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A Unitary Approach to Multi-Jet Matching @N^{n}LO

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[1] Most work done by Stefan Prestel

Introduction

- The Unitarity issue in multi-jet matching
- Reformulating CKKW(-L) $\rightarrow$ UMEPS
- Multi-jet merging to NLO $\rightarrow$ UNLOPS

The Unitary nature of Parton Showers

Start from a Born-level ME

$$\frac{d\sigma_0^{inc}}{d\phi_0} \equiv F_0|\mathcal{M}_0|^2,$$

Make it exclusive above some resolution scale $\rho_{\rm MS}$

$$\frac{d\sigma_0^{excl}}{d\phi_0} = F_0|\mathcal{M}_0|^2 \exp\left(-\int_{\rho_{\rm MS}}^{\rho_0} \alpha_s \mathcal{P}_1 d\rho dz\right) = F_0|\mathcal{M}_0|^2 \Gamma_0(\rho_0, \rho_{\rm MS})$$

Add *the first emission* above ρ_{MS}

$$\frac{d\sigma_1^{first,excl}}{d\phi_0} = F_0|\mathcal{M}_0|^2\alpha_s\mathcal{P}_1 d\rho dz \Gamma_0(\rho_0,\rho),$$

Make that exclusive

$$\frac{d\sigma_1^{excl}}{d\phi_0} = F_0|\mathcal{M}_0|^2\alpha_s\mathcal{P}_1 d\rho dz \Gamma_0(\rho_0,\rho)\Gamma_1(\rho,\rho_{\text{MS}})$$

Then add second emission and make it exclusive and so on.

This procedure is unitary to all orders in α_s.

Eg. the $\mathcal{O}(\alpha_s)$ term in the expanded no-emission probability in $d\sigma_0^{excl}$ exactly cancels with the $\mathcal{O}(\alpha_s)$ term in the integrated $d\sigma_1^{excl}$.

Looking at how partons showers are actually implemented, exact unitarity is obvious.

CKKW(-L)

Replace products of splitting functions with exact tree-level matrix elements. Eg. $\mathcal{P}_1 \rightarrow \mathcal{P}_1^{\mathrm{ME}}$. Typically generated as a separate event sample by an external ME-generator.

Multiply with no-emission probabilities from the shower, to make it "*first-inclusive*" and also exclusive.

This explicitly breaks unitarity.

The real emissions do not match the exponentiated splittings in the no-emission probability.

Not a big problem — beyond the precision of the procedure.

But it gives a irritating dependence on the merging scale. Also at very large $p_\perp$.

It has to be solved before going to NLO.

To be unitary, we need to remove from the 0-jet exclusive cross section everything we add in higher multiplicities.

In the parton shower we remove exactly the integrated *first-inclusive* 1-jet cross section.

And to make the 1-jet cross section exclusive we remove exactly the integrated *first&second-inclusive* 2-jet cross section. And so on. Why not do the same in CKKW-L?

(inspired by LoopSim)

UMEPS

Let's introduce some notation:

$$B_n = F_n |\mathcal{M}_n|^2 d\phi_n$$

Generated by a tree-level ME-generator

Assume we can project the *n*-parton state to a shower history

$$d\phi_n \to d\phi_0 \prod_{i=1}^{n} d\rho_i dz_i$$

Standard in CKKW(-L)

The *n-first-inclusive* cross section

$$B_n^f = F_n |\mathcal{M}_n|^2 d\phi_0 \prod_{i=1}^{n} \Gamma_{i-1}(\rho_{i-1}, \rho_i) d\rho_i dz_i$$

No-emission probabilities generated by the shower.

Also define the reclustered cross sections

$$B^f_{n\to n-1} = \left(\int d\rho_n dz_n \Gamma_{n-1}(\rho_{n-1},\rho_n) F_n|\mathcal{M}_n|^2\right) \times d\phi_0 \prod_{i=1}^{n-1}\Gamma_{i-1}(\rho_{i-1},\rho_i) d\rho_i dz_i$$

and

$$B^f_{n\to n-2} = \left(\int d\rho_n dz_n d\rho_{n-1} dz_{n-1} \Gamma_{n-1}(\rho_{n-1},\rho_n)\Gamma_{n-2}(\rho_{n-2},\rho_{n-1}) \times F_n|\mathcal{M}_n|^2\right) \times d\phi_0 \prod_{i=1}^{n-2}\Gamma_{i-1}(\rho_{i-1},\rho_i) d\rho_i dz_i$$

0-jet: $B_0 - B^f_{1\to0} - B^{f\downarrow}_{2\to0}$

1-jet: $B^f_1 - B^f_{2\to1}$

2-jet: $B^f_2 + B^{\mathcal{I}}_2$

$\cdots$

Recluster twice if first reclustering gives $\rho < \rho_{\text{MS}}$.

$B^{\mathcal{I}}_2$ corresponds to eg. W-strahlung processes.

Except for $B^{\mathcal{I}}_2$ we have a completely unitary process.

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UNLOPS

Now we can go on to NLO.

We have the exact exclusive NLO cross section

$$\tilde{B}_n = \bar{B}_n - B_{n+1\to n}$$

We want to overall normalization to be the 0-jet inclusive NLO cross section. So we multply everything in UMEPS with a K-factor: $K = 1 + \alpha_s(\mu_R)k(\mu_R, \mu_F)$.

We know how to take the terms in UMEPS and extract or subtract any given order in α_s.

$B_1^f\big|_{-12}$ means remove the $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha_s^2)$ terms.

$B_1^f\big|_2$ means keep only the $\mathcal{O}(\alpha_s^2)$ term.

0-jet: $B_0 - B^{f}_{1\to0} - B^{f\downarrow}_{2\to0}$

$\to \quad \tilde{B}_0 + B_0\big|_{-01} - B^{f}_{1\to0}\Big|_{-1} - B^{f\downarrow}_{2\to0}$

$\quad - \tilde{B}_{1\to0} + B^{f}_{1\to0}\Big|_{12} - B^{f}_{2\to0}\Big|_{2}$

1-jet: $B^{f}_{1} - B^{f}_{2\to1}$

$\to \quad \tilde{B}_1 + B^{f}_{1}\Big|_{-12} - B^{f}_{2\to1}\Big|_{-2}$

2-jet: $B^{f}_{2} + B^{\mathcal{I}}_{2}$

$\dots$

Everything is still unitary.
The total cross section is given by the NLO calculation.

$$
\begin{aligned}
\text{0-jet:} \quad & B_0 - B^f_{1\to0} - B^{f\downarrow}_{2\to0} \\
\to \quad & \tilde{B}_0 + B_0\big|_{-01} - B^f_{1\to0}\Big|_{-1} - B^{f\downarrow}_{2\to0} \\
& - \tilde{B}_{1\to0} + B^f_{1\to0}\Big|_{12} - B^f_{2\to0}\Big|_{2} \\
\text{1-jet:} \quad & B^f_1 - B^f_{2\to1} \\
\to \quad & \tilde{B}_1 + B^f_1\Big|_{-12} - B^f_{2\to1}\Big|_{-2} \\
\text{2-jet:} \quad & B^f_2 + B^{\mathcal{I}}_2 \\
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\text{2-jet:} \quad & B^f_2 + B^{\mathcal{I}}_2 \\
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\end{aligned}
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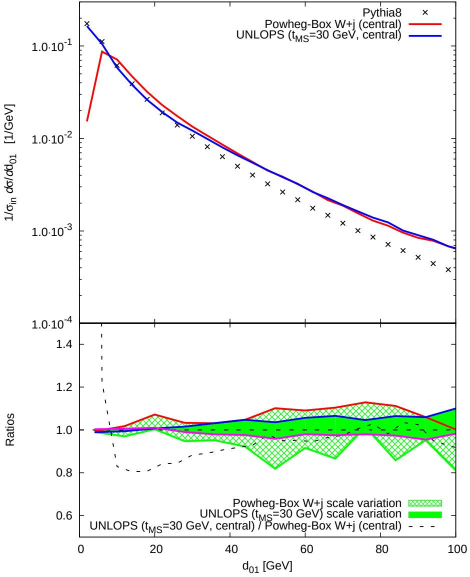
Pythia8
Powheg-Box W+j (central)
UNLOPS (t_{MS}=30 GeV, central)
$1/\sigma_{in}\ d\sigma/dd_{01}$ [1/GeV]
$1.0\cdot10^{-1}$
$1.0\cdot10^{-2}$
$1.0\cdot10^{-3}$
$1.0\cdot10^{-4}$
Ratios
1.4
1.2
1.0
0.8
0.6
Powheg-Box W+j scale variation
UNLOPS (t_{MS}=30 GeV) scale variation
UNLOPS (t_{MS}=30 GeV, central) / Powheg-Box W+j (central)
0
20
40
60
80
100
d_{01} [GeV]

- Implemented in PYTHIA8.
- PS have running α_s and PDFs, while ME's have fixed μ_R and μ_F. We can handle that.
- If someone gives us an event sample generated according to a NNLO exclusive n-jet cross section, we can use that as well.
- $B_2^{\mathcal{I}}$ can be handled if we have a PS which resums EW corrections (see Torbjörns talk on Thursday).