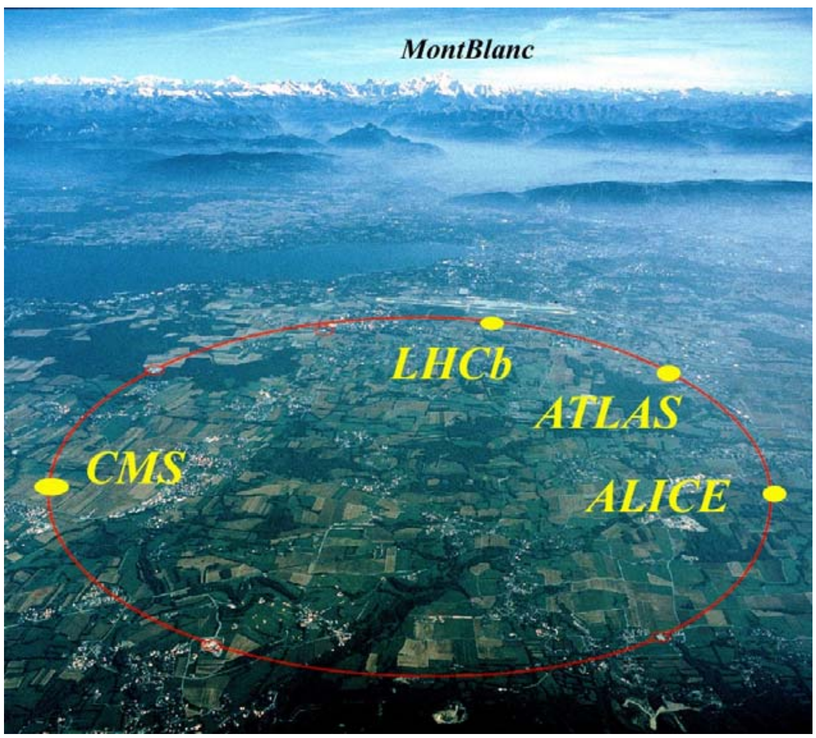
On the ubiquity of metastable vacua

Hirosi Ooguri (Caltech)



CERN Theory Institute

New Physics and the LHC

22 August 2007

Constructing models without SUSY vacua is hard.

The Witten index eliminates a large class of models.

Nelson-Seiberg theorem states that we need an exact U(1) R-symmetry.

$\because$)

Without a continuous R symmetry,
a generic superpotential always has extrema:

$$\frac{\partial W}{\partial \phi_i} = 0 \quad , \quad i = 1, \cdots, n$$

n equations, n variables

An example of model with SUSY vacua:

N=1 supersymmetric SU(Nc) gauge theory with Nf massive quarks (matter in fundamental reps).

$$\underbrace{\mathrm{rank}\,(m)}_{\text{mass matrix}} = N_F$$

$\Rightarrow$

This model has Nc vacua with unbroken supersymmetry:

$$\langle Q\tilde{Q}_{ij} \rangle = m^{-1}_{ij} \left(\det m \cdot \Lambda^{3N_c - N_F} \right)^{\frac{1}{N_c}}$$

Consistent with the Witten index computation

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Intriligator, Seiberg and Shih showed that
this model has, in addition, meta-stable vacua
(and therefore with broken supersymmetry)
when

$$N_c < N_F < \frac{3}{2} N_c \,.$$

This raises various questions:

-- How generic is this phenomenon?

-- Is it useful for model building?

-- Can we realize it in string theory?

-- How does the story change
when the gravity is turned on?

How generic is this phenomenon?

Perturbed Seiberg-Witten theories

Ookouchi, Park + H.O.
(0704.3613)

This is about N=1 theories obtained by perturbing N=2 theories with superpotentials.

Consider an arbitrary N=2 gauge theory.

Choose a generic point *p* on the Coulomb branch.

e.g. pure SU(2) theory

×	×	×
massless monopole	massless dyon	p

Perturbed Seiberg-Witten theories

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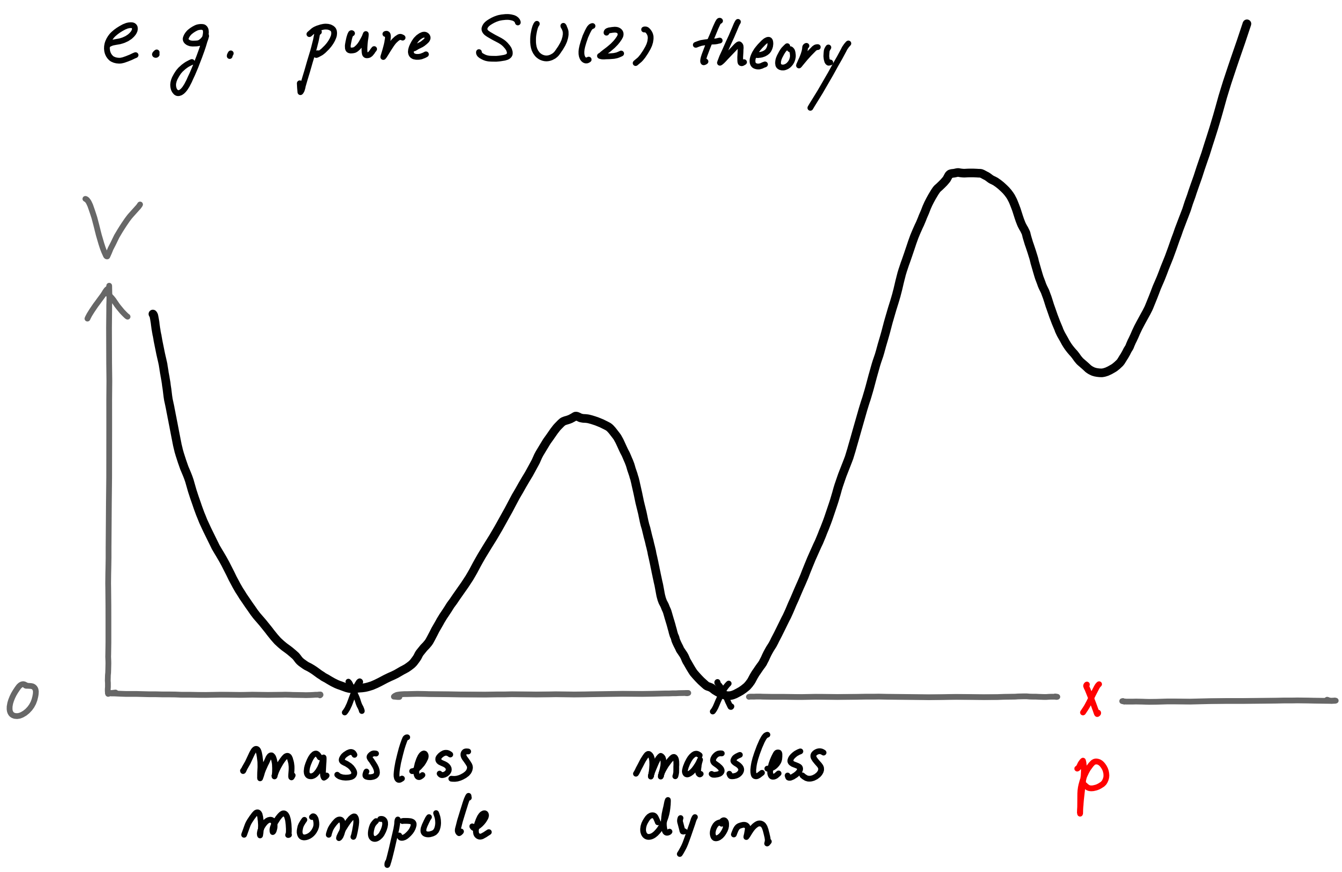
One can find a superpotential *W* which generates a meta-stable vacuum at *p*.



This follows from the positivity of the sectional curvature on the Coulomb branch.

In general, in geodesic coordinates:

$$g_{i\bar{j}}(z) = g_{i\bar{j}}(0) + R_{i\bar{j}k\bar{l}}\, z^k \bar{z}^{\bar{l}} + \cdots$$

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In general, in geodesic coordinates:

$$g_{i\bar{j}}(z) = g_{i\bar{j}}(0) + R_{i\bar{j}k\bar{l}}\, z^k \bar{z}^{\bar{l}} + \cdots$$

Choose $W = k_i z^i$, where $z^i(p) = 0$
and k_i is a constant vector.

$$V = g^{i\bar{j}}\, \partial_i W\, \bar{\partial}_{\bar{j}} \bar{W}$$

$$= g^{i\bar{j}}(0) k_i \bar{k}_{\bar{j}} + \underbrace{R^{i\bar{j}}{}_{k\bar{l}}\, k_i \bar{k}_{\bar{j}}}_{\text{positive definite}}\, z^k \bar{z}^{\bar{l}} + \cdots$$

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The positivity is a consequence of the rigid limit of special geometry:

$$g_{i\bar{j}} = \mathrm{Im}\, \partial_i \partial_j F$$

prepotential

Sectional curvature on the Coulomb branch is positive semi-definite:

$$g_{i\bar{j}} = \mathrm{Im}\, \partial_i \partial_j F$$

holomorphic

$$R_{i\bar{j}k\bar{l}} = \partial_k g_{i\bar{m}}\, \bar{\partial}_{\bar{l}}\, g_{\bar{j}n}\, g^{n\bar{m}}$$

using $\partial\bar{\partial} g_{i\bar{j}} = 0$

$$R_{i\bar{j}k\bar{l}}\; V^i \bar{V}^{\bar{j}} U^k \bar{U}^{\bar{l}}$$

$$= \left(V^i U^k \partial_k g_{i\bar{m}}\right) \left(\bar{V}^{\bar{j}} \bar{U}^{\bar{l}} \partial_{\bar{l}}\, g_{\bar{j}n}\right) g^{n\bar{m}}$$

$$\geq 0$$

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If we turn on gravity, the sectional curvature for vector multiplets in N=2 supergravity is not positive definite.

With gravity:

$$V = e^{K} \left(g^{i\bar{j}} D_i W \bar{D}_{\bar{j}} \bar{W} - 3 |W|^2 \right)$$

Denef and Douglas (hep-th/0404116) have shown that:

For flux compactification with one modulus, there are <u>no meta-stable de Sitter vacua</u> in the large complex structure region.

Note that the curvature of the moduli space is negative in this region.

Surprizingly, they also found <u>no meta-stable de Sitter vacua</u> even in the conifold region, where the curvature turns positive.

Quiver Gauge Theories

Kawano, Ookouchi + H.O.
(0704.1085)

Any quiver gauge theory with adjustable superpotentials for the adjoint fields and masses for the bifundamental fields has meta-stable vacua in some range of its parameters.

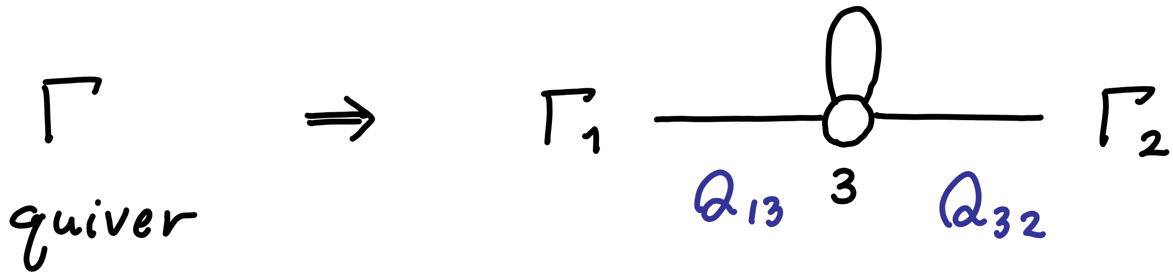
Many of these gauge theories have geometric realizations in string theory.

Douglas, Moore
(hep-th/9603167)

......

Cachzo, Katz, Vafa
(hep-th/0108120)

Consider the following situation:



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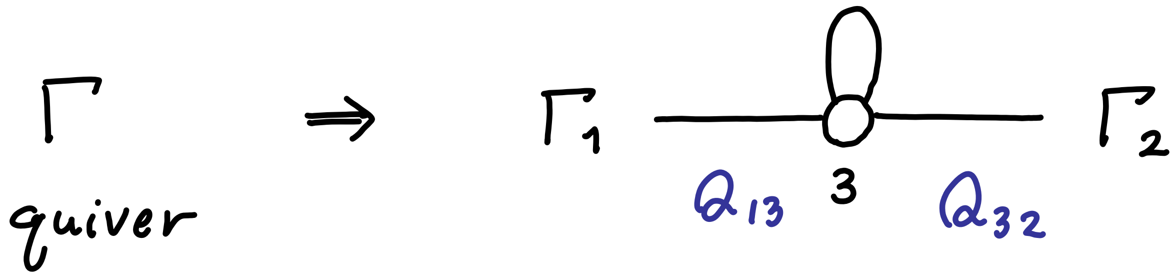

Γ_1 can be the ISS model or its variant, which has meta-stable vacua.

The meta-stable vacua are not disturbed if the interactions through the node 3 are weak.

Note: This argument would not work if we are looking for models without SUSY vacua since small perturbations may generate SUSY vacua.

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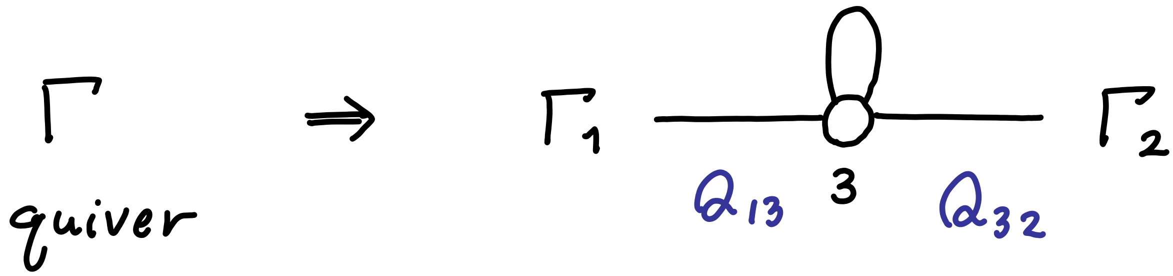

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The meta-stable vacua are not disturbed if the interactions through the node 3 are weak.

The supersymmetry breaking effect can be communicated to Γ_2 by the simple gauge mediation.

$$W_{\text{messenger}} \sim \Big(m + \underset{\substack{\uparrow \\ \langle Q_{31} Q_{13} \rangle}}{F}\,\theta^2\Big)\, Q_{32}\, Q_{23}$$

SUSY breaking will propagate through the quiver diagram.

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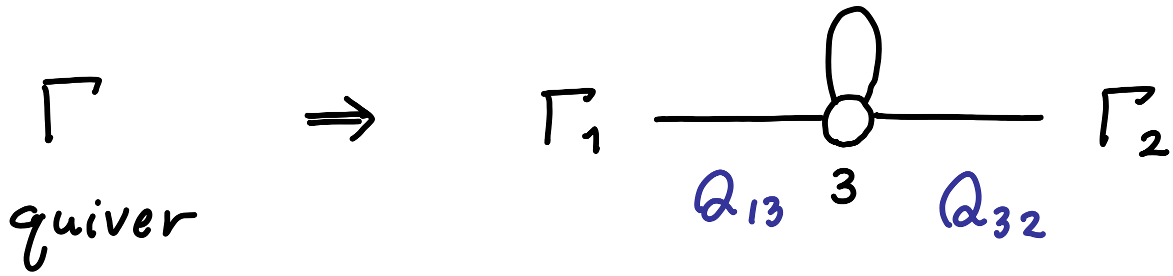

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This idea was recently applied to study meta-stable vacua in the *An* quiver theories.

Amariti, L.Girardello, Mariotti (0706.3151)

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In contrast, models without SUSY vacua are non-generic.

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In contrast, models without SUSY vacua are non-generic.

Difference between > and = .

These field theory models have frozen parameters.

When the gravity is turned on, they become dynamical.

One should worry about stabilizing them in an appropriate range.

Meta-stable vacua are known to occur even in non-supersymmetric models:

An argument based on the chiral Lagrangian and the large-N counting suggest that the SU(N) YM theory with massive quarks have N vacua and that they all become stable in the large N limit.

Dashen (1971), Witten (1980)

In large-N :

$$E_k = N^2 \cdot h\left(\frac{\theta + 2\pi k}{N}\right)$$

$$k = 0, 1, \cdots, N-1$$

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In AdS/QCD:

Maxwell action in the bulk.

$$E_k = C \cdot (\theta + 2\pi k)^2$$

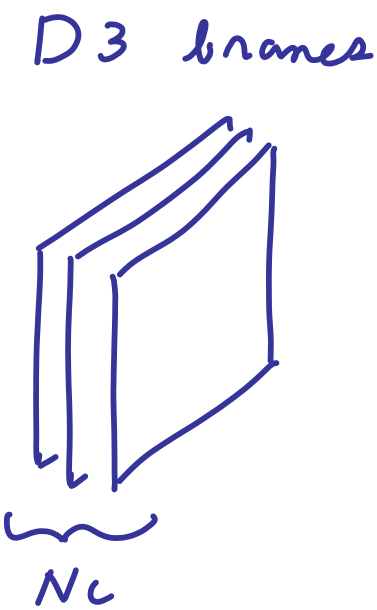
Decay rate $\sim e^{-N}$

domain-wall tension

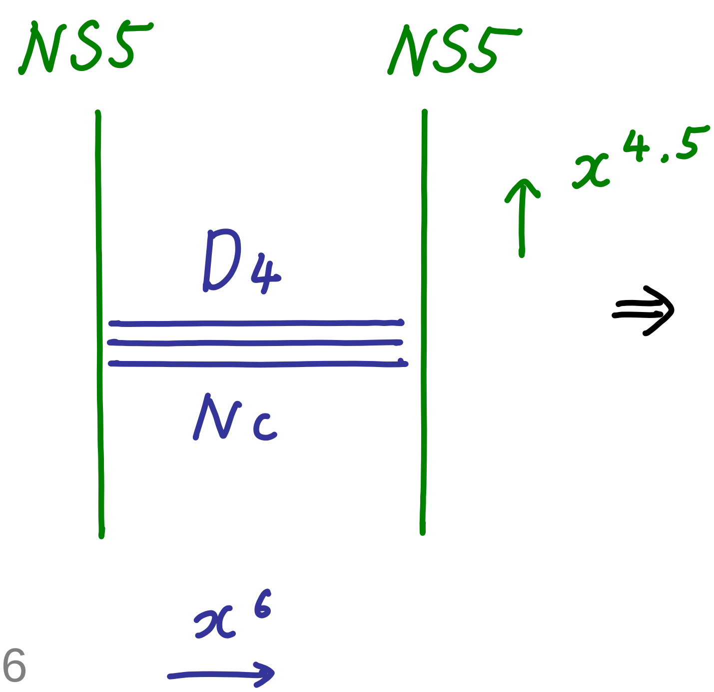
Embedding in String Theory

The meta-stable vacua can be realized on intersecting branes in string theory

(Ookouchi + H.O., hep-th/0607183
Franco, Garcia-Etxebarria, Uranga, hep-th/0607218)



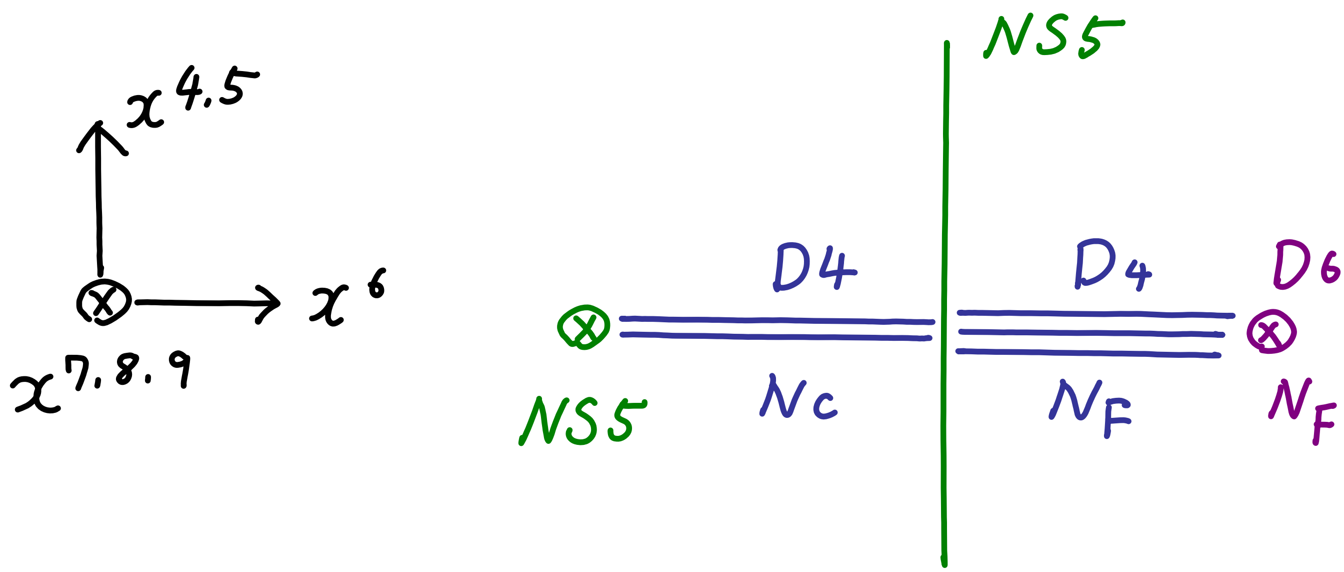
$\Rightarrow$ $\mathcal{N}=4$ super Yang-Mills
$SU(N_c)$ gauge group



$\Rightarrow$ $\mathcal{N}=2$ super Yang-Mills
$SU(N_c)$ gauge group

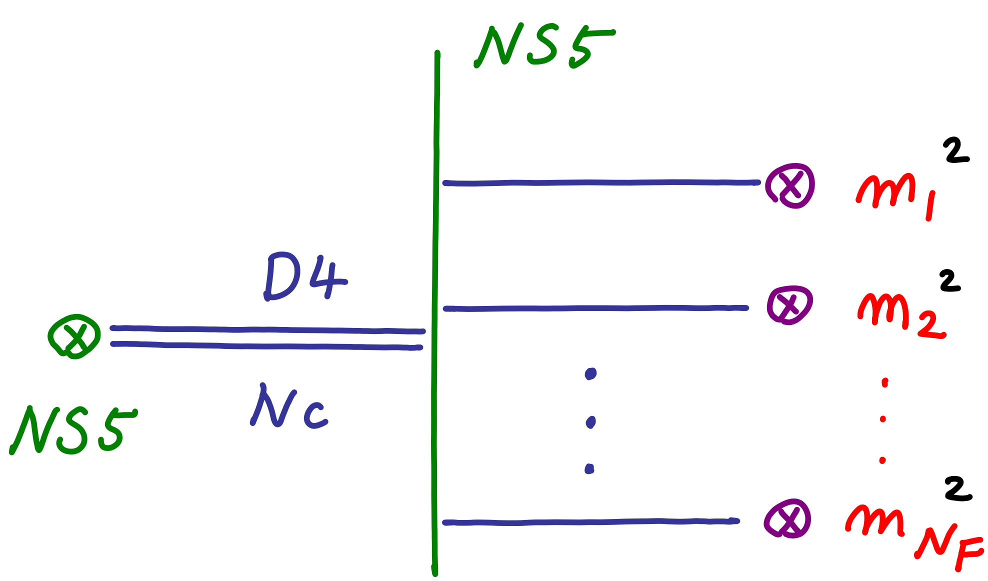
$\mathcal{N}=1$ $SU(N_c)$ gauge theory

with N_F massless quarks.

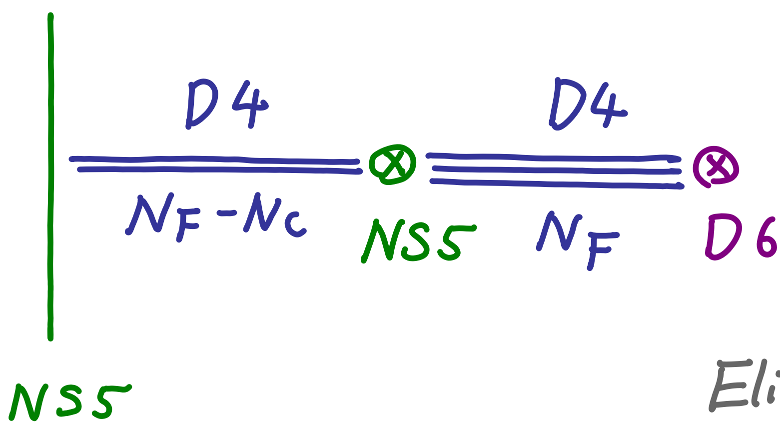


We can add quark masses by

moving D6 branes in the $x^{4,5}$ directions.



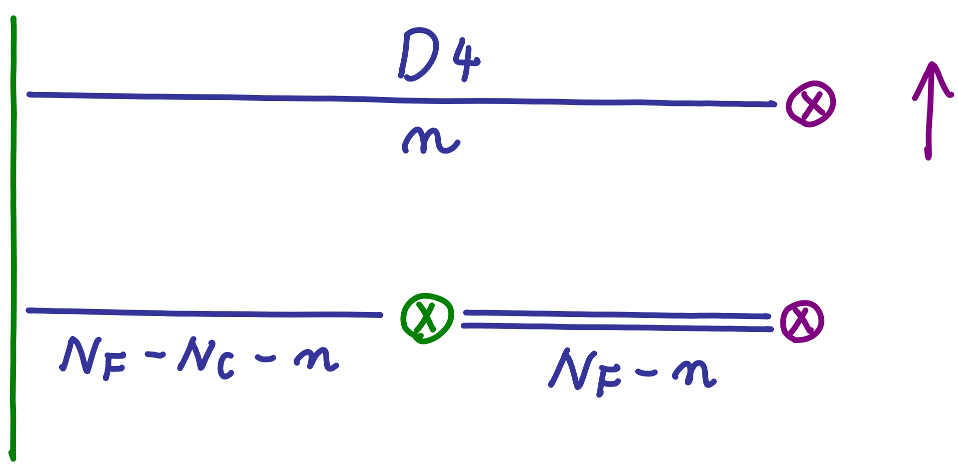
The Seiberg dual with N_F massless quarks



Elitzur, Giveon, Kutasov

We can introduce the mass matrix m by moving D6 branes.

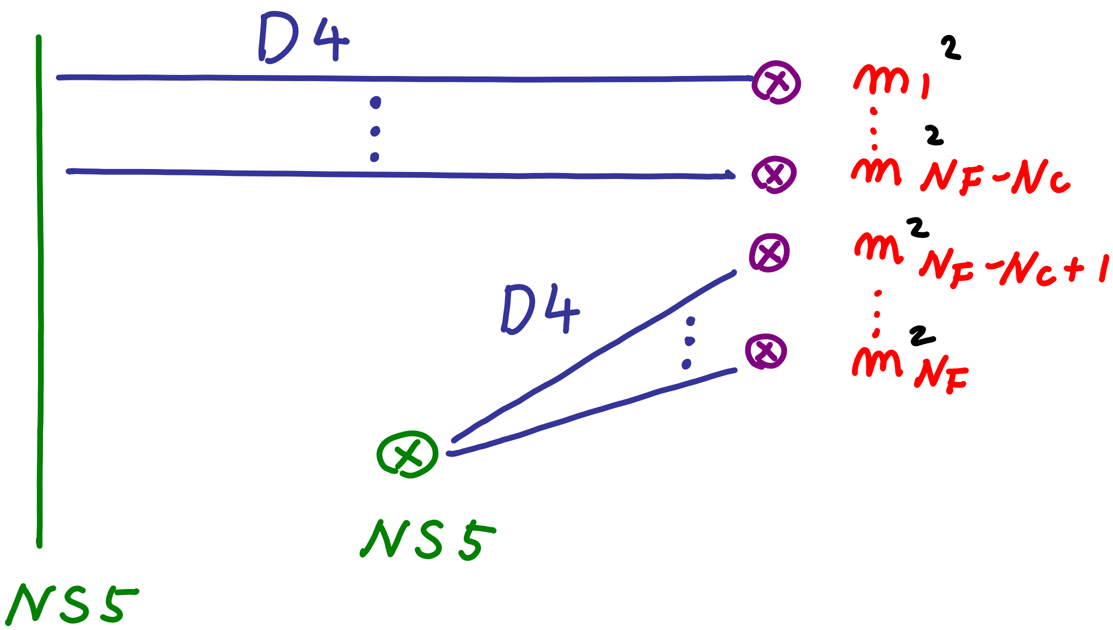
If rank $m = n \le N_F - N_c$,



What happens when rank $m > N_F - N_c$?

When $m = \begin{pmatrix} m_1 & & & \\ & m_2 & & \\ & & \ddots & \\ & & & m_{N_F} \end{pmatrix}$

with $|m_1| \geq |m_2| \geq \cdots \geq |m_{N_F}| > 0$,



Supersymmetry is broken since D4 branes are not parallel to each other.

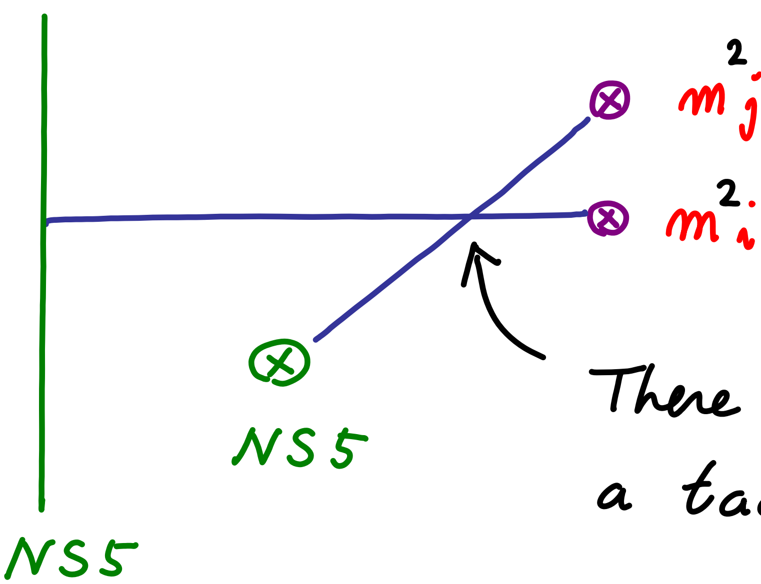
Properties of the meta-stable vacua

- vacuum energies
- local stability
- decay processes . etc

can be quantitatively described by the brane configurations.

e.g. Why do we need to order

$|m_1| \geq |m_2| \geq \cdots \geq |m_{N_F}|$?



There will be a tachyon, otherwise.

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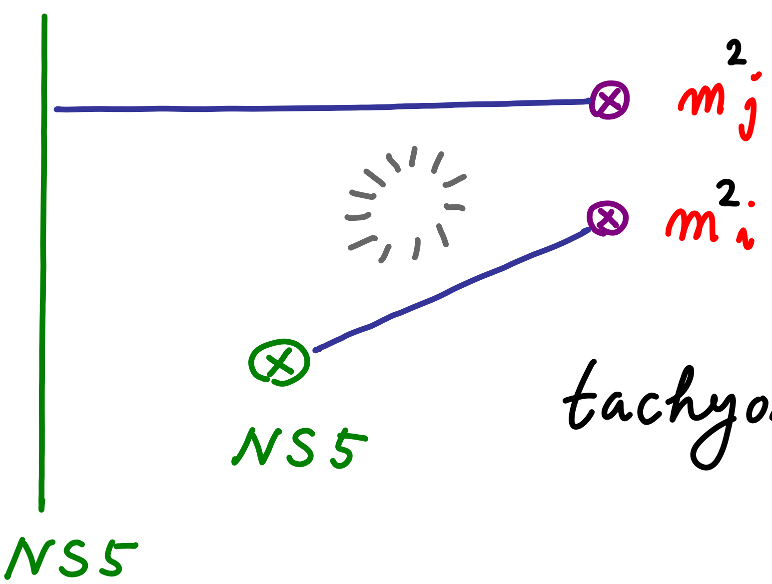

tachyon condensation

Application to Model Building

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Meta-stability requires > rather than = .

This simplifies model building.

Constructing models without SUSY vacua is hard.

-- Witten index (e.g., non-zero for SQCD)
-- Nelson-Seiberg theorem on R-symmetry

Realizing them in string theory is harder.

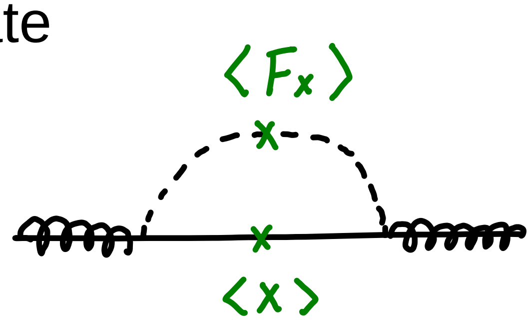
Many of the difficulties can be avoided by accepting meta-stability.

-- Witten index and Nelson-Seiberg theorem are not obstructions any more.

-- Greater flexibilities.

e.g., Superpotential can be generic.

In particular, we can break the R-symmetry and generate the gaugino masses at one-loop.



Intriligator, Seiberg, Shih:

$SU(N_c)$ super Yang-Mills
$+ \; N_F$ quarks

$\Rightarrow$

Seiberg dual for $N_c < N_F$

$SU(N_F - N_c)$ super Yang-Mills
$+ \; N_F$ quarks $(q_i, \tilde{q}_i)$
$+ \; N_F \times N_F$ singlets M_{ij}

($M = Q\tilde{Q}$ in the original model.)

This model is IR free when $N_F < \frac{3}{2} N_c$

- suitable to study vacuum structure.
- Landau pole --- UV completion by the original model.

The superpotential

$$W = \mathrm{tr}\, m^2 M \quad + \quad \mathrm{tr}\, q\tilde{q} M$$

inherited from $Q\tilde{Q}$-mass term

- $$\frac{\partial W}{\partial M} = m^2 + q\tilde{q} = 0$$

  has no solution.

- The decay rate can be made parametrically small, provided

$$m = \begin{pmatrix} m\,\mathbb{1}_{N_F-N_c} & 0 \\ 0 & \mu\,\mathbb{1}_{N_c} \end{pmatrix} \quad \text{with } m \gg \mu .$$

But, this model has an R symmetry:

$[M] = 2$. $[q] = [\tilde{q}] = 0$.

Gaugino masses are forbidden.

How about breaking the R-symmetry explicitly by a superpotential?

Kitano, Ookouchi + H.O.
(hep-ph/0612139)

$SU(N_c)$ group with N_f flavors $(Q_i, \tilde{Q}_i)$

$$W_{ISS} = \sum_{i=1}^{N_f} m_i Q_i \tilde{Q}_i$$

add

$$\delta W = \sum_{ijkl} c_{ijkl} Q_i \tilde{Q}_j Q_k \tilde{Q}_l .$$

This model can be naturally realized on D-branes at Calabi-Yau singularities.

Argurio, Bertolini, Franco, Kachru (hep-th/0703236)

We choose:

$$(m_i) = (\underbrace{m, \cdots, m}_{N_F - N_c}, \underbrace{\mu, \cdots, \mu}_{N_c})$$

appropriate c_{ijkl}

so that there is a meta-stable vacuum with unbroken global symmetry:

$$SU(N_F - N_c) \times SU(N_c) \times U(1)$$

$$\langle Q \tilde{Q} \rangle_{\cancel{SUSY}} = \begin{array}{cc} \begin{pmatrix} 0 & 0 \\ 0 & * \end{pmatrix} & \begin{array}{l} \}\, N_F - N_c \\ \}\, N_c \end{array} \end{array}$$

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This vacuum is long-lived if:

$$m \gg \mu$$

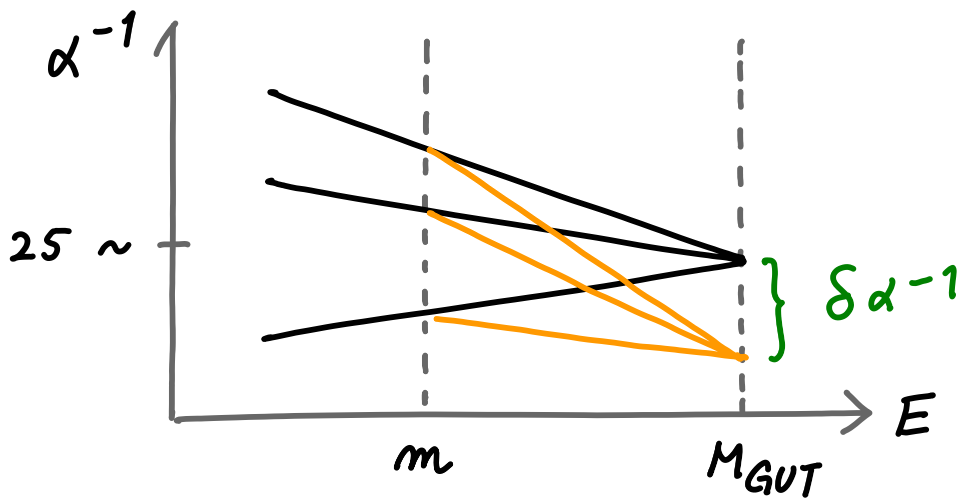
c_{ijkl} : appropriate range

Couple this to the Standard Model by:

$$SU(3) \times SU(2) \times U(1) \subset SU(N_F - N_c)$$

Masses of gauginos, scalars, and gravitino come out to be phenomenologically attractive values,

and the Landau pole problem can be avoided,



$$\delta\alpha^{-1} = \frac{2N_F - N_C}{2\pi} \log \frac{M_{GUT}}{m} < 25$$

if we choose:

$$10^8\,\mathrm{GeV} \lesssim \mu \lesssim 10^9\,\mathrm{GeV}$$

$$10^{13}\,\mathrm{GeV} \lesssim m \lesssim 10^{15}\,\mathrm{GeV}$$

C_{ijkl} : appropriate range

$(C^2 \sim m\Lambda^{-3})$

There have been model building activities making use of related ideas:

Murayama, Nomura	(hep-ph/0612186, 0701231)
Csaki, Shirman, Terning	(hep-ph/0612241)
Aharony, Seiberg	(hep-ph/0612308)
Abel, Khoze	(hep-ph/0701069)
Amariti, Girardello, Mariotti	(hep-th/0701121)
Dudas, Mourad, Nitti	(0706.1269)

.....

Conclusion

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How does the story change when the gravity is turned on?

How can we tell when they are not in the Swampland?