The importance of higher log resummation in extending CKKW to NLO

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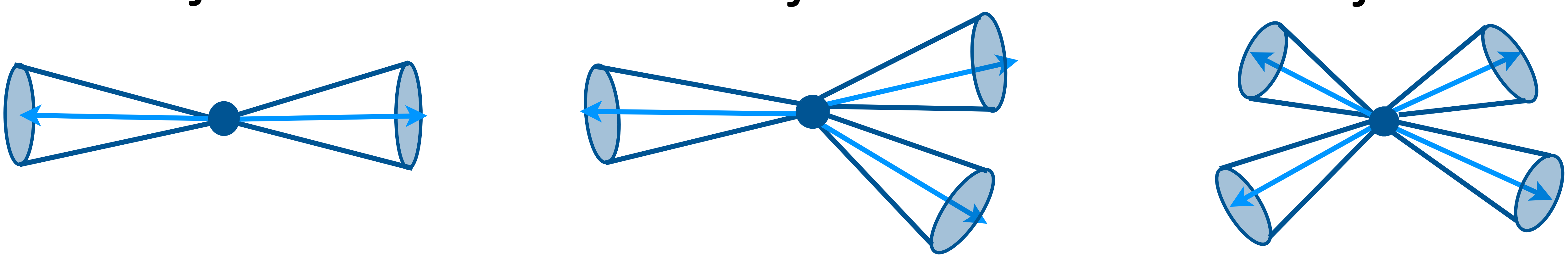
Oct 2 2012

The idea of merging is to calculate exclusive cross-sections and combine them to form an inclusive sample

For example, inclusive hadronic cross section at e^+e^- can be separated into individual jet cross-sections

2 jets 3 jets 4 jets



Many ways to define jets but all need resolution μ_{jet}

For $\mu_{jet} \ll Q$ ($\alpha_s \, \mathrm{Log}^2(\mu_{jet}/Q) \sim 1$) needs LL resummation (This is main point of CKKW paper)

What happens if we try to extend to NLO?

A simple setup using thrust illustrates all the important features

1. Choose thrust as the jet resolution variable ($\tau = 1\text{-}T$)
   $\tau < \tau_{cut} \Rightarrow$ 2 jets, $\tau > \tau_{cut} \Rightarrow$ 3+ jets

2. Imagine we only ever measure thrust on 3+ jet
   $d\sigma/d\tau = \int d\Phi_3 \; d\sigma_3/d\Phi_3 \; \delta[\, \tau - \tau(\Phi_3)\,]$

$$\sigma^{\rm incl} = \sigma_2(\tau^{\rm cut}) + \int {\rm d}\tau \, \frac{{\rm d}\sigma_3^{\rm incl}}{{\rm d}\tau} \theta(\tau > \tau^{\rm cut})$$

PT expressions for both terms contain large logarithms

$L_{cut} = \text{Log}(\tau_{cut})$ $\qquad$ $L = \text{Log}(\tau)$

Form of these logarithms well known

The structure of the large logarithms follows a well known pattern

$$\sigma^{\mathrm{incl}} = \sigma_2(\tau^{\mathrm{cut}}) + \int \mathrm{d}\tau \, \frac{\mathrm{d}\sigma_3^{\mathrm{incl}}}{\mathrm{d}\tau} \theta(\tau > \tau^{\mathrm{cut}})$$

For exclusive 2-jet rate

$$\sigma_2(\tau^{\mathrm{cut}}) \sim 1 + \alpha_s(L_{\mathrm{cut}}^2 + L_{\mathrm{cut}} + 1) + \alpha_s^2(L_{\mathrm{cut}}^4 + L_{\mathrm{cut}}^3 + L_{\mathrm{cut}}^2 + L_{\mathrm{cut}} + 1) + \ldots$$

For inclusive 3-jet rate

$$\frac{\mathrm{d}\sigma_3^{\mathrm{incl}}}{\mathrm{d}\tau} \sim \alpha_s \left[\frac{L}{\tau} + \frac{1}{\tau} + \delta(\tau) + 1\right] + \alpha_s^2 \left[\frac{L^3}{\tau} + \frac{L^2}{\tau} + \frac{L}{\tau} + \frac{1}{\tau} + \delta(\tau) + 1\right] + \ldots$$

LL resummation [all terms $(\alpha_s L_{\mathrm{cut}}^2)^n$] needed if

$$\alpha_s L_{\mathrm{cut}}^2 \sim 1$$

The structure of the large logarithms follows a well known pattern

$$\sigma^{\rm incl} = \sigma_2(\tau^{\rm cut}) + \int {\rm d}\tau \, \frac{{\rm d}\sigma_3^{\rm incl}}{{\rm d}\tau} \theta(\tau > \tau^{\rm cut})$$

Some definitions I will use

$\Delta(1,\tau)$: Sudakov Factor

$P(\tau) \equiv \frac{{\rm d} \log \Delta(1,\tau)}{{\rm d}\tau}$: Splitting function

Conservation of probability (fundamental theorem of calculus)

$$\Delta(1,\tau^{\rm cut}) + \int_{\tau^{\rm cut}}^1 {\rm d}\tau \, P(\tau) \Delta(1,\tau) = 1$$

With this notation, can illustrate what other approaches are doing

$$\sigma^{\rm incl} = \sigma_2(\tau^{\rm cut}) + \int {\rm d}\tau \, \frac{{\rm d}\sigma_3^{\rm incl}}{{\rm d}\tau} \theta(\tau > \tau^{\rm cut})$$

Original CKKW prescription

$$\sigma_2(\tau^{\rm cut}) = \sigma^{\rm LO} \Delta(1, \tau^{\rm cut})$$

$$\frac{{\rm d}\sigma_3^{\rm incl}}{{\rm d}\tau} = \frac{{\rm d}\sigma_3^{\rm LO}}{{\rm d}\tau} \Delta(1, \tau)$$

Inclusive cross-section gives

$$\begin{aligned}
\sigma^{\rm incl} &= \sigma^{\rm LO}\, \Delta(1, \tau^{\rm cut}) + \int_{\tau^{\rm cut}}^{1} {\rm d}\tau \, \frac{{\rm d}\sigma_3^{\rm LO}}{{\rm d}\tau} \, \Delta(1, \tau) \\
&= \sigma^{\rm LO}\, \Delta(1, \tau^{\rm cut}) + \sigma^{\rm LO} \int_{\tau^{\rm cut}}^{1} {\rm d}\tau \, P(\tau) \Delta(1, \tau) + \int_{\tau^{\rm cut}}^{1} {\rm d}\tau \left[\frac{{\rm d}\sigma_3^{\rm LO}}{{\rm d}\tau} - \sigma^{\rm LO} P(\tau) \right] \Delta(1, \tau) \\
&= \sigma^{\rm LO} + \mathcal{O}(\alpha_s L_{\rm cut})
\end{aligned}$$

LO accuracy for both $\sigma^{\rm incl}$ and $d\sigma_3^{\rm incl}$

With this notation, can illustrate what other approaches are doing

$$\sigma^{\text{incl}} = \sigma_2(\tau^{\text{cut}}) + \int \mathrm{d}\tau \, \frac{\mathrm{d}\sigma_3^{\text{incl}}}{\mathrm{d}\tau} \theta(\tau > \tau^{\text{cut}})$$

Powheg prescription

$$\sigma_2(\tau^{\text{cut}}) = \sigma^{\text{NLO}} \widetilde{\Delta}(1, \tau^{\text{cut}}) \qquad \frac{\mathrm{d}\sigma_3^{\text{incl}}}{\mathrm{d}\tau} = \frac{\sigma^{\text{NLO}}}{\sigma^{\text{LO}}} \frac{\mathrm{d}\sigma_3^{\text{LO}}}{\mathrm{d}\tau} \widetilde{\Delta}(1, \tau) \qquad \widetilde{P}(\tau) = \frac{1}{\sigma^{\text{LO}}} \frac{\mathrm{d}\sigma_3^{\text{LO}}}{\mathrm{d}\tau}$$

Inclusive cross-section gives

$$\begin{aligned} \sigma^{\text{incl}} &= \sigma^{\text{NLO}} \, \widetilde{\Delta}(1, \tau^{\text{cut}}) + \int_0^{\tau^{\text{cut}}} \mathrm{d}\tau \, \frac{\sigma^{\text{NLO}}}{\sigma^{\text{LO}}} \frac{\mathrm{d}\sigma_3^{\text{LO}}}{\mathrm{d}\tau} \, \widetilde{\Delta}(1, \tau) \\ &= \sigma^{\text{NLO}} \end{aligned}$$

NLO accuracy for σ^{incl}, LO accuracy for $\mathrm{d}\sigma_3^{\text{incl}}$

With this notation, can illustrate what other approaches are doing

$$\sigma^{\rm incl} = \sigma_2(\tau^{\rm cut}) + \int {\rm d}\tau \, \frac{{\rm d}\sigma_3^{\rm incl}}{{\rm d}\tau} \theta(\tau > \tau^{\rm cut})$$

MC@NLO prescription

$$\sigma_2(\tau^{\rm cut}) = \widetilde{\sigma}^{\rm NLO} \Delta(1, \tau^{\rm cut}) \qquad \frac{{\rm d}\sigma_3^{\rm incl}}{{\rm d}\tau} = \frac{{\rm d}\sigma_3^{\rm LO}}{{\rm d}\tau} - \sigma^{\rm LO} P(\tau) + \widetilde{\sigma}^{\rm NLO} P(\tau) \Delta(1, \tau)$$

$$\widetilde{\sigma}^{\rm NLO} = \sigma^{\rm LO} + \sigma^{\rm V} + \sigma^{\rm LO} \int_0^1 {\rm d}\tau \, P(\tau)$$

Inclusive cross-section gives

$$\begin{aligned} \sigma^{\rm incl} &= \widetilde{\sigma}^{\rm NLO} \Delta(1, \tau^{\rm cut}) + \int_{\tau^{\rm cut}}^1 {\rm d}\tau \left[\widetilde{\sigma}^{\rm NLO} P(\tau) \Delta(1, \tau) + \frac{{\rm d}\sigma_3^{\rm LO}}{{\rm d}\tau} - \sigma^{\rm LO} P(\tau) \right] \\ &= \widetilde{\sigma}^{\rm NLO} + \int_0^1 {\rm d}\tau \left[\frac{{\rm d}\sigma_3^{\rm LO}}{{\rm d}\tau} - \sigma^{\rm LO} P(\tau) \right] - \int_0^{\tau^{\rm cut}} {\rm d}\tau \left[\frac{{\rm d}\sigma_3^{\rm LO}}{{\rm d}\tau} - \sigma^{\rm LO} P(\tau) \right] \\ &= \sigma^{\rm NLO} - \int_0^{\tau^{\rm cut}} {\rm d}\tau \left[\frac{{\rm d}\sigma_3^{\rm LO}}{{\rm d}\tau} - \sigma^{\rm LO} P(\tau) \right] \end{aligned}$$

NLO accuracy for $\sigma^{\rm incl}$, LO accuracy for $d\sigma_3^{\rm incl}$

With this notation, can illustrate what other approaches are doing

$$\sigma^{\rm incl} = \sigma_2(\tau^{\rm cut}) + \int d\tau \, \frac{d\sigma_3^{\rm incl}}{d\tau} \theta(\tau > \tau^{\rm cut})$$

Modified MC@NLO prescription

$$\sigma_2(\tau^{\rm cut}) = \widetilde{\sigma}^{\rm NLO} \Delta(1, \tau^{\rm cut}) \qquad \frac{d\sigma_3^{\rm incl}}{d\tau} = \left[\frac{d\sigma_3^{\rm LO}}{d\tau} - \sigma^{\rm LO} P(\tau) + \widetilde{\sigma}^{\rm NLO} P(\tau)\right] \Delta(1, \tau)$$

$$\widetilde{\sigma}^{\rm NLO} = \sigma^{\rm LO} + \sigma^{\rm V} + \sigma^{\rm LO} \int_0^1 d\tau \, P(\tau)$$

Inclusive cross-section gives

$$\sigma^{\rm incl} = \widetilde{\sigma}^{\rm NLO} \Delta(1, \tau^{\rm cut}) + \int_{\tau^{\rm cut}}^1 d\tau \, \left[\widetilde{\sigma}^{\rm NLO} P(\tau) + \frac{d\sigma_3^{\rm LO}}{d\tau} - \sigma^{\rm LO} P(\tau)\right] \Delta(1, \tau)$$

$$= \sigma^{\rm NLO} - \int_0^{\tau^{\rm cut}} d\tau \, \left[\frac{d\sigma_3^{\rm LO}}{d\tau} - \sigma^{\rm LO} P(\tau)\right] + \mathcal{O}(\alpha_s^2)$$

NLO accuracy for $\sigma^{\rm incl}$, LO accuracy for $d\sigma_3^{\rm incl}$

With this notation, can illustrate what other approaches are doing

$$\sigma^{\text{incl}} = \sigma_2(\tau^{\text{cut}}) + \int \mathrm{d}\tau \, \frac{\mathrm{d}\sigma_3^{\text{incl}}}{\mathrm{d}\tau} \theta(\tau > \tau^{\text{cut}})$$

New NLO merging procedure

$$\sigma_2(\tau^{\text{cut}}) = \widetilde{\sigma}^{\text{NLO}} \Delta(1, \tau^{\text{cut}}) \qquad \frac{\mathrm{d}\sigma_3^{\text{incl}}}{\mathrm{d}\tau} = \left[\frac{\mathrm{d}\sigma_3^{\text{NLO}}}{\mathrm{d}\tau} + \frac{\mathrm{d}\sigma_3^{\text{LO}}}{\mathrm{d}\tau} \int_\tau^1 \mathrm{d}\tau' \, P(\tau') \right] \Delta(1, \tau)$$

$$\widetilde{\sigma}^{\text{NLO}} = \sigma^{\text{LO}} + \sigma^{\text{V}} + \sigma^{\text{LO}} \int_0^1 \mathrm{d}\tau \, P(\tau)$$

Inclusive cross-section gives

$$\sigma^{\text{incl}} - \sigma^{\text{MC@NLO}} = \int_{\tau^{\text{cut}}}^1 \mathrm{d}\tau \left[\frac{\mathrm{d}\sigma^{\text{NLO}}}{\mathrm{d}\tau} - \frac{\mathrm{d}\sigma^{\text{LO}}}{\mathrm{d}\tau} - \left(\sigma^{\text{NLO}} - \sigma^{\text{LO}} \right) P(\tau) \right.$$
$$\left. + \frac{\mathrm{d}\sigma_3^{\text{LO}}}{\mathrm{d}\tau} \int_\tau^1 \mathrm{d}\tau' \, P(\tau') \right] \Delta(1, \tau)$$

With this notation, can illustrate what other approaches are doing

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$$\widetilde{\sigma}^{\text{NLO}} = \sigma^{\text{LO}} + \sigma^{\text{V}} + \sigma^{\text{LO}} \int_0^1 \mathrm{d}\tau \, P(\tau)$$

Inclusive cross-section gives

$$\sigma^{\text{incl}} - \sigma^{\text{MC@NLO}} = \int_{\tau^{\text{cut}}}^1 \mathrm{d}\tau \left[\frac{\mathrm{d}\sigma^{\text{NLO}}}{\mathrm{d}\tau} - \frac{\mathrm{d}\sigma^{\text{LO}}}{\mathrm{d}\tau} - \left(\sigma^{\text{NLO}} - \sigma^{\text{LO}} \right) P(\tau) \right.$$
$$\left. + \left[\frac{\mathrm{d}\sigma_3^{\text{LO}}}{\mathrm{d}\tau} \int_\tau^1 \mathrm{d}\tau' \, P(\tau') \right] \right] \Delta(1, \tau)$$

With this notation, can illustrate what other approaches are doing

$$\sigma^{\mathrm{incl}} = \sigma_2(\tau^{\mathrm{cut}}) + \int \mathrm{d}\tau \, \frac{\mathrm{d}\sigma_3^{\mathrm{incl}}}{\mathrm{d}\tau} \theta(\tau > \tau^{\mathrm{cut}})$$

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$$\widetilde{\sigma}^{\mathrm{NLO}} = \sigma^{\mathrm{LO}} + \sigma^{\mathrm{V}} + \sigma^{\mathrm{LO}} \int_0^1 \mathrm{d}\tau \, P(\tau)$$

Inclusive cross-section gives

$$\sigma^{\mathrm{incl}} - \sigma^{\mathrm{MC@NLO}} = \int_{\tau^{\mathrm{cut}}}^1 \mathrm{d}\tau \left[\frac{\mathrm{d}\sigma^{\mathrm{NLO}}}{\mathrm{d}\tau} - \frac{\mathrm{d}\sigma^{\mathrm{LO}}}{\mathrm{d}\tau} - \left(\sigma^{\mathrm{NLO}} - \sigma^{\mathrm{LO}}\right) P(\tau) + \frac{\mathrm{d}\sigma_3^{\mathrm{LO}}}{\mathrm{d}\tau} \int_\tau^1 \mathrm{d}\tau' \, P(\tau') \right] \Delta(1, \tau)$$

$$= \mathcal{O}(\alpha_s^2 L_{\mathrm{cut}}^3)$$

With this notation, can illustrate what other approaches are doing

$$\sigma^{\rm incl} = \sigma_2(\tau^{\rm cut}) + \int \mathrm{d}\tau \, \frac{\mathrm{d}\sigma_3^{\rm incl}}{\mathrm{d}\tau} \theta(\tau > \tau^{\rm cut})$$

New NLO merging procedure

$$\sigma_2(\tau^{\rm cut}) = \widetilde{\sigma}^{\rm NLO} \Delta(1, \tau^{\rm cut}) \qquad \frac{\mathrm{d}\sigma_3^{\rm incl}}{\mathrm{d}\tau} = \left[\frac{\mathrm{d}\sigma_3^{\rm NLO}}{\mathrm{d}\tau} + \frac{\mathrm{d}\sigma_3^{\rm LO}}{\mathrm{d}\tau} \int_\tau^1 \mathrm{d}\tau' \, P(\tau') \right] \Delta(1, \tau)$$

$$\widetilde{\sigma}^{\rm NLO} = \sigma^{\rm LO} + \sigma^{\rm V} + \sigma^{\rm LO} \int_0^1 \mathrm{d}\tau \, P(\tau)$$

Inclusive cross-section gives

$$\sigma^{\rm incl} - \sigma^{\rm MC@NLO} = \int_{\tau^{\rm cut}}^1 \mathrm{d}\tau \left[\frac{\mathrm{d}\sigma^{\rm NLO}}{\mathrm{d}\tau} - \frac{\mathrm{d}\sigma^{\rm LO}}{\mathrm{d}\tau} - \left(\sigma^{\rm NLO} - \sigma^{\rm LO}\right) P(\tau) + \frac{\mathrm{d}\sigma_3^{\rm LO}}{\mathrm{d}\tau} \int_\tau^1 \mathrm{d}\tau' \, P(\tau') \right] \Delta(1, \tau)$$

$$= \mathcal{O}(\alpha_s^2 L_{\rm cut}^3)$$

Large logarithms spoil NLO accuracy for $\sigma^{\rm incl}$

Need very carefully defined P(τ) (higher log resummation)

All existing approaches sum logarithms at LL accuracy

Higher logarithmic resummation is important in its own right, but also needed to combine NLO calculations

The structure of the large logarithms follows a well known pattern

$$\sigma^{\text{incl}} = \sigma_2(\tau^{\text{cut}}) + \int \mathrm{d}\tau \, \frac{\mathrm{d}\sigma_3^{\text{incl}}}{\mathrm{d}\tau} \theta(\tau > \tau^{\text{cut}})$$

Notation for logarithmic accuracy

$$\sigma_2(\tau_{\text{cut}}) = \begin{bmatrix} 1 & & & & & & \\ \alpha L^2 & \alpha L & \alpha & & & & \\ \alpha^2 L^4 & \alpha^2 L^3 & \alpha^2 L^2 & \alpha^2 L & \alpha & & \\ \alpha^3 L^6 & \alpha^3 L^5 & \alpha^3 L^4 & \alpha^3 L^3 & \alpha^3 L^2 & \alpha L & \ldots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

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$$\sigma_2(\tau_{\rm cut}) = \begin{bmatrix} 1 & & & & & & \\ \alpha L^2 & \alpha L & \alpha & & & & \\ \alpha^2 L^4 & \alpha^2 L^3 & \alpha^2 L^2 & \alpha^2 L & \alpha & & \\ \alpha^3 L^6 & \alpha^3 L^5 & \alpha^3 L^4 & \alpha^3 L^3 & \alpha^3 L^2 & \alpha L & \ldots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

LL

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$$\sigma^{\text{incl}} = \sigma_2(\tau^{\text{cut}}) + \int \mathrm{d}\tau \, \frac{\mathrm{d}\sigma_3^{\text{incl}}}{\mathrm{d}\tau} \theta(\tau > \tau^{\text{cut}})$$

Notation for logarithmic accuracy

$$\sigma_2(\tau_{\text{cut}}) = \begin{bmatrix} 1 & & & & & & \\ \alpha L^2 & \alpha L & \alpha & & & & \\ \alpha^2 L^4 & \alpha^2 L^3 & \alpha^2 L^2 & \alpha^2 L & \alpha & & \\ \alpha^3 L^6 & \alpha^3 L^5 & \alpha^3 L^4 & \alpha^3 L^3 & \alpha^3 L^2 & \alpha L & \ldots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

NLL

The structure of the large logarithms follows a well known pattern

$$\sigma^{\mathrm{incl}} = \sigma_2(\tau^{\mathrm{cut}}) + \int \mathrm{d}\tau \, \frac{\mathrm{d}\sigma_3^{\mathrm{incl}}}{\mathrm{d}\tau} \theta(\tau > \tau^{\mathrm{cut}})$$

Notation for logarithmic accuracy

$$\sigma_2(\tau_{\mathrm{cut}}) = \begin{bmatrix} 1 & & & & & & \\ \alpha L^2 & \alpha L & \alpha & & & & \\ \alpha^2 L^4 & \alpha^2 L^3 & \alpha^2 L^2 & \alpha^2 L & \alpha & & \\ \alpha^3 L^6 & \alpha^3 L^5 & \alpha^3 L^4 & \alpha^3 L^3 & \alpha^3 L^2 & \alpha L & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

NLL’

The structure of the large logarithms follows a well known pattern

$$\sigma^{\mathrm{incl}} = \sigma_2(\tau^{\mathrm{cut}}) + \int \mathrm{d}\tau \, \frac{\mathrm{d}\sigma_3^{\mathrm{incl}}}{\mathrm{d}\tau} \theta(\tau > \tau^{\mathrm{cut}})$$

Notation for logarithmic accuracy

$$\sigma_2(\tau_{\mathrm{cut}}) = \begin{bmatrix} 1 & & & & & & \\ \alpha L^2 & \alpha L & \alpha & & & & \\ \alpha^2 L^4 & \alpha^2 L^3 & \alpha^2 L^2 & \alpha^2 L & \alpha & & \\ \alpha^3 L^6 & \alpha^3 L^5 & \alpha^3 L^4 & \alpha^3 L^3 & \alpha^3 L^2 & \alpha L & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

NNLL

The basic formula of Geneva takes a very simple form

$$\sigma^{\text{incl}} = \sigma_2(\tau^{\text{cut}}) + \int \mathrm{d}\tau \, \frac{\mathrm{d}\sigma_3^{\text{incl}}}{\mathrm{d}\tau} \theta(\tau > \tau^{\text{cut}})$$

GENEVA approach

$$\sigma_2(\tau^{\text{cut}}) = \sigma_2^{\text{F}}(\tau^{\text{cut}}) \left[\frac{\sigma_2^{\text{R}}(\tau^{\text{cut}})}{\sigma_2^{\text{RE}}(\tau^{\text{cut}})} \right]$$

$$\frac{\mathrm{d}\sigma_3^{\text{incl}}}{\mathrm{d}\tau} = \frac{\mathrm{d}\sigma_3^{\text{F}}}{\mathrm{d}\tau} \left[\frac{\frac{\mathrm{d}\sigma_3^{\text{R}}}{\mathrm{d}\tau}}{\frac{\mathrm{d}\sigma_3^{\text{RE}}}{\mathrm{d}\tau}} \right]$$

F: Fixed order   R: Resummed   RE: Expansion of resummed

If resummed to LL, the ratio becomes equal to Sudakov

By resumming to NLL’, find

$$\sigma^{\text{incl}} - \sigma_2^{\text{GENEVA}} = \mathcal{O}(\alpha_s^2 L_{\text{cut}})$$

Everything I said previously easily generalizes to fully inclusive events

For 2 different jet multiplicities can write

$$\mathrm{d}\sigma^{\mathrm{incl}} = \frac{\mathrm{d}\sigma_2(\tau^{\mathrm{cut}})}{\Phi_2}\,\mathrm{d}\Phi_2 + \frac{\mathrm{d}\sigma_3^{\mathrm{incl}}}{\mathrm{d}\Phi_3}\theta\left(\tau(\Phi_3) > \tau^{\mathrm{cut}}\right)\,\mathrm{d}\Phi_3$$

For more jets, write

$$\mathrm{d}\sigma_N = \sum_{n=N}^{M-1} \mathrm{d}\sigma_n(\mathcal{T}_n^{\mathrm{cut}}) + \mathrm{d}\sigma_M^{\mathrm{incl}}$$

Using same trick as before, can derive expressions for each term that is correct to given fixed order and given resummed order

Need to go to at least NLL' (two orders more that LL) in resummation to be able to merge two NLO calculations

Can not rely on Sudakov factors any more, but need honest resummation

GENEVA allows to do precisely that: combine fixed order resummation with higher log resummation