Real Radiation at NNLO with the STRIPPER Subtraction Scheme

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CERN, SM@LHC, 2 October 2012

Subtraction at NLO

analytic?

$$\sigma^{NLO} = \int_{m+1} \left[\left(d\sigma^R \right)_{\epsilon=0} - \left(d\sigma^A \right)_{\epsilon=0} \right] + \int_m \left[d\sigma^V + \int_1 d\sigma^A \right]_{\epsilon=0}$$

remapping?

$$\langle M_m^{(0)} | \mathbf{O} | M_m^{(0)} \rangle$$

together?

remapping (Catani-Seymour)

$$\{p\}_{m+1} \longrightarrow \{\tilde{p}\}_m$$

mapping (Frixione-Kunszt-Signer)

$$\{p\}_m,\ E_{m+1},\ \theta_{m+1},\ \phi_{m+1}$$

Subtraction at NNLO

STRIPPER (SecToR ImProved Phase space for real Radiation) MC ’10

- complete solution for real radiation in top quark pair production MC’11
- recently applied to Z -> e^+e^- γγ in QED by Boughezal, Melnikov, Petriello ’11

Proof of Concept with Top Pairs

- First ever hadron collider calculation at NNLO with more than 2 colored partons.
- First ever NNLO hadron collider calculation with massive fermions.

- published qQ → tT + X Bärnreuther, Czakon, Mitov `12
- published all fermionic reactions (qq,qq',qQ') Czakon, Mitov `12
- work on the remaining reactions (qg, gg) progressing well.

Computed. To appear.

Computed. Dominant at Tevatron

$$q\bar{q} \to t\bar{t}$$
$$q\bar{q} \to t\bar{t}g$$
$$q\bar{q} \to t\bar{t}gg$$
$$q\bar{q} \to t\bar{t}q'\bar{q}' \,, \qquad q \neq q'$$

$$gg \to t\bar{t}$$
$$gg \to t\bar{t}g$$
$$gg \to t\bar{t}gg$$
$$gg \to t\bar{t}q\bar{q}$$

$$qg \to t\bar{t}q$$
$$qg \to t\bar{t}qg$$

$$qq' \to t\bar{t}qq' \,, \qquad q \neq \bar{q}'$$
$$q\bar{q} \to t\bar{t}q\bar{q}$$

STRIPPER Ideas

About the phase space:

1. parameterization of the massless system with energies and angles modified to allow for a description of all collinear singular configurations with only two variables

2. level 1 decomposition into sectors allowing for only one type of collinear singularities

3. level 2 decomposition into sectors defining the order of singular limits

About the subtraction terms:

1. Subtraction at the endpoint derived from known soft and collinear limits of QCD amplitudes

2. No analytic integration of the subtraction terms

Phase Space

$$d\Phi_4 = \frac{d^{d-1}k_1}{(2\pi)^{d-1}2k_1^0}\frac{d^{d-1}k_2}{(2\pi)^{d-1}2k_2^0}\frac{d^{d-1}q_1}{(2\pi)^{d-1}2q_1^0}\frac{d^{d-1}q_2}{(2\pi)^{d-1}2q_2^0}(2\pi)^d\delta^{(d)}(k_1+k_2+q_1+q_2-p_1-p_2)$$

$$\begin{aligned}
p_1^\mu &= \frac{\sqrt{s}}{2}(1,0,0,1)\ ,\\
p_2^\mu &= \frac{\sqrt{s}}{2}(1,0,0,-1)\ ,\\
n_1^\mu &= \frac{\sqrt{s}}{2}\beta^2(1,0,\sin\theta_1,\cos\theta_1)\ ,\\
n_2^\mu &= \frac{\sqrt{s}}{2}\beta^2(1,\sin\phi\sin\theta_2,\cos\phi\sin\theta_2,\cos\theta_2)\ ,\\
k_1^\mu &= \hat{\xi}_1\ n_1^\mu\ ,\\
k_2^\mu &= \hat{\xi}_2\ n_2^\mu\ ,
\end{aligned}$$

$$\begin{aligned}
\hat{\eta}_{1,2} &= \frac{1}{2}(1-\cos\theta_{1,2})\ ,\\
\eta_3 &= \frac{1}{2}(1-\cos\theta_3)\\
&= \frac{1}{2}(1-\cos\phi\sin\theta_1\sin\theta_2-\cos\theta_1\cos\theta_2)\\
&= \frac{1}{2}(1-\cos(\theta_1-\theta_2)+(1-\cos\phi)\sin\theta_1\sin\theta_2)\ ,
\end{aligned}$$

$$\eta_3 = \frac{(\hat{\eta}_1-\hat{\eta}_2)^2}{\hat{\eta}_1+\hat{\eta}_2-2\hat{\eta}_1\hat{\eta}_2-2(1-2\zeta)\sqrt{\hat{\eta}_1(1-\hat{\eta}_1)\hat{\eta}_2(1-\hat{\eta}_2)}}$$

$$\zeta = \frac{1}{2}\frac{(1-\cos(\theta_1-\theta_2))(1+\cos\phi)}{1-\cos(\theta_1-\theta_2)+(1-\cos\phi)\sin\theta_1\sin\theta_2}$$

all collinear limits with only two variables

$$d\Phi_4 = d\Phi_3(p_1+p_2;k_1,k_2)\ d\Phi_2(Q;q_1,q_2)$$

$$\begin{aligned}
d\Phi_3(p_1+p_2;k_1,k_2) &= \frac{\pi^{2\epsilon}}{8(2\pi)^5\Gamma(1-2\epsilon)}s^{2-2\epsilon}\beta^{8-8\epsilon}\ (\zeta(1-\zeta))^{-\frac{1}{2}-\epsilon}\\
&\times\ (\hat{\eta}_1(1-\hat{\eta}_1))^{-\epsilon}(\hat{\eta}_2(1-\hat{\eta}_2))^{-\epsilon}\frac{\eta_3^{1-2\epsilon}}{|\hat{\eta}_1-\hat{\eta}_2|^{1-2\epsilon}}\ \hat{\xi}_1^{1-2\epsilon}\hat{\xi}_2^{1-2\epsilon}\\
&\times\ d\zeta\ d\hat{\eta}_1\ d\hat{\eta}_2\ d\hat{\xi}_1\ d\hat{\xi}_2\ .
\end{aligned}$$

$d\mu_{\eta\xi}$

Level 1 Decomposition

$$
\begin{aligned}
1 = \; & \left. \begin{array}{l} +\theta_1(k_1)\theta_1(k_2) \\ +\theta_2(k_1)\theta_2(k_2) \end{array} \right\} \text{ triple-collinear sector} \\
& \left. \begin{array}{l} +\theta_1(k_1)\theta_2(k_2)(1-\theta_3(k_1,k_2)) \\ +\theta_2(k_1)\theta_1(k_2)(1-\theta_3(k_1,k_2)) \end{array} \right\} \text{ double-collinear sector} \\
& +(\theta_1(k_1)\theta_2(k_2)+\theta_2(k_1)\theta_1(k_2))\theta_3(k_1,k_2)\} \text{ single-collinear sector}
\end{aligned}
$$

most difficult → triple-collinear sector

non-trivial only because of soft-collinear divergences → double-collinear sector

trivial, because NLO type
attach to first sector (contains same divergences) → single-collinear sector

top quark pair production

general case

$$1 = \sum_{\substack{\text{pairs} \\ i,j \in \mathcal{F}}} \sum_{\substack{k \in \mathcal{I} \cup \mathcal{F} \\ k \notin \{i,j\}}} \left[\theta_{ij,k} + \sum_{\substack{l \in \mathcal{I} \cup \mathcal{F} \\ l \notin \{i,j,k\}}} \theta_{ij,kl} \right]$$

$$d_{ij} = \left[\left(\frac{2E_i}{\sqrt{s}} \right) \left(\frac{2E_j}{\sqrt{s}} \right) \right]^{\alpha} (1-\cos\theta_{ij})^{\beta},$$

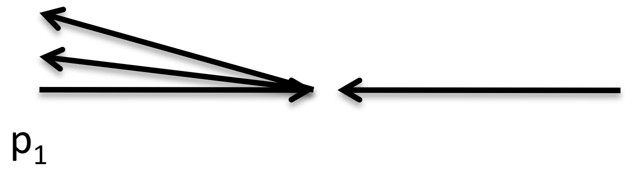
$$d_{ijk} = \left[\left(\frac{2E_i}{\sqrt{s}} \right) \left(\frac{2E_j}{\sqrt{s}} \right) \left(\frac{2E_k}{\sqrt{s}} \right) \right]^{\alpha} \left[(1-\cos\theta_{ij})(1-\cos\theta_{ik})(1-\cos\theta_{jk}) \right]^{\beta}$$

$$\theta_{ij,k} = \frac{1}{\mathcal{D}} \frac{h_{ij,k}}{d_{ijk}}, \qquad \theta_{ij,kl} = \frac{1}{\mathcal{D}} \frac{h_{i,k}}{d_{ik}} \frac{h_{j,l}}{d_{jl}}$$

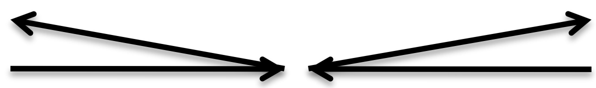
$$\mathcal{D} = \sum_{\substack{\text{pairs} \\ i,j \in \mathcal{F}}} \sum_{\substack{k \in \mathcal{I} \cup \mathcal{F} \\ k \notin \{i,j\}}} \left[\frac{h_{ij,k}}{d_{ijk}} + \sum_{\substack{l \in \mathcal{I} \cup \mathcal{F} \\ l \notin \{i,j,k\}}} \frac{h_{i,k}}{d_{ik}} \frac{h_{j,l}}{d_{jl}} \right]$$

collinear-singular configurations at NNLO

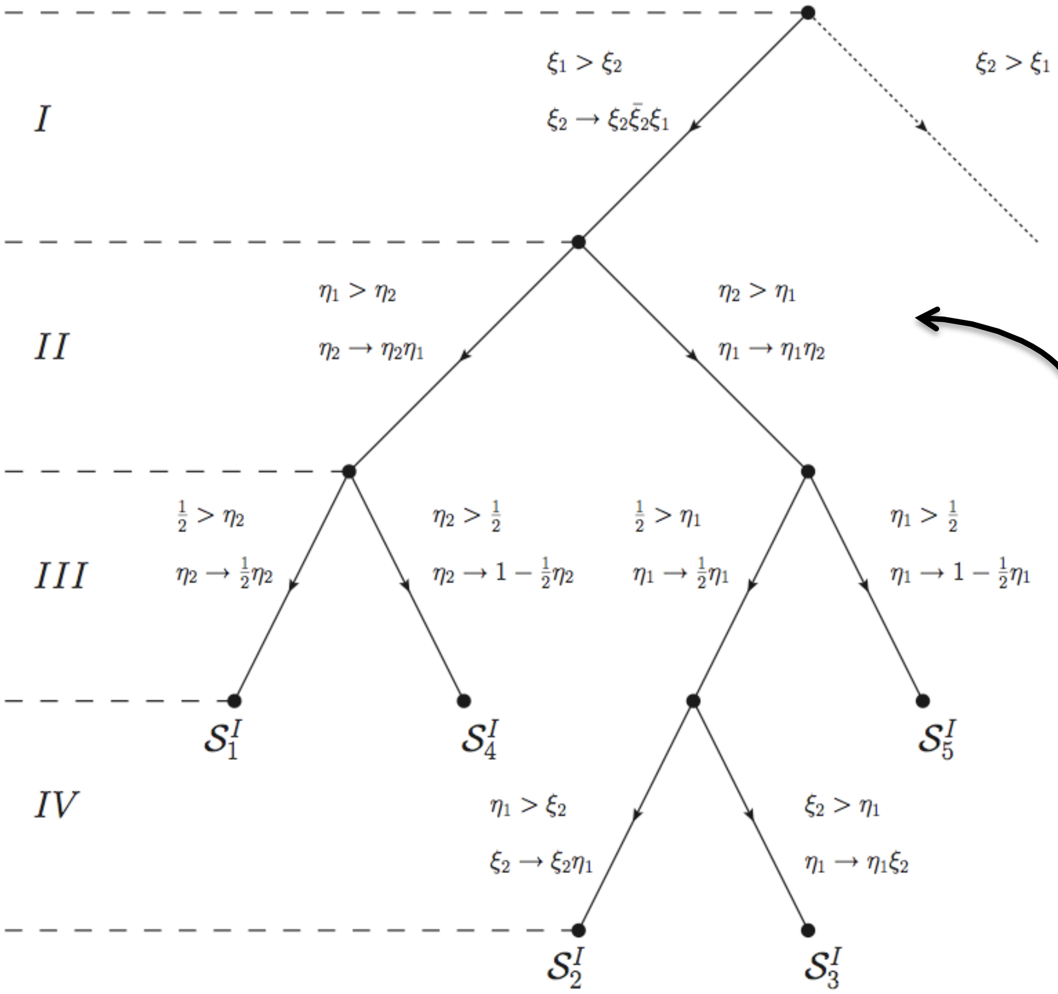
triple collinear



double collinear



Level 2 Decomposition



I) factorization of the soft singularities;

II, III) factorization of the collinear singularities;

IV) factorization of the soft-collinear singularities.

triple-collinear sector

Example from S_1^I

$$s_{156} = -\beta^2(\hat{\eta}_1\hat{\xi}_1 + \hat{\eta}_2\hat{\xi}_2 - \beta^2\hat{\xi}_1\hat{\xi}_2\eta_3)$$

$$-\frac{1}{2}\beta^2\eta_1\xi_1\left(2 + \eta_2\xi_2\bar{\xi}_2 - 2\beta^2\xi_1\xi_2\eta_{31}\bar{\xi}_2\right)$$

Subtraction Terms

$$d\mu_{\eta\xi} = \eta_1^{a_1+b_1\epsilon}\eta_2^{a_2+b_2\epsilon}\xi_1^{a_3+b_3\epsilon}\xi_2^{a_4+b_4\epsilon}\ \mu_{\mathcal{S}}^{\rm reg}\ d\eta_1 d\eta_2 d\xi_1 d\xi_2$$

$$\sigma_{\mathcal{O}} = \sum_{\mathcal{S}} \sigma_{\mathcal{O}}^{(\mathcal{S})} \qquad \sigma_{\mathcal{O}}^{(\mathcal{S})} = \int d\zeta\ d\eta_1\ d\eta_2\ d\xi_1\ d\xi_2\ d\cos\theta_Q\ d\phi_Q\ d\cos\rho_Q\ \Sigma_{\mathcal{O}}^{(\mathcal{S})}$$

$$\Sigma_{\mathcal{O}}^{(\mathcal{S})} = \frac{1}{2s}\left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^{3\epsilon} \mu_\zeta\ \mu_{\mathcal{S}}^{\rm reg}\ \mu_2\ \theta_{\mathcal{S}}\ F_J \frac{1}{\eta_1^{1-b_1\epsilon}}\frac{1}{\eta_2^{1-b_2\epsilon}}\frac{1}{\xi_1^{1-b_3\epsilon}}\frac{1}{\xi_2^{1-b_4\epsilon}}\ \mathfrak{M}_{\mathcal{S}}$$

$$\mathfrak{M}_{\mathcal{S}} = \eta_1^{1+a_1}\eta_2^{1+a_2}\xi_1^{1+a_3}\xi_2^{1+a_4}\ \overline{|\mathcal{M}_4|^2}$$

$$\int_0^1 \frac{d\lambda}{\lambda^{1-b\epsilon}} f(\lambda) \longrightarrow \int_0^1 d\lambda \left[\frac{f(0)}{b\epsilon} + \frac{f(\lambda)-f(0)}{\lambda^{1-b\epsilon}}\right]$$

apply four times

$$\Sigma_{\mathcal{O}}^{(\mathcal{S})} \longrightarrow \left[\Sigma_{\mathcal{O}}^{(\mathcal{S})}\right]$$

Subtraction Terms

$$\mathbf{X} \subseteq \{\eta_1, \eta_2, \xi_1, \xi_2\}$$

$$\lim_{\mathbf{X}\to\mathbf{0}} \mathfrak{M}_{\mathcal{S}} = g^2 \, \langle \mathcal{M}_3 | \mathbf{V} | \mathcal{M}_3 \rangle \quad \text{or} \quad \lim_{\mathbf{X}\to\mathbf{0}} \mathfrak{M}_{\mathcal{S}} = g^4 \, \langle \mathcal{M}_2 | \mathbf{V} | \mathcal{M}_2 \rangle$$

Example:

$$\mathfrak{R}_{\mathcal{S}} = \eta_1^{1+a_1} \eta_2^{1+a_2} \xi_1^{1+a_3} \xi_2^{1+a_4}$$

$$\boxed{\hat{\eta}_1 = \hat{\eta}_2 = 0}$$

$$\mathbf{V}^{ss'}_{a_1 a_5 a_6} = \lim_{\mathbf{X}\to\mathbf{0}} \mathfrak{R}_{\mathcal{S}} \frac{4\hat{P}^{ss'}_{a_1 a_5 a_6}}{s^2_{156}}$$

$$x_1 = -1 \,, \quad x_5 = \beta^2 \hat{\xi}_1 \,, \quad x_6 = \beta^2 \hat{\xi}_2 \,,$$

$$k^\mu_{\perp 1} = 0 \,, \quad k^\mu_{\perp 5} = \beta^2 \hat{\xi}_1 \sqrt{\hat{\eta}_1} \, \bar{k}^\mu_{\perp 5} \,, \quad k^\mu_{\perp 6} = \beta^2 \hat{\xi}_2 \sqrt{\hat{\eta}_2} \, \bar{k}^\mu_{\perp 6}(\hat{\eta}_1, \hat{\eta}_2) \,,$$

$$\begin{aligned} \bar{k}^\mu_{\perp 5} &= (0,0,1,0) \,, \\ \bar{k}^\mu_{\perp 6}(\hat{\eta}_1, \hat{\eta}_2) &= \frac{1}{\hat{\eta}_1 + \hat{\eta}_2 - 2(1-2\zeta)\sqrt{\hat{\eta}_1 \hat{\eta}_2}} \\ &\times \Big(0, \; 2|\hat{\eta}_1 - \hat{\eta}_2| \sqrt{\zeta(1-\zeta)}, \; 2\sqrt{\hat{\eta}_1 \hat{\eta}_2} - (\hat{\eta}_1 + \hat{\eta}_2)(1-2\zeta), \; 0\Big) \end{aligned}$$

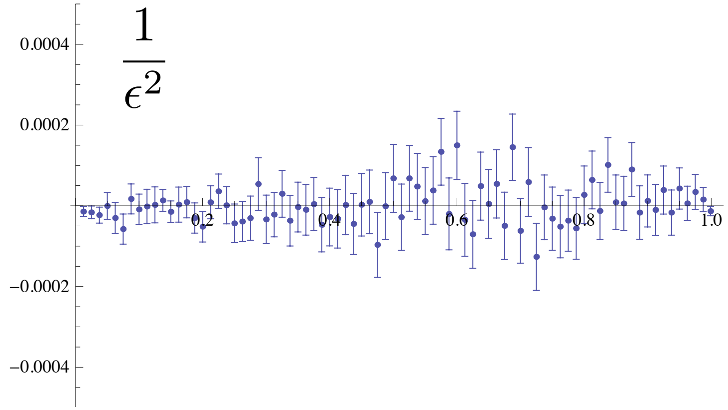
Cancellation of Divergences

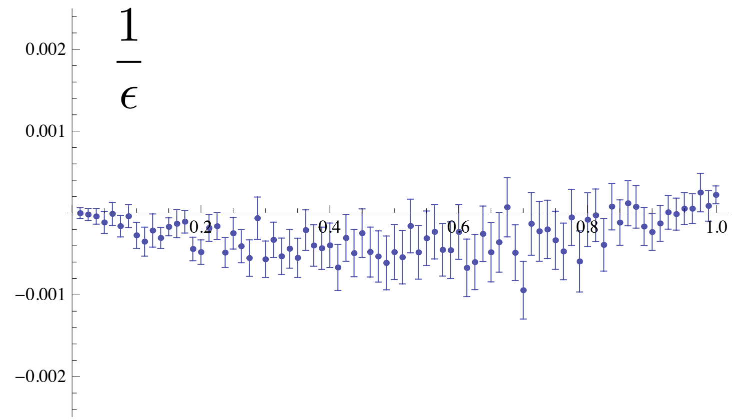
$$\text{PolesRR} = \frac{25\,\text{beta}\,(-3+\text{beta}^2)\,(-1+\text{beta}^2)}{486\,\text{ep}^4\,\pi} + \frac{1}{\text{ep}^3}\left(\frac{\text{beta}\,(-1+\text{beta}^2)\,(-4273+1566\,\text{beta}^2)}{1944\,\pi} - \frac{125\,\text{beta}\,(-3+\text{beta}^2)\,(-1+\text{beta}^2)\,\text{Log}[\text{beta}]}{243\,\pi} - \frac{25\,(-28+\text{beta}^2)\,(-1+\text{beta}^2)^2\,\text{Log}\left[\frac{1-\text{beta}}{1+\text{beta}}\right]}{3888\,\pi} + \frac{175\,\text{beta}\,(-3+\text{beta}^2)\,(-1+\text{beta}^2)\,\text{Log}\left[\frac{m^2}{s}\right]}{486\,\pi}\right);$$

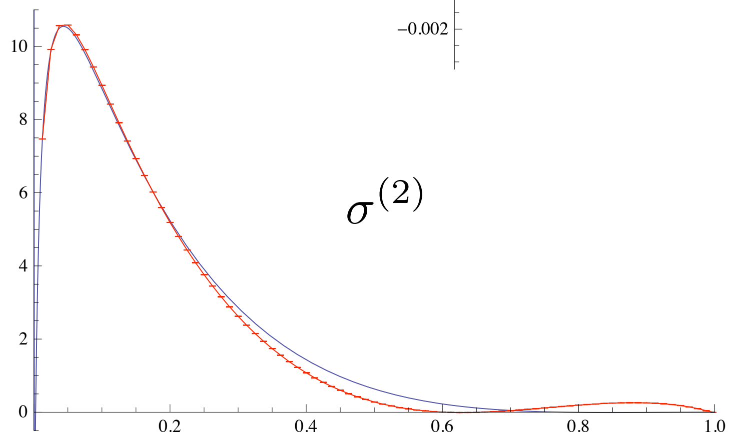
$$\text{PolesRV} = -\frac{41\,\text{beta}\,(3-4\,\text{beta}^2+\text{beta}^4)}{486\,\text{ep}^4\,\pi} + \frac{1}{\text{ep}^3}\left(-\frac{\text{beta}\,(-1+\text{beta}^2)\,(-21\,081+7724\,\text{beta}^2)}{5832\,\pi} + \frac{2\,\text{beta}\,(-3+\text{beta}^2)\,(-1+\text{beta}^2)\,\text{nl}}{243\,\pi} + \frac{47\,\text{beta}\,(-3+\text{beta}^2)\,(-1+\text{beta}^2)\,\text{Log}[\text{beta}]}{81\,\pi} + \frac{(-1+\text{beta}^2)\,(1660-1189\,\text{beta}^2+41\,\text{beta}^4)\,\text{Log}\left[\frac{1-\text{beta}}{1+\text{beta}}\right]}{3888\,\pi} - \frac{223\,\text{beta}\,(-3+\text{beta}^2)\,(-1+\text{beta}^2)\,\text{Log}\left[\frac{m^2}{s}\right]}{486\,\pi}\right);$$

$$\text{PolesVV} = \frac{8\,\text{beta}\,(-3+\text{beta}^2)\,(-1+\text{beta}^2)}{243\,\text{ep}^4\,\pi} + \frac{1}{\text{ep}^3}\left(\frac{17\,\text{beta}\,(-1+\text{beta}^2)\,(-243+89\,\text{beta}^2)}{2916\,\pi} - \frac{\text{beta}\,(3-4\,\text{beta}^2+\text{beta}^4)\,\text{nl}}{162\,\pi} - \frac{16\,\text{beta}\,(-3+\text{beta}^2)\,(-1+\text{beta}^2)\,\text{Log}[\text{beta}]}{243\,\pi} - \frac{(-1+\text{beta}^2)\,(60-29\,\text{beta}^2+\text{beta}^4)\,\text{Log}\left[\frac{1-\text{beta}}{1+\text{beta}}\right]}{243\,\pi} + \frac{8\,\text{beta}\,(-3+\text{beta}^2)\,(-1+\text{beta}^2)\,\text{Log}\left[\frac{m^2}{s}\right]}{81\,\pi}\right);$$

```
PolesRR + PolesRV + PolesVV /. nl → 0 // Expand
0
```







partonic cross section for qq -> tt+X
as function of top quark velocity

Advantages of STRIPPER

- Process independence
- Pointwise convergence
- Suitability for automation
- Elegance
- The claim: general subtraction scheme for any process at NNLO
- Notice: did not discuss single-real radiation because no special treatment needed