MINLO

Multiscale Improved NLO

Giulia Zanderighi
University of Oxford & STFC

Work done in collaboration with Keith Hamilton and Paolo Nason
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Benefits of NLO

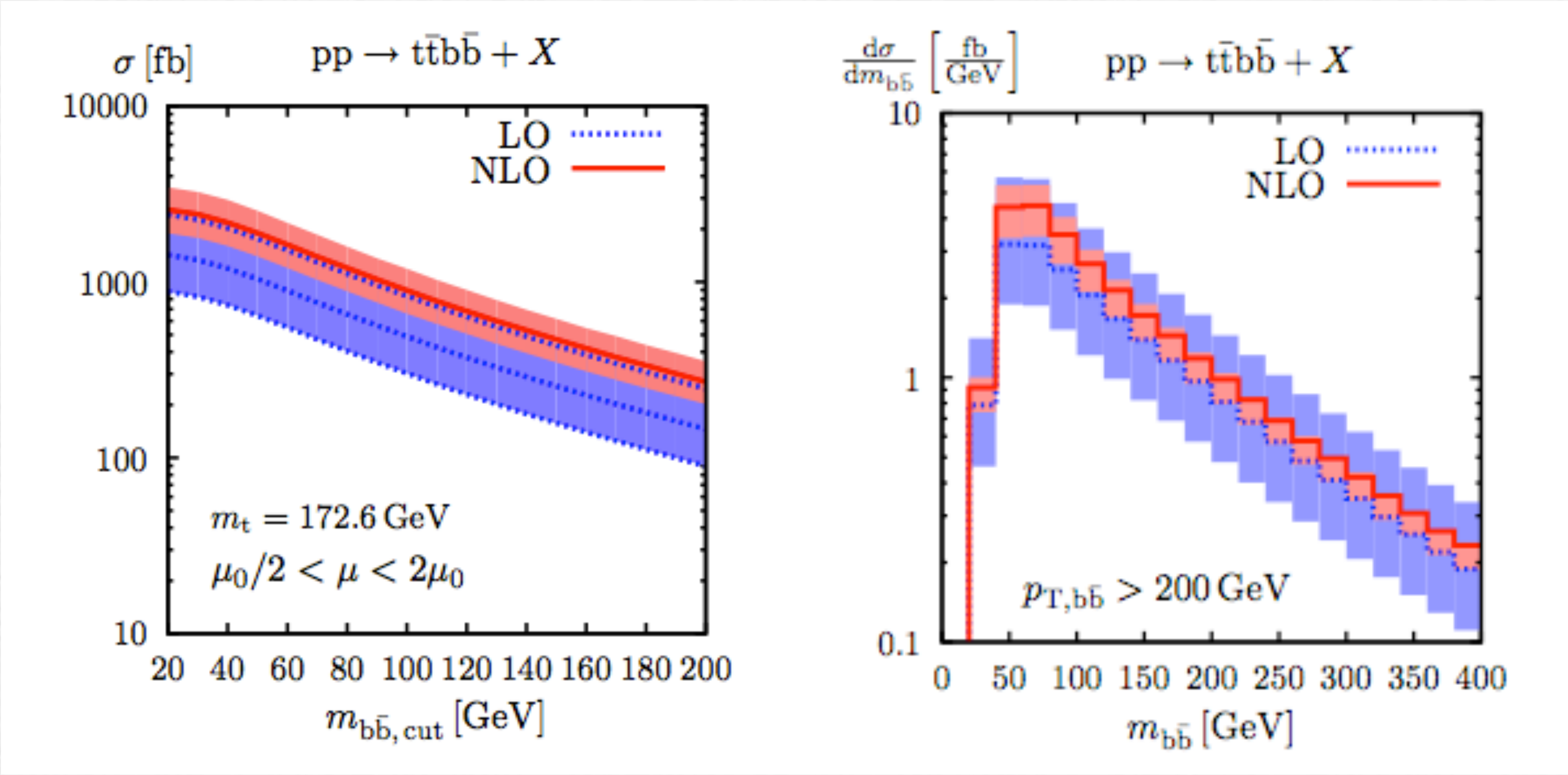
- Leading Order is often a quite crude approximation
  - normalization can float arbitrarily up and down by just changing α_s (more so, the more jets in the final state)
  - poor control on shapes of distributions (***but BSM searches rely heavily on a solid control of shapes to extrapolate backgrounds from control regions to signal regions***)
  - poor description of jets, no any internal substructure (1jet=1parton) ***(but lots of recent work uses jet-substructure)***
- Next-to-Leading order is just "a better approximation" to data: experience at LEP and Tevatron teaches us that it is so. This is also manifest in reduced theory uncertainties, often estimated by varying renormalization and factorization scales

BUT: even at NLO the scale choice is an issue and different choices can lead to a different picture/contrasting conclusions

Scale choice at NLO

Example where a scale choice leads to a different picture at NLO

Bredenstein et al. 0905.0110, 1006.2653



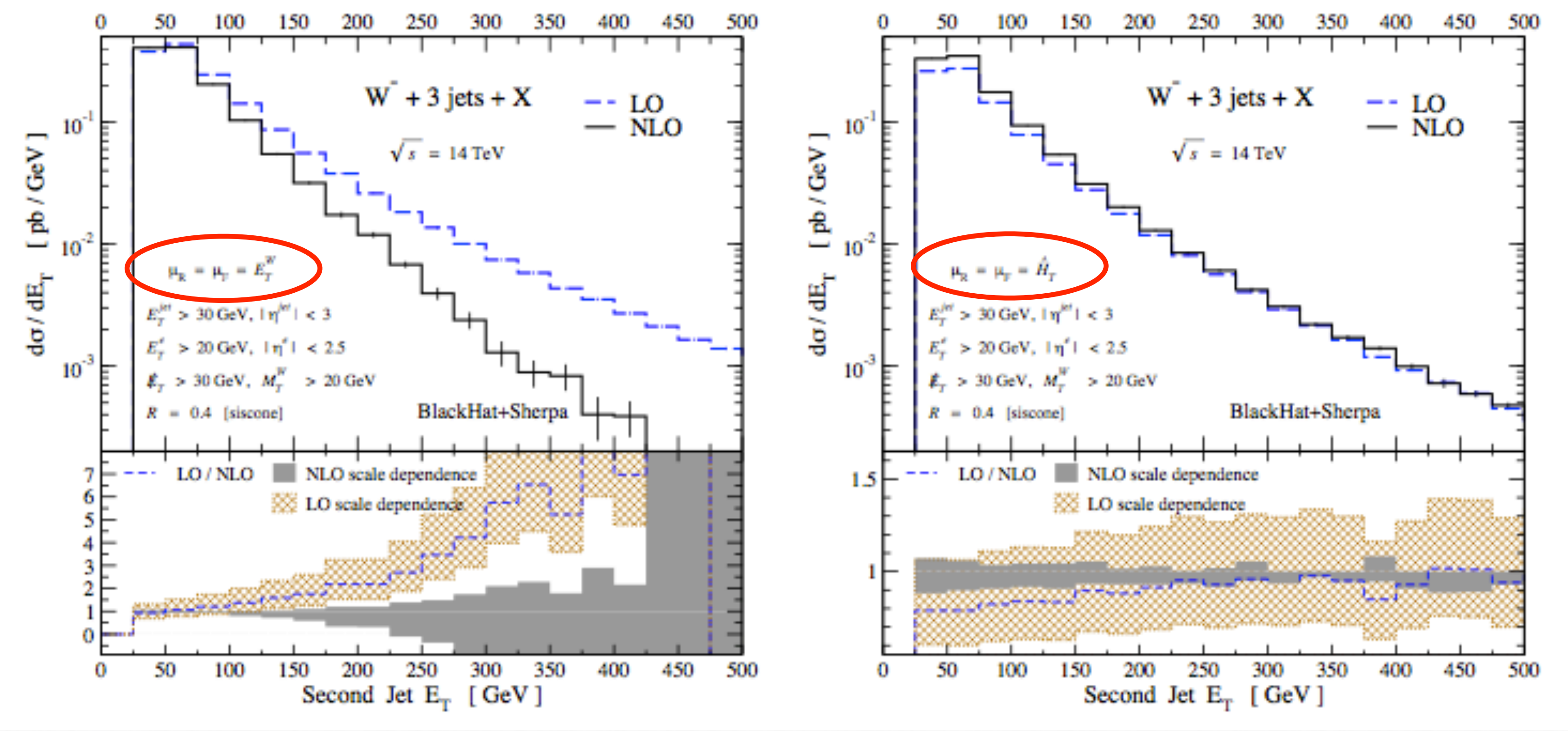
$\mu_0 = m_t + m_{bb,cut}/2$ $\mu_0^2 = m_t\sqrt{p_{t,b}p_{t,\bar{b}}}$

ttbb important background to ttH with H → bb. Whether or not we can control this background to better than 20% makes a crucial difference ***(ttH is unique to measure the ttH Yukawa coupling)***

Scale choice at NLO

Example where a scale choice leads to a different picture at NLO

Bern et al. 0907.1984



W+ multi-jet processes are important backgrounds to SUSY searches at high transverse energies

Could quote many more examples. In general the problem is more severe as the number of jets increases (as more scales come into play)

H + 1jet example

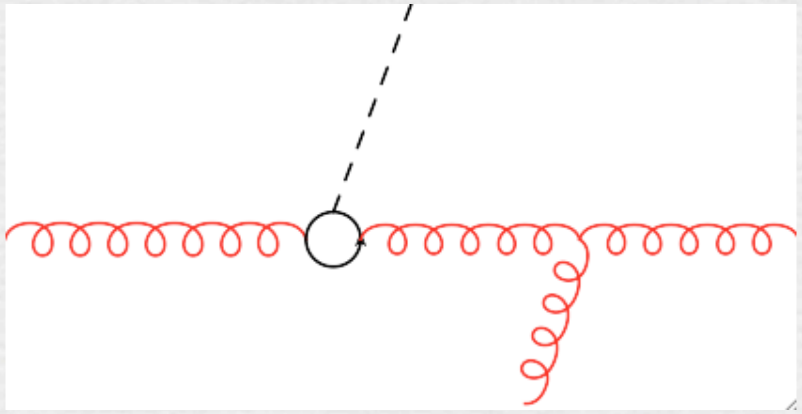
But even in very simple processes the "best scale choice" is not clear.

Simple example: H+1 jet production

The jet has most likely very small transverse momentum, and so has the Higgs. Therefore two very different scales are present m_H and $p_{t,H} \sim p_{t,j}$

One can argue in favour of a scale choice

of the type $\alpha_s(M_H)^2\, \alpha_s(p_{T,H})$ or of the type $\alpha_s(p_{T,H})^3$ or even $\alpha_s(M_H)^3$



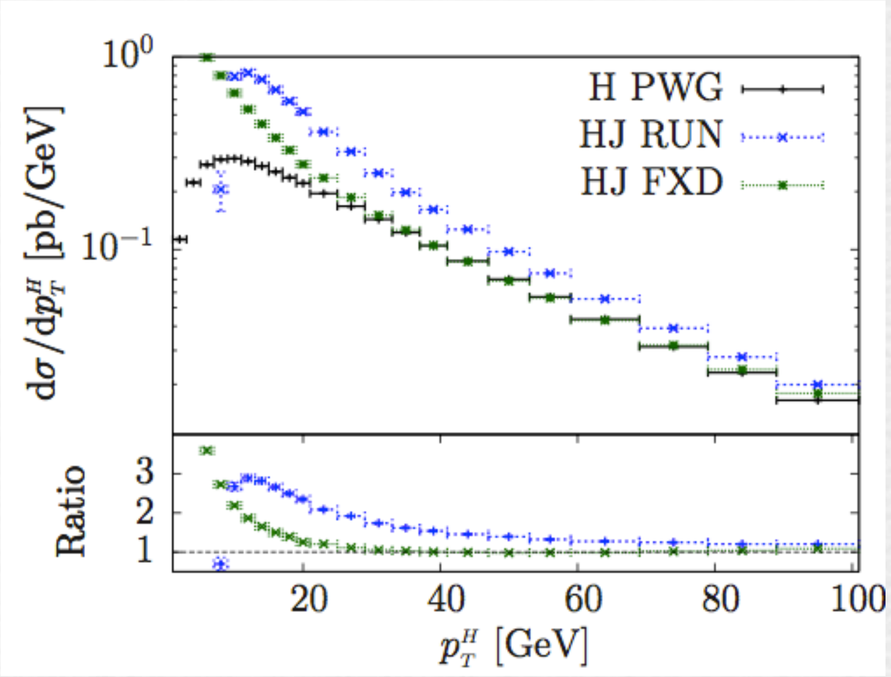
These scales lead to incompatible results

(but we'll show that there is no incompatibility once Sudakov form factors are properly taken into account)

Scale choice in H+1 jet

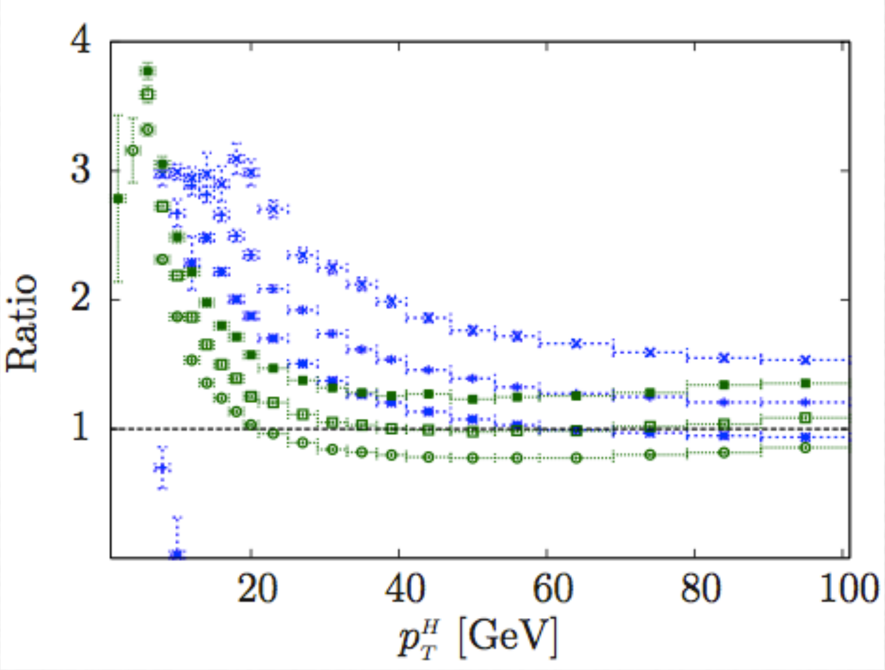
- H PWG: gluon-gluon fusion inclusive Higgs in POWHEG+PYTHIA
- HJ RUN: H+1jet in PWG+PYTHIA with running scale $\mu_R=\mu_F=p_{T,H}$
- HJ FXD: H+1jet in PWG+PYTHIA with fixed scale $\mu_R=\mu_F=M_H$

Spectrum at default scale choice



☛ HJ results differ already at moderate $p_{T,H}$

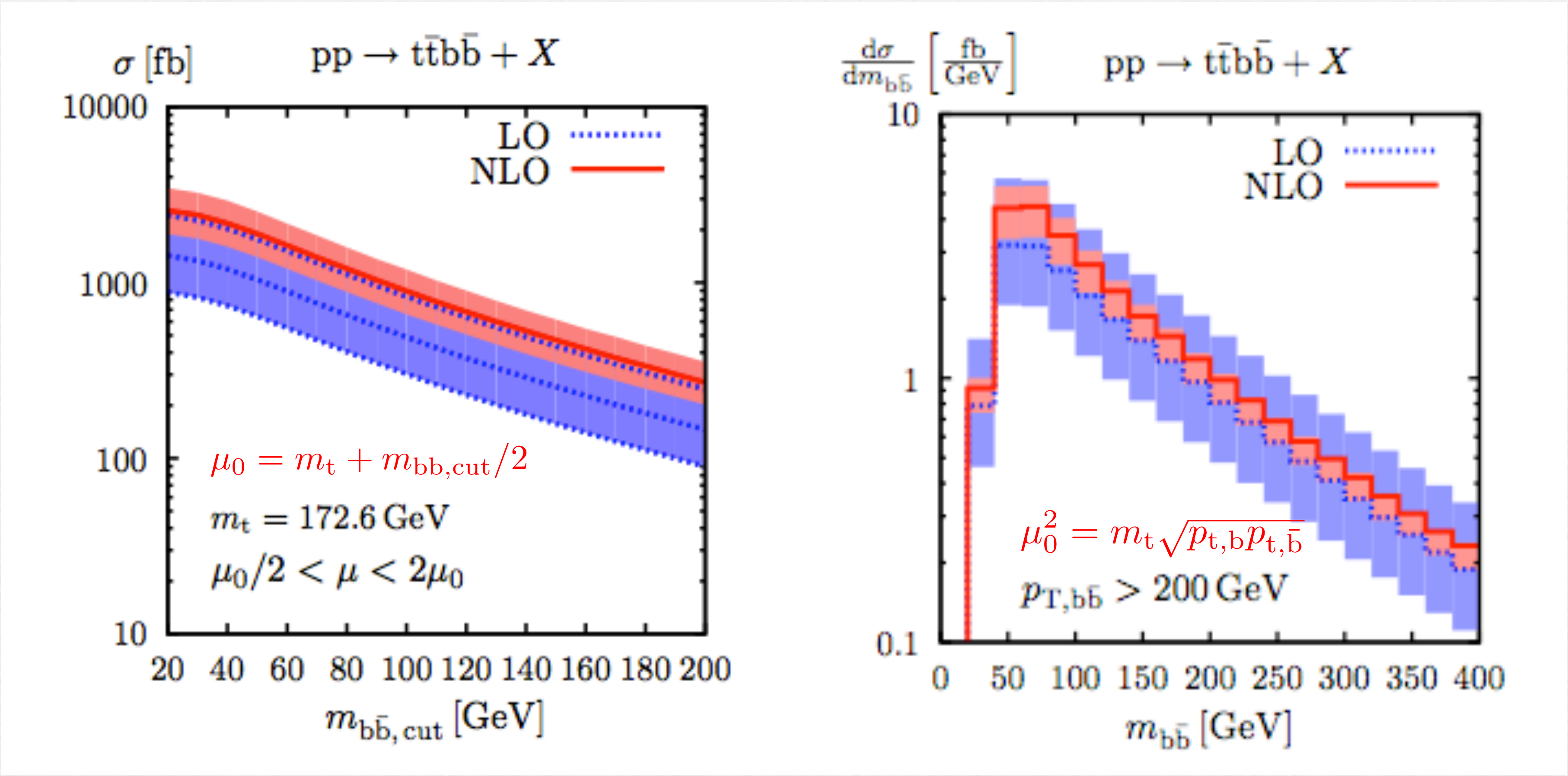
Ratio over inclusive production



☛ scale bands do not overlap

Scale choice at NLO

Often a “good scale” is determined ***a posteriori***, either by requiring NLO corrections to be small, or by looking where the sensitivity to the scale is minimized



bad scale

good scale

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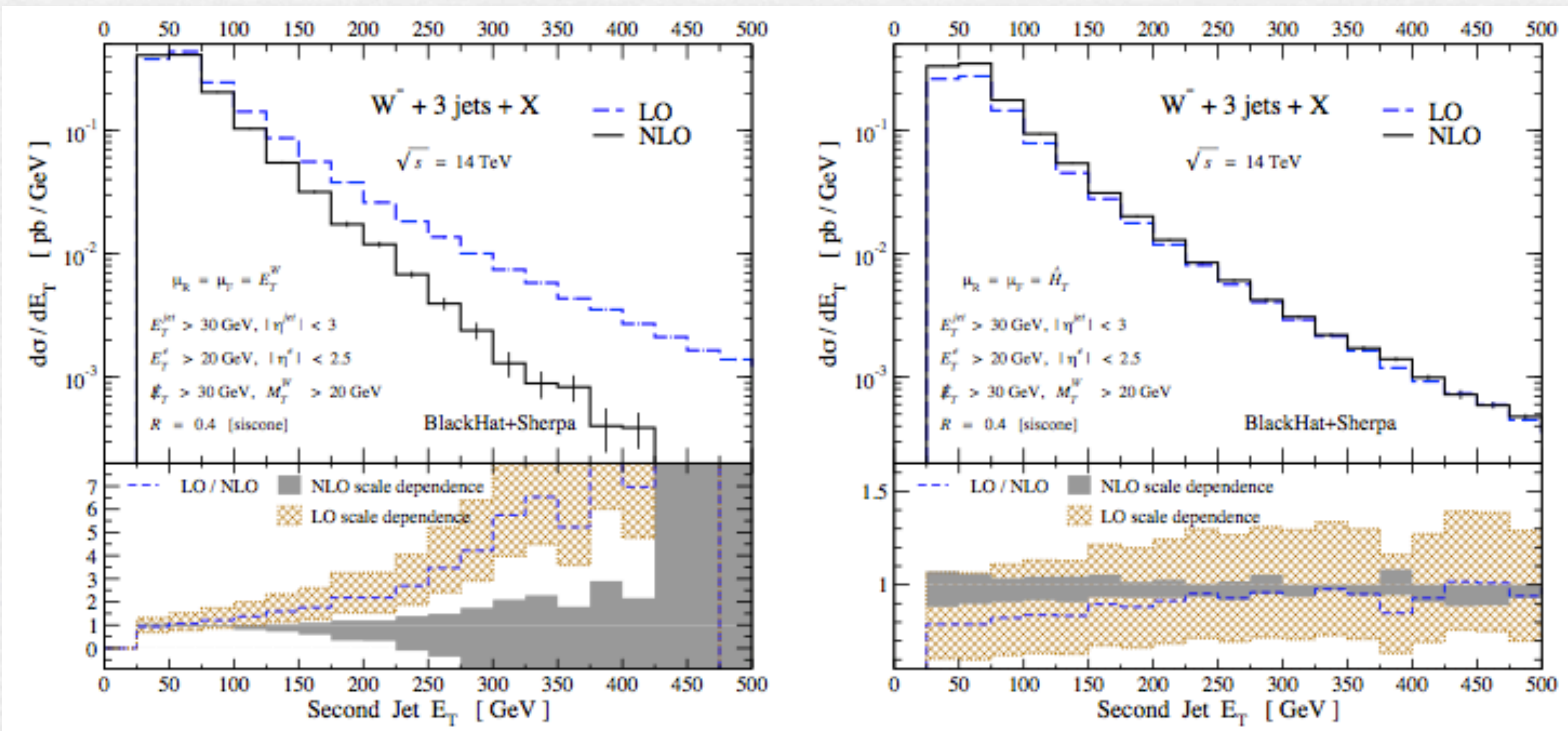

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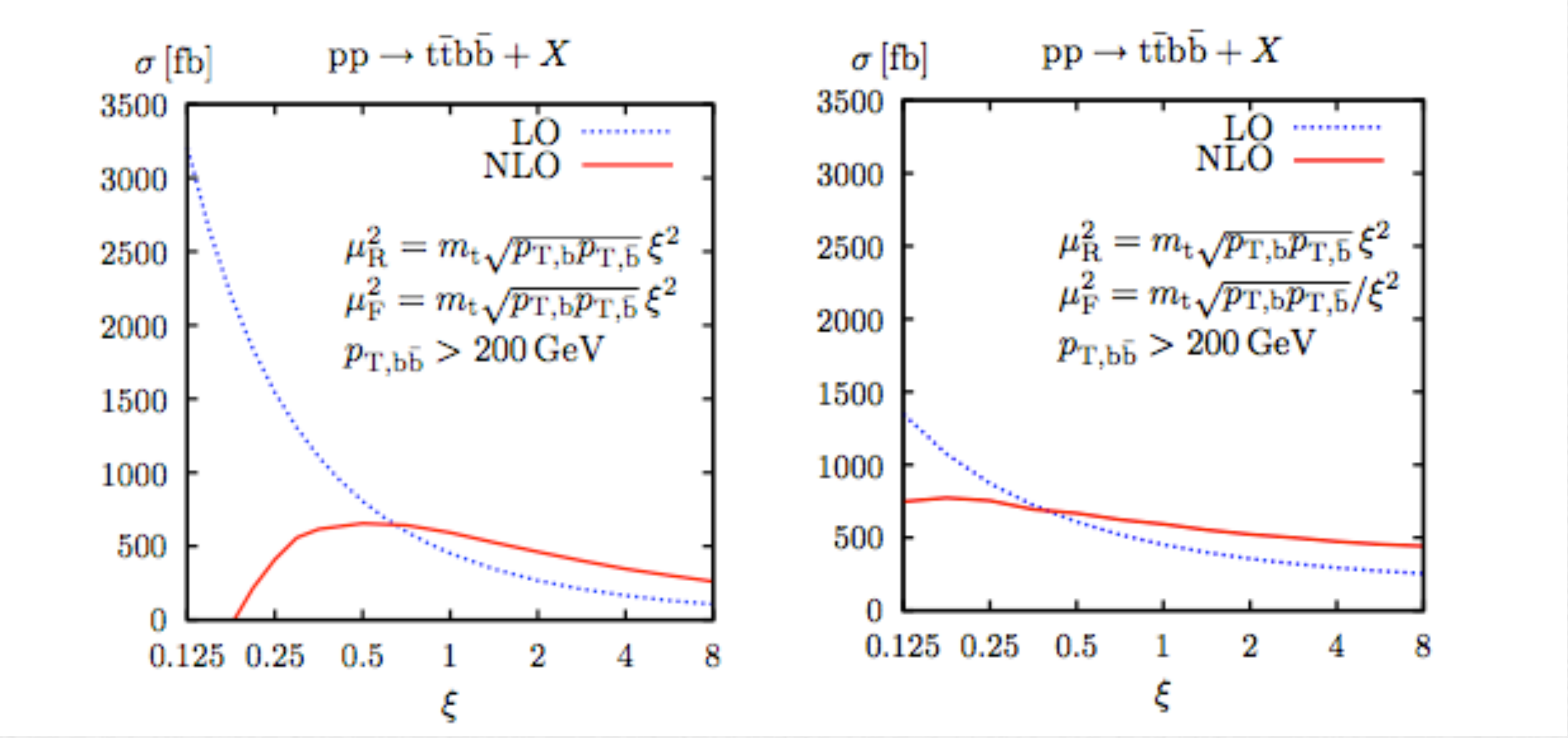


good range

good range

Scale choice at NLO

Often a "good scale" is determined ***a posteriori***, either by requiring NLO corrections to be small, or by looking where the sensitivity to the scale is minimized

Reason: bad scale ➠ large logs ➠ large NLO and large scale dependence

But we also know that large NLO ~~➠~~ bad scale choice, since NLO corrections can have a "genuine" physical origin
(new channels opening up, Sudakov logarithms, color factors, large gluon flux ...)

Furthermore, double logarithmic corrections can never be absorbed by a choice of scale (single log). So a "stability criterion" can be misleading.

Scale prescriptions

- Principal of Minimal Sensitivity (PMS): choose the central scale so that the NLO correction vanishes. Then the scale dependence is minimized. PMS would work well if scale logarithms were the only source of large logarithms, but we know they are not.
- Brodsky, Lapage, Mackenzie (BML) suggested to resum as much as possible all β-function terms in NLO calculations (i.e. exactly in QED, n_f terms in QCD)
- Brodsky & Giustino: extend this to the Principal of Maximal Conformality: resum all non-conformal terms ($\beta \neq 0$) (PMC)
- ....

But, hard to see how the associated theory uncertainty can be reliably estimated in the presence of spurious (forced) compensation mechanisms

Scale prescriptions

We are not trying to follow a similar approach here, instead we'll argue that a scale choice should be discussed in conjunction with Sudakov form factors

N.B.
We know that the use of scale variation to asses theory uncertainties has serious limitations
(e.g. it does not work in conformal invariant theories, it has no value in QED where photon polarization effects can be resummed exactly ...).
In QCD it often works well in practice and it is simple. That is why it has become a standard.

Scale choice at LO

LO calculations in matrix elements generators that follow the CKKW procedure are quite sophisticated in the scale choice:

they use optimized/local scales at each vertex and Sudakov form factors at internal/external lines

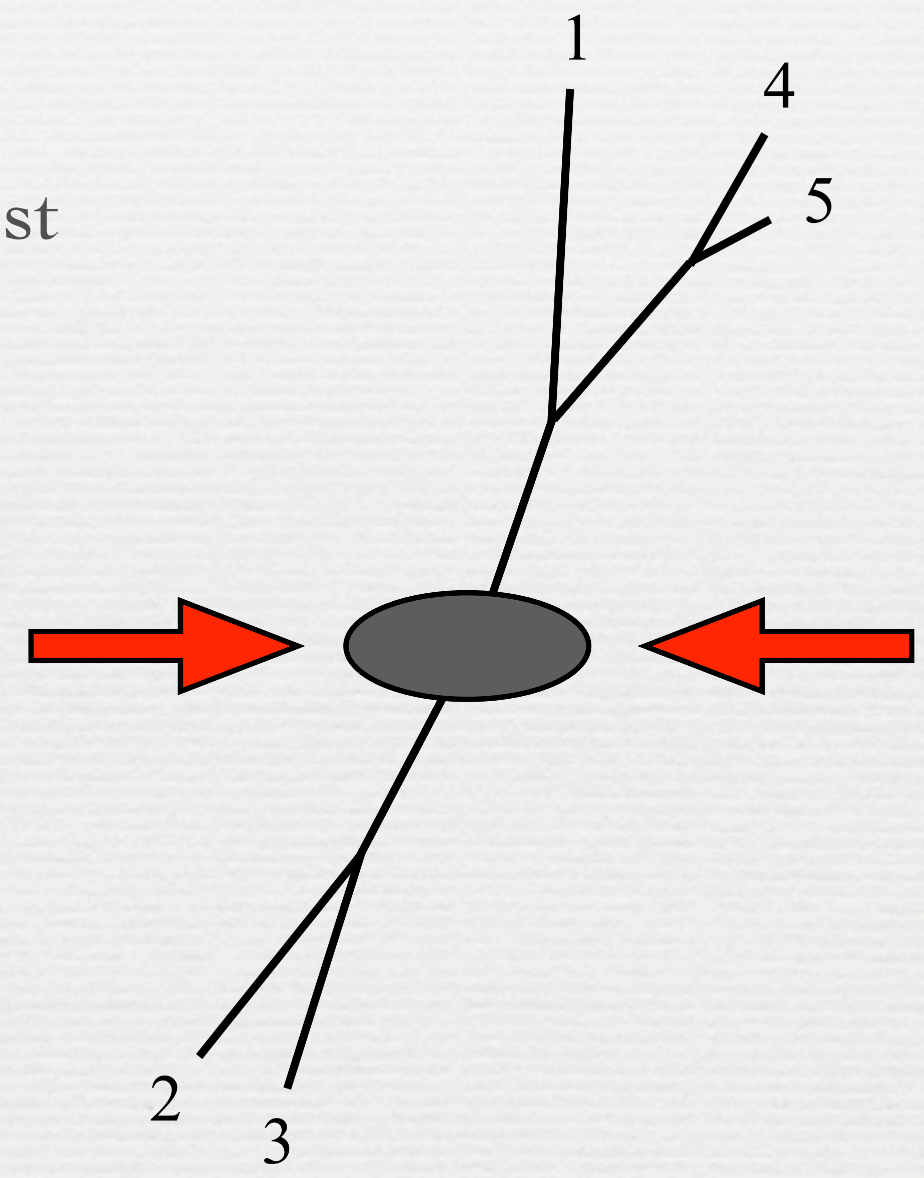
Catani,Krauss, Kuehn, Webber '01
extension to hh collisions *Krauss '02*

Reminder:
a Sudakov form factor encodes the probability of evolving from one scale to the next without branching

Recap of CKKW

The *CKKW* prescription in brief:

- use the k_t algorithm to reconstruct the most likely branching history



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The *CKKW* prescription in brief:

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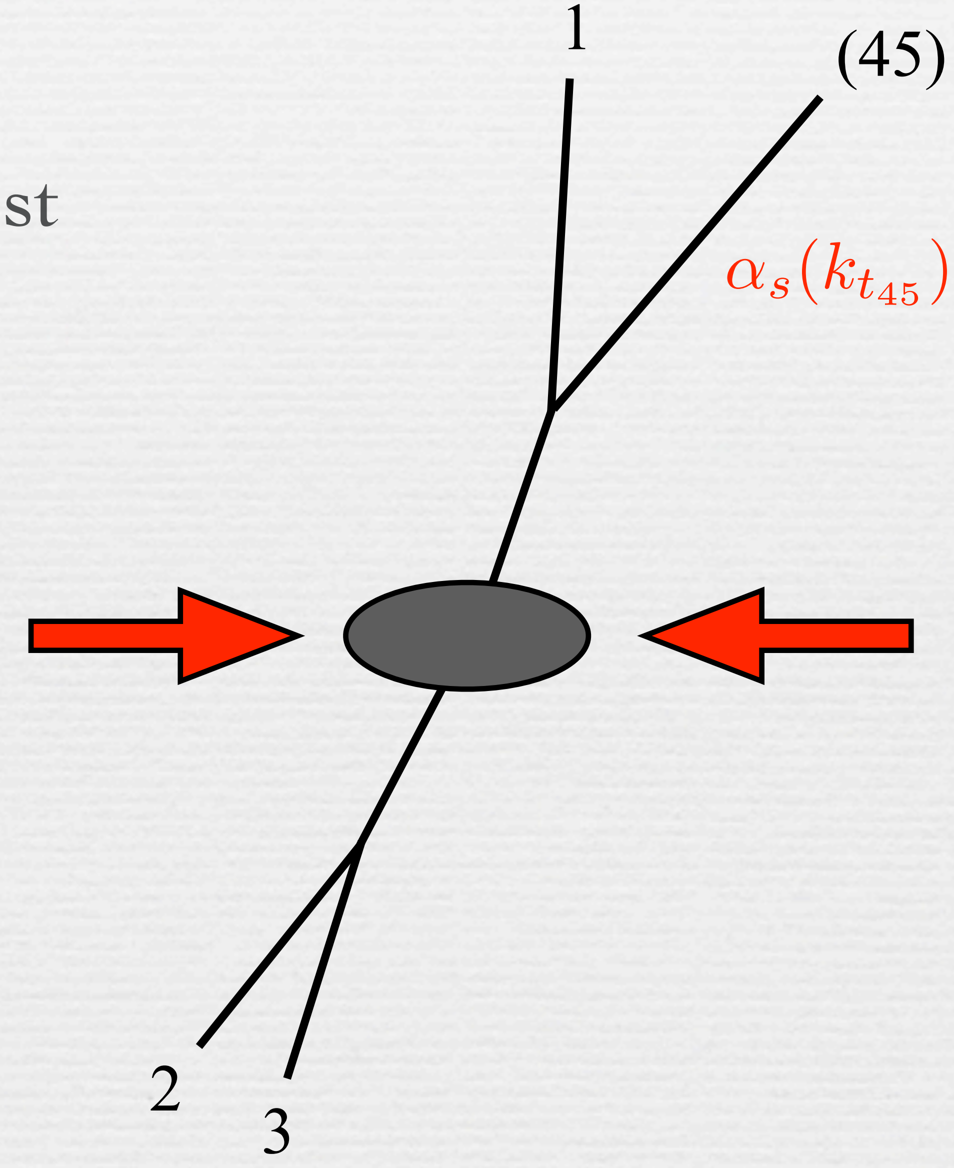

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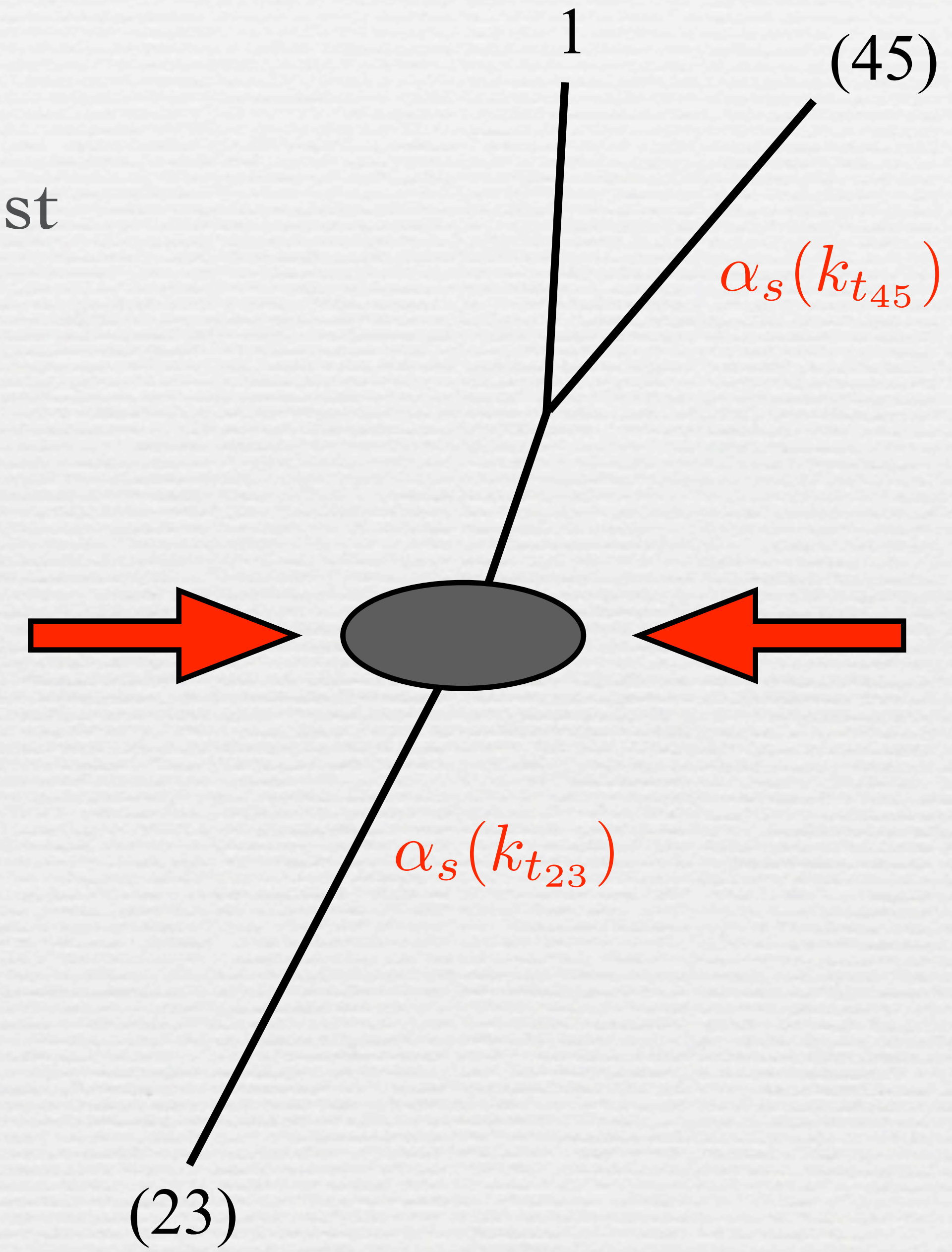

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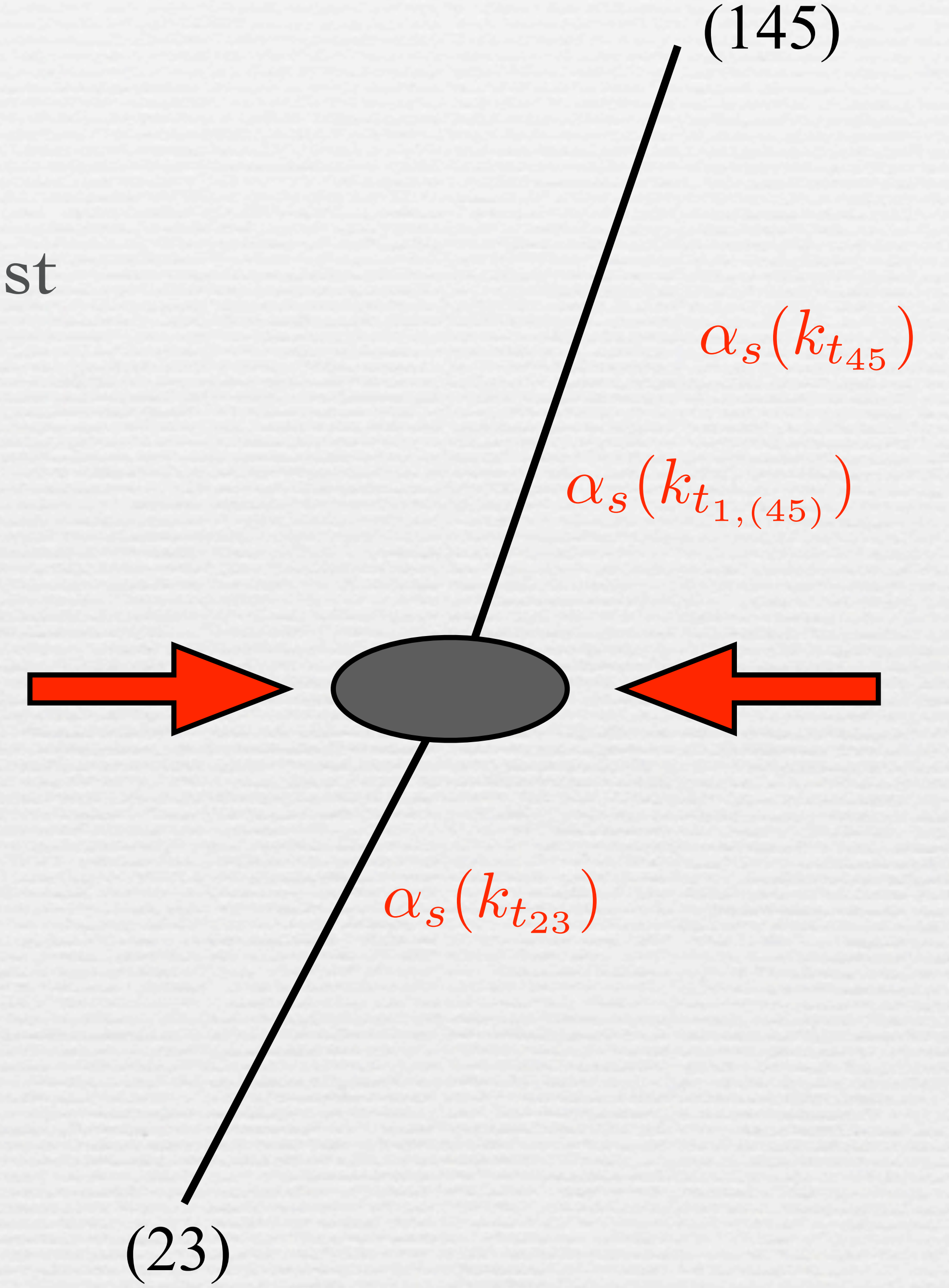

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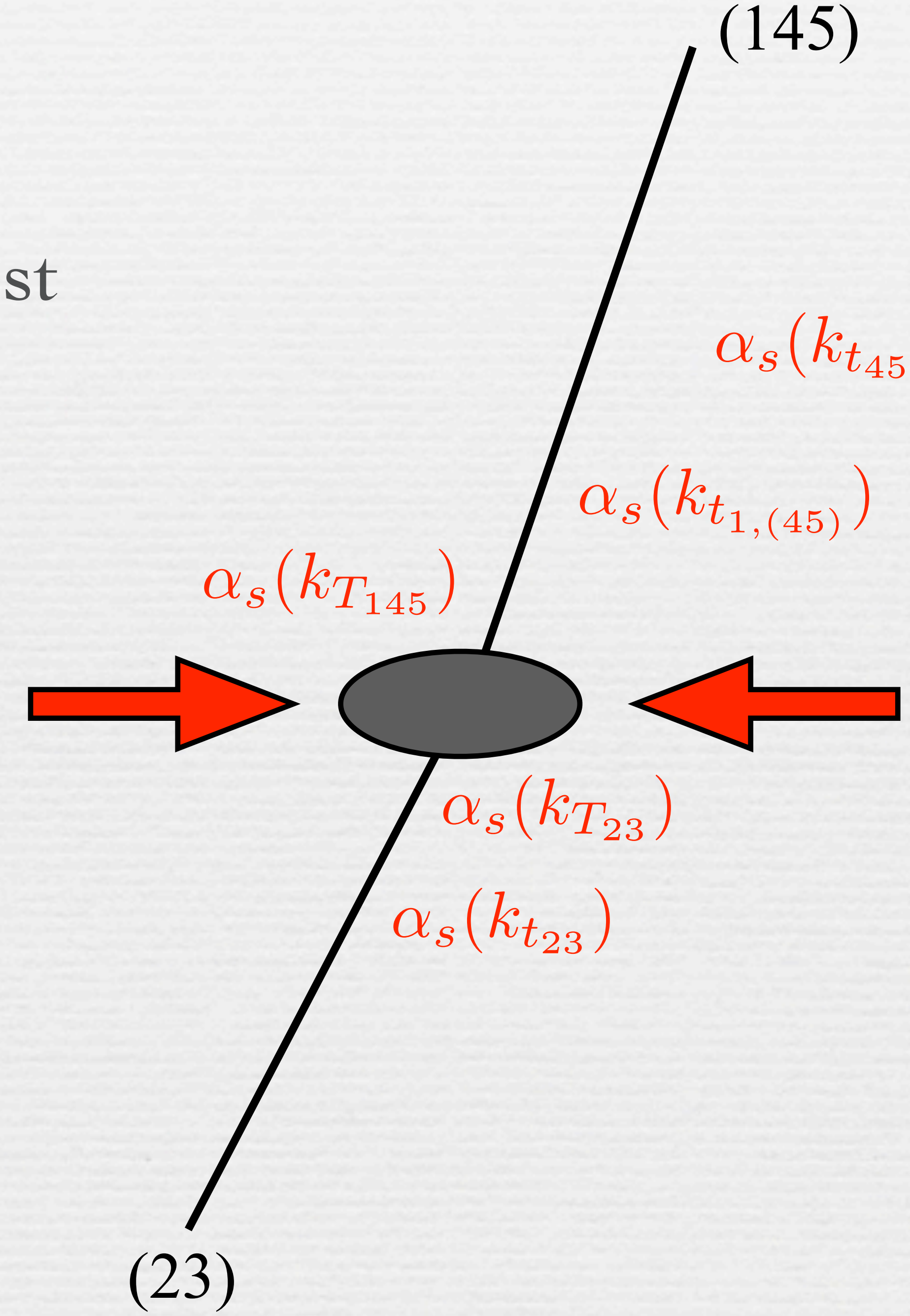

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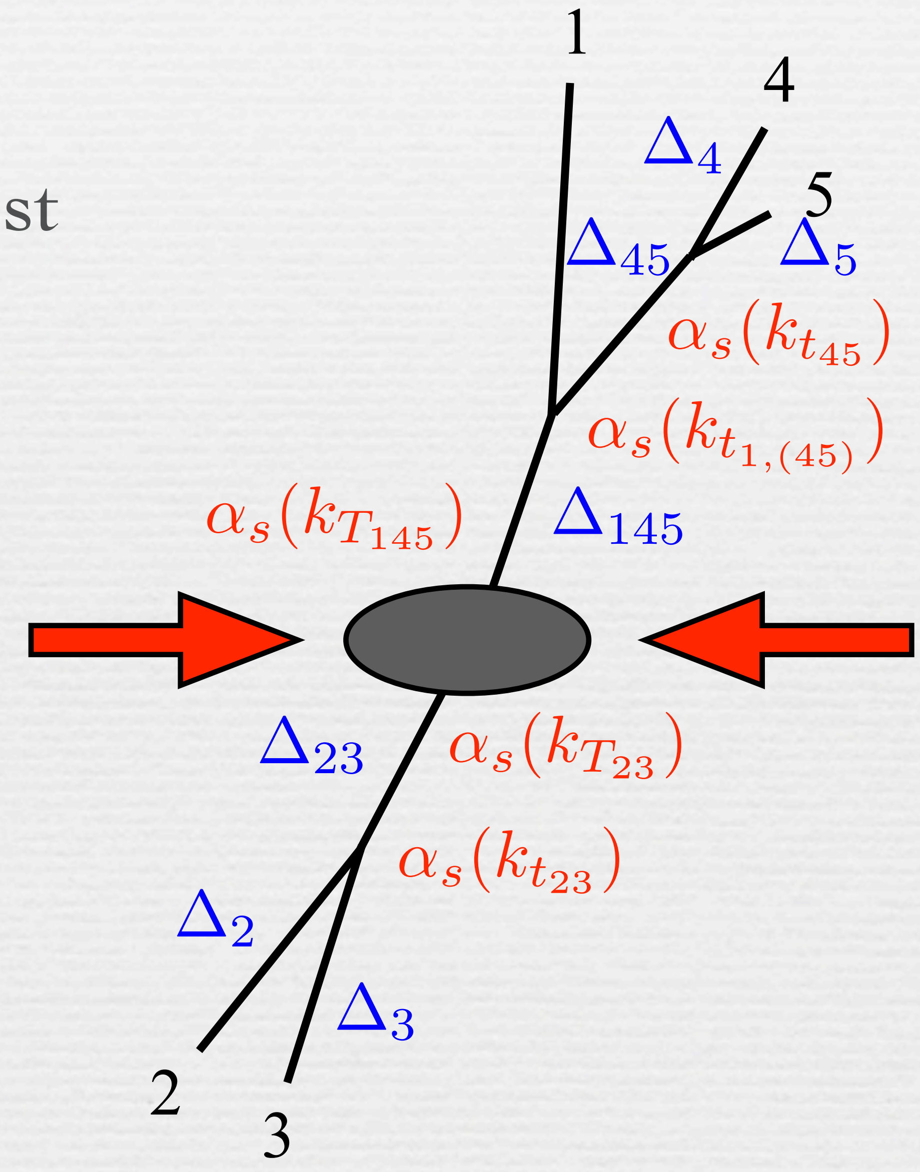
- evaluate each α_s at the local transverse momentum of the splitting
- for each internal line include a Sudakov form factor $\Delta_{ij}=D(Q_0,Q_i)/D(Q_0,Q_j)$ that encodes the probability of evolving from scale Q_i to scale Q_j without emitting. For external lines include $\Delta_i=D(Q_0,Q_i)$



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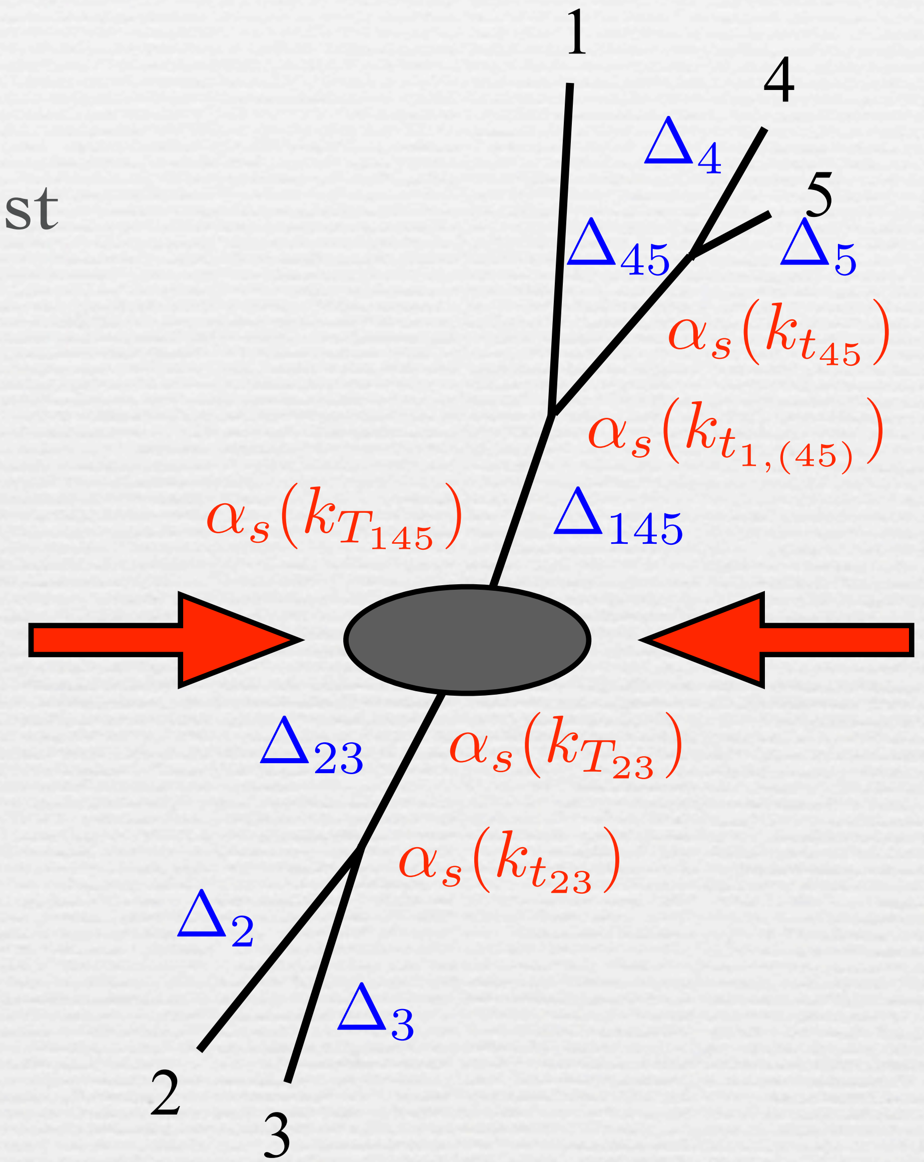

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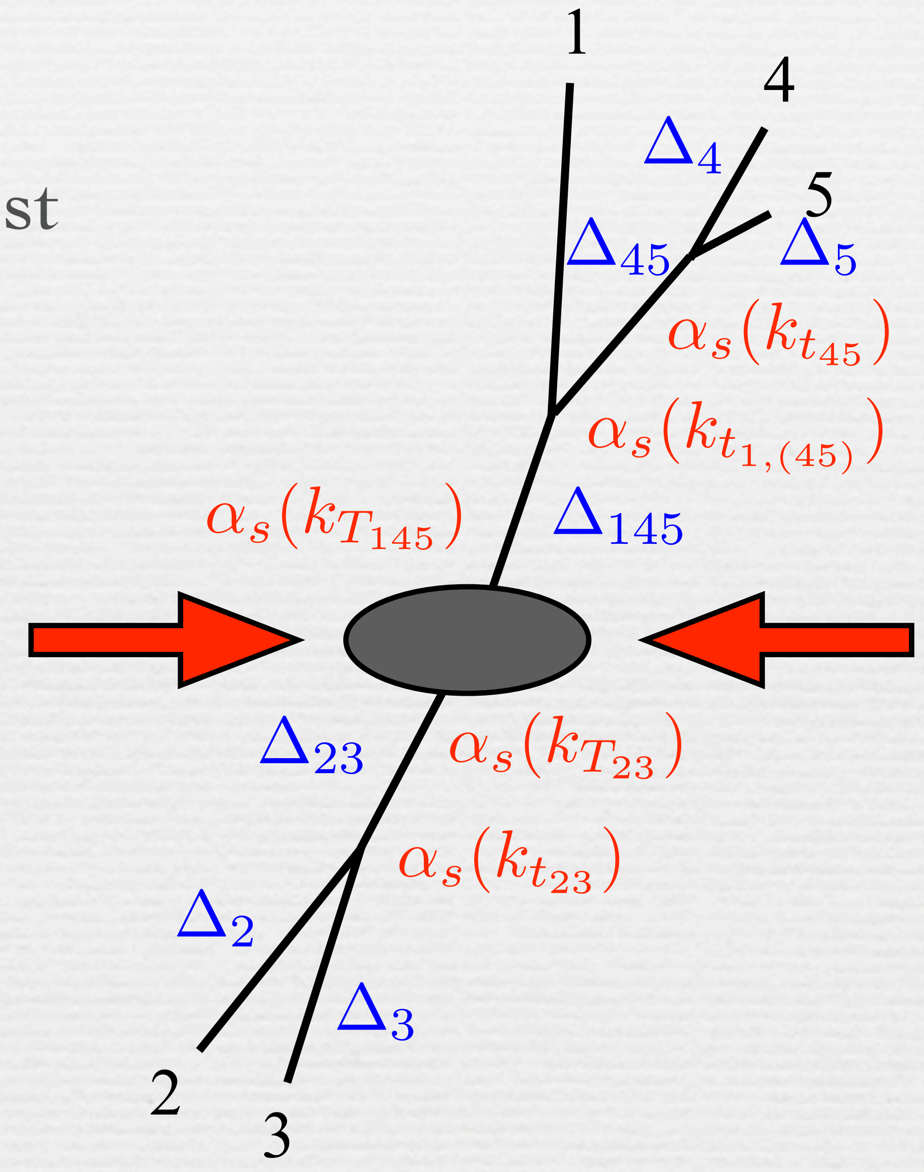


Scale choice intervened with inclusion of Sudakov form factors

Aim of this work

The goal: formulate a procedure to compute the actual NLO corrections to matrix-element style LO calculations with Sudakov form factors, such that the procedure to fix the scale is unbiased and decided ***a priori***

In particular, we focus on processes involving **many scales** (e.g. X +multi-jet production) and on soft/collinear branchings, i.e. we focus on the region where it is more likely that associated jets are produced.

Two observations

1. A generic NLO cross-section has the form

$$\alpha_{\mathrm{S}}^{n}\left(\mu_R\right)B+\alpha_{\mathrm{S}}^{n+1}\left(\mu_R\right)\left(V\left(Q\right)+nb_0\log\frac{\mu_R^2}{Q^2}B\left(Q\right)\right)+\alpha_{\mathrm{S}}^{n+1}\left(\mu_R\right)R$$

   Adopting CKKW scales at LO, this becomes naturally

$$\alpha_s(\mu_1)\ldots\alpha_s(\mu_n)B+\alpha_s^{n+1}(\mu_R')\left(V(Q)+b_0\log\frac{\mu_1^2\ldots\mu_n^2}{Q^{2n}}B\right)+\alpha_s^{n+1}(\mu_R'')R$$

   and the scale choices μ_R' and μ_R'' are irrelevant for the scale cancelation. This can be achieved for instance by evaluating the virtual term at a single scale equal to the geometric average of the Born scales

2. Sudakov corrections included at LO via the CKKW procedure lead to NLO corrections that need to be subtracted to preserve NLO accuracy

Arbitrariness

When trying to extend the CKKW procedure to NLO there is arbitrariness in

1. the arguments of α_s in the real and virtual term
2. the exact definition of the subtraction terms of the NLO terms in the Born Sudakovs
3. whether or not to include Sudakovs in the real and virtual

Our guiding principle is that the virtues of the CKKW result at leading order are maintained once radiative corrections are included

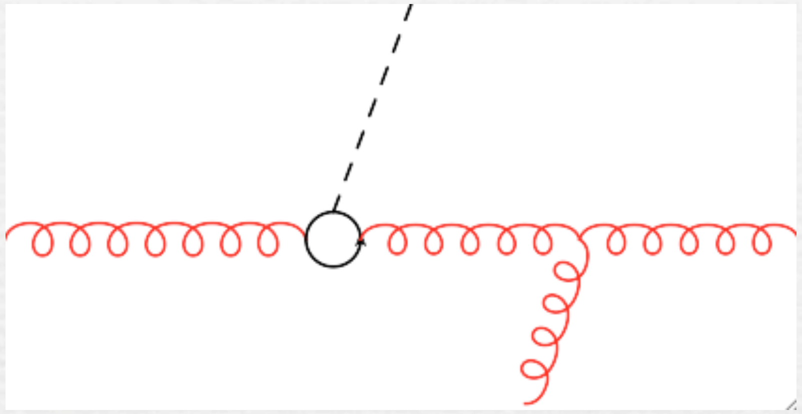
This method is completely new, for simplicity we suggested one definite choice among various options, more experience will tell if other options are better.

The MINLO method

1. Find the CKKW clustering scales $q_1 < \ldots < q_n$ (and $q_0 < q_1$ for the real term). Fix the hard scale of the process Q to the system invariant mass after clustering. Set Q_0 to q_1 (inclusive on radiation below q_1)
2. Evaluate the n coupling constants at the scales q_i (times a scale factor to probe scale variation)
3. Set μ_R in the virtual to the geometric average of these scales and μ_F to the softest scale q_1 s
4. Include Sudakov form factors for Born and NLO terms (for the real only after the first branching)
5. Subtract the NLO bit present in the CKKW Sudakov of the Born
6. The $(n+1)^{th}$ power of α_s in the real and virtual is evaluated at the arithmetic average of the n α_s in the Born term (corrections can be thought of as additive at each vertex)

Back to H+1j example

How is the mismatch between using $\alpha_s(M_H)^2\,\alpha_s(p_{T,H})$ or $\alpha_s(p_{T,H})^3$ addressed when Sudakov form factors are included?



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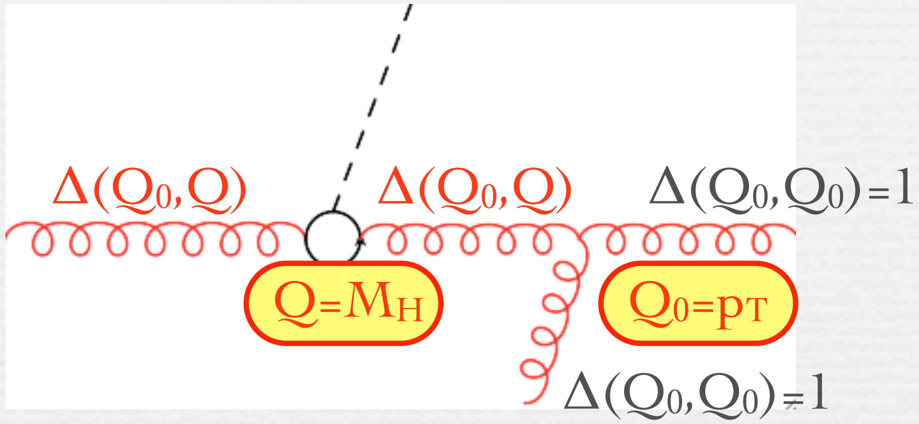

MINLO/CKKW procedure suggests to use $\alpha_s(M_H)^2\,\alpha_s(p_{T,H})$ supplemented by Sudakov form factors. We have two Sudakov factors, giving

$$F = \alpha_s^2(M_H)\alpha_s(p_T)\left\{\exp\left[-\frac{C_A}{\pi b_0}\left\{\log\frac{\log\frac{Q^2}{\Lambda^2}}{\log\frac{Q_0^2}{\Lambda^2}}\left(\frac{1}{2}\log\frac{Q^2}{\Lambda^2}-\frac{\pi b_0}{C_A}\right)-\frac{1}{2}\log\frac{Q^2}{Q_0^2}\right\}\right]\right\}^2$$

NLL Sudakov

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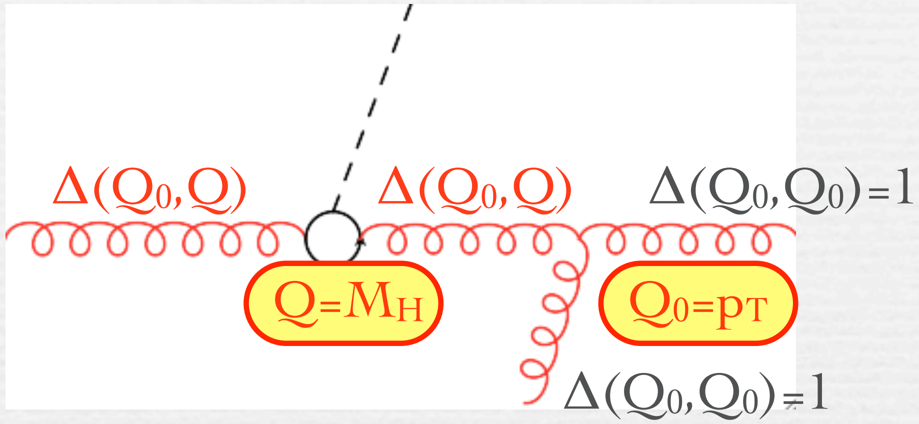

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NLL Sudakov

Which is equivalent to

$$\alpha_s^3(p_T)\exp\left[-\frac{C_A}{\pi b_0}\left\{\log\frac{\log\frac{Q^2}{\Lambda^2}}{\log\frac{Q_0^2}{\Lambda^2}}\log\frac{Q^2}{\Lambda^2}-\log\frac{Q^2}{Q_0^2}\right\}\right]$$

LL Sudakov

So, both choices are "fine" but one should not forget double logarithms in the Sudakov form factors, which are more important than the single logs in the scale choice.

Properties of MINLO

MINLO satisfies the following requirements

- the result is accurate at NLO, i.e. the scale dependence is NNLO
- the accuracy in the Sudakov region is Leading Log (LL) or better, according to the form of the Sudakov used
- the smooth behaviour of the CKKW scheme in the singular regions is preserved
  - X+n-jet cross-sections are finite even without jet cuts (do not need expensive generation cuts or Born suppression factors)
  - X+n-jet cross-sections reproduce the inclusive cross-section accurate to LO (NB is finite rather than divergent!)
- the procedure is simple to implement in any NLO calculation, i.e. the improvement requires only a very modest amount of work

It is then interesting to see how the method fares in practice

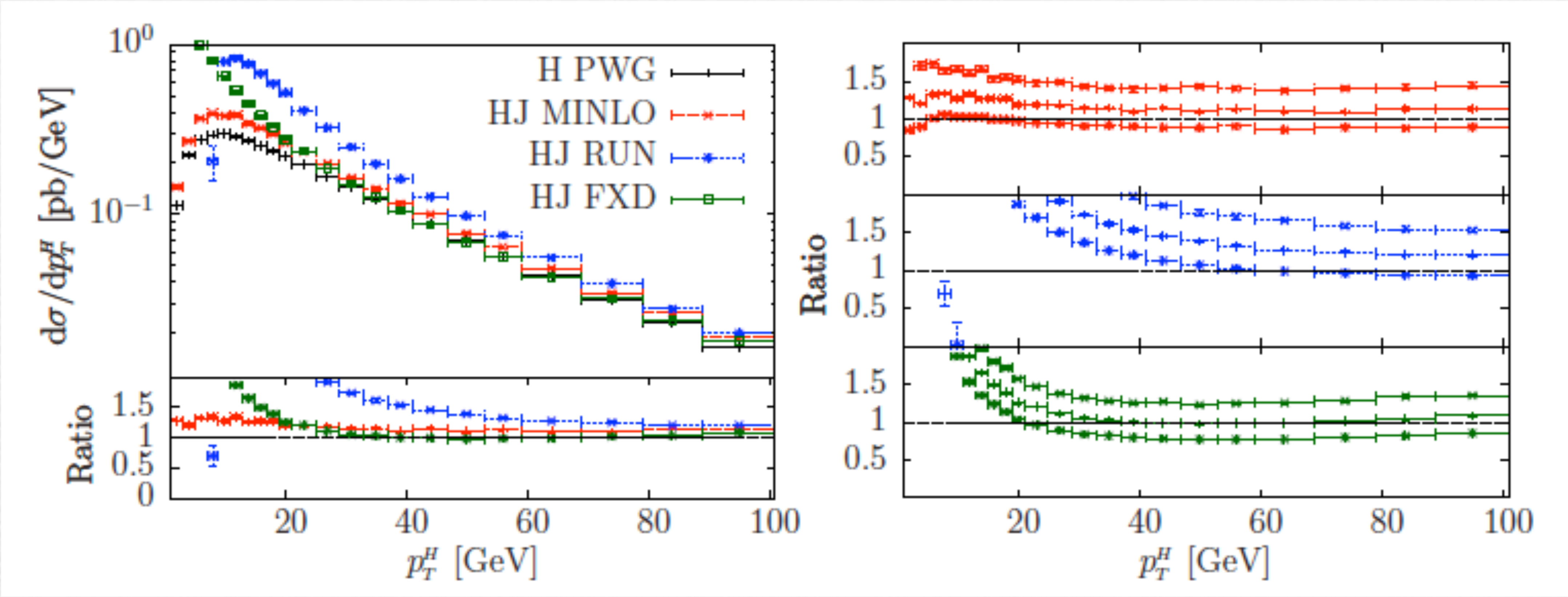
Phenomenology

To asses how the method fares in practice, we considered the following processes

- H+1jet, H+2jets, Z+1jet, Z+2jets. We implemented the latter process ourselves in POWHEG using the automated MadGraph4 interface (available thanks to Rikkert Frederix) and taking virtual corrections from MCFM.
- we compare the MINLO predictions to standard NLO results with a number of common scales used for these processes
- we compare the MINLO predictions with POWHEG results with (n-1) jets

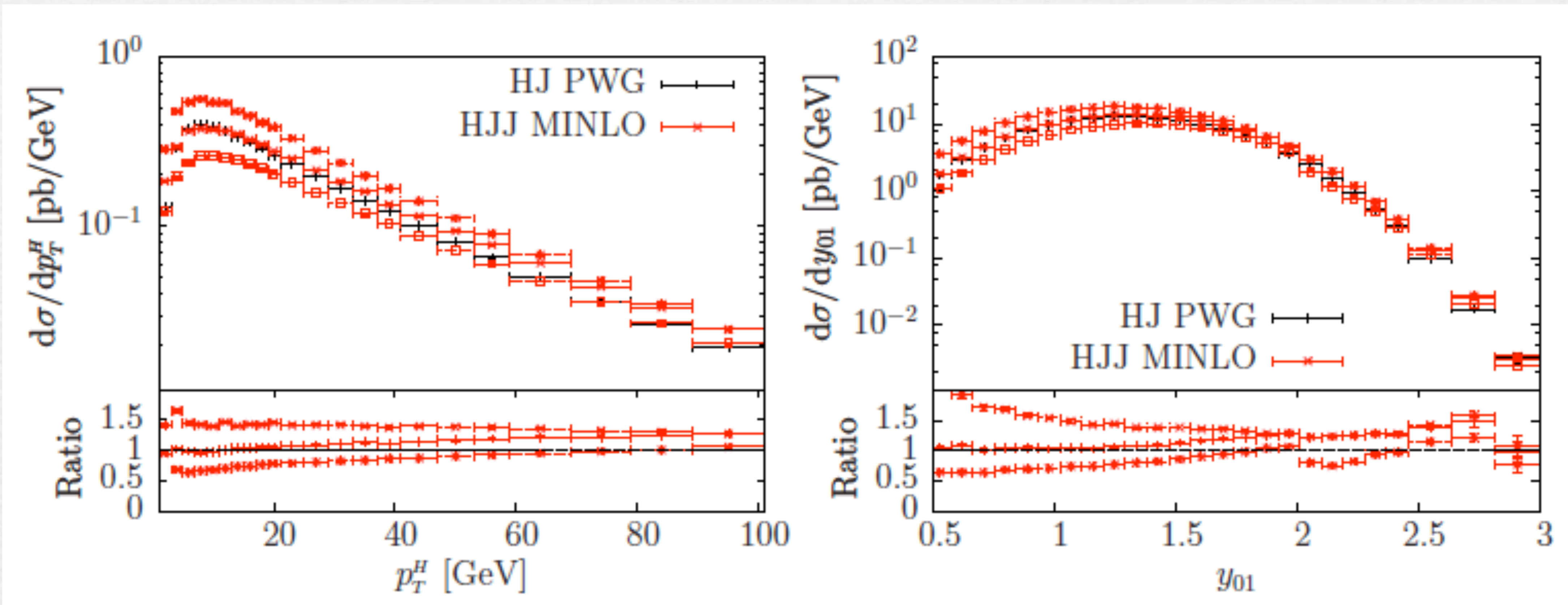
We use a standard LHC setup, but since MINLO includes Sudakov form factors, **we do not need to impose any jet cut**. We generated hundreds of distributions, I'll just show few simple examples here.

H+1jet



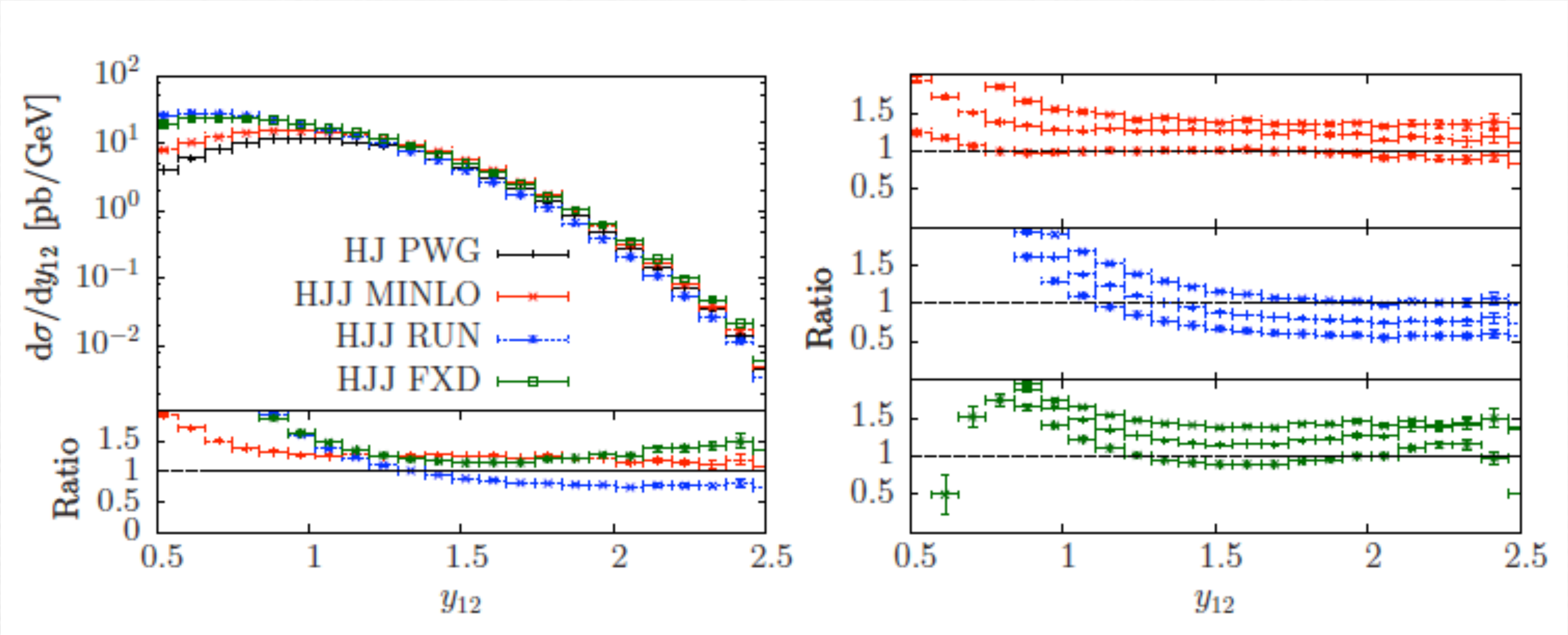
- MINLO mimics POWHEG all the way down to very small $p_{T,H}$ where standard H+1j order results diverge
- MINLO uncertainty band compatible with POWHEG all the way down to low transverse momenta
- MINLO more compatible with fixed rather than running scales (surprising? No, running scale misses Sudakov)

H+2jets



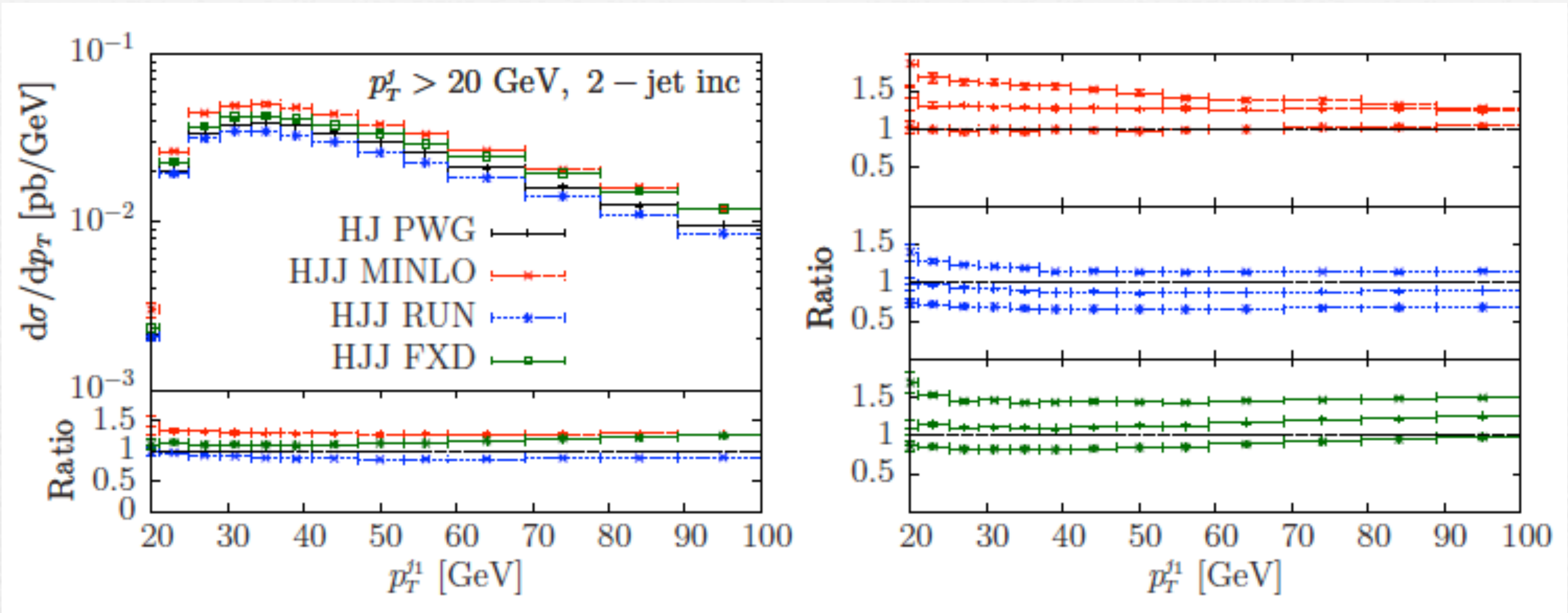
- without cuts impossible to compare to Standard NLO
- again, MINLO uncertainty band compatible with POWHEG all the way down to low transverse momenta

H+2jets



- for the jet-resolution parameter y_{12} both MINLO and standard NLO are predictive, and mostly agree well at large merging scales
- at small scales MINLO agrees better with POWHEG and has a better scale stability
- standard NLO have unphysical behavior at small scales

H+2jets



- running scale (H_T) is outside the band of MINLO
- using $H_T/2$ leads to much better agreement
- $H_T/2$ has become the preferred scale because it leads to an improved scale stability
- the MINLO result confirms, independently, this choice

Conclusions

MINLO is a simple procedure to assign scales and Sudakov form factors in NLO calculations to account for distinct kinematical scales. It can be thought of an extension to NLO of the CKKW procedure. Key features are

- results are well-behaved in the Sudakov region, where standard NLO calculations break down
- away from the Sudakov region, results are accurate at NLO
- MINLO agrees better with NLO using higher scale choices like $H_T/2$. Tempting to interpret this as due to the fact that large scales (smaller couplings) compensate the lack of a genuine Sudakovs
- first results look very promising, more to come
- the procedure is simple to implement NLO calculations, i.e. it requires only a very modest amount of work, just try it out ...