



# Team Austria: RISC

## Research Institute for Symbolic Computation Johannes Kepler University Linz (JKU)

Research leader: Carsten Schneider

RISC JKU–Institute: J. Ablinger, M. Kauers, P. Paule (educational board),  
C. Raab, M. Round, C. Schneider

RISC Software Company: W. Freiseisen (coordinator), P. Heinzlreiter,  
A. Kondratyev, M. Krieger, P. Stadelmeyer, R. Stainko

# General role in the training network of young researchers

## 1. As research partner: Bringing in top level expertise in computer algebra and special function algorithms

- ▶ Individual training in new technologies (1:1 instructions, seminars)
- ▶ Dissemination of knowledge (schools, meetings)
- ▶ Development of new methods and their application in particle physics

Spin off: New research group “Computer Algebra for QFT” at RISC

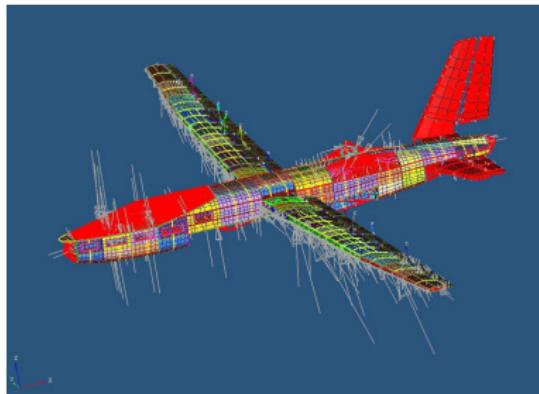
## 2. As industrial partner: Training in industrial projects

- ▶ Software engineering skills
- ▶ Industrial team work
- ▶ Modern programming tools in industrial applications

# Training of young researchers in industrial projects

First round of students:

- ▶ Radomir Sevillano: October 15 - December 15, 2012
- ▶ Michael Ochman : October 15 - December 15, 2012

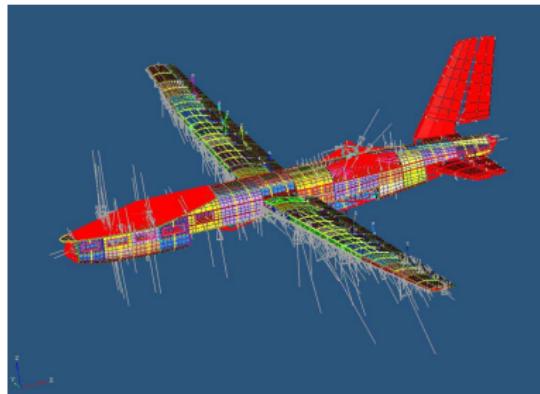


Design of airplanes with minimal mass under constraints such as stability (customer: EADS)

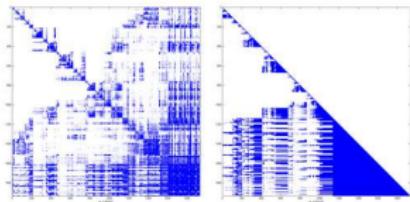
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FEM  
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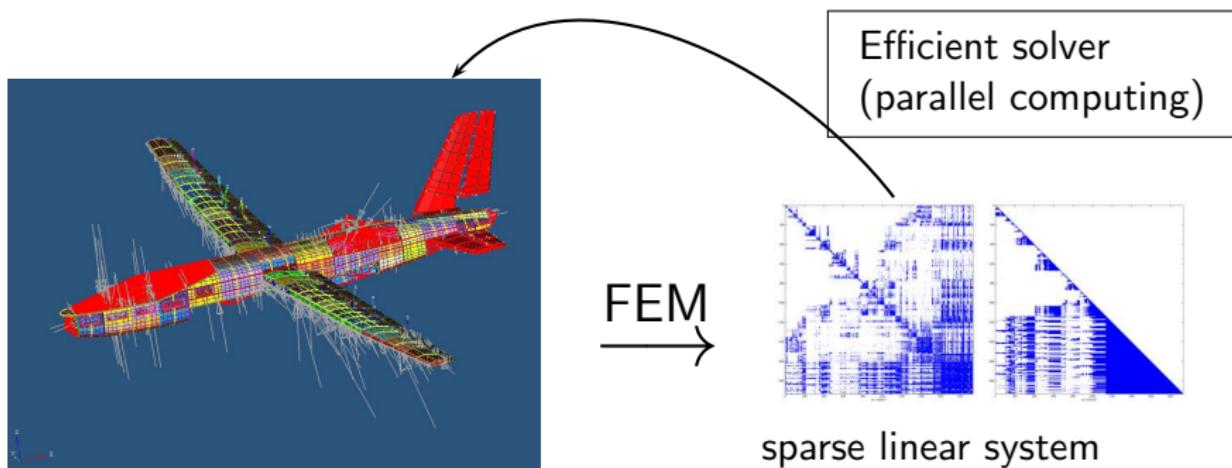
sparse linear system

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Languages:  
C,C++,Fortran

# Involved young scientists (in close cooperation with DESY)

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- ▶ 2007–2011 at RISC
- ▶ 8 month ESR at DESY
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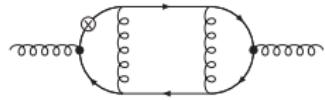
## 4. Alexander Hasselhuhn

- ▶ 2009–2012 at DESY
- ▶ 2 years ESR at RISC (October 2012–2014)

# New computer algebra packages for Feynman integrals

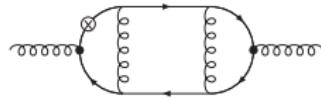
1. **HarmonicSums** ([ESR J. Ablinger](#)): a Mathematica package for harmonic sums, S-sums, cyclotomic sums and cyclotomic S-sums as well as iterated integrals such as harmonic polylogarithms, multiple polylogarithms and cyclotomic polylogarithms.
2. **EvaluateMultiSums** ([C. Schneider](#)): a Mathematica package for computing multiple nested sums.
3. **SumProductson** ([C. Schneider](#)): a Mathematica package for dealing with large scale sums.
4. **MultIntegrate** ([ESR J. Ablinger](#)): a Mathematica package for computing multiple integrals.
5. **RhoSum** ([ESR M. Round](#)): a Mathematica package for a Refined HOlonomic SUmmation Machinery based on difference field theory.

# Tools in action



(massive 3–loop ladder graph with operator insertion)

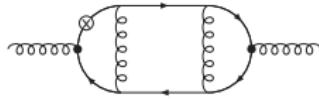
# Tools in action



A. Hasselhuhn

$$\begin{aligned}
 &= \frac{C_3}{(N+1)(N+2)} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{l=2}^{N+2} \binom{N+2}{l} \sum_{j=2}^l \binom{l}{j} \left\{ \right. \\
 &\sum_{k=1}^j \binom{j}{k} \sum_{r=0}^{l-k} \binom{l-k}{r} (-1)^{l+j+k+r} \frac{B\left(k, m+1 + \frac{\varepsilon}{2}\right) \Gamma(k+r+m+n+\frac{\varepsilon}{2})}{\Gamma(m+1)\Gamma(n+1)\Gamma(k+r+\frac{\varepsilon}{2})} \frac{B\left(r+l-1, n+1 + \frac{\varepsilon}{2}\right)}{(N+3-j)} \frac{B\left(k+m - \frac{\varepsilon}{2}, r+1+n-\frac{\varepsilon}{2}\right)}{(k+r+1+m+n-\varepsilon)} \\
 &+ \sum_{r=0}^{l-j} \binom{l-j}{r} (-1)^{l+j+r} \frac{B\left(r+l-1, n+1 + \frac{\varepsilon}{2}\right) \Gamma(j+r+m+n+\frac{\varepsilon}{2})}{\Gamma(m+1)\Gamma(n+1)\Gamma(j+r+\frac{\varepsilon}{2})} \frac{B\left(j, m+1 + \frac{\varepsilon}{2}\right) B\left(j+m - \frac{\varepsilon}{2}, r+1+n-\frac{\varepsilon}{2}\right)}{(j+r+1+m+n-\varepsilon)(N+3-j)} \left. \right\}
 \end{aligned}$$

# Tools in action



A. Hasselhuhn

$$\frac{C_3}{(N+1)(N+2)} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{l=2}^{N+2} \binom{N+2}{l} \sum_{j=2}^l \binom{l}{j} \left\{ \right.$$

$$\begin{aligned} & \sum_{k=1}^j \binom{j}{k} \sum_{r=0}^{l-k} \binom{l-k}{r} (-1)^{l+j+k+r} \frac{B\left(k, m+1+\frac{\varepsilon}{2}\right) \Gamma(k+r+m+n+\frac{\varepsilon}{2})}{\Gamma(m+1)\Gamma(n+1)\Gamma(k+r+\frac{\varepsilon}{2})} \frac{B\left(r+l-1, n+1+\frac{\varepsilon}{2}\right)}{(N+3-j)} \frac{B\left(k+m-\frac{\varepsilon}{2}, r+1+n-\varepsilon\right)}{(k+r+1+m+n-\varepsilon)} \\ & + \sum_{r=0}^{l-j} \binom{l-j}{r} (-1)^{l+j+r} \frac{B\left(r+l-1, n+1+\frac{\varepsilon}{2}\right) \Gamma(j+r+m+n+\frac{\varepsilon}{2})}{\Gamma(m+1)\Gamma(n+1)\Gamma(j+r+\frac{\varepsilon}{2})} \frac{B\left(j, m+1+\frac{\varepsilon}{2}\right) B\left(j+m-\frac{\varepsilon}{2}, r+1+n-\frac{\varepsilon}{2}\right)}{(j+r+1+m+n-\varepsilon)(N+3-j)} \end{aligned} \left. \right\}$$

|| symbolic summation (ESR M. Round)

$$\begin{aligned} & \frac{C_3}{(N+1)(N+2)(N+3)} \left\{ \frac{1}{6} S_1^3(N) + \frac{N^2+12N+16}{2(N+1)(N+2)} S_1(N)^2 + \frac{4(2N+3)}{(N+1)^2(N+2)} S_1(N) \right. \\ & + 2 \left[ -2^{N+3} + 3 - (-1)^N \right] \zeta_3 + \left[ \frac{3N^2+40N+56}{2(N+1)(N+2)} - \frac{1}{2} S_1(N) \right] S_2(N) \\ & - (-1)^N S_{-3}(N) + \frac{8(2N+3)}{(N+1)^3(N+2)} - \frac{3N+17}{3} S_3(N) - 2(-1)^N S_{-2,1}(N) - (N+3) S_{2,1}(N) \\ & \left. + 2^{N+4} S_{1,2} \left( \frac{1}{2}, 1; N \right) + 2^{N+3} \boxed{S_{1,1,1} \left( \frac{1}{2}, 1, 1; N \right)} \right\} + O(\varepsilon) \end{aligned}$$

$$\boxed{S_{1,1,1} \left( \frac{1}{2}, 1, 1; N \right)} = \sum_{i=1}^N \frac{\left(\frac{1}{2}\right)^i \sum_{j=1}^i \frac{1}{j}}{i}$$

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|| asymptotic expansion algorithms (ESR J. Ablinger)

$$\begin{aligned}
& 2^{-N} \left( + \frac{541}{N^6} - \frac{75}{N^5} + \frac{13}{N^4} - \frac{3}{N^3} + \frac{1}{N^2} - \frac{1}{2N} \right) (\ln(N) + \gamma)^2 \\
& + 2^{-N-3} \left( - \frac{114686}{5N^6} + \frac{44099}{15N^5} - \frac{1372}{3N^4} + \frac{266}{3N^3} - \frac{20}{N^2} \right) (\ln(N) + \gamma) \\
& + 2^{-N} \left( + \frac{541}{N^6} - \frac{75}{N^5} + \frac{13}{N^4} - \frac{3}{N^3} + \frac{1}{N^2} - \frac{1}{2N} \right) \zeta(2) + \frac{3\zeta(3)}{4} \\
& + 2^{-N-9} \left( \frac{69280576}{45N^6} - \frac{1582096}{9N^5} + \frac{69184}{3N^4} - \frac{3264}{N^3} + \frac{256}{N^2} \right) + O\left(\frac{1}{2^N N^7}\right)
\end{aligned}$$

# Scientific environment for young researcher

Senior researchers: M. Kauers, P. Paule, C. Schneider (numerous projects)

Accompanying networks: Doctorate Program, applied for SFB

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## Longterm Cooperations:

- ▶ DESY, Zeuthen, Germany: Johannes Blümlein
- ▶ INRIA, Paris, France: Frédéric Chyzak
- ▶ University of Ljubljana, Slovenia: Marko Petkovšek
- ▶ North Carolina State University: Michael F. Singer
- ▶ Pennsylvania University, University Park, USA: George E. Andrews
- ▶ Rutgers University, New Brunswick, USA: Doron Zeilberger
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## RISC PhD Curriculum (most relevant courses): 14

- ▶ Symbolic Summation I,II; Computer Analysis
- ▶ Special Functions I,II
- ▶ Computer Algebra I,II; Gröbner basis I,II; Symbolic Linear Algebra
- ▶ Combinatorics Seminar, Seminar Symbolic Summation for Particle Physics
- ▶ Analytic Combinatorics, Algebraic Combinatorics
- ▶ **RISC Summer** (Sequence of conferences each summer)

# RISC-DESY workshop at RISC, Austria (May 7–8, 2012)

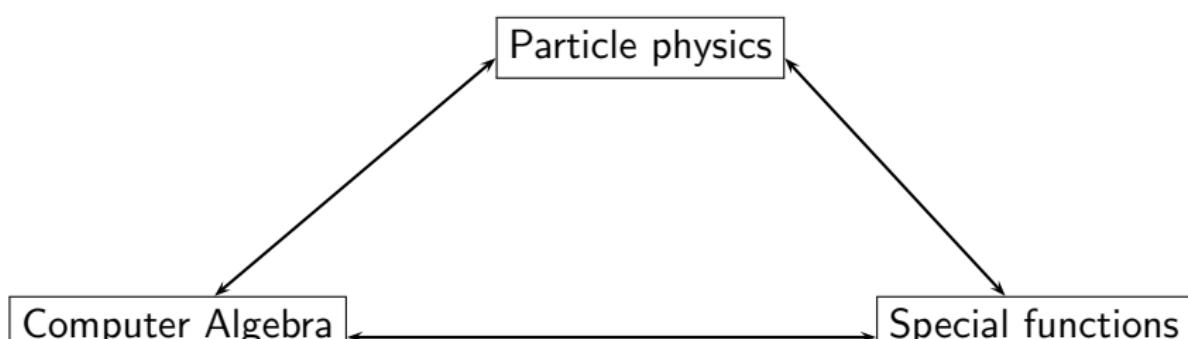
## Computer algebra/special functions:

1. ESR J. Ablinger (RISC/DESY): Computer algebra algorithms for special functions in QFT
2. M. Hoffman (U.S. Naval Academy): Hopf algebra structures on quasi-shuffle algebras
3. F. Johansson (RISC): Fast Special Function Computations with FLINT
4. M. Kauers (RISC): Fine-Tuning Zeilberger's Algorithm
5. V. Pillwein (RISC): Symbolic Computation for Special Functions Inequalities
6. ESR C. Raab (RISC): Symbolic Computation of Linear Relations of Parameter Integrals
7. ESR M. Round (RISC): Refined Holonomic Multi-Summation Applied to QCD Graphs
8. C. Schneider (RISC): Computer algebra and elementary particle physics: a successful story
9. F. Wissbrock (DESY): The hyperlogarithm method for non-divergent 3-loop integrals

## Physics:

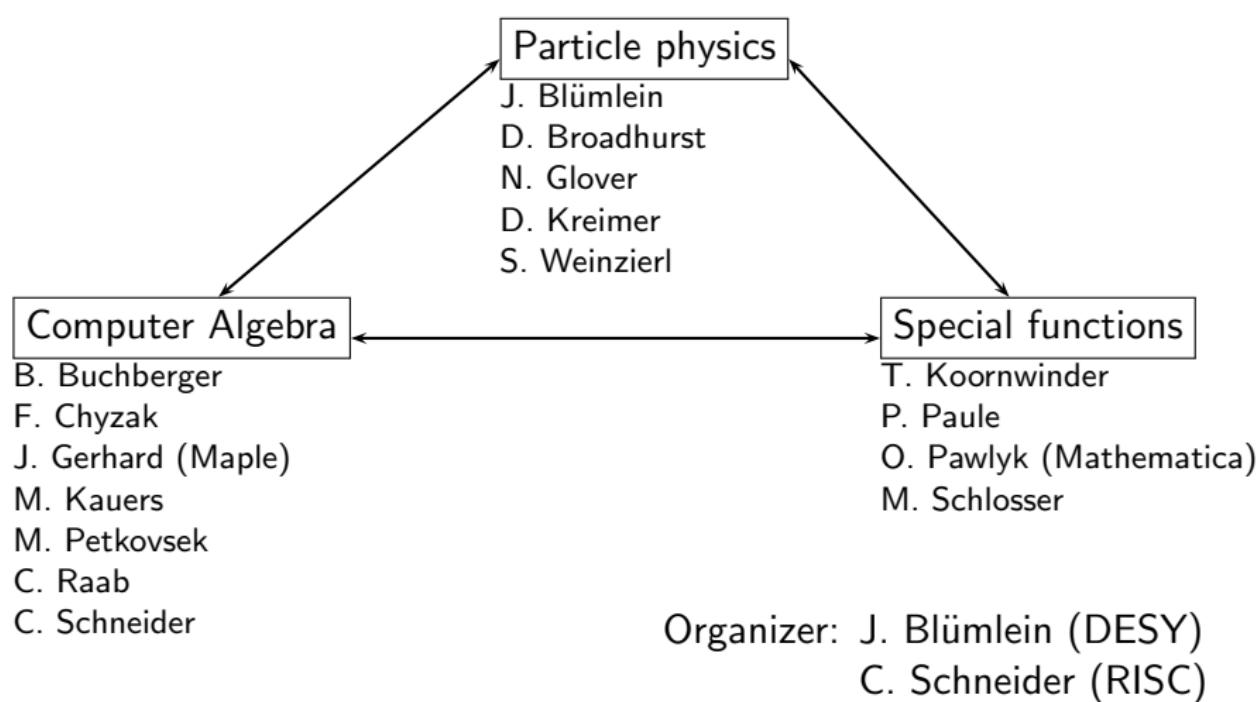
1. I. Bierenbaum (DESY): A short introduction to the calculation of Feynman diagrams using Feynman parameters, Mellin-Barnes integrals and nested sums
2. J. Blümlein (DESY): Solving single mass Feynman Integrals at 2 and 3 Loops
3. A. de Freitas (DESY): Integrals in 2-loop massive OMEs with external massive legs
4. ESR A. Hasselhuhn (DESY): Ladder and gluonic contributions at 3-loops
5. A. Hoang (University Vienna): Summation of renormalization and renormalon logarithms in jet production and precision determination of  $\alpha_s$  at NNNLL order
6. Tord Riemann (DESY): Simple Feynman diagrams and simple sums

# LHCPhenoNet school (July 9-13, 2012; Austria): Summation, Integration and Special Function in QFT



Organizer: J. Blümlein (DESY)  
C. Schneider (RISC)

# LHCPhenoNet school (July 9-13, 2012; Austria): Summation, Integration and Special Function in QFT



# LHCPhenoNet school (July 9-13, 2012; Austria): Summation, Integration and Special Function in QFT



- ▶ Interdisciplinary school (speakers and participants)
- ▶ Fruitful discussions after the talks
- ▶ About 42 participants (21 young researchers)
- ▶ Springer book in preparation (introductory lectures/handbook of new methods and technologies)

# (Interdisciplinary) Publications

## Computer algebra/special functions

1. M. Kauers, C. Schneider. Proc. of ISSAC 2011, Anton Leykin (ed.), pp. 201-208. 2011.
2. [J. Ablinger](#), J. Blümlein, C. Schneider. J. Math. Phys. 52(10), pp. 1-52. 2011.
3. J. Blümlein, [A. Hasselhuhn](#), C. Schneider Evaluation of Multi-Sums for Large Scale Problems. PoS(RADCOR2011)32, pp. 1-9. 2012.
4. [J. Ablinger](#). J. Kepler University Linz. PhD Thesis. April 2012.
5. J. Blümlein, S. Klein, C. Schneider, [F. Stan](#). J. Symbolic Comput. 47, pp 1267-1289, 2012.

## Particle physics

1. [J. Ablinger](#), J. Blümlein, S. Klein, C. Schneider, [F. Wissbrock](#). To appear in Proc. of 19th International Workshop On Deep-Inelastic Scattering And Related Subjects, 2011.
2. [J. Ablinger](#), J. Blümlein, S. Klein, C. Schneider, [F. Wissbrock](#). Nucl. Phys. B, 844, pp. 26-54. 2011.
3. [J. Ablinger](#), J. Blümlein, [A. Hasselhuhn](#), S. Klein, C. Schneider, [F. Wissbrock](#). PoS(RADCOR2011)31, pp. 1-8. 2012.
4. [J. Ablinger](#), [J. Blümlein](#), [A. Hasselhuhn](#), S. Klein, C. Schneider, [F. Wissbrock](#). Nuclear Physics B. 864, pp. 52-84, 2012.
5. J. Blümlein, [A. Hasselhuhn](#), S. Klein, C. Schneider. To appear in Nuclear Physics B, in press. 2012.