Validity of the soft approximation

and

the impact of higher order corrections in Higgs production in gluon fusion

Marco Bonvini

DESY Hamburg

CERN, TH/LPCC Institute on SM at the LHC, October 4, 2012

Work in collaboration with:

Stefano Forte, Giovanni Ridolfi + Richard Ball, Simone Marzani

Goal of the talk

I will present a formalism to systematically answer the questions:

1. Is soft-gluon resummation needed?
2. Is soft-gluon resummation advisable? (Are soft terms dominant?)

[MB, Forte, Ridolfi 2012 (and 2010)]

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I will also present:

- a prediction of N^3LO Higgs cross-section (gg channel only)
  [Ball, MB, Forte, Marzani, Ridolfi (work in progress)]

Higgs production cross-section: notations

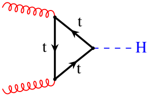
$$\sigma(\tau) = \tau\,\sigma_0 \int_\tau^1 \frac{dz}{z}\,\mathscr{L}\left(\frac{\tau}{z}\right) C(z,\alpha_s), \qquad \tau = \frac{m_H^2}{s}, \qquad z = \frac{m_H^2}{\hat{s}}$$

I concentrate on the gluon fusion channel: $\mathscr{L} = \mathscr{L}_{gg}$

Partonic cross section

$$\hat{\sigma}(z) = z\,\sigma_0 \underbrace{\left[\delta(1-z) + \alpha_s C^{(1)}(z) + \alpha_s^2 C^{(2)}(z) + \dots\right]}_{\text{coefficient function } C(z,\alpha_s)}$$

σ_0 is the LO partonic cross-section (order α_s^2):



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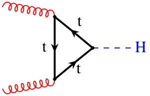

The questions 1., 2., 3. refer to the hadronic cross-section $\sigma(\tau)$, but the approximations apply to the partonic coefficient function $C(z,\alpha_s)$.

It is **not** straightforward to answer!

Saddle point argument

$$\frac{\sigma(\tau)}{\tau\,\sigma_0} = \int_\tau^1 \frac{dz}{z}\, \mathscr{L}\left(\frac{\tau}{z}\right) C(z, \alpha_s)$$

The soft region $z \to 1$ is always included in the integration region.
When is the $z \sim 1$ region actually relevant/dominant?

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The integral is dominated by the values of N in the proximity of the saddle point $N = N_0$:

$$\log\frac{1}{\tau} = -\frac{d}{dN}\log\mathscr{L}(N)\Big|_{N_0} - \frac{d}{dN}\log C(N,\alpha_s)\Big|_{N_0}$$

[MB, Forte, Ridolfi 2010]

Then we rephrase the questions:

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3. Is the pointlike (large-m_t) approximation adequate?

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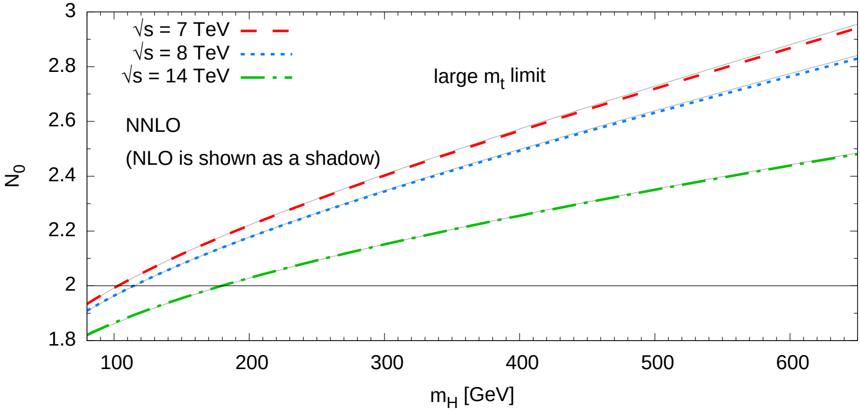
Hadron level kinematics $m_H, \sqrt{s}$ (or τ) $\quad\Rightarrow\quad$ saddle point N_0

Position of the saddle point

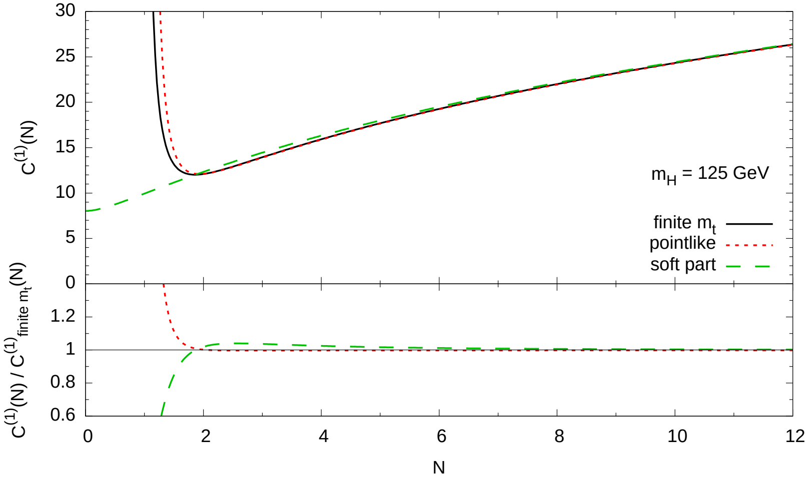
A unique real saddle N_0 always exists.



Note that these curves **do not depend on the perturbative order.**
This is due to the fact that they are mainly determined by the PDFs.

Partonic comparison

$$C(z,\alpha_s) = \delta(1-z) + \alpha_s\, C^{(1)}(z) + \alpha_s^2\, C^{(2)}(z) + \alpha_s^3\, C^{(3)}(z) + \dots$$



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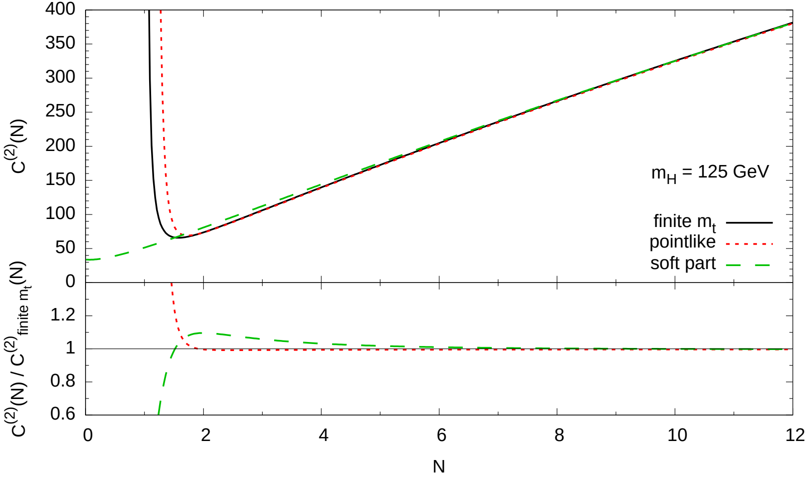

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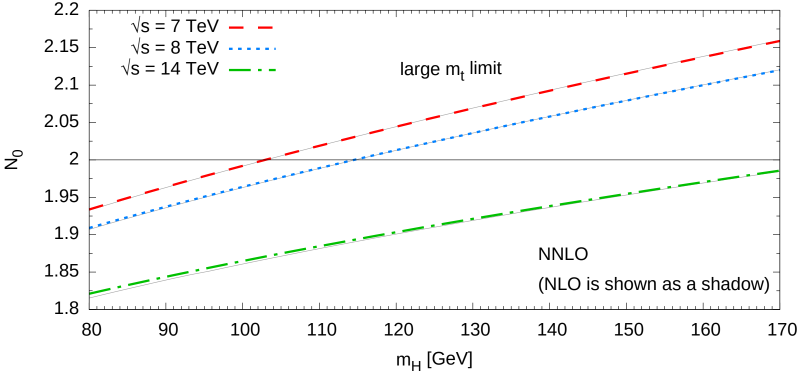
Based on all-order considerations, we can argue that
soft terms dominate for $N \gtrsim 2$

(and, a fortiori, the pointlike approx has to be good at least in the same region)

Example of conclusions one may draw



Example of conclusions one may draw

Assuming $m_H = 125$ GeV

- $N_0 \simeq 2.06$ for LHC 7 TeV
- $N_0 \simeq 2.03$ for LHC 8 TeV
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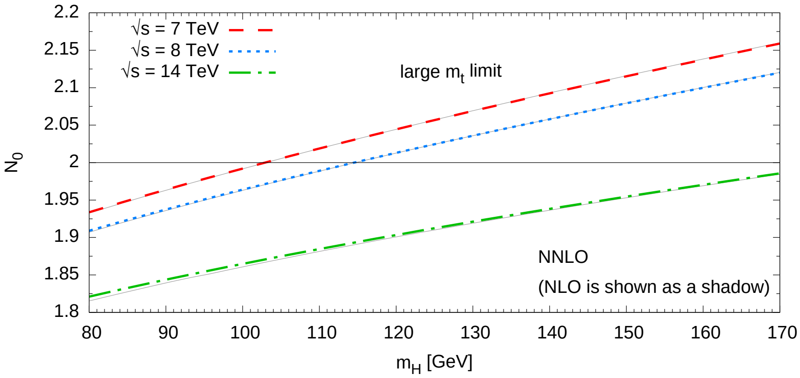

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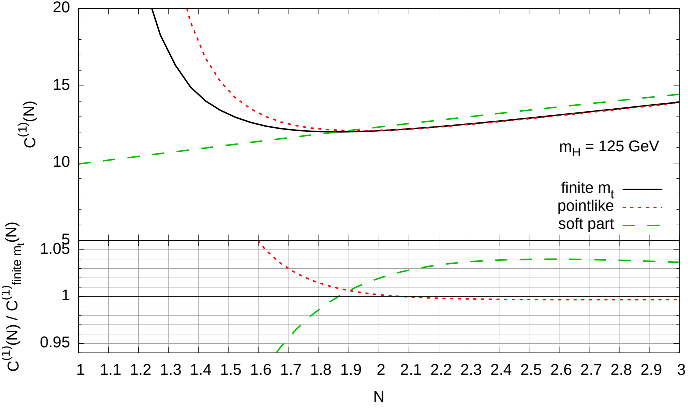

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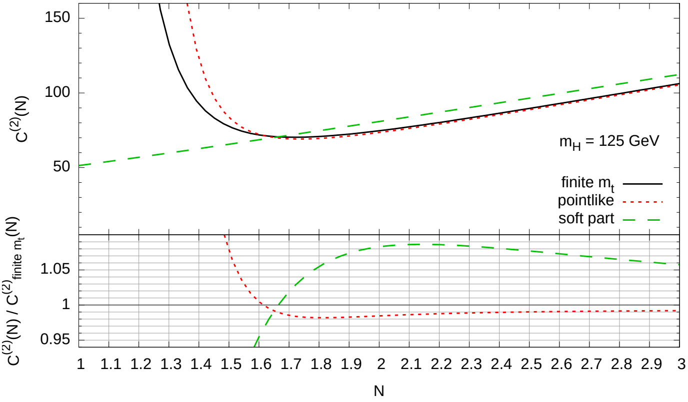

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The loss-of-perturbativity condition is $\alpha_s \log^2 N \gtrsim 1$ which gives $N \gtrsim 10$

The saddle point is never so large! $\Rightarrow$ Resummation never needed.

Soft-gluon resummation can just be used as a (useful!) tool to predict higher order terms.

Predictive power of our method

- We have a solid method to establish the dominant partonic region (saddle point)
- The saddle point is independent on the perturbative order and on other details $\rightarrow$ property of the considered subprocess
- Based on all-order considerations, we can argue that the partonic region in which soft terms dominate is the region $N \gtrsim 2$
- Then, when $N_0 \gtrsim 2$, we can predict higher orders using soft terms coming from soft-gluon resummation

N^3LO prediction for Higgs production

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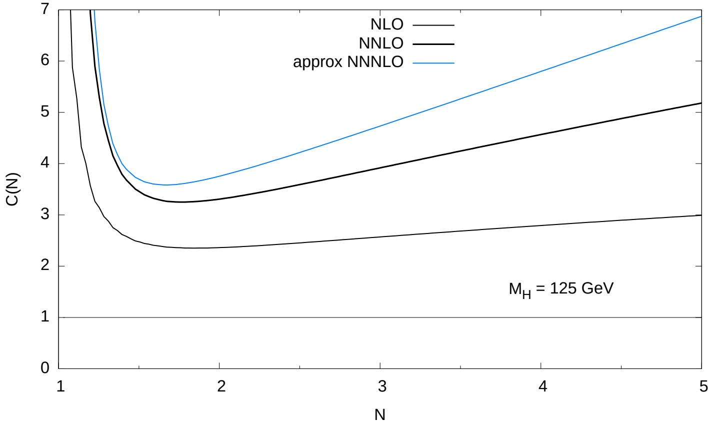

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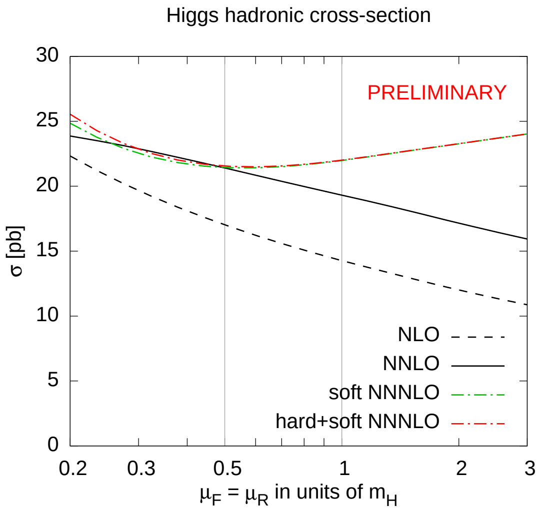

Conclusions

- The saddle point method is fast, efficient and it works!
- It is a tool to easily check the validity of the soft (and other) approximations
- Approximate prediction for Higgs cross-section at N^3LO

Backup slides

Discussion on what is SOFT — 1

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The last option allows to include all the terms $\alpha_s^n \log^{2n-1}(1-z)$ in the soft terms (equivalent to the collinear improvement of [Krämer, Laenen, Spira 1997], [Catani, de Florian, Grazzini 2001], [Catani, de Florian, Grazzini, Nason 2003])

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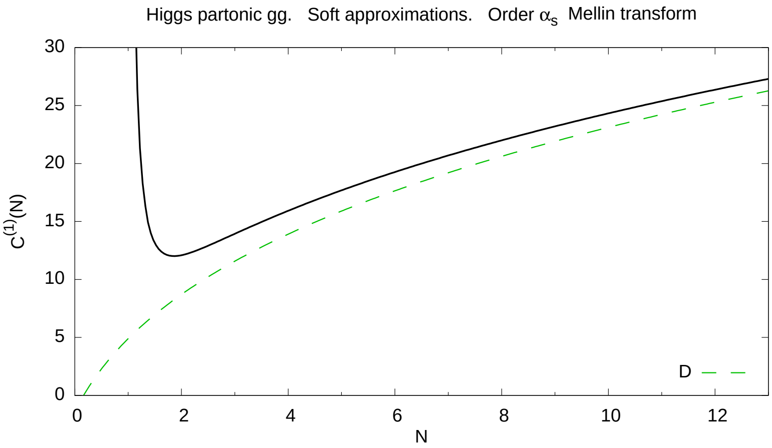

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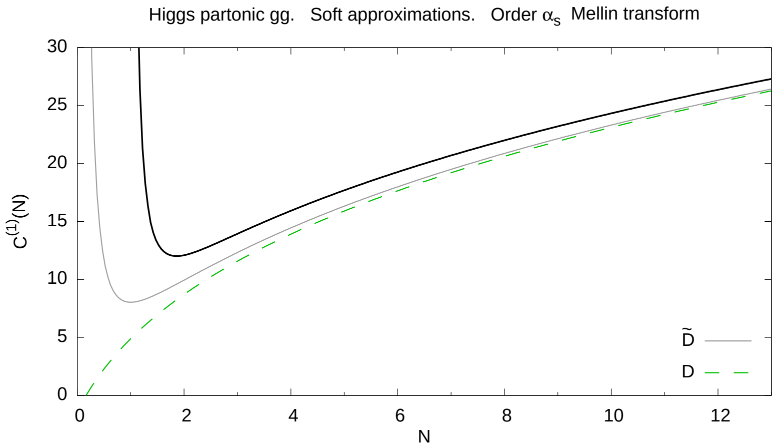

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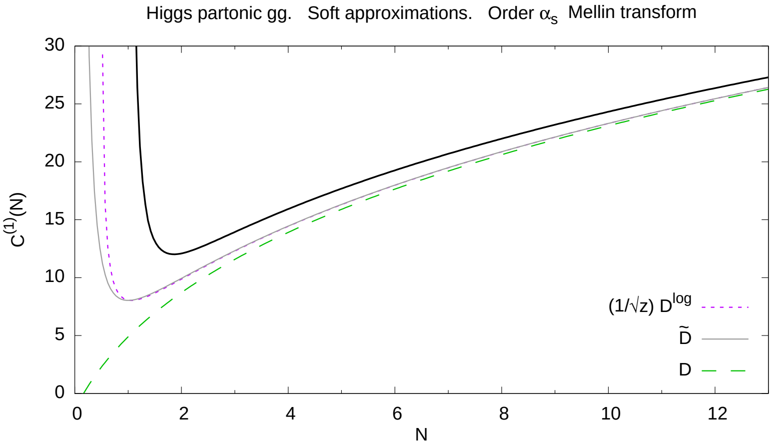

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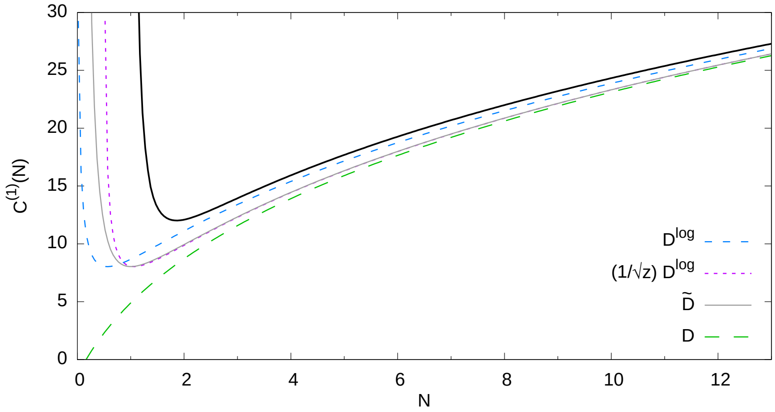
Higgs partonic gg. Soft approximations. Order α_s Mellin transform



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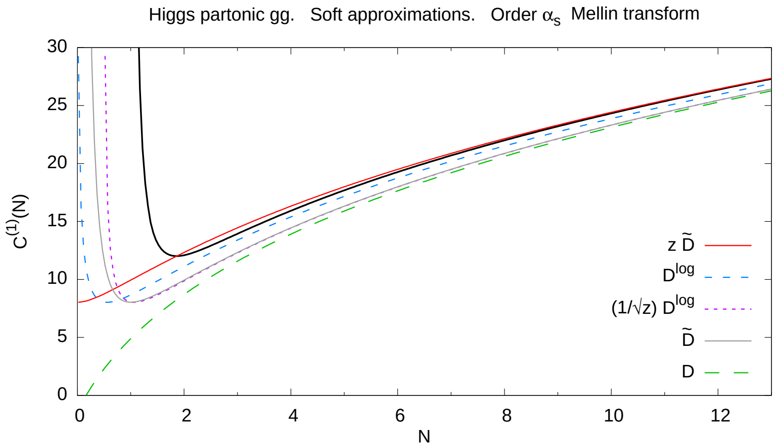

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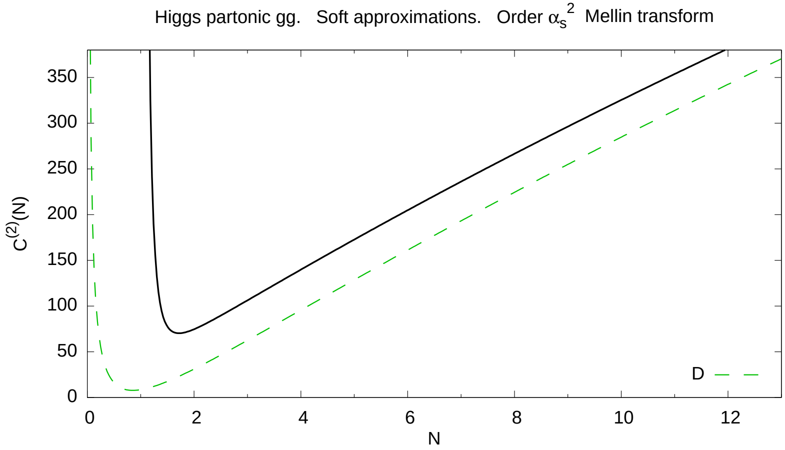

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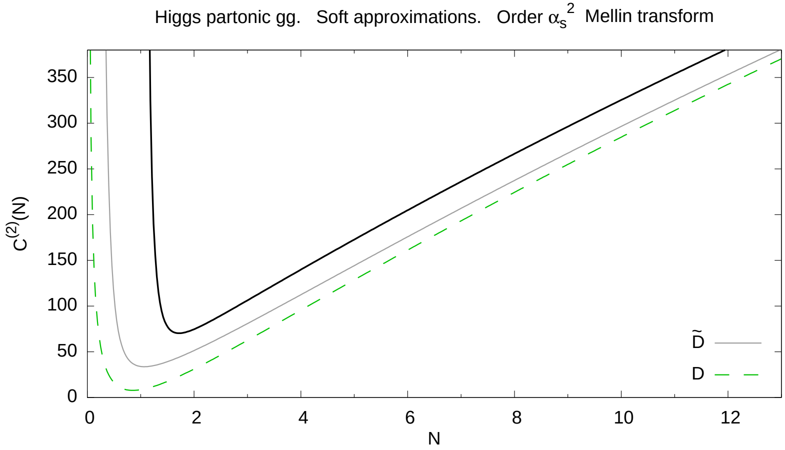
$$\mathcal{D}_k = \left[\frac{\log^k(1-z)}{1-z}\right]_+ \qquad \tilde{\mathcal{D}}_k = \left[\frac{\log^k \frac{1-z}{\sqrt{z}}}{1-z}\right]_+' \qquad \mathcal{D}_k^{\log} = \left[\frac{\log^k \log\frac{1}{z}}{\log\frac{1}{z}}\right]_+ + c_k\delta(1-z)$$



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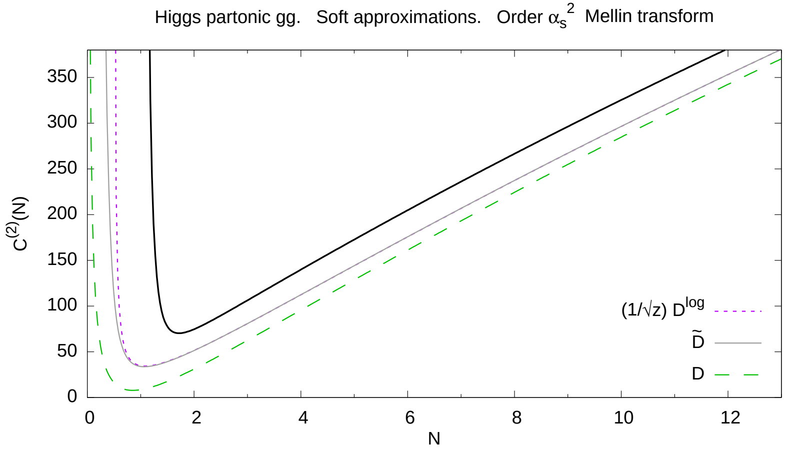

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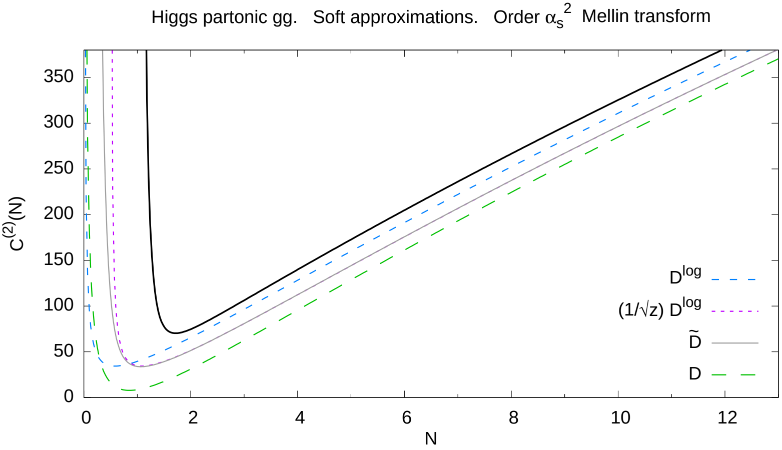

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