Signatures of Gravitational Fixed Points at High Energy Colliders

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motivation

- **physics of classical gravity**

  Einstein's theory $G_N = 6.67 \times 10^{-11} \frac{m^3}{\text{kg}\, s^2}$

  $G_N \approx$ const. on length scales between $\sim 10^{-2} - 10^{28}\,\text{cm}$

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- **physics of quantum gravity**

  Planck length $\ell_{\mathrm{Pl}} = \left(\frac{\hbar G_N}{c^3}\right)^{1/2} \approx 10^{-33}\,\mathrm{cm}$

  expect large corrections to G_N at scales $\ll \ell_{\mathrm{Pl}}$

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- **low-scale quantum gravity**

  presence of n compact extra dimensions of size $\sim L$ imply

$$M_D^2 \sim M_{\rm Pl}(L\,M_D)^n$$

  allows for a fundamental Planck scale $M_D = \mathcal{O}$**(1TeV)** $\ll M_{\rm Pl}$

  potential signatures at high energy colliders

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  potential signatures at high energy colliders

- **renormalisation group for gravity**

  scale-dependence of gravitational couplings $G_N \to G_N(\mu)$

  non-trivial RG fixed point at short distances?

  asymptotic safety

renormalisation group scaling

- **RG scaling of gravitational coupling**

dimensionless coupling $$g(\mu) = Z_N(\mu)^{-1} \cdot G_N \cdot \mu^{D-2}$$

anomalous dimension $$\eta_N = -\frac{\mathrm{d}\ln Z_N}{\mathrm{d}\ln\mu}$$

RG running $$\frac{\mathrm{d}g}{\mathrm{d}\ln\mu} = (D - 2 + \eta_N)\, g$$

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- **fixed points**

  Gaussian: $g = 0$ **dominates GR**

  non-Gaussian: $\eta_N = 2 - D$ **may dominate deep UV (deep IR)**

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UV fixed point implies **weakly coupled gravity** at high energies

$$\mu \to \infty : \qquad G(\mu) \to g_* \mu^{2-D} \ll G_N$$

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  IR fixed point implies **strongly coupled gravity** at low energies

$$\mu \to 0 : \qquad G(\mu) \to g_* \mu^{2-D} \gg G_N$$

renormalisation group scaling

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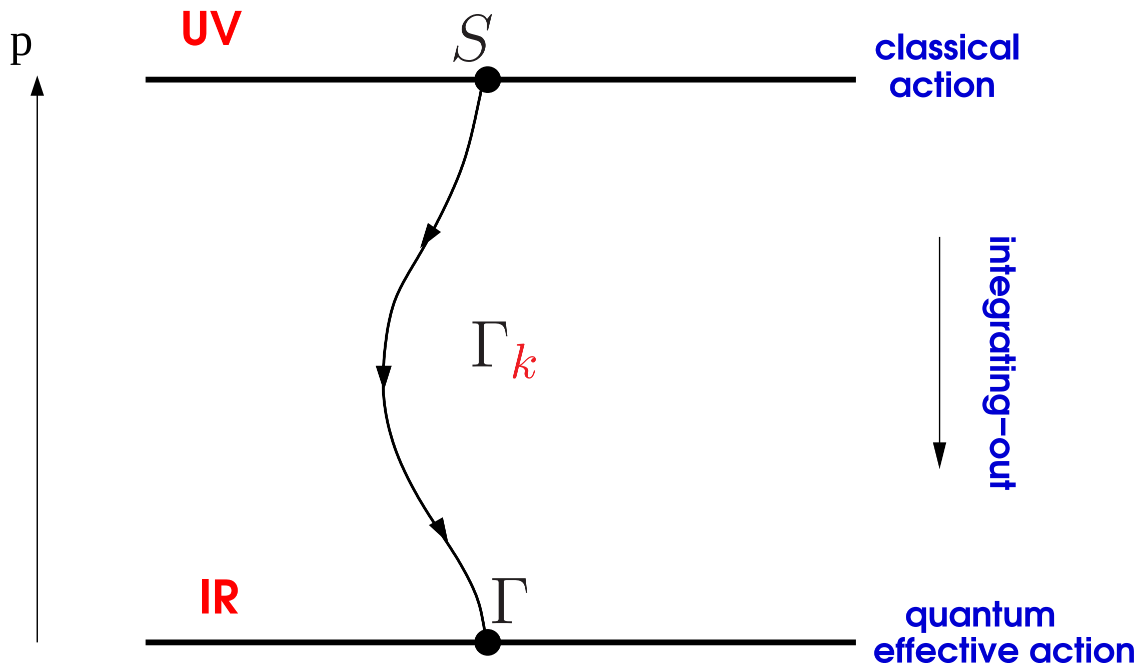
  non-Gaussian: $\eta_N = 2 - D$ **may dominate deep UV (deep IR)**

- **asymptotic safety of quantum gravity** (Weinberg '79)

  UV fixed point allows for non-perturbative definition of QG

Wilson's renormalisation group

- **integrating-out momentum degrees of freedom**



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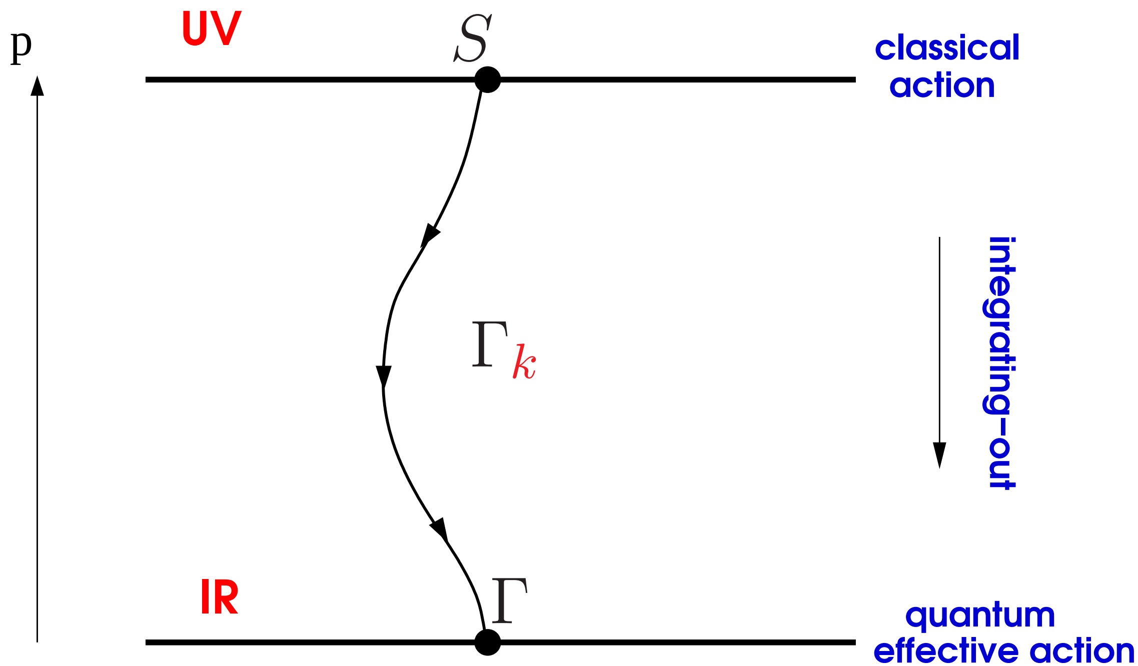
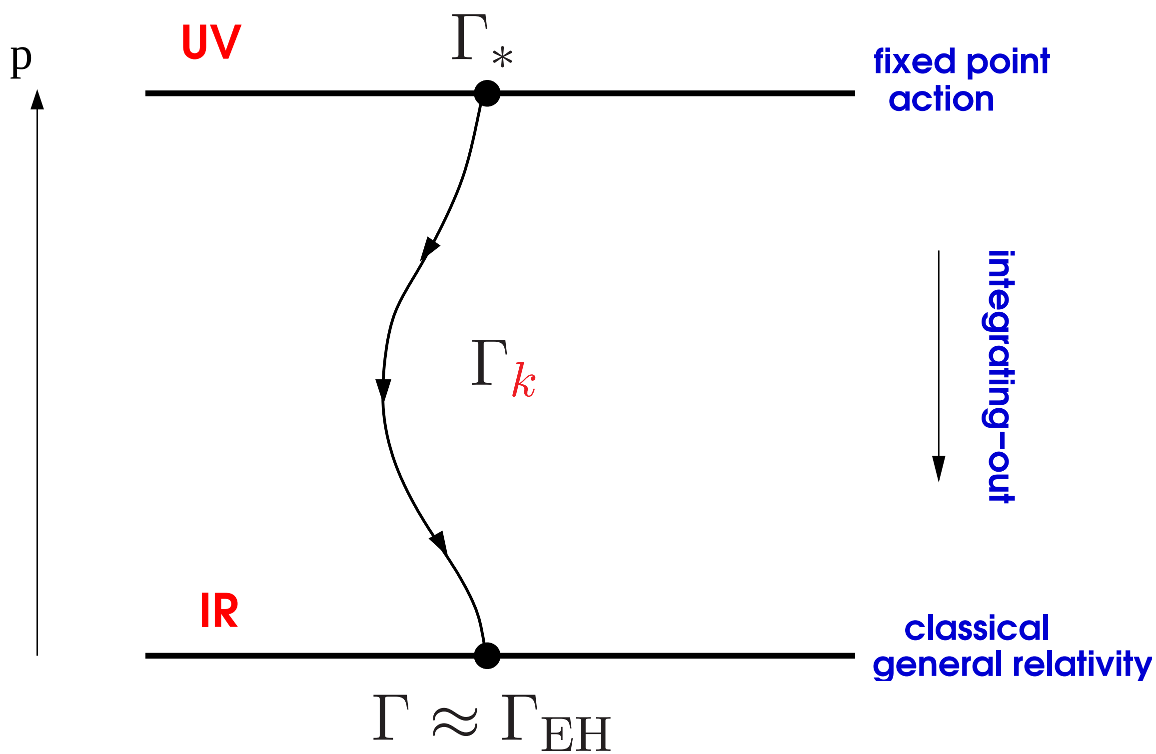


- **QCD: signatures of confinement** (Pawlowski, DL, Nedelko, Smekal '03)

Wilson's renormalisation group

- **for quantum gravity**



Wilson's renormalisation group

- **effective action**

$$\Gamma_k = \frac{1}{16\pi G_k} \int \sqrt{g}\,(\Lambda_k + R + \cdots) + S_{\mathrm{matter},k} + S_{\mathrm{gf},k} + S_{\mathrm{ghosts},k}$$

up to now: $\sqrt{g},\ \sqrt{g}R,\ \sqrt{g}R^2, \cdots,\ \sqrt{g}R^8$, **matter fields**

mainly four dimensions

Reuter (1996), Souma (1999), Lauscher, Reuter (2001), Reuter, Saueressig (2001), DL (2003), Percacci, Perini (2003), Bonnano, Reuter (2004), Bonanno (2005), Lauscher, Reuter (2005), Percacci (2005), Fischer, DL (2006)

Codello, Percacci (2006), Codello, Percacci, Rahmede (2007)

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- **wilsonian RG flow**

$$k\frac{\mathrm{d}}{\mathrm{d}k}\Gamma_k[g_{\mu\nu}] = \frac{1}{2}\,\mathrm{Tr}\,\left[\left(\Gamma_k^{(2)}[g_{\mu\nu}] + R_k\right)^{-1} k\frac{\mathrm{d}R_k}{\mathrm{d}k}\right]$$

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- **running couplings**

  projection of $k\partial_k\Gamma_k$ onto $\sqrt{g},\ \sqrt{g}R,\ \sqrt{g}R^2$, $\cdots$

Einstein-Hilbert theory

$$\beta_g = (D-2+\eta)\, g \qquad \eta = \frac{g\, b_1(\lambda)}{1+g\, b_2(\lambda)}$$

$$\beta_\lambda = (-2+\eta)\lambda + g(a_1 - \eta\, a_2) \qquad \lambda_k = \Lambda_k/k^2$$

$$a_1 = \frac{D(D-1)(D+2)}{2(1-2\lambda)} + \frac{D(D+2)}{1-2\alpha\lambda} - 2D(D+2)$$

$$a_2 = \frac{D(D-1)}{2(1-2\lambda)} + \frac{D}{1-2\alpha\lambda}$$

$$\begin{aligned} b_1 = & -\frac{1}{3}(1+\frac{2}{D})(D^3+6D+12) \\ & -\frac{(D+2)(D^3-4D^2+7D-8)}{(D-1)(1-2\lambda)^2} + \frac{D(D+2)(D^3-2D^2-11D-12)}{12(D-1)(1-2\lambda)} \\ & -\frac{2(D+2)(\alpha D^2-2\alpha D-D-1)}{D(1-2\alpha\lambda)^2} + \frac{(D+2)(D^2-6)}{6(1-2\alpha\lambda)} \end{aligned}$$

$$\begin{aligned} b_2 = & -\frac{D^3-4D^2+7D-8}{(D-1)(1-2\lambda)^2} + \frac{(D+2)(D^3-2D^2-11D-12)}{12(D-1)(1-2\lambda)} \\ & -\frac{2(\alpha D^2-2\alpha D-D-1)}{D(1-2\alpha\lambda)^2} + \frac{(D+2)(D^2-6)}{6D(1-2\alpha\lambda)} \end{aligned}$$

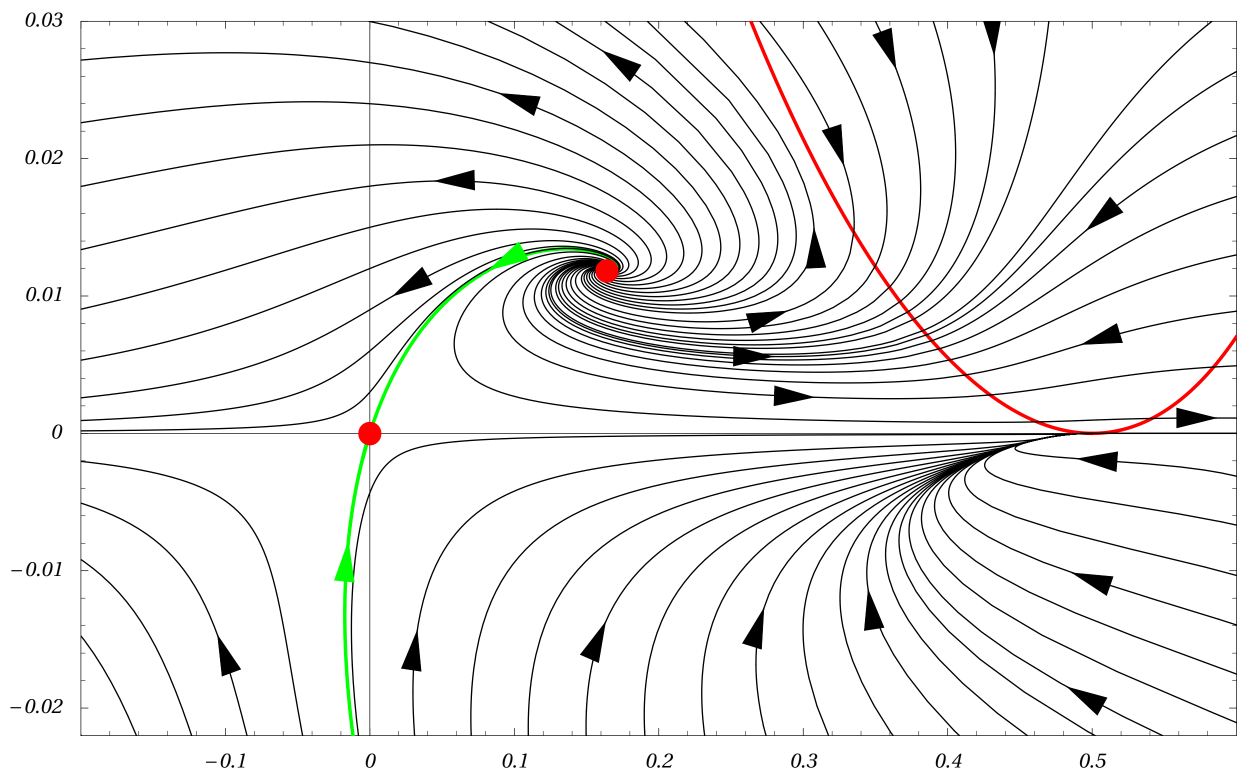
(DL '03)

fixed points and phase diagram

with Peter Fischer (U Aachen)
PLB (2006) hep-th/0602203, hep-th/0606135

- **phase diagram in four dimensions**

$$g = G_k\, k^2$$



$$\lambda = \Lambda_k / k^2$$

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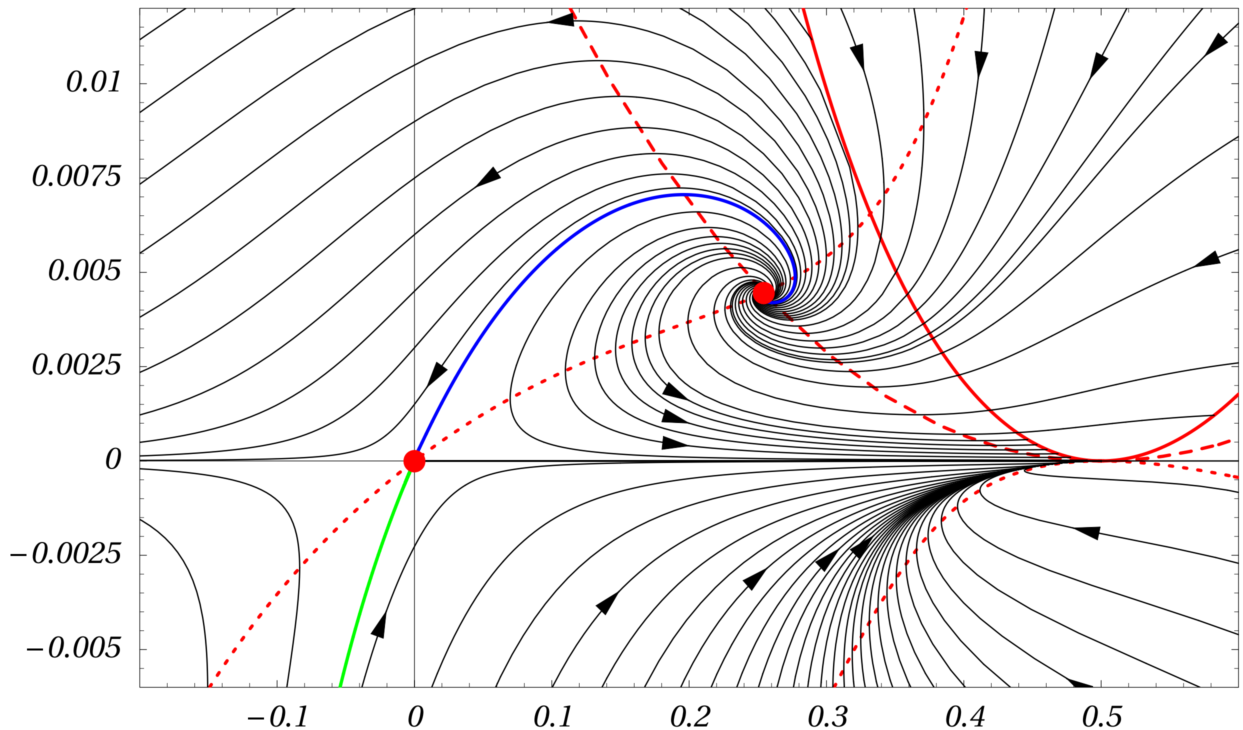
PLB (2006) hep-th/0602203, hep-th/0606135

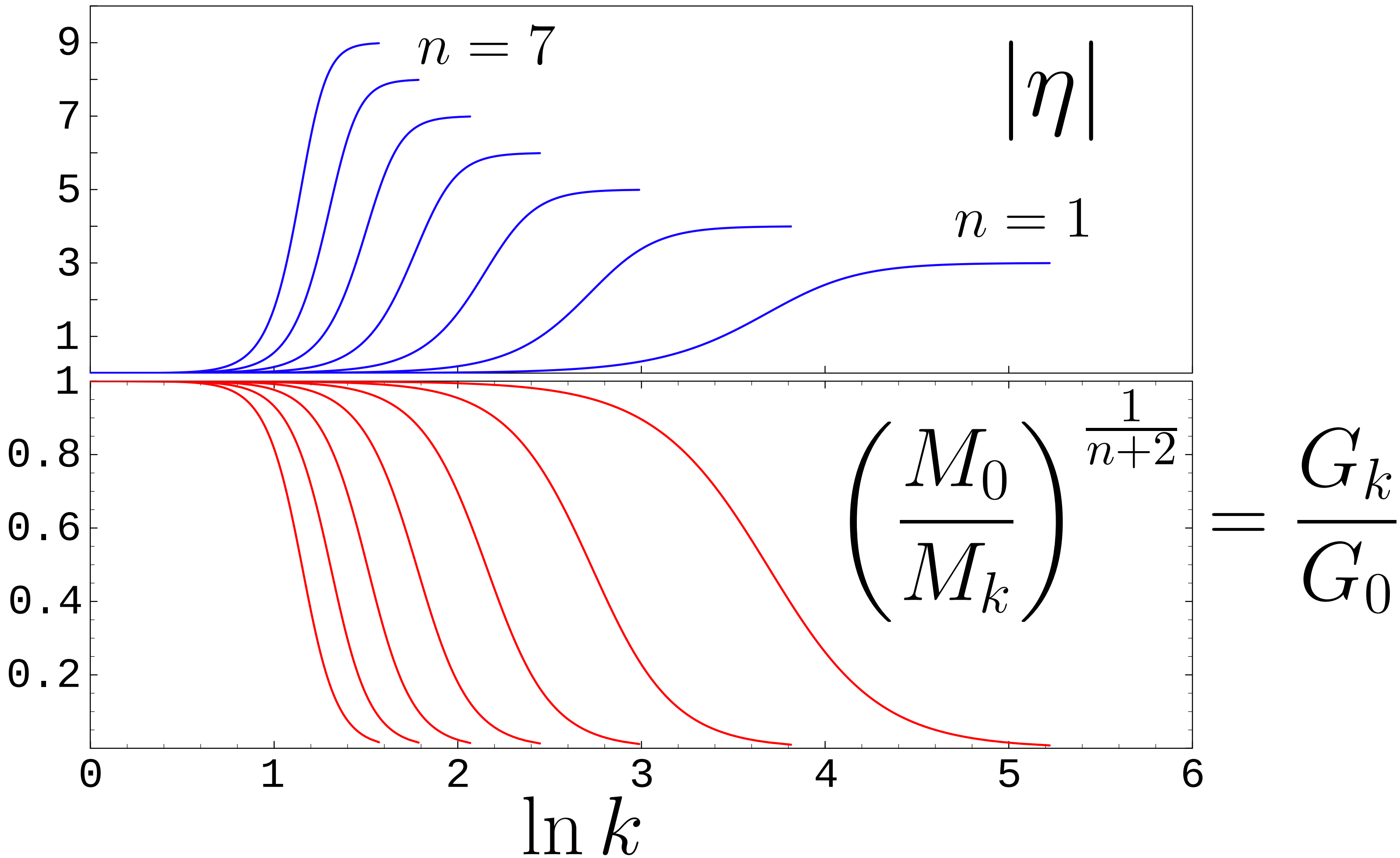
- **phase diagram in six dimensions**

$$g = G_k\, k^4$$



$$\lambda = \Lambda_k / k^2$$

running couplings and Planck scale



UV safe gravity in many dimensions

- **D-dimensional Einstein Hilbert theory**

  unique UV fixed point (DL ’03)

$$\lambda_* = \frac{D^2 - D - 4 - \sqrt{2D(D^2 - D - 4)}}{2(D-4)(D+1)}$$

$$g_* = \Gamma(\tfrac{D}{2}+2)(4\pi)^{D/2-1}\frac{(\sqrt{D^2 - D - 4} - \sqrt{2D})^2}{2(D-4)^2(D+1)^2}$$

  RG connected with perturbative infrared regime

  cutoff and gauge-fixing independence

- **signatures**

  vicinity of Planck scale: gravity becomes weak

collider signatures of quantum gravity

- **real gravitons**

  graviton production via **p p $\longrightarrow$ jet + G**

  signature: missing energy

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- **mini-black holes**

  black hole production and decay

  signature: many body final states

gravitational Drell-Yan with effective theory

- **effective theory** Giudice, Rattazzi, Wells ('98)

  effective dim-8 operator for Drell-Yan lepton production

$$\sim \frac{1}{M_D^{n+2}} \int_0^\infty \mathrm{dm_{kk}} \frac{\mathrm{m_{kk}}^{n-1}}{s + \mathrm{m_{kk}}^2}$$

  UV divergent for $n \geq 2$.

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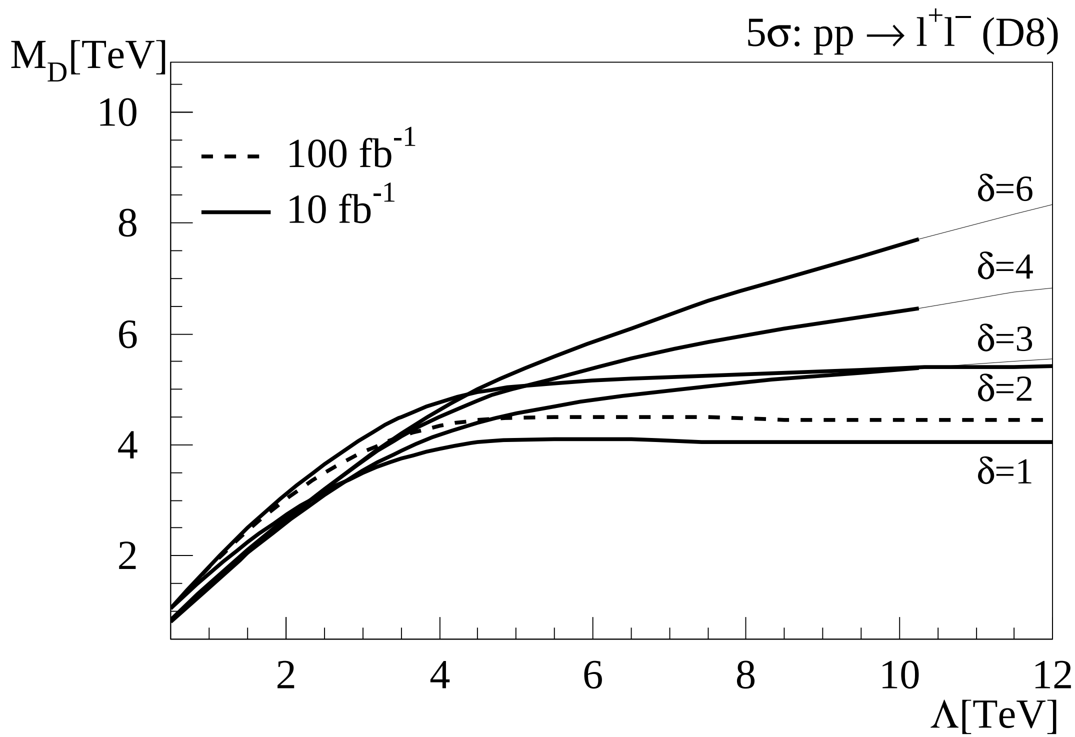
gravitational Drell-Yan with effective theory

- **effective theory + Monte Carlo simulations**

Giudice, Plehn, Strumia ('04)



gravitational Drell-Yan with UV fixed point

- **renormalisation group improvement** DL, Plehn ('07)

  effective dim-8 operator for Drell-Yan lepton production

$$\mathcal{S}_{\rm IR} = \frac{1}{M_D^{n+2}} \int_0^\Lambda {\rm dm} \frac{{\rm m}^{n-1}}{s + {\rm m}^2} \approx \frac{S_{n-1}}{n-2} \frac{1}{M_D^4} \left(\frac{\Lambda}{M_D} \right)^{n-2}$$

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$$\mathcal{S}_{\rm UV} = \frac{1}{M_D^{n+2}} \int_\Lambda^\infty {\rm dm} \frac{{\rm m}^{n-1} \Lambda^{n+2}}{(s + {\rm m}^2)^{n/2+2}} \approx \frac{S_{n-1}}{4} \frac{1}{M_D^4} \left(\frac{\Lambda}{M_D} \right)^{n-2}$$

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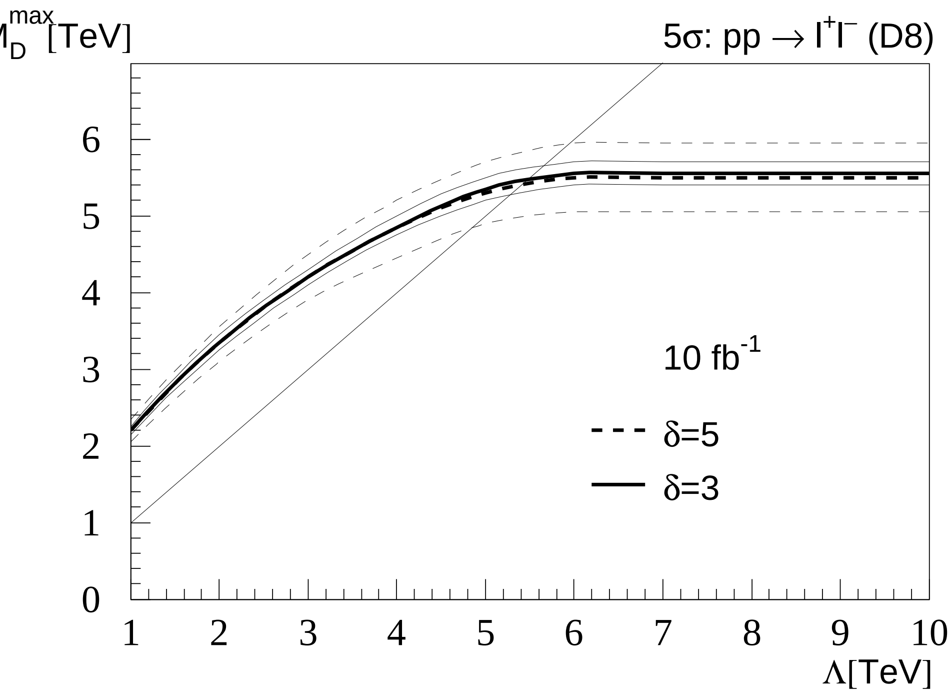
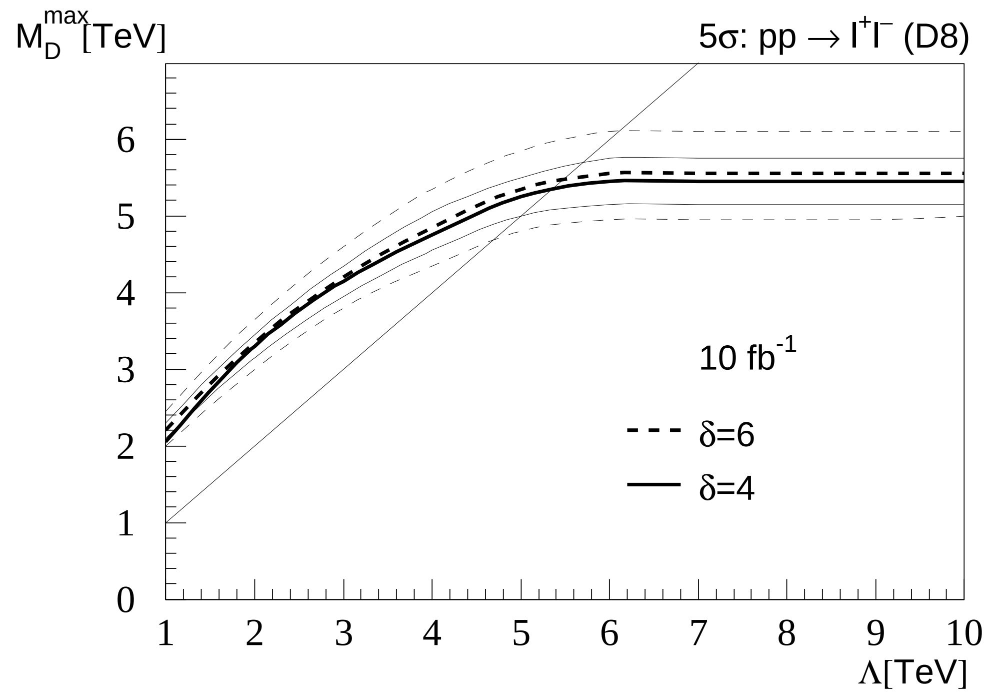
independently of Λ, we observe

$$\mathcal{S} = \mathcal{S}_{\rm IR} + \mathcal{S}_{\rm UV} = \left(1 + \tfrac{n-2}{4}\right) \mathcal{S}_{\rm IR}\,, \qquad \mathcal{S}_{\rm IR}/\mathcal{S}_{\rm UV} = 4/(n-2)$$

gravitational Drell-Yan with UV fixed point

- **renormalisation group + Monte Carlo simulation** DL, Plehn ('07)





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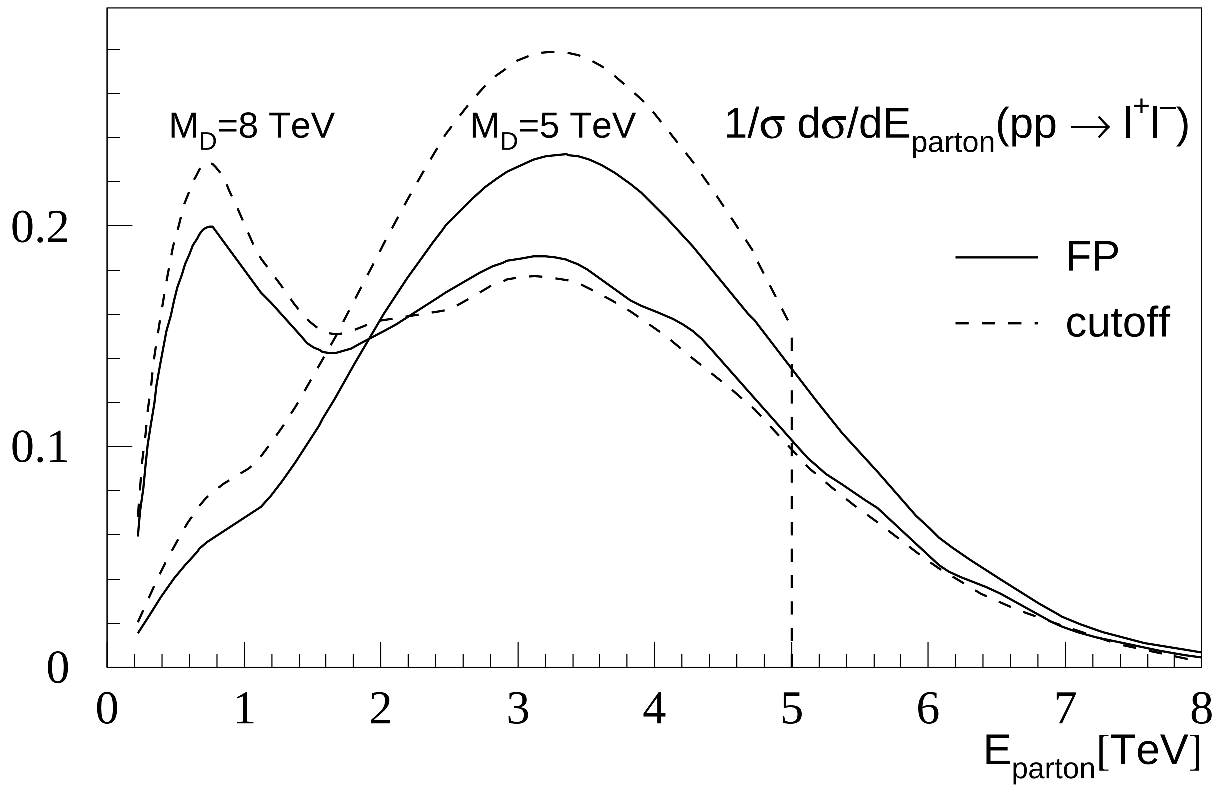

production cross sections: comparison

σ[fb]	$n = 3$		
M_D	2 TeV	5 TeV	8 TeV
$a)$	2270	1.41	0.0317
$b)$	408	1.24	0.0317
$b')$	407	1.23	0.031
$b'')$	270	1.13	0.0316
$c)$	173	0.72	0.0204
$d)$	129	0.55	0.0184

$a)$ fixed point scaling, leading term in s/M_D^2

$b)$ fixed point scaling, leading term in s/M_D^2 until $s = M_D^2$, large-s once $s > M_D^2$

$b')$ same as $b)$ without interference terms

$b'')$ fixed point scaling, sharp cutoff for large-s at $\Lambda_s = M_D$

$c)$ effective theory, leading term in s/M_D^2 with sharp cutoff at $\Lambda_{\rm kk} = \Lambda_s = M_D$

$d)$ leading term in s/M_D^2 with sharp cutoff at $\Lambda_{\rm kk} = M_D$ times form factor $F(s)$ from fixed point scaling

summary and outlook

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  gravity may be asymptotically safe!

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- **collider phenomenology**

  finite Drell-Yan production rates DL, Plehn (2007)

  many more observables in reach e.g. Hewett, Rizzo (2007), Koch (2007)

Workshop Announcement

Renormalisation Group and Effective Field Theories

Durham, IPPP, 27 - 29 Sep 2007

Topics include:

- **quantum gravity**
- **QCD at strong coupling**
- **fermionic theories, cold atoms, BEC**
- **dynamical phenomena**

For details, see **www.pact.cpes.sussex.ac.uk/ dl79/workshop07.html**

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