The Matrix Element Method at NLO

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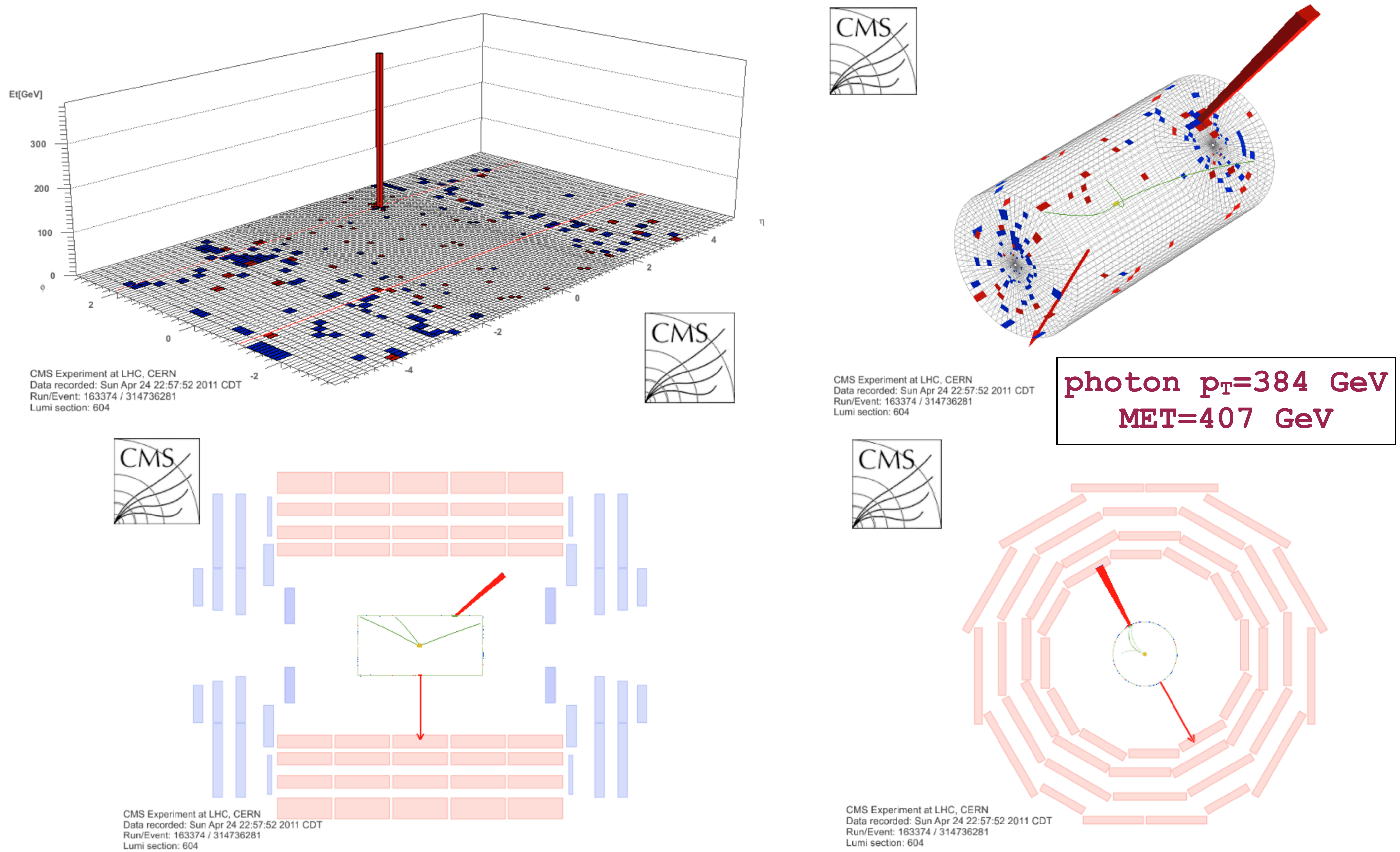
MEM: Motivation, hunting for signals



Suppose we see an interesting event at the LHC, how can we describe how likely it is to be from the SM or something new?

Introducing the Matrix Element Method.

- One experimental method is to assign events weights based upon the assumption that they arose from a given theory model.

$$P(\mathbf{x}|\Omega) = \frac{1}{\sigma}\int dx_1\, dx_2 d\mathbf{y}\, \frac{f(x_1)f(x_2)}{x_1 x_2 s}|\mathcal{M}_\Omega(\mathbf{y})|^2 W(\mathbf{x},\mathbf{y})$$

- With the full data set we can calculate a weight for each event and for a variety of models (or parameter values) and calculate likelihoods

$$\mathcal{L}(\mathbf{x}|\Omega) = f(N) \prod_{i=1,N} \mathcal{P}(\mathbf{x_i}|\Omega).$$

- Maximising this likelihood yields the best fit value between data and theory.

Pros and cons of the method.

- Clean separation between theory and experimental inputs
- Utilizes full ME.
- Many potential applications.
- Ripe for parallelisation

- Computationally expensive
- Need for simplifications:
  - Transfer function form
  - LO ME elements

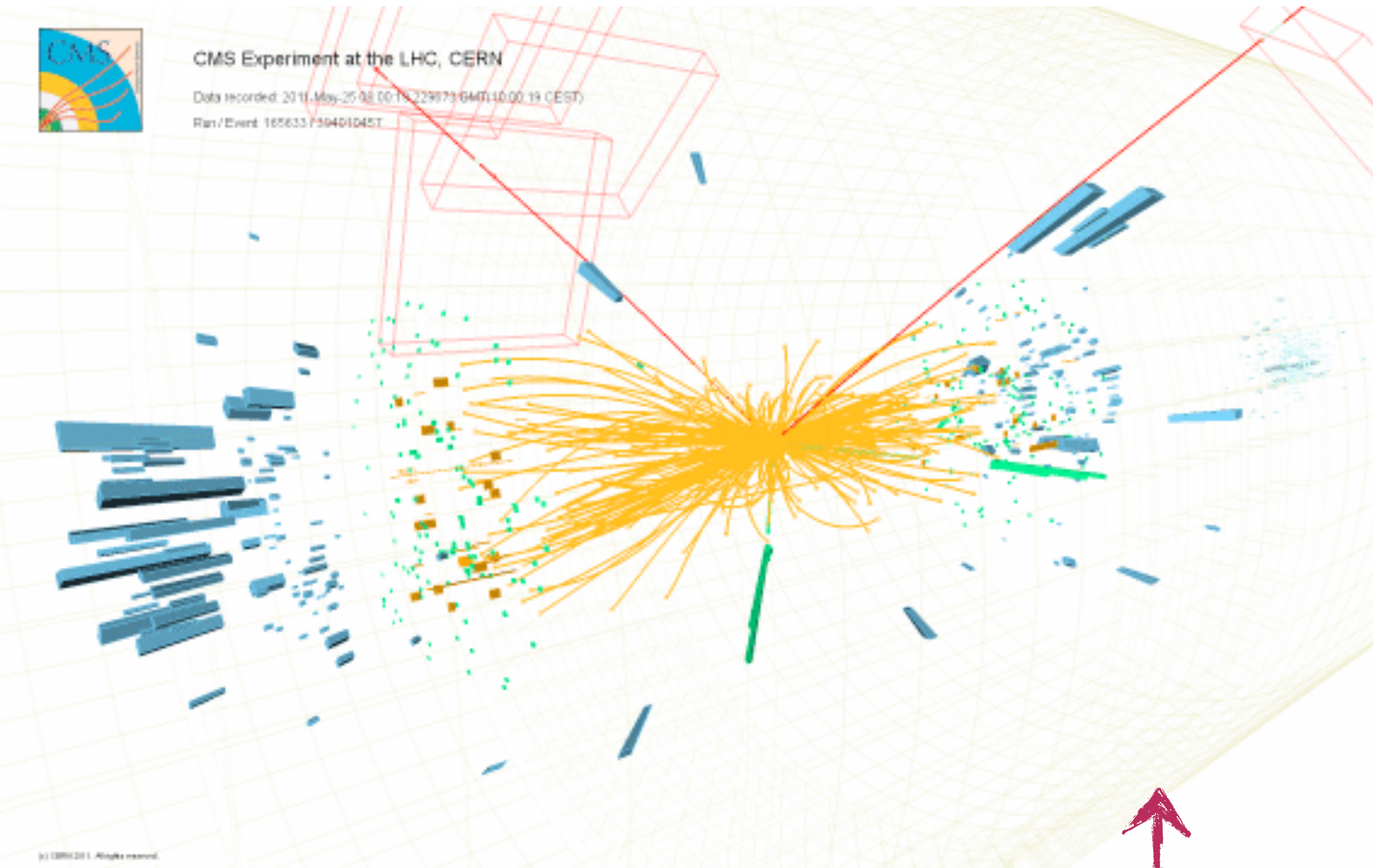
$$\mathcal{P}(\mathbf{x}|\Omega) = \frac{1}{\sigma_\Omega^{LO}} \int dx_a dx_b \, d\mathbf{y} \sum_{ij} \frac{f_i(x_a) f_j(x_b)}{x_a x_b s} \, \mathcal{B}_\Omega^{ij}(p_a, p_b, \mathbf{y}) W(\mathbf{x}, \mathbf{y}) \ .$$

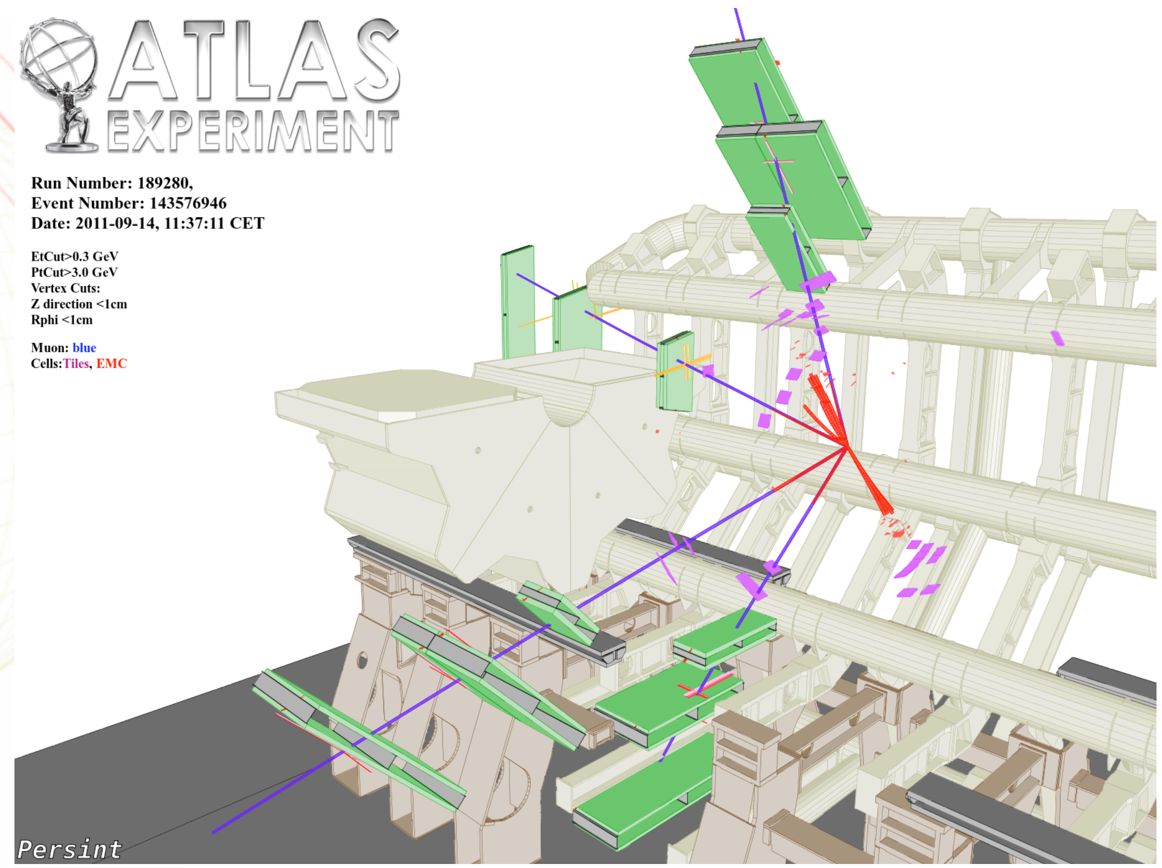
Theory input

Experimental input

The Matrix Element Method: LO definition.

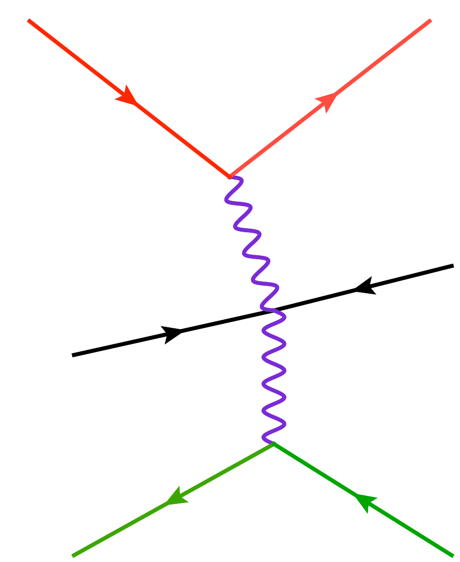
Experimental events versus fixed order weights.





Our aim is to turn this

Into this



Mapping Data to Born

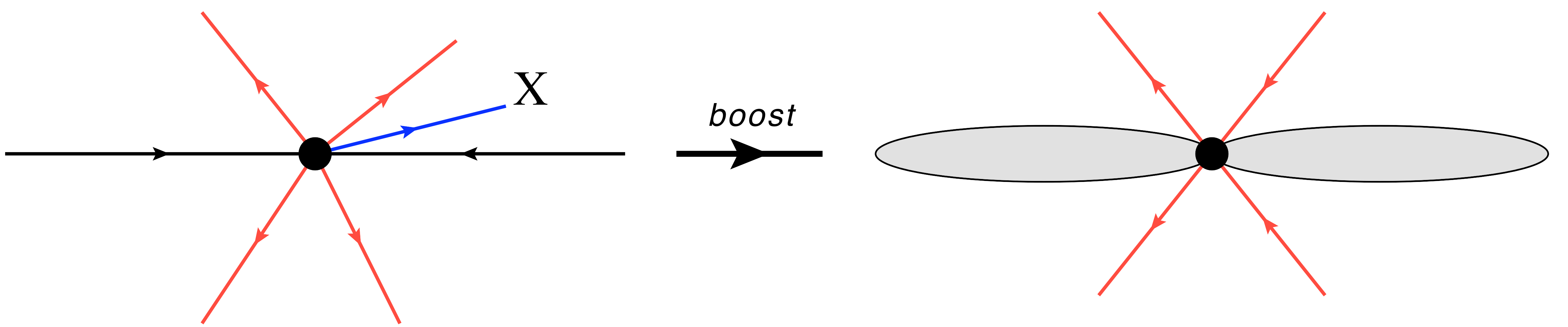
- Consider the production of an electorweak final state Q, on the experimental side this is measured as the desired final state, plus some addition recoil X.

$$Q + X$$

- We wish to model this as,

$$p_a + p_b \rightarrow Q$$

- One obvious mechanism to remove the excess recoil is to boost it into the initial state. I.e. we boost our final state Q such that its conserves momentum in the transverse plane. This has the obvious advantage of preserving all Lorentz invariants associated with our final state.



Getting to the MEM frame

- The only a priori requirement that our boost must fulfill is that it conserves transverse momentum

$$p_i^\mu = \Lambda^\mu{}_\nu(X)\,\tilde{p}_i^\nu \qquad \text{with} \qquad \sum_{i=1}^{n} p_i^x = \sum_{i=1}^{n} p_i^y = 0 \;.$$

- This transformation is not unique, there is freedom in the definition of the longitudinal (p_z, E) components

- The longitudinal components specify the parton fractions,

$$x_a - x_b = \frac{2}{\sqrt{s}}\left(\sum_{i=1}^{n} p_i^z\right) \;, \qquad x_a + x_b = \frac{2}{\sqrt{s}}\left(\sum_{i=1}^{n} E_i\right)$$

- So in other words, our boosts do not fix xa and xb uniquely only the product.

$$\delta(x_a x_b s - Q^2)$$

The LO MEM

- We define the “boost function” as,

$$\mathcal{L}_{ij}(s_{ab}, x_l, x_u) = \int dx_a dx_b \, \frac{f_i(x_a) f_j(x_b)}{x_a x_b s} \, \delta(x_a x_b s - s_{ab})$$
$$= \int_{x_l}^{x_u} dx_a \, \frac{f_i(x_a) f_j(s_{ab}/(s x_a))}{s x_a s_{ab}} \ ,$$

- Then our (LO) weight is defined as

$$\mathcal{P}(\mathbf{x}|\Omega) = \frac{1}{\sigma_\Omega^{LO}} \mathcal{L}_{ij}(s_{ab}, x_l, x_u) \mathcal{B}_\Omega^{ij}(p_a, p_b, \mathbf{x}) \ .$$

- We now have a mechanism for defining LO probability distributions. The two remaining caveats are : implementation of cuts and the definition of the bounds of integration on x.....

The complication of cuts!

- Cross sections and events are defined in the lab frame, we want to perform our calculation in the MEM frame => Need a map for fiducial cuts.

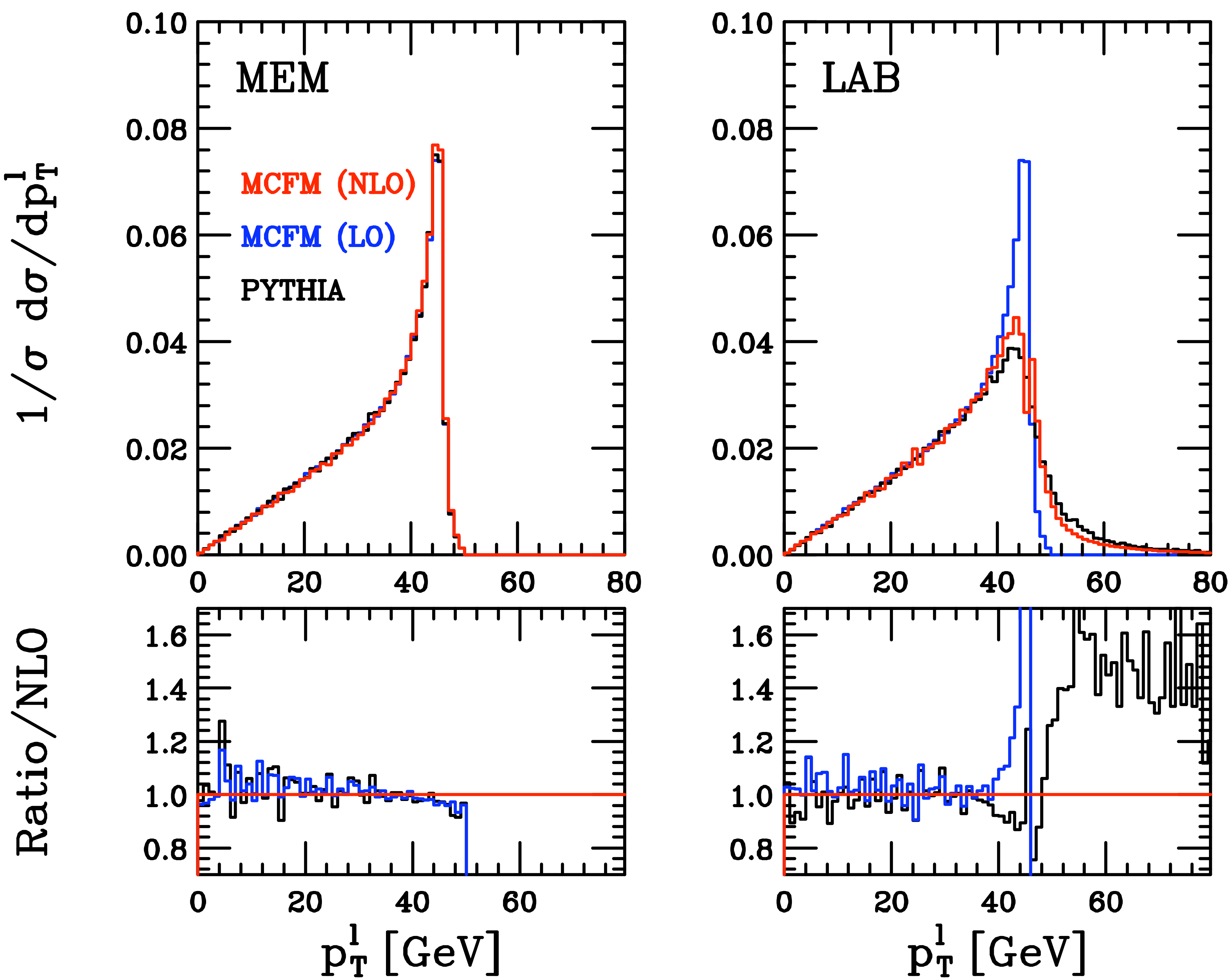
- This is defined by

$$p_T^{lab,i} = \sqrt{\frac{s_{ai}s_{ib}}{s_{ab}}}\,, \quad \eta^{lab,i} = \frac{1}{2}\log\left(\frac{x_a^2 s}{s_{ab}}\frac{s_{ib}}{s_{ai}}\right).$$

- Note pT is defined in terms of invariants, rapidity is boost dependent. In fact cuts on rapidity actually fix the upper and lower bounds on the boost integration,

$$\mathcal{L}_{ij}(s_{ab}, x_l, x_u) = \int_{x_l}^{x_u} dx_a \, \frac{f_i(x_a) f_j(s_{ab}/(s x_a))}{s x_a s_{ab}}$$

Inside the MEM frame.



Basic example: Z=>2l

Compare the shapes of the pT distribution in the two frames for 3 different theory predictions.

Since the MEM frame naturally removes recoil, the three predictions become similar, NLO corrections are small and of order 10%.

The Matrix Element Method: NLO definition.

MEM at NLO

- Our goal is thus to define the NLO cross section in terms of a single identified Born final state.

$$\frac{d\,\sigma_{\Omega}^{NLO}(\mathbf{x})}{d\mathbf{x}} = R_{\Omega}(\mathbf{x}) + V_{\Omega}(\mathbf{x})\ .$$

- Where R and V represent the real and virtual pieces.

- The above can be used to define unique NLO weights for an exclusive event.

- To accomplish our goal we will use a Forward Branching Phase Space (FBPS) generator 1106.5045 (Giele, Stavenga, Winter), hep-ph/9302225 (Giele, Glover, Kosower).

- (Note how our definition above is similar to those found in NLO+PS definitions, but here we are focussing on just NLO).

Virtual Corrections

- As usual the virtual pieces contain two types of term, a pure loop diagram and integrated subtraction terms to cancel the singularities.

- $$V_\Omega(\mathbf{x}) = \mathcal{L}_{ij}(s_{ab}, x_l, x_u)\Big(\mathcal{B}^{ij}_\Omega(p_a, p_b, \mathbf{x}) + \mathcal{V}^{ij}_\Omega(p_a, p_b, \mathbf{x})\Big) + \sum_{m=0}^{2}\int dz \Big(\mathcal{D}_m(z, \mathbf{x}) \otimes \mathcal{L}_m(z, s_{ab}, x_l, x_u)\Big)_{ij} \mathcal{B}^{ij}_\Omega(p_a, p_b, \mathbf{x}).$$

- The second term here represents these subtractions and contains an additional integral over the convolution between the subtraction terms and the PDFs.

$$\mathcal{L}_0 = \mathcal{L}, \quad \mathcal{L}_1 = \int_{x_l}^{x_u} dx_a \frac{f_i(x_a/z) f_j(s_{ab}/(s x_a))}{z s x_a s_{ab}}, \quad \mathcal{L}_2 = \int_{x_l}^{x_u} dx_a \frac{f_i(x_a) f_j(s_{ab}/(z s x_a))}{z s x_a s_{ab}}.$$

The Forward Branching Phase Space Generator

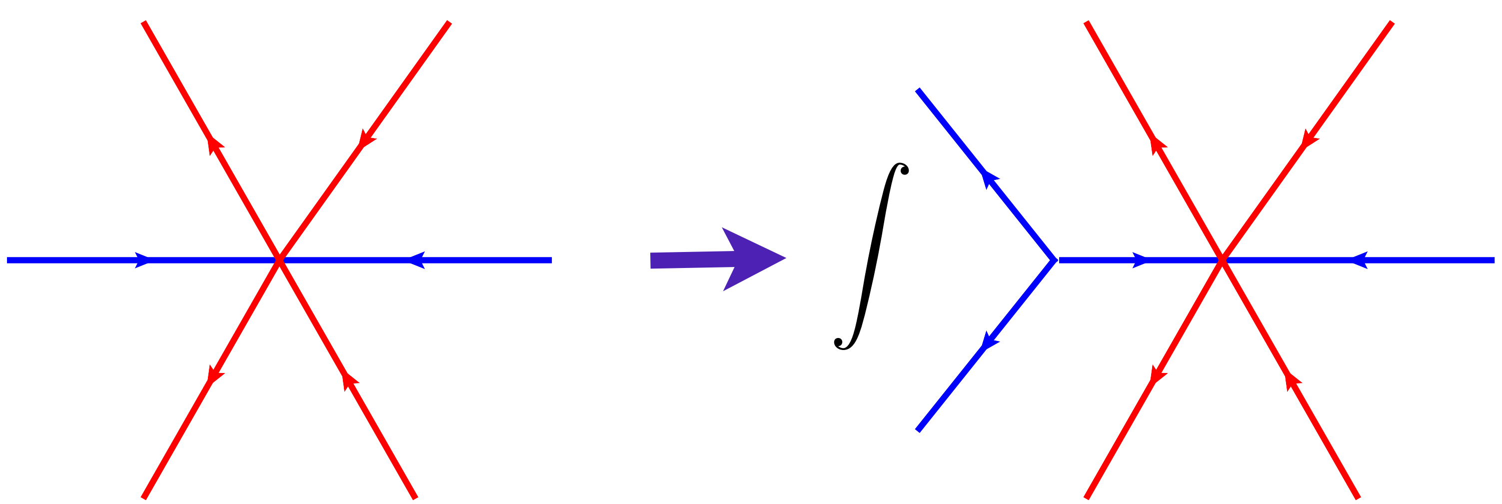
- Mathematically we need to factorize the real phase space into the following,

$$d\Phi(p_a + p_b \to Q + p_r) = d\,\Phi(\widehat{p}_a + \widehat{p}_b \to Q) \times d\,\Phi_{\mathrm{FBPS}}(p_a, p_b, p_r) \times \theta_{\mathrm{veto}}$$

- Then Q is identified with the observed final state, from this we derive the form of the FBPS integration

$$d\,\Phi_{\mathrm{FBPS}}(p_a, p_b, p_r) = \frac{1}{(2\pi)^3}\left(\frac{\widehat{s}_{ab}}{s_{ab}}\right) d\,t_{ar} d\,t_{rb} d\,\phi\ ,$$

- We then explicitly integrate out these quantities for each event.



Issues with dipoles

- MCFM, uses Catani-Seymour (hep-ph/9605323) dipoles. The idea is to map a real phase space point to a Born PS point multiplied by a Dipole function.

$$\mathcal{V}_{ijk}(p_a, p_b, p_r, Q) = D_{ijk}(p_a, p_b, p_r)|\mathcal{M}(\tilde{p}_a, \tilde{p}_b, \tilde{Q})|^2$$

- This occurs via transformation of the real PS point, involving an emitter, emitted and spectator particles (ijk). For the case when both emitter and spectator are initial state particles this transformation proceeds as follows,

$$\begin{aligned}
\widetilde{p}_{ar} &= x_{a,r}\, p_a \ , \\
x_{r,ab} &= \frac{s_{ab} + s_{ar} + s_{rb}}{s_{ab}} \ . \\
\tilde{p}_b &= p_b \\
\tilde{Q} &= \Lambda Q
\end{aligned}$$

- Where the LT on Q ensures momentum conservation

Dipoles cont......

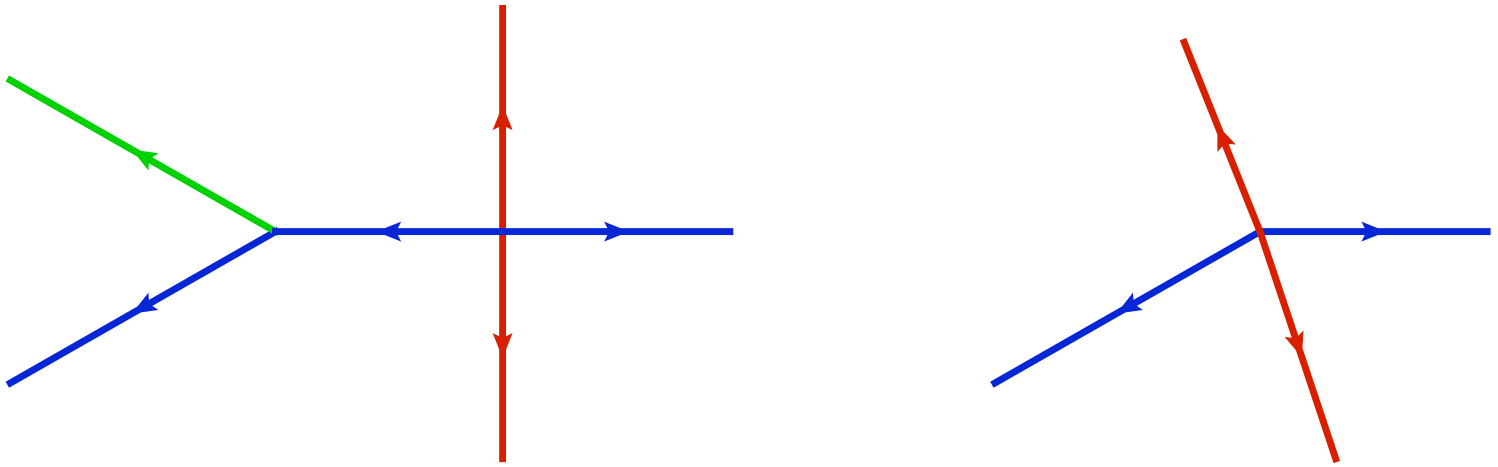
- This transformation is a disaster in the MEM frame, since it breaks our holy rule....

- *"Thou shalt not modify a final state phase space point once it has been rendered in the MEM frame."*

- A much more pious transformation is,

$$\widetilde{p}_{ar} = x_{a,r}\,\hat{p}_a \ ,$$
$$x_{r,ab} = \frac{s_{ab} + s_{ar} + s_{rb}}{s_{ab}} \ .$$

- We still need to do a LT, but now only in the longitudinal direction.



Real Corrections.

- We are now in a position to define our real corrections specified for a specific Born input.

$$R_\Omega(\mathbf{x}) = \int d\,\Phi_{\mathrm{FBPS}}(p_a, p_b, p_r)\Big(\mathcal{L}_{ij}(s_{ab}, x_l, x_u)\mathcal{R}^{ij}_\Omega(p_a, p_b, \mathbf{x}, p_r) - \sum_m \mathcal{L}_{ij}(s_{ab}, x_l^m, x_u^m)D^m(p_a, p_b, p_r)\mathcal{B}^{ij}_\Omega(\hat{p}_a, \hat{p}_b, \mathbf{x})\Big).$$

- The take home message being that we use the FBPS to integrate out allowed partonic emissions (up to some optional jet-veto definition). To ensure finiteness we use dipole subtractions to remove the soft+collinear divergences, and we had to slightly modify the normal CS dipoles to do this.

- Other than that its not too dissimilar to a regular NLO calculation!!

The MEM at NLO.

- We now have everything we need to define the MEM at NLO,

$$\mathcal{P}(\mathbf{x}|\Omega) = \frac{1}{\sigma_{\Omega}^{NLO}}\left(V_{\Omega}(\mathbf{x}) + R_{\Omega}(\mathbf{x})\right)$$

- Note that the real and virtual are both defined for the observed Born topology x.

- This method is not adding events x+jet into the MEM as some might imagine the NLO MEM should do. If there is a jet in the final state then the Born is x+jet not x!!

- We have implemented this into a new code NLOME based upon MCFM (Campbell, Ellis, CW).

Example H=>ZZ=>4l

- Use SHERPA to generate 4 lepton events with NLO+PS.

- Define

$$\mathcal{L}_{S+B}(\mu, N) = \frac{e^{-\mu}\mu^N}{N!}\prod_{i=1}^{N}\mathcal{P}(\mathbf{x_i}|S = m_H)\ ,$$

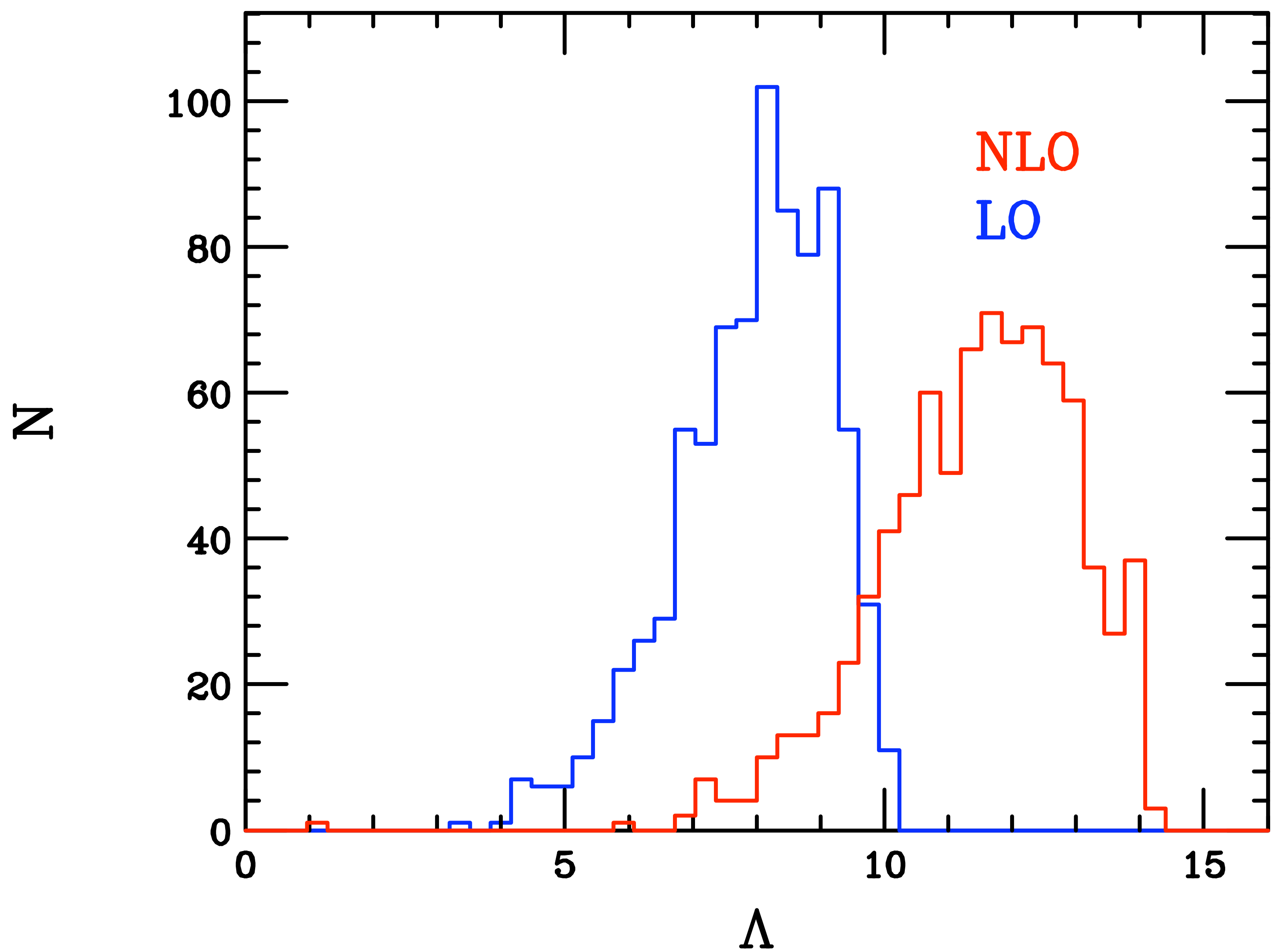
$$\mathcal{P}^{LO}(\mathbf{x}_i|S = m_H) = \frac{1}{(\sigma_S^{LO} + \sigma_B^{LO})}\Big(B_S(\mathbf{x}_i) + B_B(\mathbf{x}_i)\Big)\ ,$$

$$\mathcal{P}^{NLO}(\mathbf{x}_i|S = m_H) = \frac{1}{\left(\sigma_S^{NLO} + \sigma_B^{NLO}\right)}\Big(V_S(\mathbf{x}_i) + V_B(\mathbf{x}_i) + R_S(\mathbf{x}_i) + R_B(\mathbf{x}_i)\Big)\ .$$

$$\Lambda = \log(\mathcal{L}_B/\mathcal{L}_{S+B})$$

- Here μ denotes the expected number of events (for a given signal +background hypothesis (with a fixed background expectation of 200) and N is the actual number of observed events.

Example H=>ZZ=>4l

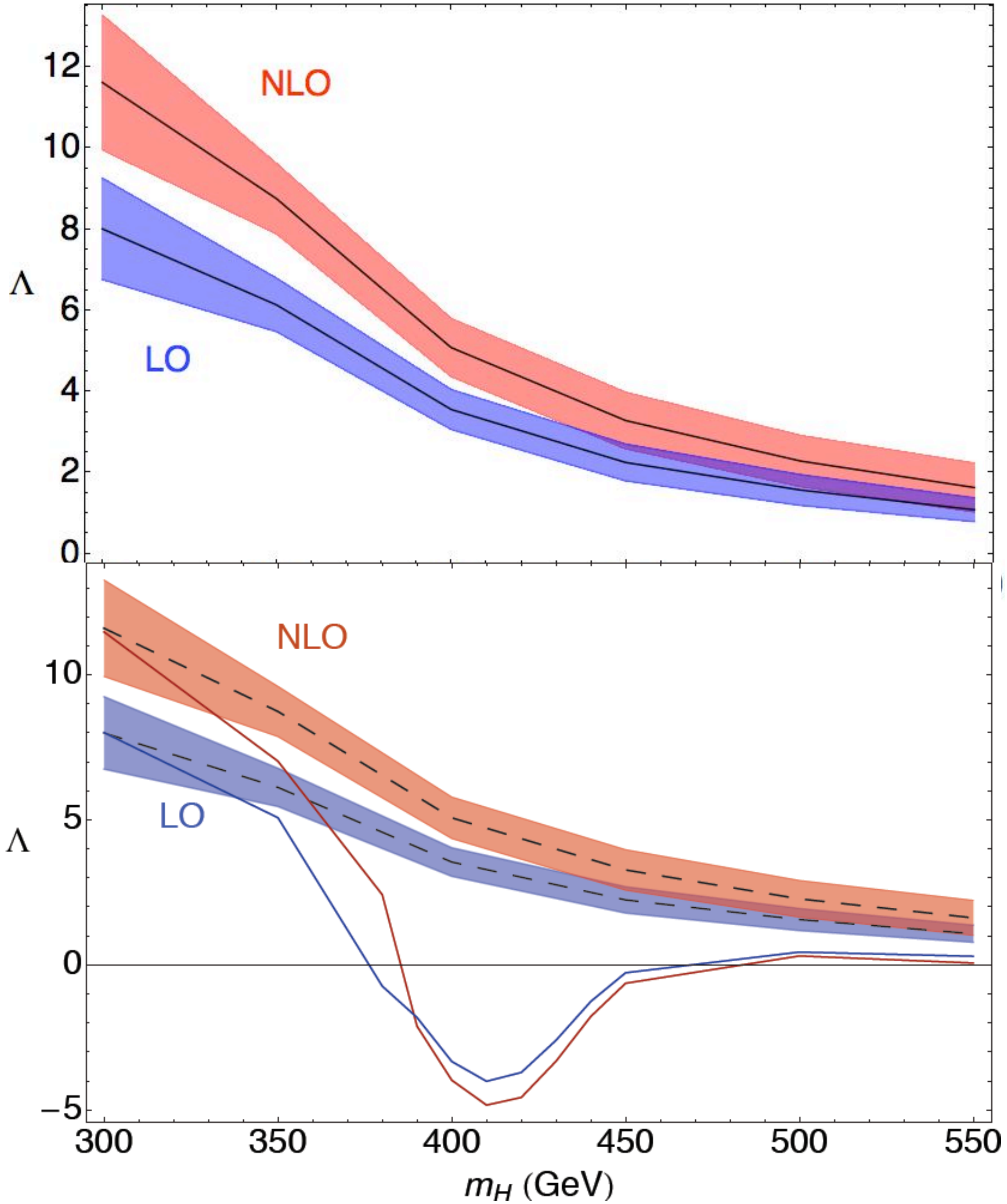


We generate pseudo experiments with no signal and proceed to set limits.

This plot shows results from around 1000 p-experiments at LO and NLO, for a hypothesis of mh=300 GeV.

Note in this example I'm going to keep mh > 300 to safely neglect experimental resolution effects.

Example H=>ZZ=>4l



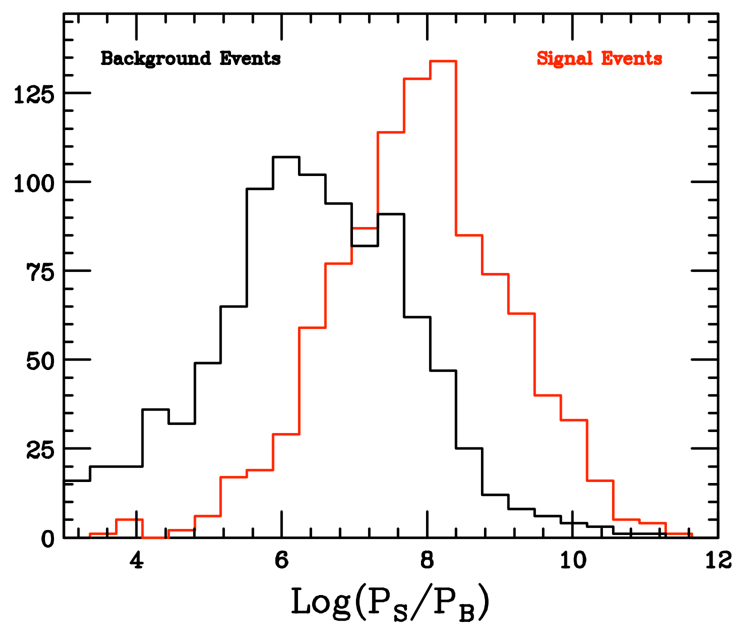
Calculate the expected limit from our p-experiments then the standard deviation to get measure of the spread.

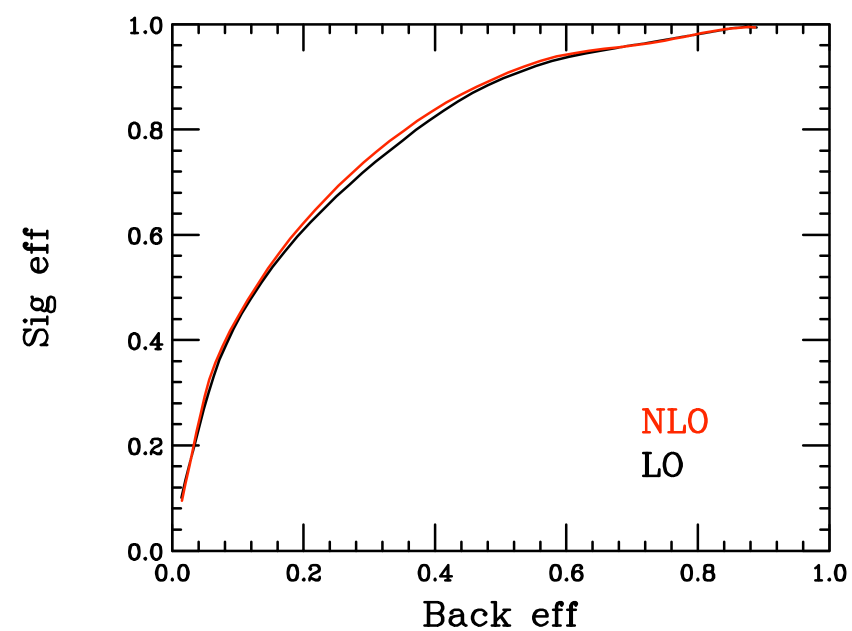
Top plot shows expected results in the presence of no Higgs. The bottom plot indicates an experiment with an injected signal at mh=425 GeV.

Future plans.

- Have currently implemented leptons, MET and photons plans to release a public code for EW studies, Higgs, anomalous couplings.

- Idea is to use these weights as kinematic discriminates for general processes (cf. Gao, Gritsan, Guo, Melnikov, Schulze, Tran and De Rujula, Lykken, Pierini, Rogan, Spiropulu) for ZZ=4l applications. Shown below is the code with full detector effects included for mh=125 and background events with 120 < m4l < 130 GeV.





- Beyond that we would like to include jets although this will require more serious alterations to MCFM ......

Conclusions.

- We developed an extension of the Matrix Element Method which is accurate to NLO.
- We work in a special frame in which experimental data is rendered into Born input and we calculate NLO corrections in this frame.
- We have tested the method on several EW processes including MET (although I didnt have time to discuss MET today).
- The method can separate signal like events from background like events based upon the kinematics in the MEM frame.
- The EW code should be available soon (ish), although some serious computer power is needed for events with large amounts of missing ET.........