

Start date: 1 September 2011

Home country: Austria

Host country: Germany

## Educational Background:

- ▶ studies in technical mathematics, J. Kepler University Linz; graduated in March 2009
- ▶ title of the thesis: A computer Algebra Toolbox for Harmonic Sums Related to Particle Physics.
- ▶ since June 2009: PhD studies at the Research Institute for Symbolic Computation (RISC)

- ▶ host institute: DESY-Zeuten
- ▶ supervisor: Johannes Blümlein
- ▶ co-supervisor: Carsten Schneider
- ▶ project: WP4 Technology Innovations

## Present status:

- ▶ finished PhD in May
- ▶ PostDoc position at the Research Institute for Symbolic Computation (RISC)

# Deliverables and Milestones

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Two Mathematica Packages:

- ▶ **MultiIntegrate** allows to compute multi-dimensional integrals over hyperexponential integrands in terms of (generalized) harmonic sums. This package uses variations and extensions of the multivariate Almkvist-Zeilberger algorithm.
- ▶ **HarmonicSums** allows to deal with nested sums such as harmonic sums, S-sums, cyclotomic sums and cyclotomic S-sums as well as iterated integrals such as harmonic polylogarithms, multiple polylogarithms and cyclotomic polylogarithms in an algorithmic fashion.

Our strategy to evaluate integrals of the form

$$\int_{u_d}^{o_d} \dots \int_{u_1}^{o_1} F(n; x_1, \dots, x_d) dx_1 \dots dx_d,$$

where  $F(n; x_1, \dots, x_d)$  is a hyperexponential function is:

- ▶ compute a recurrence for the integrand  $F(n; x_1, \dots, x_d)$
- ▶ use the recurrence for the integrand to derive a recurrence for the integral
- ▶ solve the recurrence

# MultiIntegrate

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$$\text{In[1]:= mAZIntegrate}\left[\frac{(1+x1*x2)^n}{(1+x1)^\epsilon}, n, \{\{x1, 0, 1\}, \{x2, 0, 1\}\}\right]$$

$$\text{Out[1]= } \frac{\sum_{\iota_1=1}^n \frac{1}{\iota_1-\epsilon+1}}{-n-1} - \frac{2 \sum_{\iota_1=1}^n \frac{2^{\iota_1}}{-\iota_1+\epsilon-1}}{(n+1)2^\epsilon} + \frac{2^\epsilon-2}{(n+1)(\epsilon-1)2^\epsilon}$$

$$\text{In[2]:= mAZIntegrate}\left[\frac{(1+x1+x2+x1*x2)^n}{(1+x1)^\epsilon}, n, \{\{x1, 0, 1\}, \{x2, 0, 1\}\}\right]$$

$$\text{Out[2]= } \frac{2^{-2n-\epsilon} (208 - 83 2^{2+n} + 63 2^{1+2n} - 2^{4+\epsilon} + 7 2^{2+n+\epsilon} - 13 2^{2n+\epsilon})}{(1+n)(1+n-\epsilon)}$$

$$\text{In[3]:= mAZIntegrate}\left[\frac{(1+x1+x2+x1*x2)^n}{(1+x1)^\epsilon}, n, \{\{x1, 0, \frac{1}{2}\}, \{x2, 0, 2\}\}\right]$$

$$\text{Out[3]= } \frac{2^{-2n-\epsilon} (208 - 83 2^{2+n} + 63 2^{1+2n} - 2^{4+\epsilon} + 7 2^{2+n+\epsilon} - 13 2^{2n+\epsilon})}{(1+n)(1+n-\epsilon)}$$

# MultiIntegrate

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The following integral occurs in the direct computation of a 3-loop diagram of the ladder-type:

$$\int_0^1 \int_0^1 \left( \frac{(s(x-1) + t(u-1) + 1)^n}{(w-1)(z-1)(sx-s+tu-t-u+1)(sx-s+tu-t-x+1)} \right. \\ \left. + \frac{1}{(z-1)(sx-s+tu-t-x+1)} \frac{(z(-s+tu-t+1) + x((s-1)z+1))^n}{-swx + sw + sxz - sz - tuw + tuz + tw - tz + uw - u - w - xz + x + z} \right. \\ \left. + \frac{1}{(w-1)(sx-s+tu-t-u+1)} \frac{(u((t-1)w+1) - w(s(-x)+s+t-1))^n}{swx - sw - sxz + sz + tuw - tuz - tw + tz - uw + u + w + xz - x - z} \right) du dx$$

# MultiIntegrate

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The following integral occurs in the direct computation of a 3-loop diagram of the ladder-type:

$$\begin{aligned}
 & \int_0^1 \int_0^1 \left( \frac{(s(x-1) + t(u-1) + 1)^n}{(w-1)(z-1)(sx-s+tu-t-u+1)(sx-s+tu-t-x+1)} \right. \\
 & \quad + \frac{1}{(z-1)(sx-s+tu-t-x+1)} \frac{(z(-s+tu-t+1) + x((s-1)z+1))^n}{-swx + sw + sxz - sz - tuw + tuz + tw - tz + uw - u - w - xz + x + z} \\
 & \quad + \frac{1}{(w-1)(sx-s+tu-t-u+1)} \frac{(u((t-1)w+1) - w(s(-x)+s+t-1))^n}{swx - sw - sxz + sz + tuw - tuz - tw + tz - uw + u + w + xz - x - z} \left. du dx \right) \\
 & = \frac{1}{(w-1)(z-1)(s+t-1)} \left( S_{1,1} \left( -\frac{(s+t-1)w(z-1)}{sw-sz+z-1}, \frac{sw-sz+z-1}{z-1}; n \right) - S_{1,1} \left( -\frac{(s+t-1)w(z-1)}{sw-sz+z-1}, \frac{z(sw-sz+z-1)}{w(z-1)}; n \right) \right. \\
 & \quad - S_{1,1} \left( -\frac{(s+t-1)w(z-1)}{sw-sz+z-1}, \frac{(t-1)(sw-sz+z-1)}{(s+t-1)(z-1)}; n \right) + S_{1,1} \left( \frac{s+t-1}{t-1}, (1-t)w; n \right) + S_{1,1} \left( \frac{s+t-1}{t-1}, -\frac{(s-1)(t-1)}{s+t-1}; n \right) \\
 & \quad + S_{1,1} \left( -\frac{(s+t-1)w(z-1)}{sw-sz+z-1}, \frac{(sw-sz+z-1)(tz-1)}{(s+t-1)w(z-1)}; n \right) - S_{1,1} \left( \frac{s+t-1}{t-1}, 1-t; n \right) + S_{1,1} \left( \frac{s+t-1}{s-1}, -\frac{(s-1)(t-1)}{s+t-1}; n \right) \\
 & \quad + S_{1,1} \left( \frac{(s+t-1)(w-1)z}{(t-1)w-tz+1}, \frac{-tw+w+tz-1}{w-1}; n \right) - S_{1,1} \left( \frac{s+t-1}{t-1}, -\frac{(t-1)(sw-1)}{s+t-1}; n \right) + S_{1,1} \left( \frac{s+t-1}{s-1}, (1-s)z; n \right) \\
 & \quad - S_{1,1} \left( \frac{(s+t-1)(w-1)z}{(t-1)w-tz+1}, \frac{(s-1)(-tw+w+tz-1)}{(s+t-1)(w-1)}; n \right) - S_{1,1} \left( \frac{s+t-1}{s-1}, 1-s; n \right) - S_{1,1} \left( \frac{s+t-1}{s-1}, -\frac{(s-1)(tz-1)}{s+t-1}; n \right) \\
 & \quad - S_{1,1} \left( \frac{(s+t-1)(w-1)z}{(t-1)w-tz+1}, \frac{w((t-1)w-tz+1)}{(w-1)z}; n \right) + S_{1,1} \left( \frac{(s+t-1)(w-1)z}{(t-1)w-tz+1}, -\frac{(sw-1)((t-1)w-tz+1)}{(s+t-1)(w-1)z}; n \right) \\
 & \quad - S_{1,1}(1, (1-s)z; n) + S_{1,1}(1, z - sz; n) + S_2((1-s)z; n) - S_{1,1}(1, (1-t)w; n) + S_{1,1}(1, w - tw; n) - S_2((-s - t + 1)w; n) \\
 & \quad \left. - S_2((-s - t + 1)z; n) + 2S_2(-s - t + 1; n) - S_2(1 - s; n) + S_2((1 - t)w; n) - S_2(1 - t; n) \right)
 \end{aligned}$$

# HarmonicSums

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The package HarmonicSums offers functions to

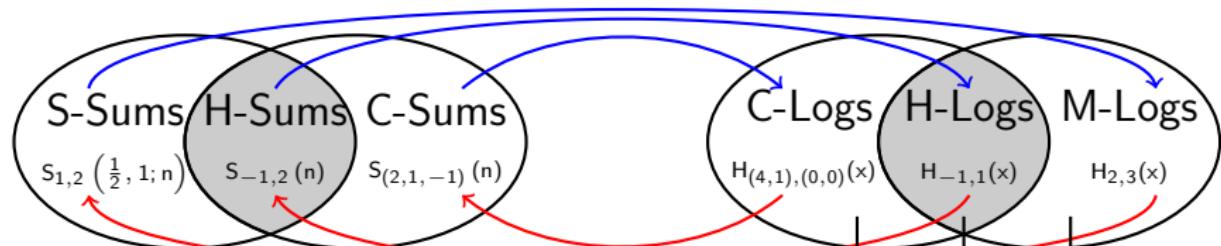
- ▶ find algebraic and structural relations of harmonic sums and their generalizations
- ▶ compute the inverse Mellin transform of harmonic sums and their generalizations, this leads to harmonic polylogarithms and their generalizations
- ▶ find algebraic and structural relations of harmonic polylogarithms and their generalizations
- ▶ apply algebraic and structural relations to harmonic sums
- ▶ calculate the asymptotic expansion of harmonic sums and their generalizations
- ▶ perform several other tasks not mentioned in this talk (see, e.g., my PhD thesis, April 2012)

# HarmonicSums

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integral representation (inv. Mellin transform)



Mellin transform



power series expansion

# HarmonicSums

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In[4]:= **SExpansion[S[-1, 3, n], n, 10]**

Out[4]=

$$\begin{aligned} & (-1)^n \left( -\frac{1}{4n^3} + \frac{5}{8n^4} - \frac{5}{8n^5} - \frac{5}{16n^6} + \frac{31}{24n^7} + \frac{133}{96n^8} - \frac{169}{24n^9} - \frac{163}{16n^{10}} \right) + \\ & \frac{3 \ln 2 z3}{4} + \\ & (-1)^n \left( \frac{1}{2n} - \frac{1}{4n^2} + \frac{1}{8n^4} - \frac{1}{4n^6} + \frac{17}{16n^8} - \frac{31}{4n^{10}} \right) z3 - \frac{19 z2^2}{40} \end{aligned}$$

In[5]:= **GetApproximation[S[-1,3,n], {-2.5, 2}]**

Out[5]=  $-0.795096 - 0.105476 i$

In[6]:= **HLimit[n\*(S[2, n] - z2 - S[2, 2, n] + 7\*z2^2/10), n]**

Out[6]=  $-1 + z2$

# Overview of Training

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## Attended Conferences and Workshops:

- ▶ German-Japanese Workshop, Zeuthen
- ▶ LHCphenoNet Annual Meeting 2012, Durham
- ▶ LL2012: Loops and Legs in Quantum Field Theory, Wernigerode
- ▶ RISC-DESY workshop
- ▶ Symbolic Computation and its Application, Aachen
- ▶ member of the local organizers of LHCPhenoNet School on Integration, Summation and Special Functions in QFT.

# Publications and Talks

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- ▶ J. Ablinger, J. Blümlein, A. Hasselhuhn, S. Klein, C. Schneider, F. Wissbrock  
New Heavy Flavor Contributions to the DIS Structure Function  $F_2(x, Q^2)$  at  $O(\alpha_s^3)$ . In: Proceedings of RADCOR 2011, Proceedings of 10th International Symposium on Radiative Corrections (Applications of Quantum Field Theory to Phenomenology) PoS(RADCOR2011)31, pp. 1-8. 2012. ISSN 1824-8039.
- ▶ J. Ablinger  
Computer Algebra Algorithms for Special Functions in Particle Physics. J. Kepler University Linz. PhD Thesis. April 2012.
- ▶ J. Ablinger, J. Blümlein, A. Hasselhuhn, S. Klein, C. Schneider, F. Wissbrock  
Massive 3-loop Ladder Diagrams for Quarkonic Local Operator Matrix Elements. Nuclear Physics B 864, pp. 52-84. 2012. ISSN: 0550-3213. arXiv:1206.2252v1 [hep-ph].
- ▶ J. Ablinger  
Almkvist Zeilberger Algorithm and (Generalized) Harmonic Sums.. May 17, 2012. Contributed talk at Symbolic Computation and its Applications 2012
- ▶ J. Ablinger  
Generalizations of Harmonic Sums and Polylogarithms and the Package HarmonicSums.. March 21, 2012. Contributed talk at LHCPhenoNet Annual Meeting.
- ▶ J. Ablinger, J. Blümlein, S. Klein, C. Schneider, F. Wissbrock  
The  $O(\alpha_s^3)$  Massive Operator Matrix Elements of  $O(n_f)$  for the Structure Function  $F_2(x, Q^2)$  and Transversity. Nucl. Phys. B 844, pp. 26-54. 2011. ISSN: 0550-3213.
- ▶ J. Ablinger, J. Blümlein, C. Schneider  
Harmonic Sums and Polylogarithms Generated by Cyclotomic Polynomials. J. Math. Phys. 52(10), pp. 1-52. 2011. ISSN 0022-2488. [arXiv:1007.0375 [hep-ph]].
- ▶ J. Ablinger, J. Blümlein, S. Klein, C. Schneider, F. Wissbrock  
3-Loop Heavy Flavor Corrections to DIS with two Massive Fermion Lines. In: 19th International Workshop On Deep-Inelastic Scattering And Related Subjects (DIS 2011), 2011. American Institute of Physics (AIP).