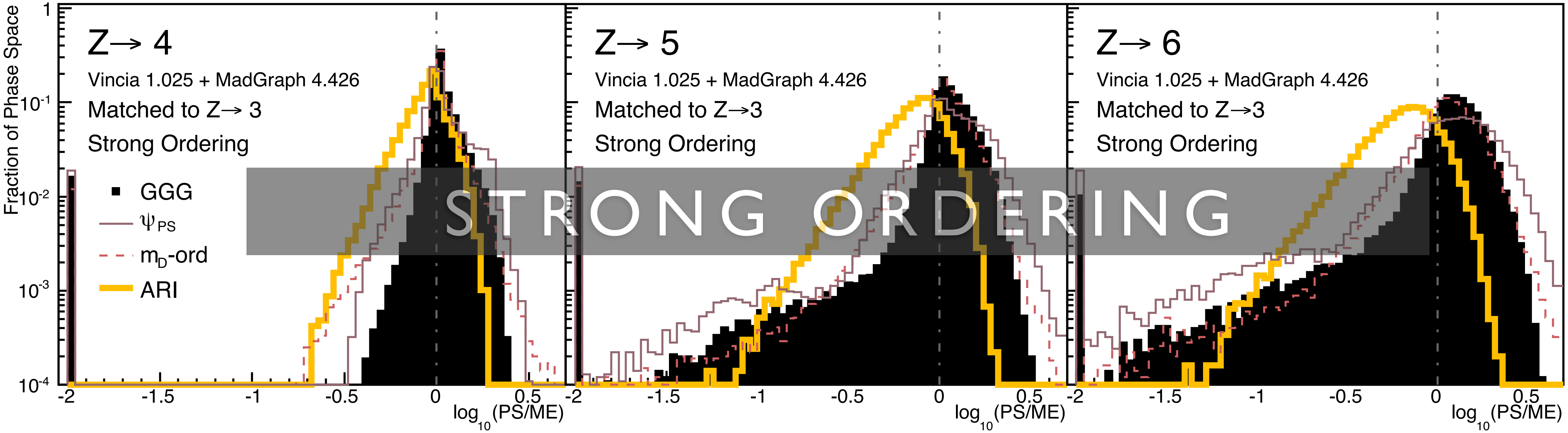
$\hat{p}_\perp^2$ last branching
$p_\perp^2$ current branching



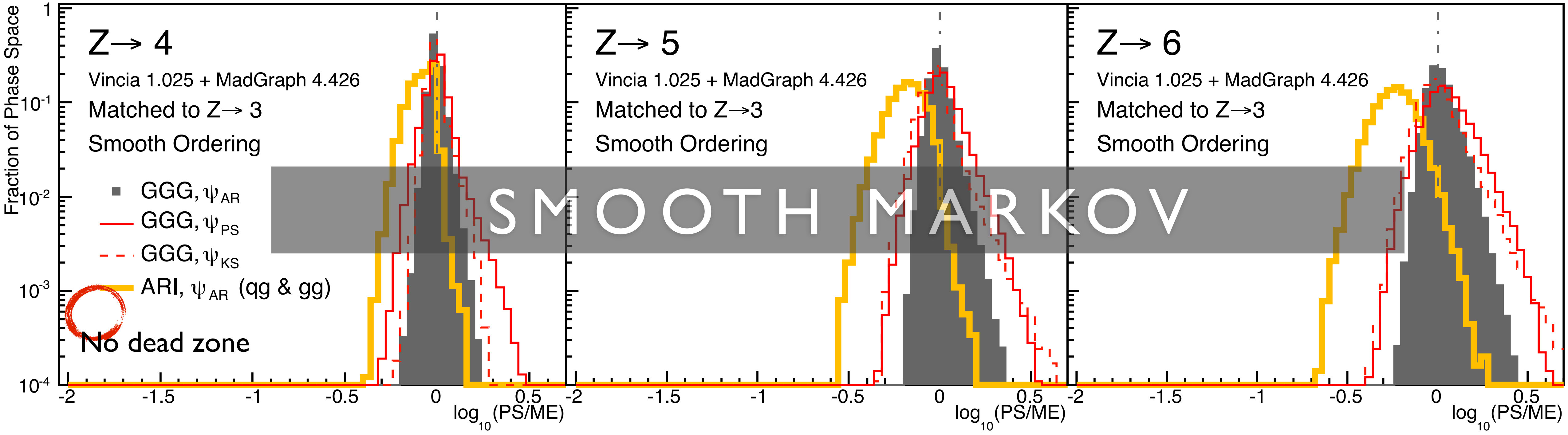


→ Better Approximations

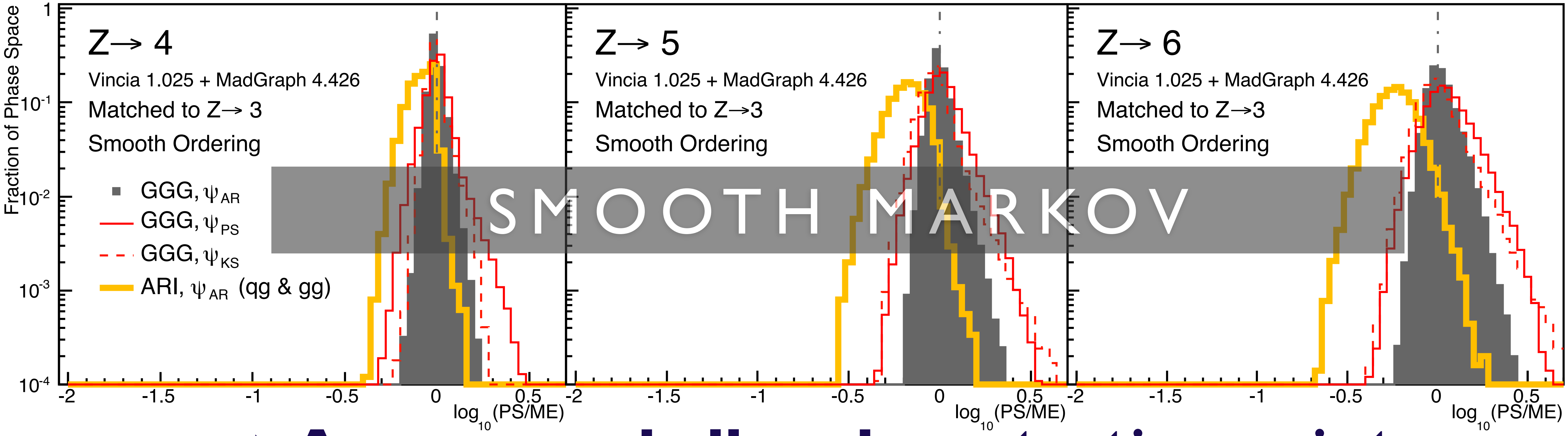
Distribution of **$\log_{10}(PS_{LO}/ME_{LO})$** (inverse ~ matching coefficient)



Leading Order, Leading Color, Flat phase-space scan, over **all of phase space** (no matching scale)

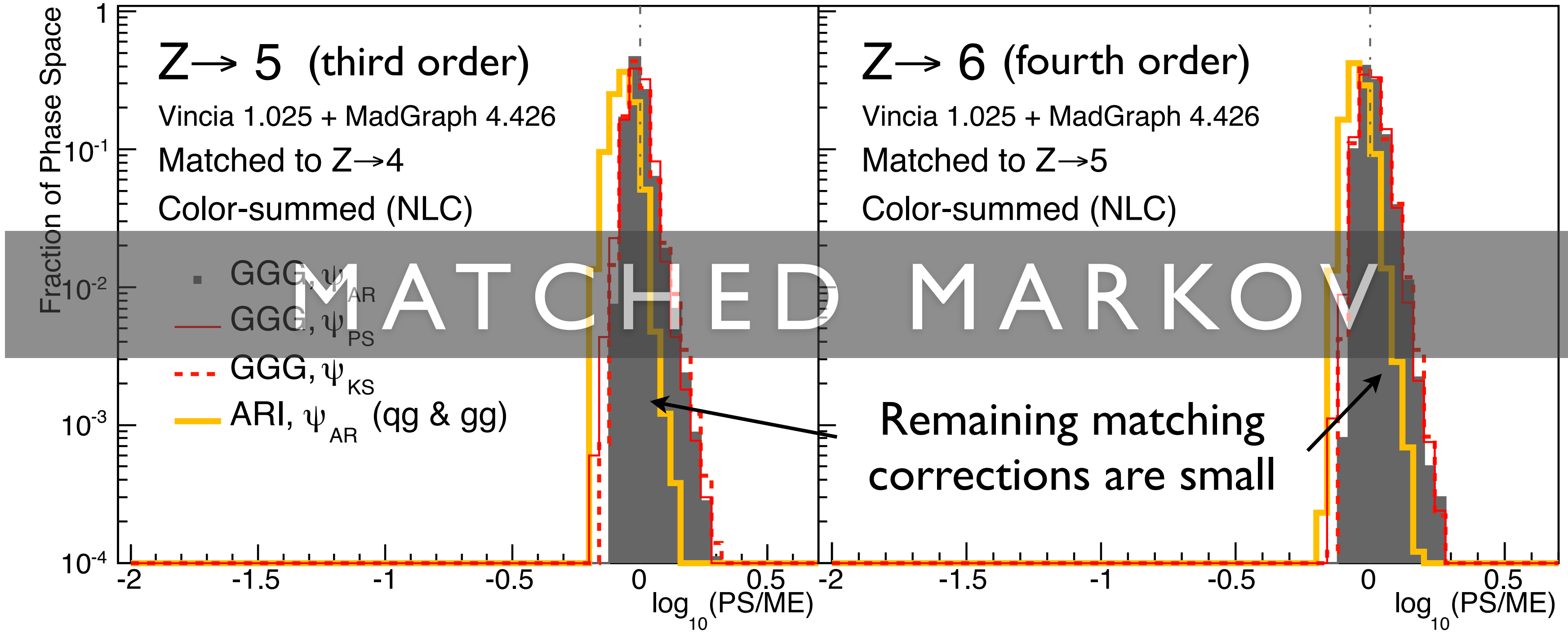


+ Matching (+ full colour)



SMOOTH MARKOV

→ **A very good all-orders starting point**



MATCHED MARKOV

Remaining matching corrections are small

IR Singularity Operators

Gehrmann, Gehrmann-de Ridder, Glover, JHEP 0509 (2005) 056

$q\bar{q} \to qg\bar{q}$ antenna function

$$X^0_{ijk} = S_{ijk,IK}\,\frac{|\mathcal{M}^0_{ijk}|^2}{|\mathcal{M}^0_{IK}|^2}$$

$$A_3^0(1_q,3_g,2_{\bar{q}}) = \frac{1}{s_{123}}\left(\frac{s_{13}}{s_{23}}+\frac{s_{23}}{s_{13}}+2\frac{s_{12}s_{123}}{s_{13}s_{23}}\right)$$

Integrated antenna

$$\mathcal{P}oles\left(\mathcal{A}_3^0(s_{123})\right) = -2\mathbf{I}^{(1)}_{q\bar{q}}\left(\epsilon, s_{123}\right)$$

$$\mathcal{F}inite\left(\mathcal{A}_3^0(s_{123})\right) = \frac{19}{4}\,.$$

$$\mathcal{X}^0_{ijk}(s_{ijk}) = \left(8\pi^2\left(4\pi\right)^{-\epsilon}e^{\epsilon\gamma}\right)\int \mathrm{d}\Phi_{X_{ijk}}\,X^0_{ijk}.$$

Singularity Operators

$$\mathbf{I}^{(1)}_{q\bar{q}}\left(\epsilon,\mu^2/s_{q\bar{q}}\right) = -\frac{e^{\epsilon\gamma}}{2\Gamma\left(1-\epsilon\right)}\left[\frac{1}{\epsilon^2}+\frac{3}{2\epsilon}\right]\mathrm{Re}\left(-\frac{\mu^2}{s_{q\bar{q}}}\right)^{\epsilon}$$

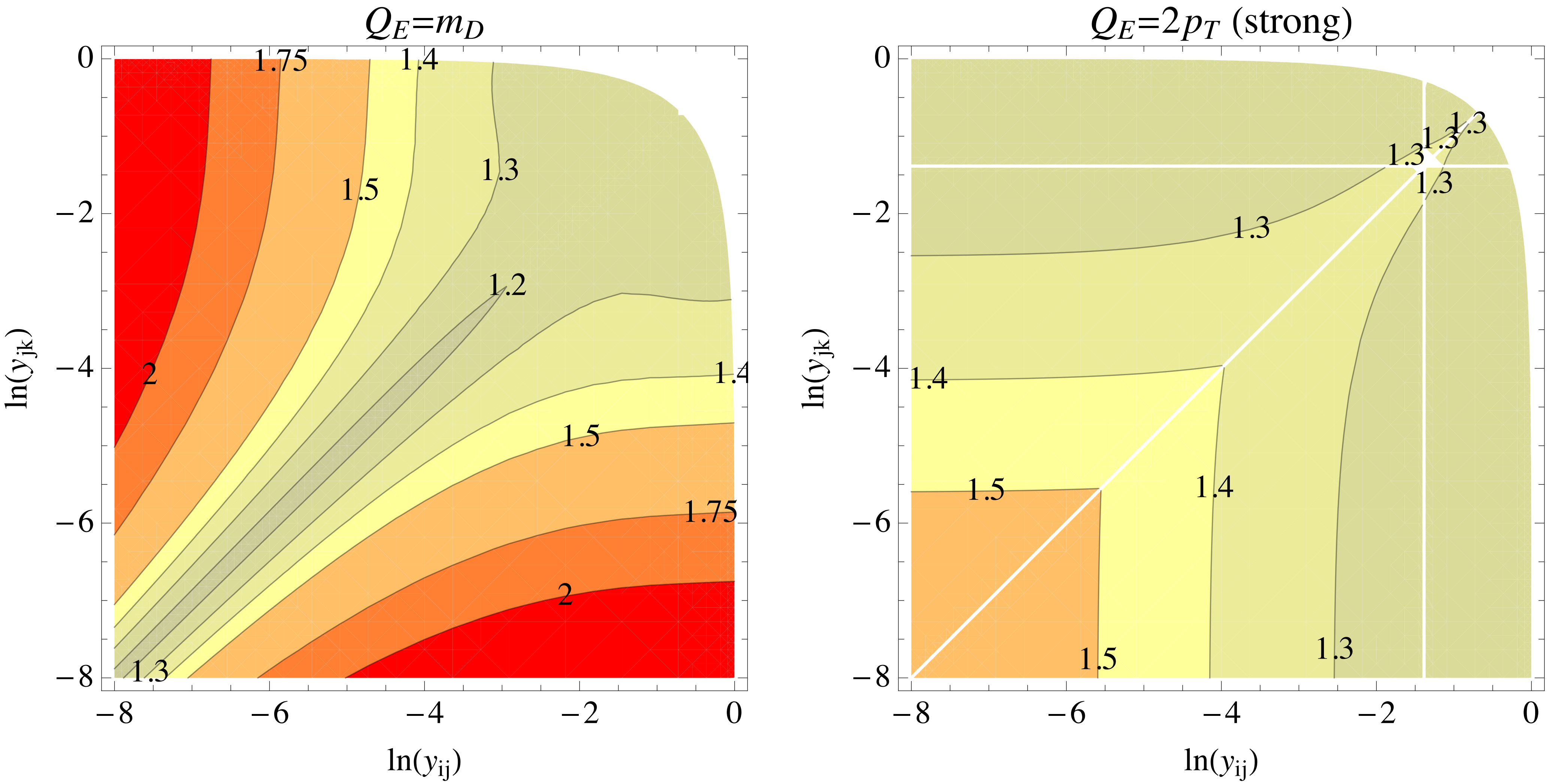
$$\mathbf{I}^{(1)}_{qg}\left(\epsilon,\mu^2/s_{qg}\right) = -\frac{e^{\epsilon\gamma}}{2\Gamma\left(1-\epsilon\right)}\left[\frac{1}{\epsilon^2}+\frac{5}{3\epsilon}\right]\mathrm{Re}\left(-\frac{\mu^2}{s_{qg}}\right)^{\epsilon}$$ for qg→qgg

$$\mathbf{I}^{(1)}_{qg,F}\left(\epsilon,\mu^2/s_{qg}\right) = \frac{e^{\epsilon\gamma}}{2\Gamma\left(1-\epsilon\right)}\frac{1}{6\epsilon}\mathrm{Re}\left(-\frac{\mu^2}{s_{qg}}\right)^{\epsilon}$$ for qg→qq’q’

Loop Corrections

The choice of evolution variable (Q)

Variation with $\mu_R = m_D = 2 \min(s_{ij},s_{jk})$



Parameters: $\alpha_S(M_Z) = 0.12$, $\Lambda_{QCD} = \Lambda_{CMW}$