Direct numerical integration at two- and three-loops

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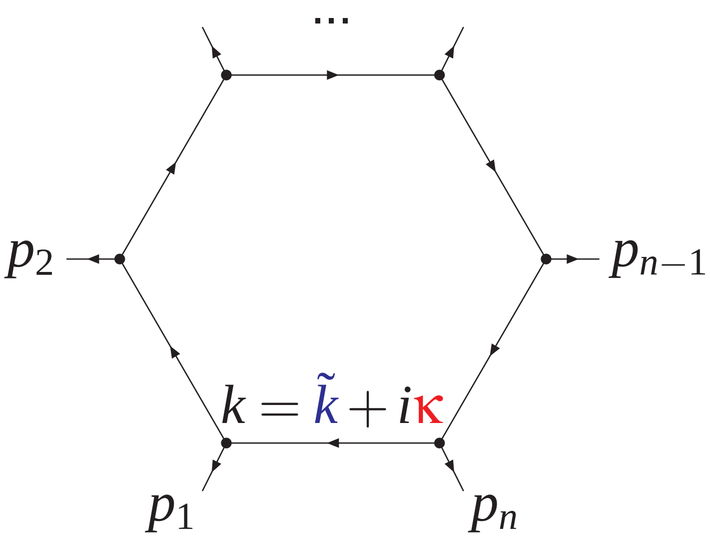
Numerical approach at NLO

- Subtraction terms

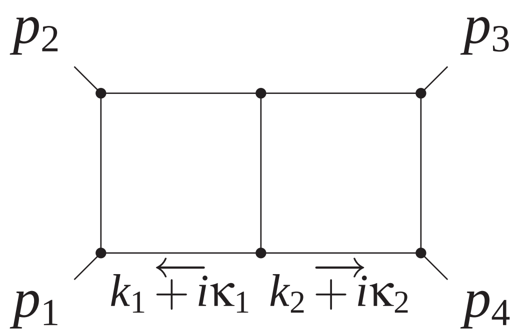
- Contour deformation

$$\int \frac{d^4k}{(2\pi)^4} f(k) \quad = \quad \int \frac{d^4\tilde{k}}{(2\pi)^4} \left| \frac{\partial k^\mu}{\partial \tilde{k}^\nu} \right| f(k(\tilde{k}))$$

κ vanishes whenever one loop momentum becomes soft.



Beyond one-loop

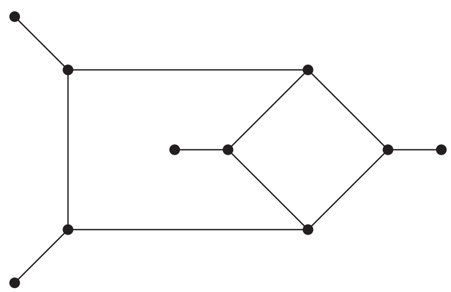


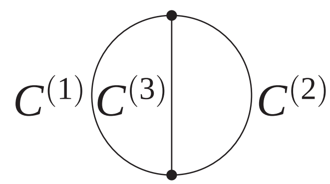
We have:

- 2 independent loop momenta
- 3 inequivalent cycles

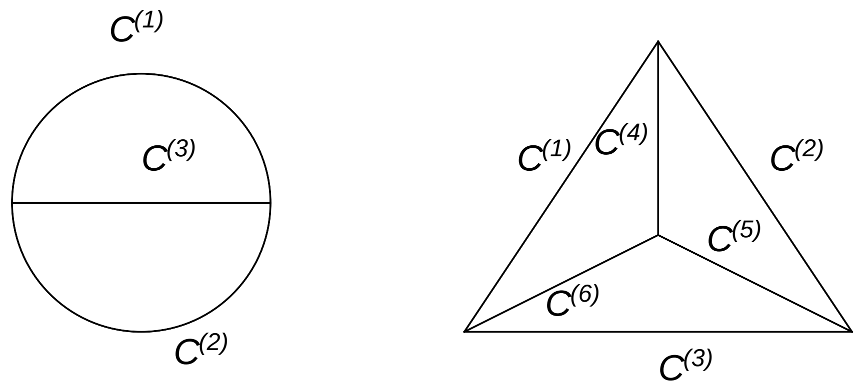
Chain diagrams

The momenta of the propagators in the same chain differ only by a linear combination of the external momenta.





Two and three loop chain diagrams

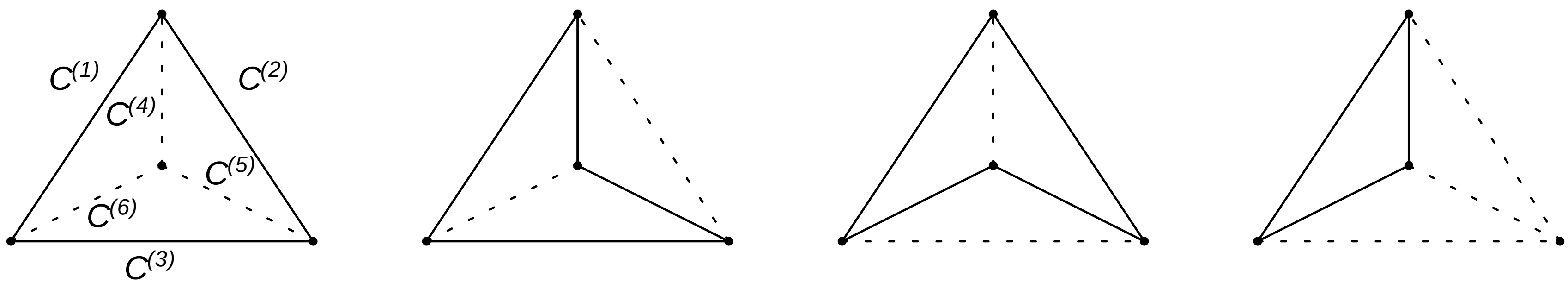


κ_i obtained as the sum of all deformation vectors for cycles containing propagator i.

Two-loop example:

$$\begin{aligned}\kappa_1 &= \kappa^{(12)} + \kappa^{(13)},\\ \kappa_2 &= \kappa^{(12)} + \kappa^{(23)},\end{aligned}$$

Three loops



$$
\begin{aligned}
\kappa_1 &= \kappa^{(123)} + \kappa^{(146)} + \kappa^{(1256)} + \kappa^{(1345)}, \\
\kappa_2 &= \kappa^{(123)} + \kappa^{(245)} + \kappa^{(1256)} + \kappa^{(2346)}, \\
\kappa_3 &= \kappa^{(123)} + \kappa^{(356)} + \kappa^{(1345)} + \kappa^{(2346)}.
\end{aligned}
$$

Preliminary results

Comparison with analytical result (no internal masses, external legs off-shell)

- two- and three-loop propagator corrections
- two- and three-loop vertex functions (planar and non-planar)
- ladder diagrams (double box, triple box)

In addition:

- Two-loop six-point functions