

The hard to soft
Pomeron transition
in small x DIS data
using optimal
renormalization

Clara Salas

Regge th. & QCD

Theoretical app.

H.O. corrections & IR

Running coupling

Ren. scheme & scale

Numerical analysis

Comments

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Clara Salas

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September 20th, Mid-Term meeting LHC-phenonet, Ravello

with M. Hentschinski, A. Sabio Vera

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The group

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Outline

① Regge theory & QCD: the Pomeron intercept

② Theoretical approach

- Higher order corrections & IR: collinear improved resummation
- Running coupling effects
- Choice of renormalization scheme & scale

③ Numerical analysis

④ Conclusions

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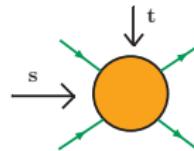
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The low x limit: Regge theory

Regge (high energy) limit:



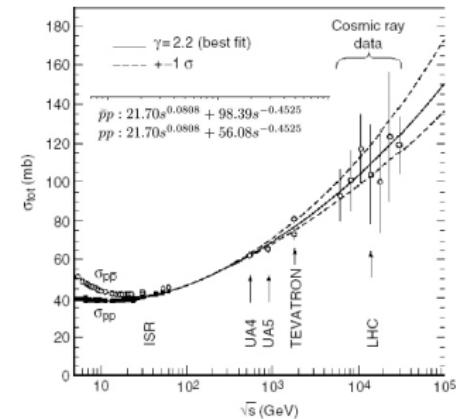
$$s \gg |t|, Q^2 \gg \Lambda_{\text{QCD}}^2$$

$$\alpha_s(Q^2) \ll 1$$

$$x \approx |t|/s \ll 1$$

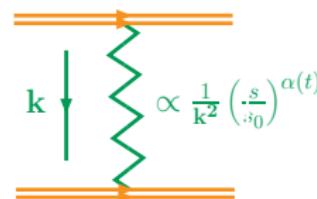
Regge theory prediction:

$$A(s, t) \sim s^{\alpha(t)} \Rightarrow \sigma(s) \sim s^{\alpha(0)-1}$$



Explanation by Regge principles:

Reggeization



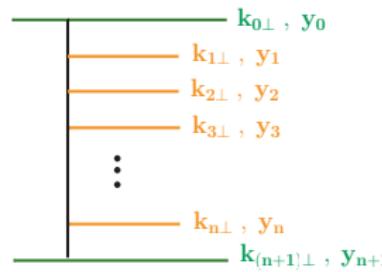
Exchange of “something” in the t -channel carrying the Q.N. of the vacuum:

SOFT POMERON: $\lambda_P \equiv \alpha(0) - 1 \simeq 0.08$

QCD and the Regge limit: $s \gg |t|$

Ladder-type diagrams:

$2 \rightarrow 2 + n$



- Multi Regge Kinematics:

Impose strong ordering in rapidity:

$$y_1 \gg y_2 \gg \dots \gg y_n$$

- Dominated by: $\bar{\alpha}_s \log(s/s_0)$

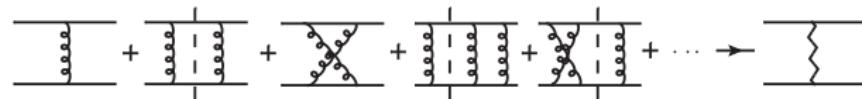
$$\sum_{n=0}^{\infty} \bar{\alpha}_s^n \log^n(s/s_0) , \quad \bar{\alpha}_s \log(s/s_0) \sim 1$$

Need of resummation: ($Y = \log(s/s_0)$)

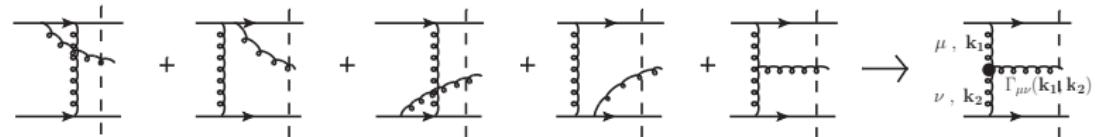
$$\sum_{n=0}^{\infty} \chi^n \bar{\alpha}_s^n \int_0^Y dy_1 \int_0^{y_1} dy_2 \dots \int_0^{y_{n-1}} dy_n = \sum_{n=0}^{\infty} \frac{(\chi \bar{\alpha}_s Y)^n}{n!} = e^{\chi \bar{\alpha}_s Y}$$

QCD and gluon reggeization

Virtual contributions



Real emissions:



- $\Gamma_{\mu\nu}(\mathbf{k}_1, \mathbf{k}_2)$ is the **Lipatov effective vertex**, gauge invariant.

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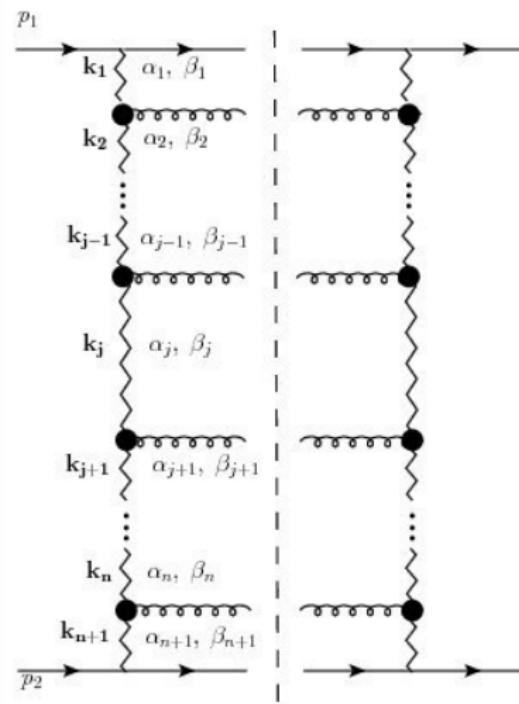
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The gluon ladder: QCD Pomeron



BFKL evolution equation & hard pomeron

- Mellin transform of the imaginary part of the amplitude

$$\omega f(\omega, \mathbf{k}_1, \mathbf{k}_2) = \delta^2(\mathbf{k}_1 - \mathbf{k}_2) + \mathcal{K}^{\text{BFKL}} \otimes f(\omega, \mathbf{k}_1, \mathbf{k}_2)$$

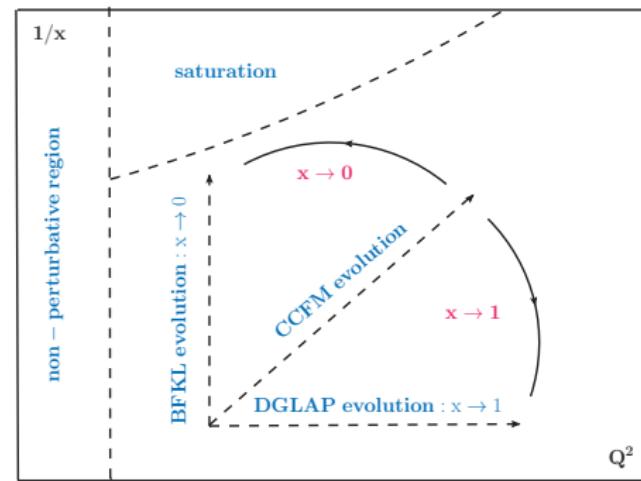
- Eigenstate equation: $\mathcal{K} \phi_i(\mathbf{k}) = \lambda_i \phi_i(\mathbf{k})$, $\mathcal{K} |n, \nu\rangle = \omega |n, \nu\rangle$
Solution using Saddle Point Approximation:

$$\omega = \bar{\alpha}_s \chi_0(\nu = 0)$$

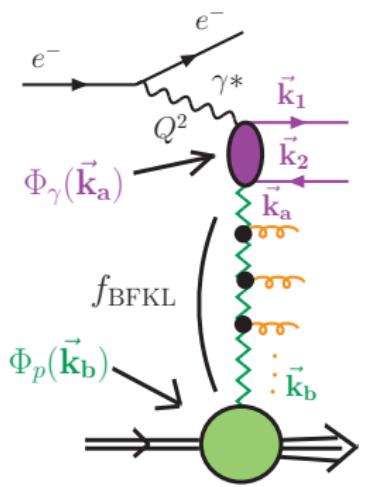
$$\chi_0(\nu) = 4 \log(2) - 14\zeta(3)\nu^2 + \dots$$

- Leading Order Pomeron intercept: $\lambda_P^{\text{LO}} \simeq 4\bar{\alpha}_s \log(2) \simeq 0.5$
- Next to Leading Order Pomeron intercept: $\lambda_P^{\text{NLO}} \simeq 0.3$
- Soft Pomeron: $\lambda_P^{\text{soft}} \simeq 0.08$

Perturbative QCD

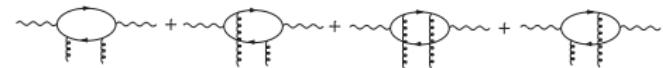


Equation	x -limit	Resummation	Energy region
DGLAP	$x \rightarrow 1$	$\log(Q^2/\mu^2)$	very perturbative QCD
BFKL	$x \rightarrow 0$	$\log(1/x)$	close to non-pert. QCD
CCFM	any x	both	interplay

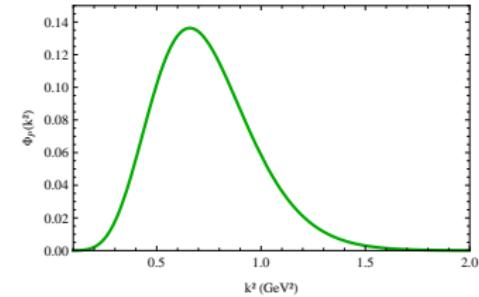


Physical setup

- Photon Impact Factor, $\Phi_\gamma(\vec{k}_a)$:



- Proton Impact Factor, $\Phi_p(\vec{k}_b)$:

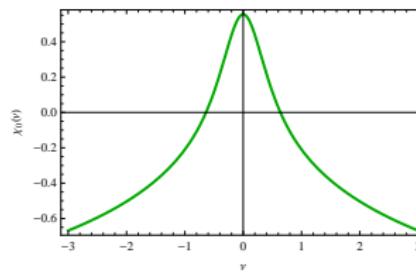


Cross section:

$$F_2(x, Q^2) = \frac{F_c}{(2\pi)^4} \int \frac{d^2 \mathbf{k}_a}{\mathbf{k}_a^2} \int \frac{d^2 \mathbf{k}_b^2}{\mathbf{k}_b} \Phi_\gamma(\mathbf{k}_a) f(x, \mathbf{k}_a, \mathbf{k}_b) \Phi_p(\mathbf{k}_b)$$

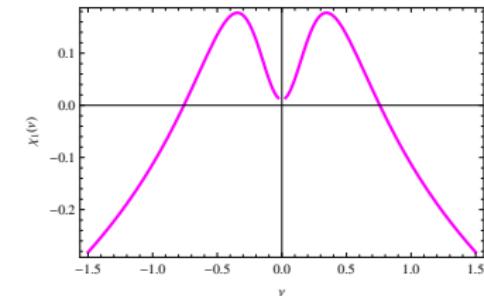
Problems with NLL accuracy

Leading Order BFKL kernel



Saddle Point App.: $\nu = 0$

Next to Leading Order BFKL kernel



Saddle Point Approximation: $\nu \neq 0$

Problematic term:

$$\int_{-\infty}^{\infty} d\nu \cdots \cos \left[\nu \log \left(\frac{Q^2}{Q_0^2} \right) \right] \text{ when } Q^2 \gg Q_0^2$$

IR problem & higher order corrections

- DIS-like choice: $s_0 = Q^2 \Rightarrow \left(\frac{s}{s_0}\right)^\omega = \left(\frac{1}{x}\right)^\omega$
- Symmetric (Regge like) scale: $s_0 = \sqrt{\mathbf{q}^2 \mathbf{k}^2}$

$$\begin{aligned} f(s, \mathbf{k}, \mathbf{q}) &= \frac{1}{2\pi\sqrt{\mathbf{q}^2 \mathbf{k}^2}} \int \frac{d\omega}{2\pi i} \int \frac{d\gamma}{2\pi i} \left(\frac{\mathbf{q}^2}{\mathbf{k}^2}\right)^{\gamma-\frac{1}{2}} \left(\frac{s}{\sqrt{\mathbf{q}^2 \mathbf{k}^2}}\right)^\omega \frac{1}{\omega - \bar{\alpha}_s \chi_0(\gamma)} \\ &= \frac{1}{2\pi\mathbf{q}^2} \int \frac{d\omega}{2\pi i} \int \frac{d\gamma}{2\pi i} \left(\frac{\mathbf{q}^2}{\mathbf{k}^2}\right)^\gamma \left(\frac{s}{\mathbf{q}^2}\right)^\omega \frac{1}{\omega - \bar{\alpha}_s \chi_0\left(\gamma - \frac{\omega}{2}\right)} \end{aligned}$$

This leads to big **double logs** (\mathbf{k} -space) or **poles** (γ -space) when $\gamma \rightarrow 0, 1$ (collinear limit), incompatible with DGLAP evolution.

- Possible solution: $\chi_0(\gamma) \rightarrow \chi_0(\gamma + \omega/2)$

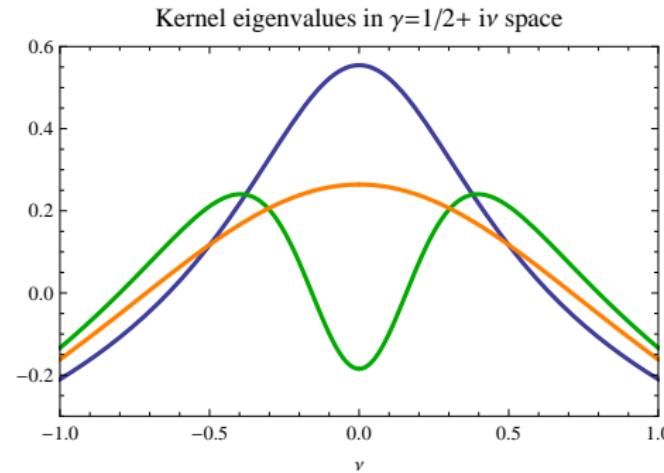
In DIS limit,

$$\omega = \bar{\alpha}_s(1 + A\bar{\alpha}_s) [2\psi(1) - \psi(\gamma + B\bar{\alpha}_s) - \psi(1 - \gamma + \omega + B\bar{\alpha}_s)]$$

[Salam (1998), hep-ph/9806482], [Sabio Vera (2005), hep-ph/0505128]

Collinear improved resummation

- This ω -shift gets rid of the double logs inconsistent with DGLAP (cancels the γ^3 and $(1 - \gamma)^3$ poles of the NLL kernel)
- Consistent with the NLL BFKL solution, changing only higher order terms



NLL: Running coupling effects

- Kernel: [Lipatov & Kotikov (2000), hep-ph/0004008]

$$\hat{\mathcal{K}}^{\text{NLL}} = \bar{\alpha}_s \chi_0 + \bar{\alpha}_s^2 \left\{ \chi_1 + \frac{\beta_0}{8N_c} \chi_0 \left[i(\overleftarrow{\partial}_v - \overrightarrow{\partial}_v) + \log(\mu^2) + i \frac{\chi'_0}{\chi_0} \right] \right\}$$

- BFKL equation and gluon Green's function:

$$\langle \hat{f}_\omega \rangle = \frac{1}{\omega - \langle \hat{\mathcal{K}} \rangle} \Rightarrow F_2 \propto \int \frac{d\omega}{2\pi i} dv \phi_{\gamma^*}(v) \left[\frac{1}{\omega} \sum_{j=0}^{\infty} \frac{\mathcal{K}^j}{\omega^j} \right] \phi_p(v) e^{\omega Y}$$

$$F_I(x, Q^2) = \mathcal{D} \int \frac{d\gamma}{2\pi i} x^{-\chi(\gamma)} c_I(\gamma) c_P(\gamma) \left\{ 1 + \bar{\alpha}_s^2 \log\left(\frac{1}{x}\right) \frac{\beta_0}{8N_c} \chi_0(\gamma) \right. \\ \left. - \log\left(\frac{Q^2 Q_0^2}{\mu^4}\right) - \psi(\delta - \gamma) - M_I(\gamma) \right\}$$

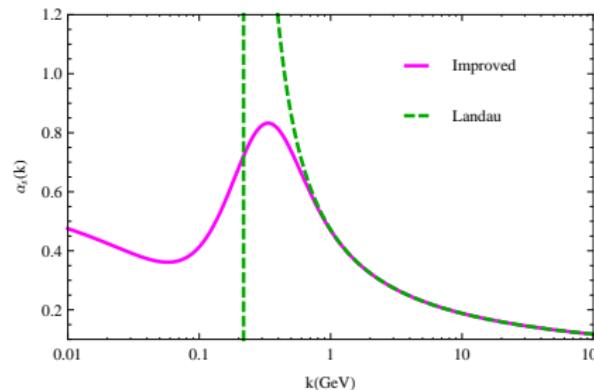
- Assumption about the running scale:

$$\bar{\alpha}_s - \bar{\alpha}_s^2 \frac{\beta_0}{8N_c} \log\left(\frac{Q^2 Q_0^2}{\mu^4}\right) \longrightarrow \bar{\alpha}_s(Q Q_0)$$

A model for the running

Running coupling **analytical in the infrared** and compatible with power corrections to jet observables:

$$\bar{\alpha}_s(\mathbf{k}^2) = \frac{4N_c}{\beta_0} \left(\frac{1}{\ln \frac{\mathbf{k}^2}{\Lambda_{QCD}^2}} + 125 \frac{(\Lambda_{QCD}^2 + 4\mathbf{k}^2)}{(\Lambda_{QCD}^2 - \mathbf{k}^2) \left(4 + \frac{\mathbf{k}^2}{\Lambda_{QCD}^2} \right)^4} \right)$$

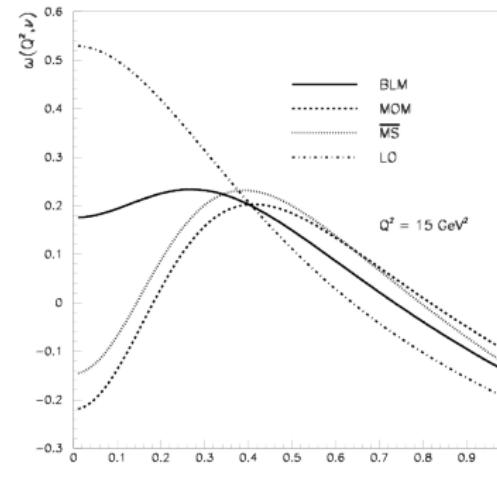


[B. Webber (1998) hep-ph/9805484]

Choice of renormalization scheme & scale

- **MOM scheme:** non abelian physical renormalization scheme based on the existence of an IR fixed point.
- **BLM scale:** redefinition of the running coupling absorbing all terms with β_0 dependence: resummation of its vacuum polarization effects.

[Brodsky, Fadin, Kim, Lipatov, Pivovarov (1999) [hep-ph/9901229](#)]



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About the fit

- Experimental values:

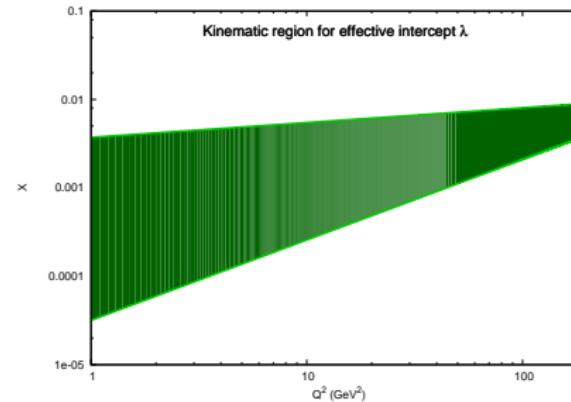
fit $F_2(x_{ij}, Q_i^2) = C(Q_i^2) x_{ij}^{-\lambda_j(Q_i^2)}$

and average $\lambda(Q_i^2) = \langle \lambda_j(Q_i^2) \rangle$ for $\{x_{ij}, x_{ij}^{\min}, x_{ij}^{\max}\}$

- Cut in x : $x < 0.01$

- Theoretical prediction: $\lambda^{\text{th}}(Q^2) = \frac{\partial F_2(x, Q^2)}{\partial \log(1/x)} / F_2(x, Q^2)$

- Smooth x boundaries:



Impact factors

Photon impact factor: LO simplest version:

$$\int \frac{d^2 q}{q^2} \Phi_I(q, Q^2) \left(\frac{q^2}{Q^2} \right)^{\gamma-1} = \alpha \bar{\alpha}_s \pi^4 \sum_{q=1}^{n_f} e_q^2 \frac{\Omega_I(\nu)}{\nu + \nu^3} \operatorname{sech}(\pi\nu) \tanh(\pi\nu),$$

with $\Omega_2 = (11 + 12\nu^2)/8$ and $\Omega_L = \nu^2 + 1/4$.

Proton impact factor:

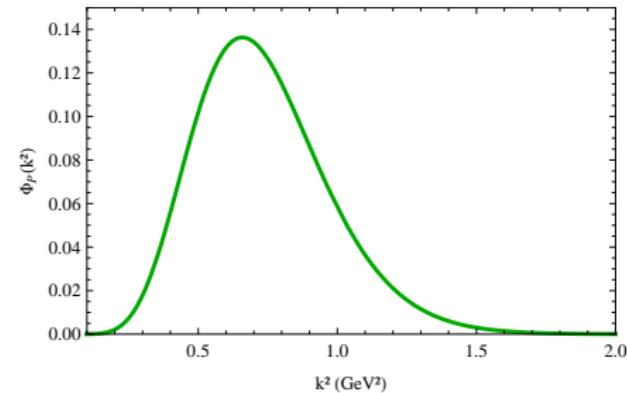
$$\phi_p(\mathbf{k}^2) = N \left(\frac{\mathbf{k}^2}{Q_0^2} \right)^\delta e^{-\mathbf{k}^2/Q_0^2}$$

Best fit:

$$\delta = 8.4,$$

$$Q_0 = 0.28 \text{ GeV},$$

$$\text{with } \Lambda = 0.21 \text{ GeV}$$



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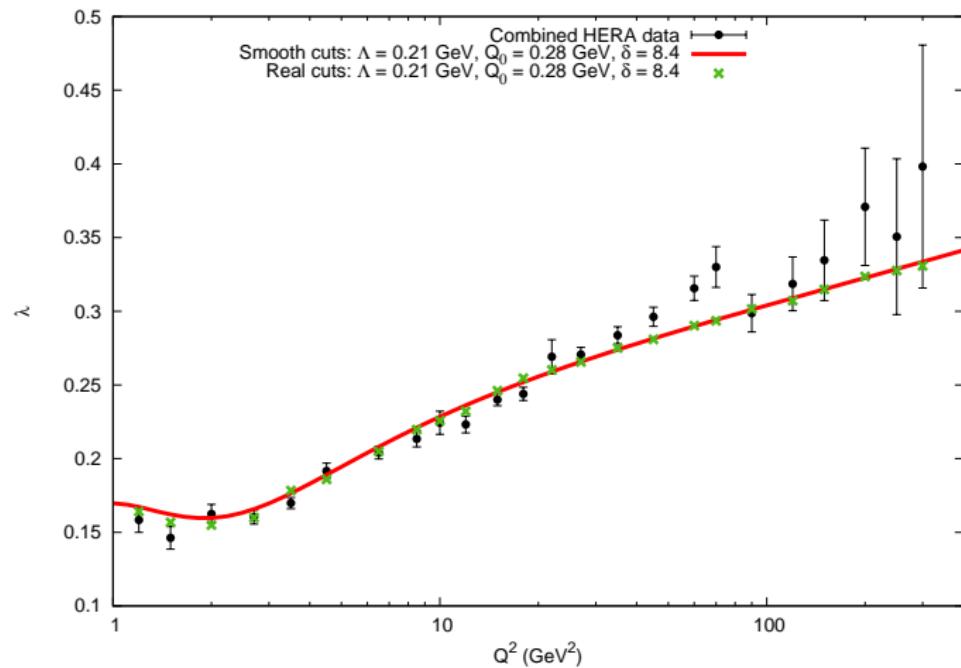
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[Hentschinski, Sabio Vera, Salas, arXiv:1209.1353]

Comments and conclusions

- The collinear improved resummation is needed to fit the data
- Use of BLM renormalization scale & MOM scheme
Better description of the IR
- Model for the running coupling with IR freezing
- Theoretical uncertainties:
 - choice of energy scale
 - choice of differential operator
 - what comes into the running?
- Work in progress...
 - Include quark masses [White, Peschanski, Thorne, hep-ph/0606169v1]
 - Include saturation effects to improve behavior at small Q^2
 - Include color threshold
 - NLO photon impact factor [Balitsky, Chirilli (2012) hep-ph/1207.3844]
 - Numerically exact implementation with Monte Carlo code