Transverse PDFs at NNLO

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Outline

1. q_T resummation at hadron colliders
2. Transverse parton distribution functions at next-to-next-to-leading order
3. Status

Hadron colliders

⇒ extract σ_{process}, $\frac{d\sigma_{\text{process}}}{dv}$, ...

e.g. for $p\bar{p} \to Z + X$ differential in q_T

⇒ results depend on many mass scales s, M_Z, q_T, ... can be of very different size.

Perturbation theory

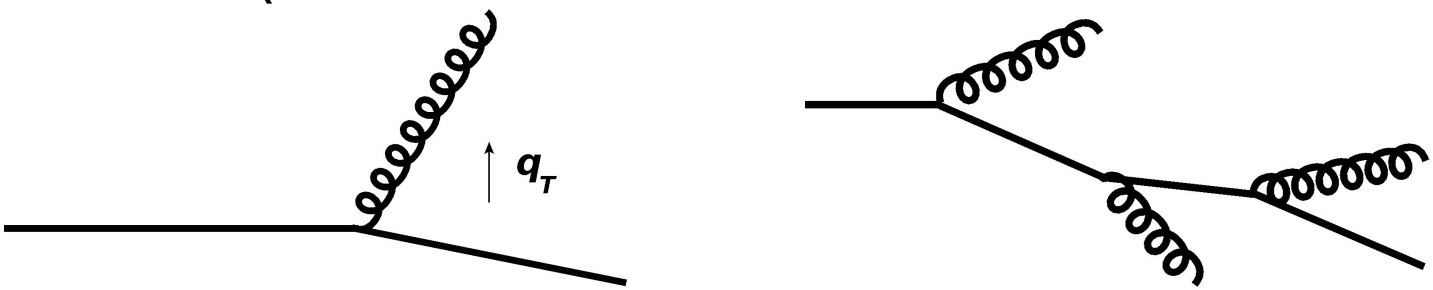
- perturbative expansion:

$$d\sigma = \sum_{n=0}^{\infty} \alpha_s^n d\sigma^{(n)}$$

  fixed order: stop at e.g. $n = 2$ (NNLO).
- For each power in α_s find large logarithms of scale ratios, which
  - spoil convergence,
  - need to be resummed to all orders in α_s.

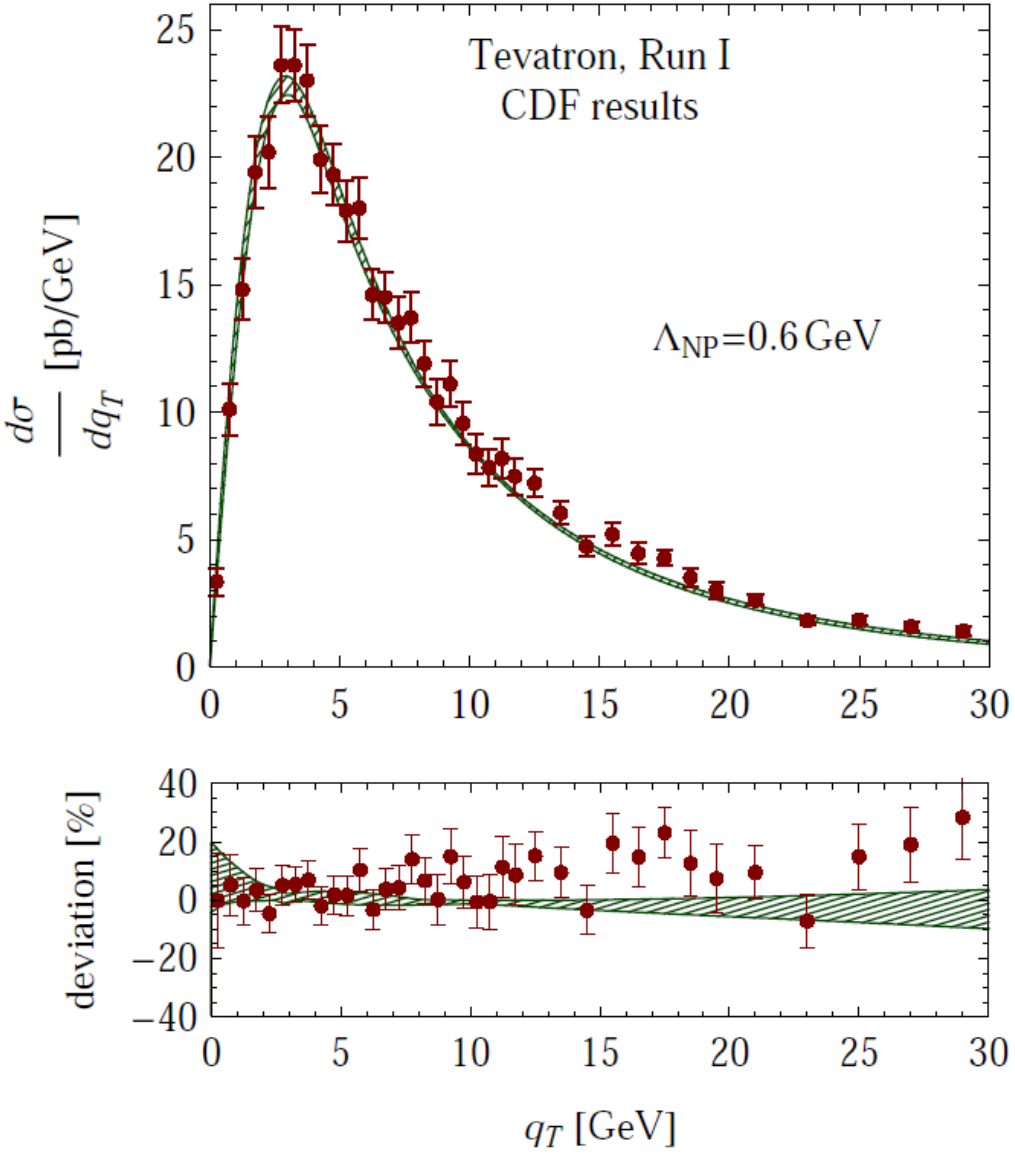
Large logarithms

Momenta of emitted particle soft, collinear
$\Rightarrow$ up to $\log^2 \frac{q_T^2}{Q^2}$ for each power α_s.



Due to specific structure, factorize, can account for them to all orders in α_s (resum).

q_T spectrum, $p\bar{p} \to Z + R$



resummed result

for large $q_T \to$ fixed order

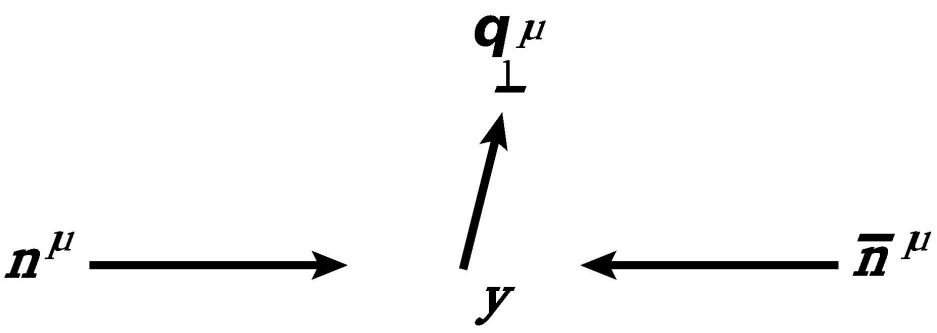
for small $q_T \to$ resummation

phenomenologically important, e.g. analog $W \to$ mass

[Becher, Neubert, Wilhelm, 2011]

Notation

Directions of incoming hadrons and outgoing vector boson:



- With $n^2, \bar{n}^2 = 0,\ n \cdot \bar{n} = 2; \qquad -q_\perp^2 = q_T^2 > 0.$
- Decompose $v^\mu = (\bar{n}v)\frac{n^\mu}{2} + (nv)\frac{\bar{n}^\mu}{2} + v_\perp^\mu\,.$

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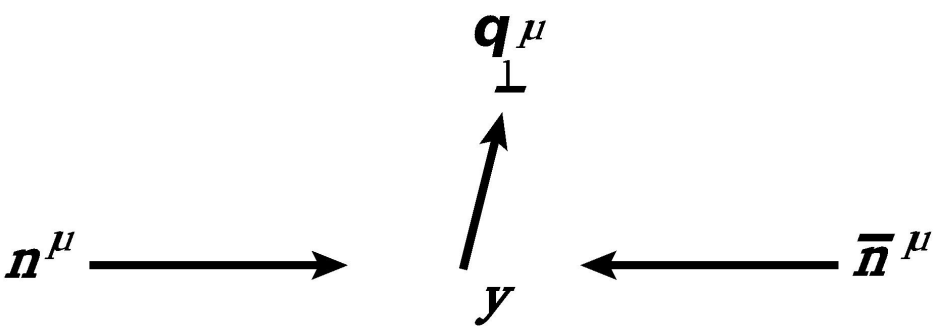


- With $n^2, \bar{n}^2 = 0,\ n \cdot \bar{n} = 2;\qquad -q_\perp^2 = q_T^2 > 0.$
- Decompose $v^\mu = (\bar{n}v)\frac{n^\mu}{2} + (nv)\frac{\bar{n}^\mu}{2} + v_\perp^\mu\,.$
- Distinguish fields with different scaling in $\lambda = \frac{q_T}{M}$:

hard-collinear:	$p_{h.c.}$	$= (\bar{n}p, np, p_\perp) \sim M(1, \lambda^2, \lambda)$
hard-anticollinear:	$p_{\overline{h.c.}}$	$\sim M(\lambda^2, 1, \lambda)$
soft:	p_s	$\sim M(\lambda, \lambda, \lambda)$

Factorized differential cross section

At $\Lambda \ll q_T \ll M$ factorization formula [Becher, Neubert, 2010]:

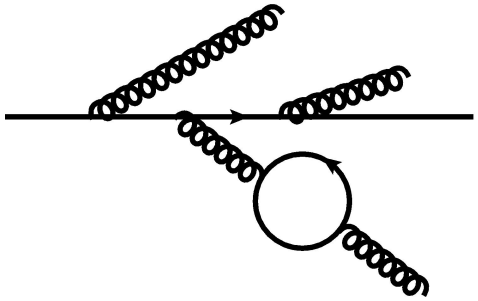
$$\frac{d^2\sigma_{N_1N_2\to V+X}}{dq_T^2\,dy} \sim \left|C_V(-M^2,\mu)\right|^2 \int d^2x_\perp\, e^{-i\,q_\perp\cdot x_\perp} \sum_q g_q^2$$

$$\times \left[\mathcal{B}_{q/N_1}(z_1, x_T^2, \mu)\bar{\mathcal{B}}_{\bar{q}/N_2}(z_2, x_T^2, \mu) + (q \leftrightarrow \bar{q})\right]_{M^2} + \mathcal{O}\left(\frac{q_T^2}{M^2}\right),$$

- C_V is the hard function, $\mathcal{B}$ are TPDFs.
- Compare to $d\sigma = \hat{\sigma}_{12}\phi_1\phi_2$.
- q_T dependent; Fourier space.
- $z_{1,2} = \sqrt{\tau}e^{\pm y}$, $\tau = (M^2 + q_T^2)/s$.

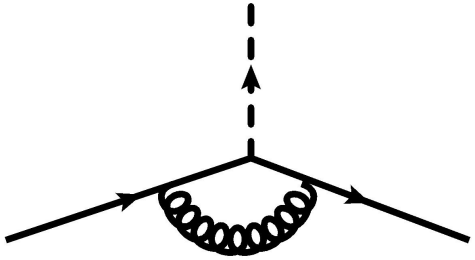
Factorized differential cross section, pictorial

$\mathcal{B}_{q/N_1}(z_1, x_T^2, \mu)$



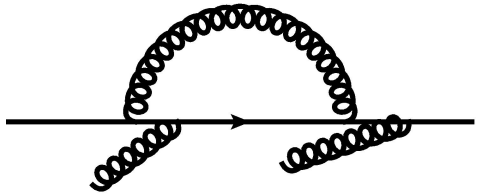
'transverse PDF'

$C_V(-M^2, \mu)$



'hard function'

$\bar{\mathcal{B}}_{\bar{q}/N_2}(z_2, x_T^2, \mu)$



'transverse PDF'

$n^\mu \longrightarrow$

h.c.

$\longleftarrow \bar{n}^\mu$

$\overline{\text{h.c.}}$

Operator definition of (T)PDF

Operator definition of TPDF (quark case) in SCET

$$\mathcal{B}_{q/N}(z, x_T^2, \mu) = \frac{1}{2\pi}\int \mathrm{d}t\; e^{-izt\bar{n}\cdot k}\, \langle N(k)|\, \bar{\chi}(t\bar{n} + x_\perp)\frac{\bar{\not{n}}}{2}\chi(0)\, |N(k)\rangle \ .$$

Compare to usual collinear PDF

$$\phi_{q/N}(z, \mu) = \frac{1}{2\pi}\int \mathrm{d}t\; e^{-izt\bar{n}\cdot k}\, \langle N(k)|\, \bar{\chi}(t\bar{n})\frac{\bar{\not{n}}}{2}\chi(0)\, |N(k)\rangle \ .$$

Above $\bar{\chi}(x) = \bar{\xi}(x)W(x)$, with the hard-collinear quark field $\xi(x)$ and the hard-collinear Wilson line $W(x)$.

For hard-anticollinear case correspondingly with $n \leftrightarrow \bar{n}$.

Replace above hadron N by parton $p \Rightarrow$ perturbatively calculable $\mathcal{B}_{ip}$ and ϕ_{jp}.

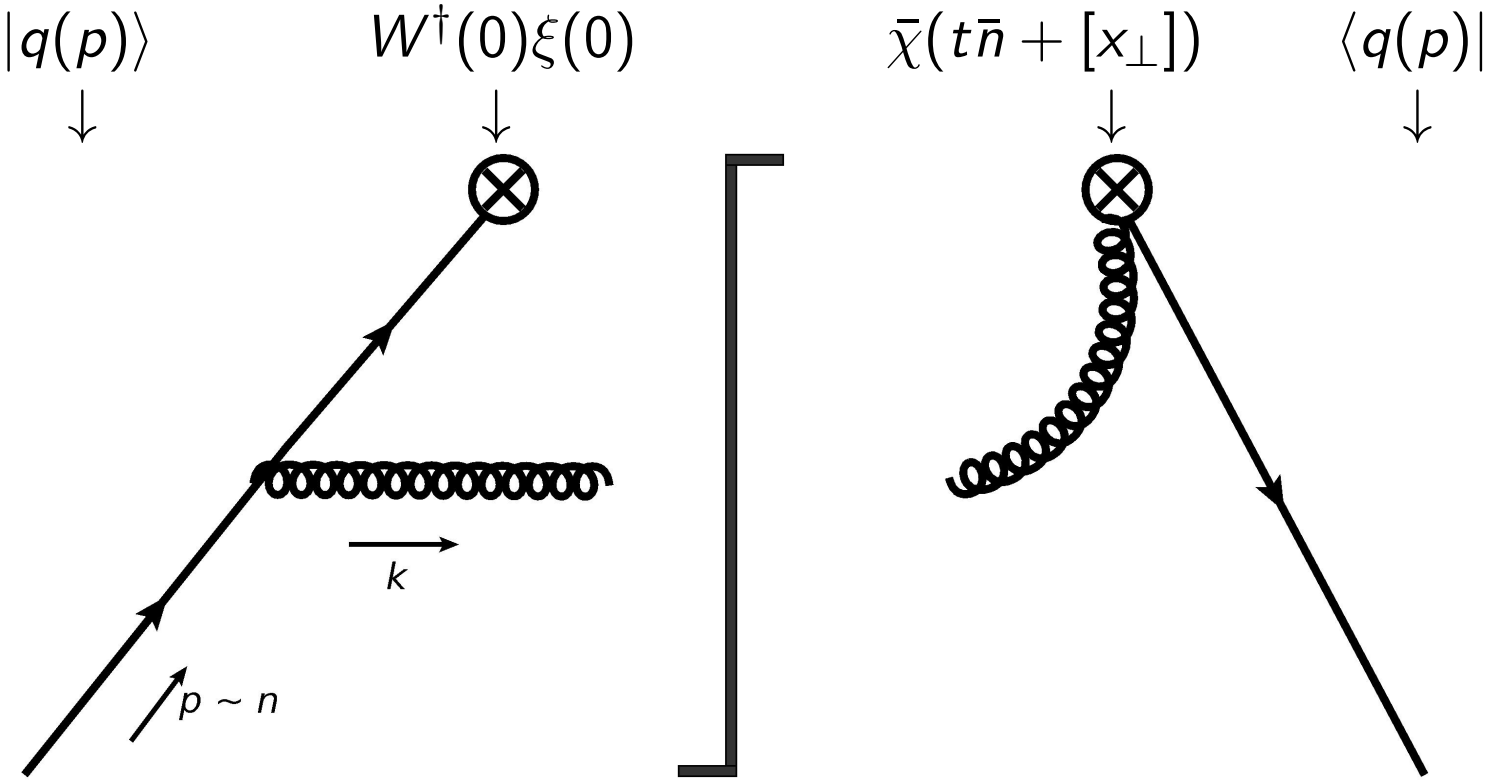
Matching kernel

Matching kernel $\mathcal{I}_{i\leftarrow j}$ relates $\mathcal{B}$ and ϕ

$$\mathcal{B}_{i/N}(\rho, x_T^2, \mu) = \sum_j \int_\rho^1 \frac{dz}{z} \mathcal{I}_{i\leftarrow j}(z, x_T^2, \mu) \phi_{j/N}(\rho/z, \mu) + \mathcal{O}(\Lambda^2 x_T^2)$$

- Compare to $\frac{d\phi}{d\log\mu^2} = P \otimes \phi$ (DGLAP).
- For $x_T^{-2} \gg \Lambda^2_{\mathrm{QCD}}$, $\mathcal{I}_{i\leftarrow j}$ can be determined from $\mathcal{B}_{ip}$ and ϕ_{pj} (perturbatively).
- Here discuss $q \to q$.

Example diagram at NLO



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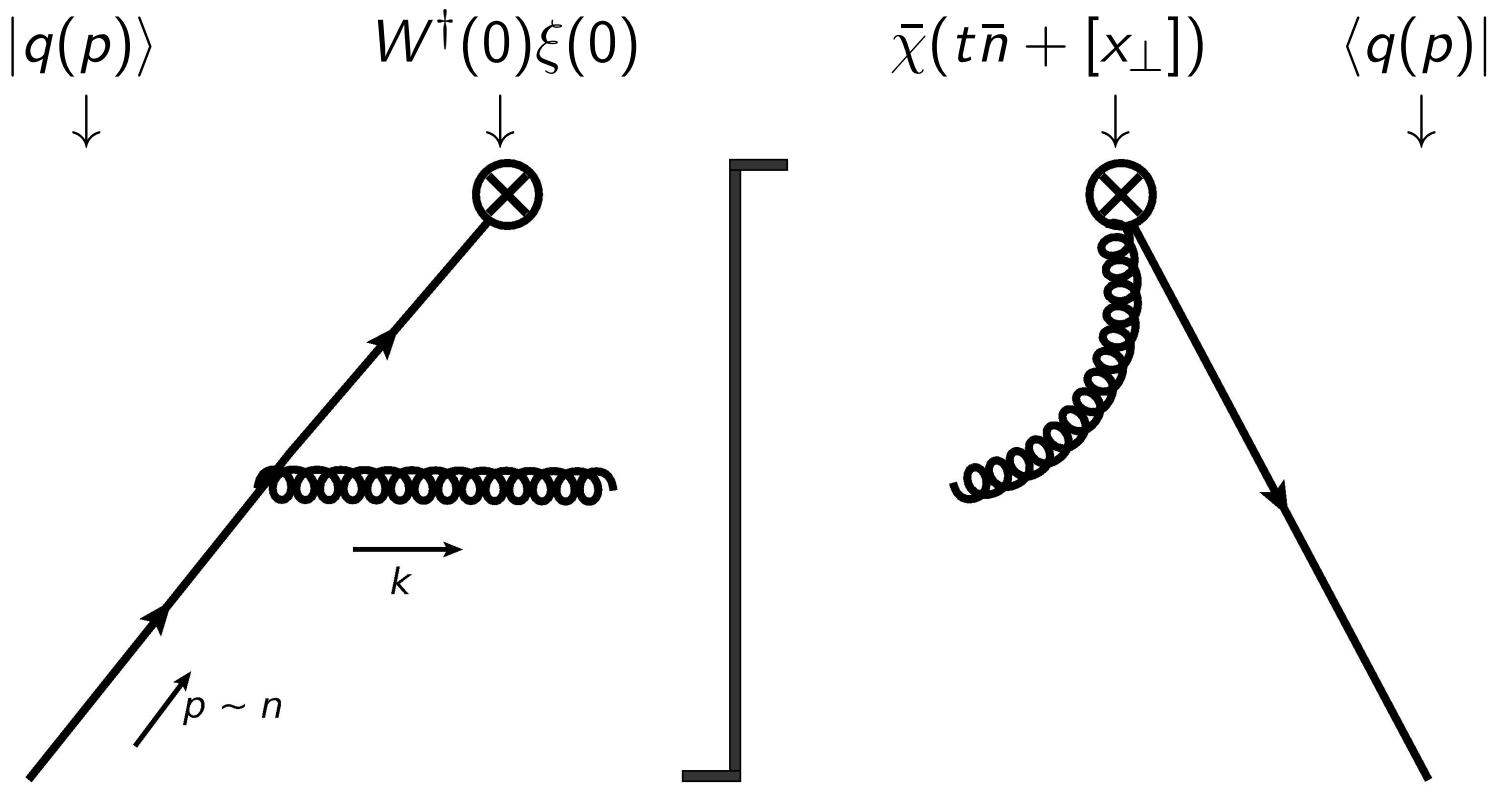
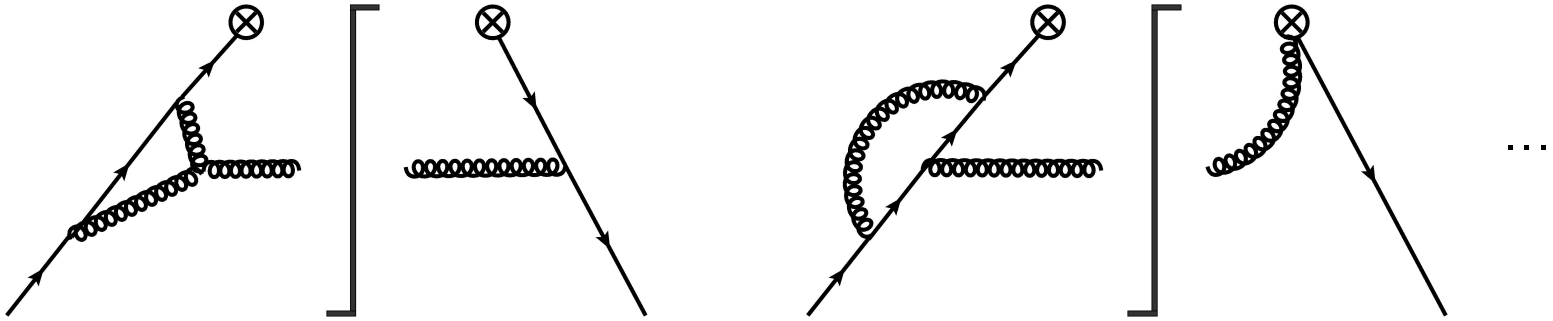


$$\frac{\alpha_s}{4\pi}\left(\mathcal{B}_{qq}^{(1)},\phi_{qq}^{(1)}\right)=\int\frac{\mathrm{d}^d k}{(2\pi)^d}(2\pi)\delta^+(k^2)\delta(\bar{n}\cdot[k-(1-z)p])\left(e^{ik_T\cdot x_T},1\right)\mathcal{M}$$

Contribution from purely virtual diagrams cancel for $\mathcal{I}$.

Example diagrams at NNLO

Virtual-Real contribution:



Example diagrams at NNLO

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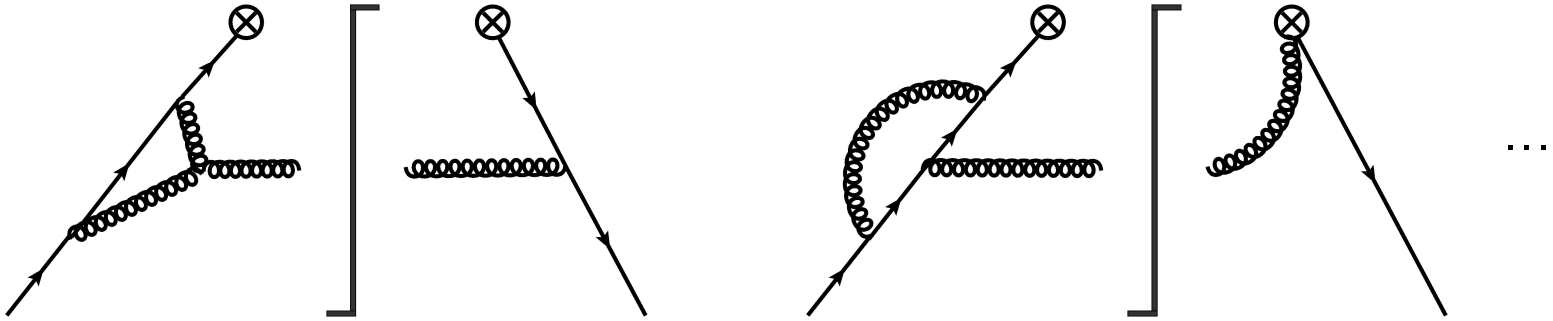
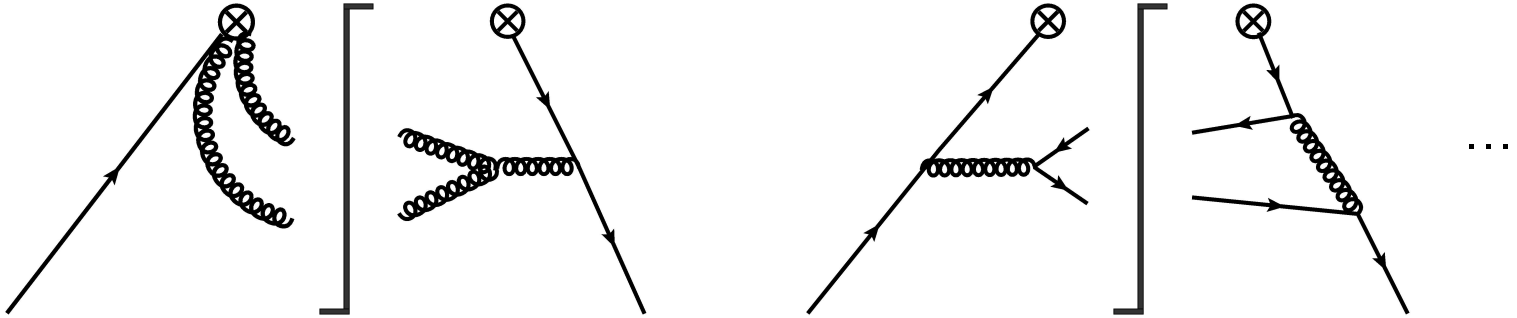


Real-Real contribution:



Analytic regularization

- Integrals contain poles, not regularized in dimensional regularization. Definition of $\mathcal{B}$ problematic.

$\Rightarrow$ Introduce additional regularization.

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Analytic regularization

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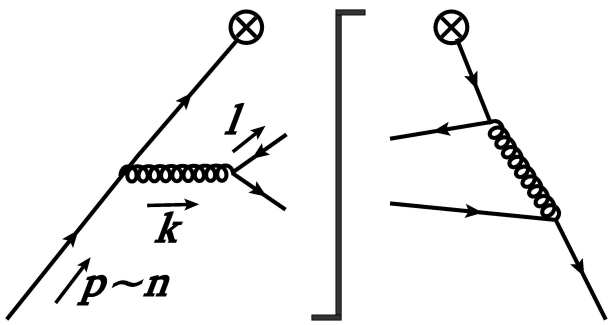
- Various ways possible. All of them so far applied only at NLO, not proven that applicable at NNLO. Introduce new complications to calculation.
- Chose variant of Smirnov's 'analytic regulators' [Becher, Bell, 2011]:
  - $\Rightarrow$ For each emitted particle (with momentum k) multiply integrand by $\left(\frac{\nu}{n \cdot k}\right)^{\alpha}$.
- Advantage: No new denominators.
  No soft contribution.
  Can expand in α (for NNLO only need up to α^0).

Double real emission



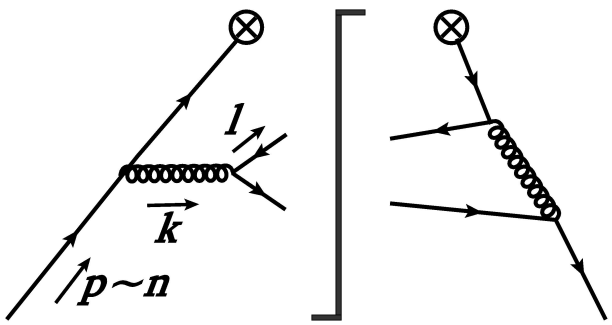
Denominators:

$n \cdot l, n \cdot (k - l), n \cdot k,$
$\bar{n} \cdot l, \bar{n} \cdot (k - l), \bar{n} \cdot k, \bar{n} \cdot (p - l),$
$k^2, (p - k)^2$

$$\left(\frac{\alpha_s}{4\pi}\right)^2 \left(\mathcal{B}^{(2)}_{RR}, \phi^{(2)}_{RR}\right) = \int \frac{d^d k}{(2\pi)^d} \delta\left(\bar{n}\cdot[k-(1-z)p]\right) \left(e^{ik_T \cdot x_T}, 1\right)$$

$$\times \int \frac{d^d l}{(2\pi)^{d-2}} \delta^+(l^2) \delta^+((k-l)^2)\, \mathcal{M}\,,$$

Double real emission



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$$\times \int \frac{d^d l}{(2\pi)^{d-2}} \delta^+(l^2) \delta^+((k-l)^2)\, \mathcal{M}\,,$$

Non-standard integral: Non-integer denominators; presence of n, $\bar{n}$ in denominators and δ-fct; frame dependent $e^{ik_T \cdot x_T}$.
$\Rightarrow$ Many standard methods not well applicable.

Example integral for double real emission

- Use 3 δ-functions.
- Choose cms frame of k and a parameterization of vectors.
- Perform k_T integral ($2-2\epsilon$ dim., by now simple) and -2ϵ-dim. angular integral.
- Of remaining 3 integrals, identify 1 as ${}_2F_1$:

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$$\sim \int_0^1 \mathrm{d}y \int_0^1 \mathrm{d}t\, y^{1-a_2-a_3-a_5-\epsilon}(1-y)^{1-a_1-a_2-a_6-\epsilon}\left[1-(1-z)(1-y)\right]^{-a_7}$$
$$\times (1-t)^{1-2a_2-2\epsilon}\,{}_2F_1\left(1-a_2-\epsilon,1-a_2-2\epsilon;1-\epsilon;t\right)\Big[t^{-a_3-\epsilon}\left\{1-y(1-t)\right\}^b$$
$$+(1-t)^{-a_1-\epsilon}\left\{1-(1-y)(1-t)\right\}^b\Big]\ ,\quad b=-2+a_1+a_2+a_3+2\epsilon\,.$$

exponent	a_1	a_3	a_5	a_6
integer part	n_1	n_3	n_5	n_6
regulator part	α	α	$-\epsilon-m\alpha$	ϵ ,

$m=2,0$

Anomaly

- $\mathcal{B}$ has poles in new regulator α.
- Must consider $\mathcal{B}\bar{\mathcal{B}}$.
- Independent of regularization.
  Poles in α cancel.
- Remaining poles in ϵ remove by $\overline{MS}$ renormalization.
- However, resulting expression depends on hard scale M^2
  $\rightarrow$ anomaly.
- Can be refactorized [Becher, Neubert, 2010].

Refactorization

Refactorize $\mathcal{B}\bar{\mathcal{B}}$:

$$[\mathcal{B}_{ip_1}(z_1, x_T^2, \mu)\bar{\mathcal{B}}_{\bar{i}p_2}(z_2, x_T^2, \mu)]_{M^2} = \left(\frac{x_T^2 M^2}{4e^{-2\gamma_e}}\right)^{-F_{i\bar{i}}(x_T^2,\mu)} \times B_{ip_1}(z_1, x_T^2, \mu) B_{\bar{i}p_2}(z_2, x_T^2, \mu)\,.$$

- Defines TPDFs B, independent of hard scale M^2. F parameterizes M^2 dependence.

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- Defines TPDFs B, independent of hard scale M^2. F parameterizes M^2 dependence.
- In analogy to $\mathcal{I}$ define I by

$$B_{i/N}(\rho, x_T^2, \mu) = \sum_j \int_\rho^1 \frac{dz}{z} I_{i\leftarrow j}(z, x_T^2, \mu)\phi_{j/N}(\rho/z, \mu) + \mathcal{O}(\Lambda_{\mathrm{QCD}}^2 x_T^2)\,.$$

Status

- Definitions & relations presented allow extraction of the functions I & F as well as $B = I \otimes \phi$.
- We performed the NNLO calculation of $I_{q \leftarrow q}(z_1, x_T^2, \mu)$ & $F_{q\bar{q}}(x_T^2, \mu)$ using 'analytic' regularization.
- All poles in α canceled $\Rightarrow$ analytic regularization applicable at NNLO ✓(first time explicitly found).

RG evolution & application

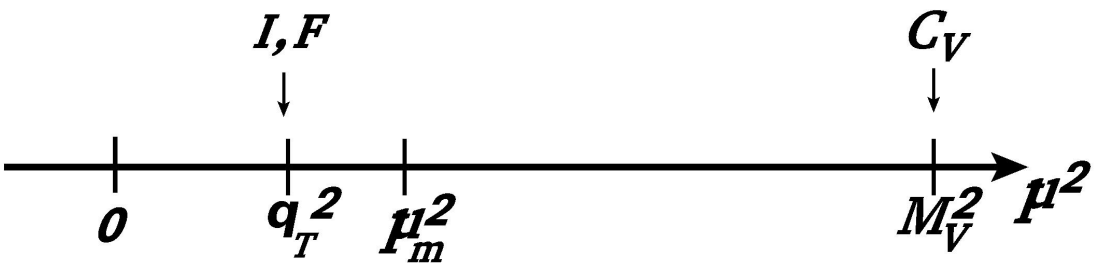
§ Once have factorized result, s.t. each function depends only on its typical mass M_{typical} and μ,

→ can consistently determine each function in fixed order perturbation theory at $\mu \sim M_{\text{typical}}$,

→ subsequently use RG equation to evolve to common scale μ_m.

⇒ Resums logarithms.

→ All ingredients at NNLO ⇒ NNLO+NNNLL precision.



Rephrased factorization formula

At $\Lambda \ll q_T \ll M$ rephrased factorization formula [Becher, Neubert, 2010]:

$$\frac{d^3\sigma}{dM^2 dq_T^2 dy} = \frac{\alpha^2}{3N_c M^2 s} \sum_{i,j=q,\bar{q},g} \sum_q e_q^2 \int_{\xi_1}^1 \frac{dz_1}{z_1} \int_{\xi_2}^1 \frac{dz_2}{z_2} \int d^2x_\perp \, e^{-iq_\perp \cdot x_\perp}$$

$$\times \left[\widetilde{C}_{q\bar{q}\leftarrow ij}(z_1,z_2,x_T^2,M^2,\mu)\phi_{i/N_1}(\xi_1/z_1,\mu)\phi_{j/N_2}(\xi_2/z_2,\mu) + (q\leftrightarrow\bar{q}) \right],$$

where

$$\widetilde{C}_{q\bar{q}\leftarrow ij}(z_1,z_2,x_T^2,M^2,\mu)$$

$$= |C_V(-M^2,\mu)|^2 \left(\frac{x_T^2 M^2}{4e^{-2\gamma_e}}\right)^{-F_{q\bar{q}}(x_T^2,\mu)} I_{q\leftarrow i}(z_1,x_T^2,\mu) I_{\bar{q}\leftarrow j}(z_2,x_T^2,\mu) \,.$$

Relation to formula by Collins, Soper, Sterman

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compare this to the one of [Collins, Soper, Sterman, 1985]:

$$= \exp\Big\{ -\int_{\mu_b}^{M^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \Big[\log\frac{M^2}{\bar{\mu}^2} A\big(\alpha_s(\bar{\mu})\big) + B\big(\alpha_s(\bar{\mu})\big)\Big]\Big\}$$

$$\times\, C_{qi}\big(z_1,\alpha_s(\mu_b)\big) C_{\bar{q}j}\big(z_2,\alpha_s(\mu_b)\big)\,,$$

x_T dependence via $\mu_b = b_0 x_T^{-1}$ $(b_0 = 2e^{-\gamma_e})$.

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$$\times\, C_{qi}(z_1,\alpha_s(\mu_b)) C_{\bar{q}j}(z_2,\alpha_s(\mu_b))\,,$$

x_T dependence via $\mu_b = b_0 x_T^{-1}$ $(b_0 = 2e^{-\gamma_e})$.

Relations:

- $C_{qi}(z,\alpha_s(\mu_b)) = \big|C_V(-\mu_b^2,\mu_b)\big|\, I_{q/i}(z,x_T^2,\mu_b)\,,$
- A & B related to F and RG evolution of F & C_V.

Dictionary CSS vs BN

Using $b_0 = 2e^{-\gamma_e}$, $\mu_b = b_0 x_T^{-1}$ and $\bar{x}_T = b_0 \bar{\mu}^{-1}$:

$$C_{qi}(z, \alpha_s(\mu_b)) = \left| C_V(-\mu_b^2, \mu_b) \right| I_{q/i}(z, x_T^2, \mu_b)\,,$$

$$A(\alpha_s(\bar{\mu})) = \Gamma^F_{\text{cusp}}(\alpha_s) - \bar{\mu}^2 \frac{dF_{q\bar{q}}(\bar{x}_T^2, \bar{\mu})}{d\bar{\mu}^2}\,,$$

$$B(\alpha_s(\bar{\mu})) = 2\gamma^q(\alpha_s) + F_{q\bar{q}}(\bar{x}_T^2, \bar{\mu}) - \bar{\mu}^2 \frac{d \log |C_V(-\bar{\mu}^2, \bar{\mu})|^2}{d\bar{\mu}^2}\,,$$

where Γ^F_{cusp} and γ^q appear in RG evolution of C_V, F & I, e.g.:

$$\frac{d}{d\log\mu} C_V(-M^2, \mu) = \left[\Gamma^F_{\text{cusp}}(\alpha_s) \log \tfrac{-M^2}{\mu^2} + 2\gamma^q(\alpha_s)\right] C_V(-M^2, \mu)\,,$$

Checks

- ✓ $F_{q\bar{q}}$ agrees with literature [Becher, Neubert 2010].
- ✓ $I_{q\leftarrow q}$: correct RG evolution [Curci, Furmanski, Petronzio, 1980].
- ✓ Using known C_V^{DY} compared our $\widetilde{C}_{q\bar{q}\leftarrow q\bar{q}}$ to related quantity in [Catani, Cieri, de Florian, Ferrera, Grazzini 2012].

Outlook and conclusions

- For phenomenology at small q_T need resummation and TPDFs.
- Definition & calculation suffer from additional singularities.
- NNLO calculation completed for $q \to q$ using analytic regularization.
- Remaining parton combinations will follow.
- In combination with hard functions, will improve theory accuracy of $d\sigma$ at small q_T to NNLO+NNNLL for processes with color neutral final state: single and multiple vector/Higgs boson production, new color neutral particles...

THE END

RG evolution equations

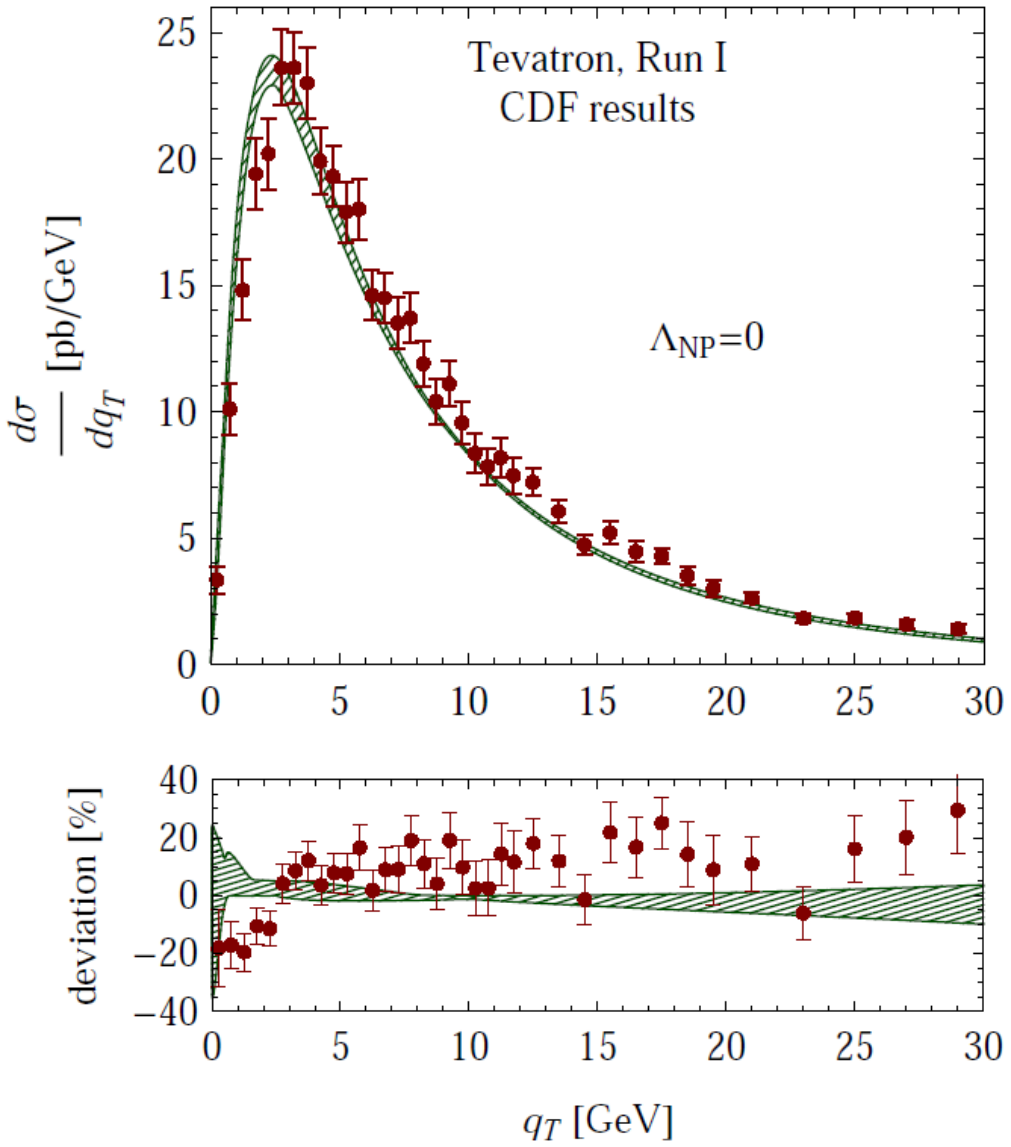
$$\frac{d}{d\log\mu} C_V(-M^2,\mu) = \left[\Gamma^F_{\rm cusp}(\alpha_s)\log\frac{-M^2}{\mu^2} + 2\gamma^q(\alpha_s)\right] C_V(-M^2,\mu)\,,$$

$$\frac{d}{d\log\mu} F_{q\bar{q}}(x_T^2,\mu) = 2\Gamma^F_{\rm cusp}(\alpha_s(\mu))\,,$$

$$\frac{d}{d\log\mu} I_{q\leftarrow q}(z,x_T^2,\mu) = \left[\Gamma^F_{\rm cusp}(\alpha_s)\log\frac{x_T^2\mu^2}{4e^{-2\gamma_e}} + 2\gamma^q(\alpha_s)\right] I_{q\leftarrow q}(z,x_T^2,\mu) - \sum_k 2\int_z^1 \frac{d\xi}{\xi} I_{q\leftarrow k}(\xi,x_T^2,\mu) P_{k\leftarrow q}(z/\xi,\mu)\,,$$

where $P_{k\leftarrow q}$ are DGLAP kernels from RG evolution of usual PDFs. Taken together, differential cross section RG invariant.

q_T spectrum, $p\bar{p} \to Z + R$



resummed result

Apart from PDFs no non-perturbative effects included.

[Becher, Neubert, Wilhelm, 2011]