

Progress in NNLO Process Involving Higgs plus Jets

LHCPhenoNet Mid-Term Meeting

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LHCphenonet



Motivation

- Why NNLO?

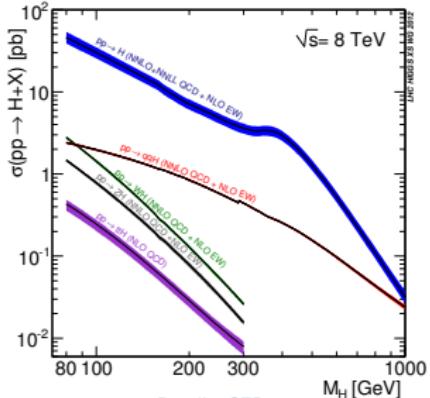
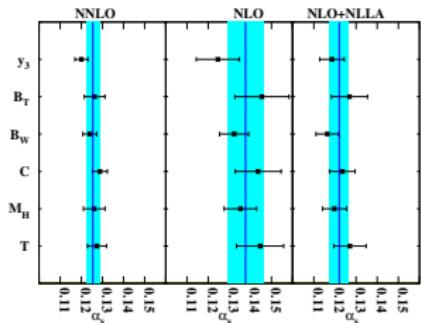
- Impose stronger constraints on the SM
- Improve the scale dependence
- Better description of hard scattering
 - Better description of the initial state
 - Better description of final-state jets

- Why Higgs production?

- Understanding the Higgs production rate
- Measurements of Higgs properties
- Gluon fusion to Higgs (dominant process at 130GeV)

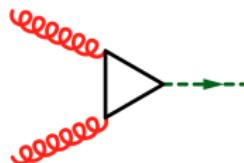
- Why Higgs + jets?

- Nonzero transverse momentum for Higgs decay (new jet topology)
- Better understanding for jet vetoes to enhance $H \rightarrow b\bar{b}$ signal from background

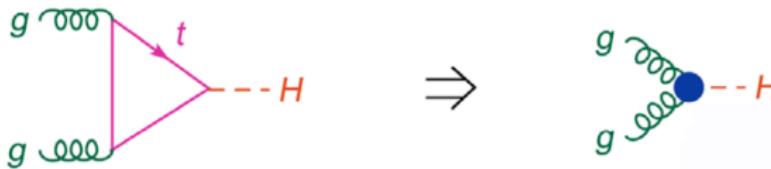


The Higgs Model

- Higgs production via gluon fusion through a quark loop



- In the large top mass limit, we have the effective interaction



- The effective interaction term in Lagrangian

$$\mathcal{L}_H^{int} = \frac{C}{2} H \operatorname{Tr} G_{\mu\nu} G^{\mu\nu}, \quad C = \frac{\alpha_s}{6\pi V} (1 + \mathcal{O}(\alpha_s))$$

Wilczek, Shifman, Vainshtein, Zakharov 1970's

- Compact expression for $H + 5$ parton amplitudes have been calculated using BCFW method

Higher Order Cross Sections in Perturbation Theory

- Renormalized cross sections contain infra-red (IR) singularities:
 - Loop integrations result in Laurent expansion of $\epsilon = D - 4$
 - Kinematic poles form phase space integration of real emission $s_{ij} = (p_i + p_j)^2 \rightarrow 0$
- Kinoshita-Lee-Nauenberg (KLN) theorem:
 - IR singularities cancel when summed over degenerate initial and final states
- *Problem*
 - Need singularities cancellation in the same form of ϵ expansion
 - Disallow numerical phase space integration

Cross Section Structure at NLO

- Structure of NLO H + jet cross section

$$\begin{aligned} d\sigma_{NLO} &= \int [\langle \mathcal{M}^{(0)} | \mathcal{M}^{(0)} \rangle]_{H+4} d\Phi_{H+2} \\ &\quad + \int [\langle \mathcal{M}^{(0)} | \mathcal{M}^{(1)} \rangle + \langle \mathcal{M}^{(1)} | \mathcal{M}^{(0)} \rangle]_{H+3} d\Phi_{H+1} \\ &= \int_{d\Phi_{H+2}} d\sigma_{NLO}^R + \int_{d\Phi_{H+1}} d\sigma_{NLO}^V \end{aligned}$$

- tree level 2→2+H amplitudes** L. Dixon, N. Glover, V. Khoze; V. Del Duca, A. Frizzo, F. Maltoni
- 1-loop 2→1+H amplitudes** D. de Florian, M. Grazzini, Z. Kunszt; V. Ravindran, J. Smith, W.L. van Neerven; C. F. Berger, V. Del Duca, L. Dixon

NLO Unresolved Factorization

- Single Unresolved Tree Factorization

- From the study of singular limits of the cross section
- Squared colour-ordered amplitudes obey **universal factorization**
- Soft gluon emission, $p_j \rightarrow 0$ Eikonal factor

$$|\mathcal{M}_{m+1}^0(\dots i, j, k, \dots)|^2 \rightarrow S_{ijk} |\mathcal{M}_m^0(\dots \tilde{I}, \tilde{K}, \dots)|^2$$

$$Soft^0(i, j, k) = \frac{\langle ik \rangle}{\langle ij \rangle \langle jk \rangle}$$

- Collinear limit $p_j \parallel p_k$

$$|\mathcal{M}_{m+1}^0(\dots i, j, k, l \dots)|^2 \rightarrow \frac{P_{jk \rightarrow \tilde{K}}^0}{s_{jk}} |\mathcal{M}_m^0(\dots i, \tilde{K}, l \dots)|^2$$

$$Split_-^0(g_a^+, g_b^+) = \frac{1}{\sqrt{z(1-z)} \langle ab \rangle}$$

$$Split_-^0(g^\pm, g^\pm), \quad Split_\pm^0(q^\pm, \bar{q}^\pm), \quad Split_-^0(q^+, g^\pm), \quad Split_-^0(g^\pm, \bar{q}^+)$$

NLO Subtraction

$$\begin{aligned} d\hat{\sigma}_{NLO} = & \int_{d\Phi_{H+2}} (d\sigma_{NLO}^R - d\sigma_{NLO}^{R,S}) \\ & + [\int_{d\Phi_{H+2}} d\sigma_{NLO}^{R,S} + \int_{d\Phi_{H+1}} d\sigma_{NLO}^V] \end{aligned}$$

- Then explicit ϵ -poles in loop integral cancel the analytically integrated kinematic poles
- Each line above is IR safe and ready for numerical evaluation
- Factorize the kinematic poles in NLO subtraction terms
 - Dipole subtraction S. Catani, M. Seymour
 - \mathcal{E} -prescription S. Frixione, Z. Kunszt, A. Signer
 - Antenna subtraction D. Kosower; J. Campbell, M. Cullen, N. Glover; A. Daleo, D. Maitre; A. Gehrmann-De Ridder, T. Gehrmann, N. Glover

Cross Section Structure at NNLO

- Structure of NNLO H + jet cross section

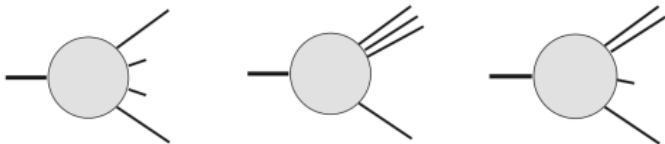
$$\begin{aligned} d\sigma_{NNLO} &= \int [\langle \mathcal{M}^{(0)} | \mathcal{M}^{(0)} \rangle]_{H+5} d\Phi_{H+3} \\ &\quad + \int [\langle \mathcal{M}^{(0)} | \mathcal{M}^{(1)} \rangle + \langle \mathcal{M}^{(1)} | \mathcal{M}^{(0)} \rangle]_{H+4} d\Phi_{H+2} \\ &\quad + \int [\langle \mathcal{M}^{(1)} | \mathcal{M}^{(1)} \rangle + \langle \mathcal{M}^{(2)} | \mathcal{M}^{(0)} \rangle + \langle \mathcal{M}^{(0)} | \mathcal{M}^{(2)} \rangle]_{H+3} d\Phi_{H+1} \\ &= \int_{d\Phi_{H+3}} d\sigma_{NNLO}^{RR} + \int_{d\Phi_{H+2}} d\sigma_{NNLO}^{RV} + \int_{d\Phi_{H+1}} d\sigma_{NNLO}^{VV} \end{aligned}$$

- tree level 2→3+H amplitudes** L. Dixon, N. Glover, V. Khoze; V. Del Duca, A. Frizzo, F. Maltoni;
- 1-loop 2→2+H amplitudes** C. F. Berger, V. Del Duca, L. Dixon; S. Badger, N. Glover, P. Mastrolia, C. Williams; S. Badger, R. Ellis, C. Williams
- 2-loop 2→1+H amplitudes** T. Gehrmann, M. Jaquier, N. Glover, A. Koukoutsakis

NNLO Unresolved Factorization

- Double real unresolved factorization

- Double soft
- Triple collinear
- Soft and collinear



Factorization holds \rightarrow new universal functions

$$S_{abcd} \quad S_{d,abc} \quad P_{ijk \rightarrow \tilde{K}} \quad \tilde{P}_{ijk \rightarrow \tilde{K}}$$

- Single unresolved loop factorization (Amplitude level)

- Collinear limits

$$\begin{aligned} \mathcal{M}_{m+1}^1(\dots i^{\lambda_i}, j^{\lambda_j}, \dots) &\xrightarrow{i||j} \sum_{\lambda=\pm} \left(\text{Split}_\lambda^0(i^{\lambda_i}, j^{\lambda_j}) \mathcal{M}_m^1(\dots P_I, \dots) \right. \\ &\quad \left. + \text{Split}_\lambda^1(i^{\lambda_i}, j^{\lambda_j}) \mathcal{M}_m^0(\dots P_I, \dots) \right) \end{aligned}$$

- Soft limits

$$\begin{aligned} \mathcal{M}_{m+1}^1(\dots i, j, k, \dots) &\xrightarrow{j \text{ soft}} \text{Soft}^0(i, j, k) \mathcal{M}_m^1(\dots i, k, \dots) \\ &\quad + \text{Soft}^1(i, j, k) \mathcal{M}_m^0(\dots i, k, \dots) \end{aligned}$$

NNLO Subtraction

$$\begin{aligned} d\hat{\sigma}_{NNLO} = & \int_{d\Phi_{H+3}} (d\sigma_{NNLO}^{RR} - d\sigma_{NNLO}^S) \\ & + \int_{d\Phi_{H+2}} (d\sigma_{NNLO}^{RV} - d\sigma_{NNLO}^T) \\ & + \int_{d\Phi_{H+1}} (d\sigma_{NNLO}^{VV} - d\sigma_{NNLO}^U) \end{aligned}$$

- Each line above have unique singular behaviour
 - Double-Real line has implicit singularities in certain phase space region
 - Real-Virtual line has implicit poles & explicit poles up to $1/\epsilon^2$ from loop integral
 - Double-Virtual line has explicit poles up to $1/\epsilon^4$ from loop integral
- Factorize the kinematic poles in NNLO subtraction terms
 - \mathcal{E} -prescription S. Frixione, M. Grazzini, V. Del Duca, G. Somogyi, Z. Trocsanyi
 - q_T subtraction (NNLO) S.Catani, M.Grazzini
 - Antenna subtraction (NNLO) A. Gehrmann-De Ridder, T. Gehrmann, N. Glover,

Antenna Subtraction Method

- Construct counterterm to mimic singular limits

$$d\Phi_m(\{p_m\}) \sim d\Phi_n(\{\tilde{p}_n\}) \cdot d\Phi_X(\{p_X\})$$

$$d\sigma_{NNLO}^S \sim \underbrace{X(\{p_x\})}_{\text{antenna}} \cdot \overbrace{|\mathcal{M}(\{\tilde{p}_n\})|^2}^{\text{reduced ME}} \cdot \underbrace{\mathcal{J}(\{\tilde{p}_n\})}_{\text{jet function}}$$

- Analytically integrate antenna function over antenna phase space

$$\int_{d\Phi_{m+2}} d\sigma_{NNLO}^S \sim \underbrace{\int_{d\Phi_x} X(\{p_x\})}_{\text{singular } \epsilon} \underbrace{\int_{d\Phi_n} |\mathcal{M}(\{\tilde{p}_n\})|^2 \mathcal{J}(\{\tilde{p}_n\})}_{\text{finite}}$$

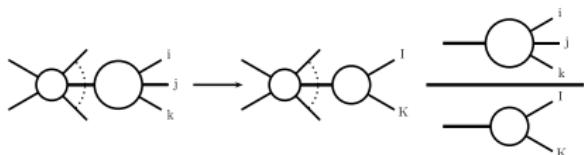
Phase Space Factorization

- Factorize a sub space $\{p_X\}$ from the $m + 2$ parton phase space $\{p_{m+2}\}$ using mapping operator that

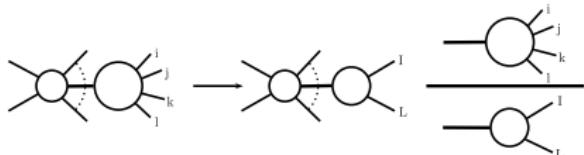
$$\mathcal{O}(\{p_{m+2}\}) \rightarrow \{\widetilde{p_n}\}$$

where $\{p_n\}$ is the redefined mementa

- $n = m + 1$ (NLO mapping) $d\Phi_{m+2}(\{p_{m+2}\}) = d\Phi_{m+1}(\{\widetilde{p_{m+1}}\}) \cdot d\Phi_X(\{p_X\})$
- $n = m$ (NNLO mapping) $d\Phi_{m+2}(\{p_{m+2}\}) = d\Phi_m(\{\widetilde{p_m}\}) \cdot d\Phi_X(\{p_X\})$



(a) FF PS Factorization at NLO



(b) FF PS Factorization at NNLO

Figure: Phase space factorisation example

- Redefined two composite momenta (on-shell and momentum conseration)

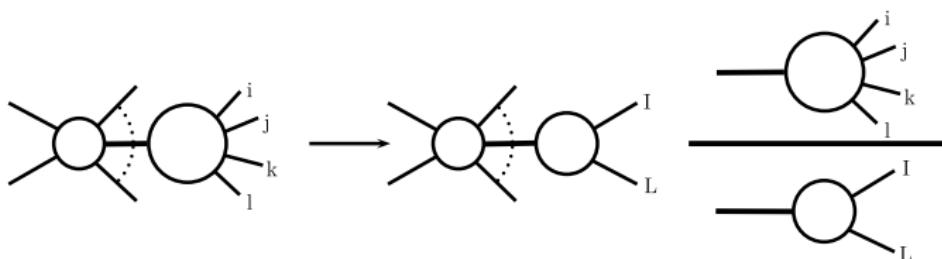
$$p_I^2 = p_K^2 = p_L^2 = 0$$

$$p_I^\mu + p_K^\mu = p_i^\mu + p_j^\mu + p_k^\mu$$

$$p_I^\mu + p_L^\mu = p_i^\mu + p_j^\mu + p_k^\mu + p_l^\mu$$

Phase-Space Splitting for $d\sigma_{NNLO}^{RR,S}$

- Final-Final (FF) phase-space splitting



- momentum mapping:

$$p_I^\mu = xp_i^\mu + r_1 p_j^\mu + r_2 p_k^\mu + z p_l^\mu$$

$$p_L^\mu = (1-x)p_i^\mu + (1-r_1)p_j^\mu + (1-r_2)p_k^\mu + (1-z)p_l^\mu$$

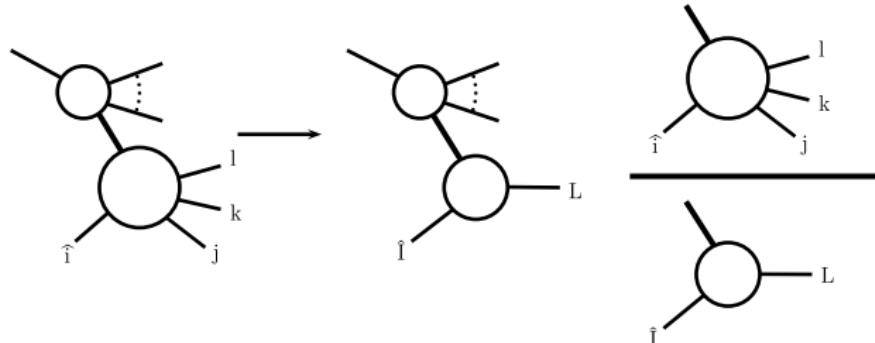
- phase-space factorisation:

$$\begin{aligned} d\Phi_{m+2}(p_a, \dots, p_i, p_j, p_k, p_l, \dots, p_{m+2}) &= d\Phi_m(p_a, \dots, p_I, p_L, \dots, p_{m+2}) \\ &\quad d\Phi_{X_{ijkl}}(p_i, p_j, p_k, p_l) \end{aligned}$$

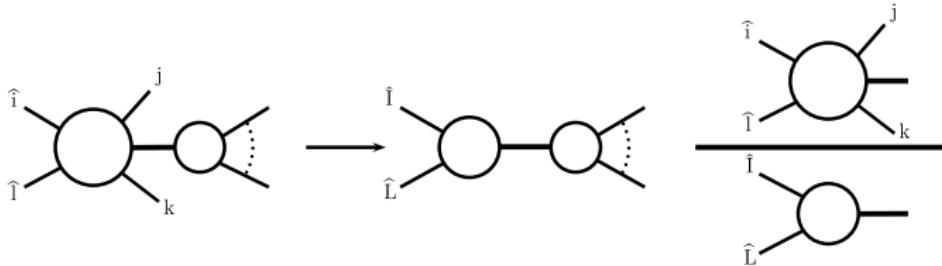
D. A. Kosower, 03

Phase-Space Splitting for $d\sigma_{NNLO}^{RR,S}$

- Initial-Final (IF) phase-space splitting

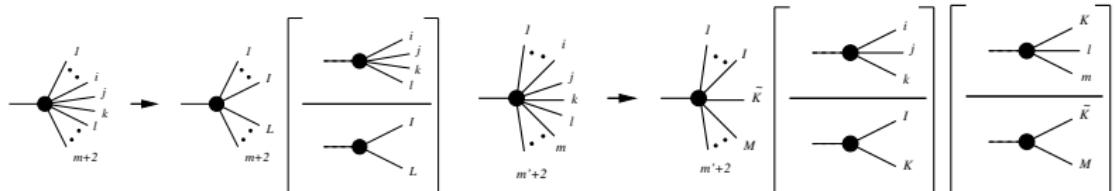


- Initial-Initial (II) phase-space splitting

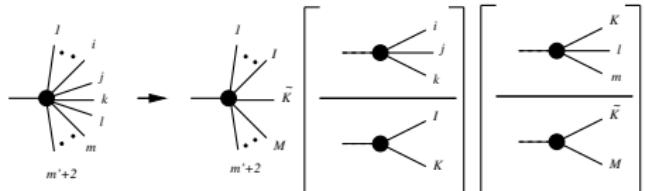


Matrix Element Splitting to Antenna Functions

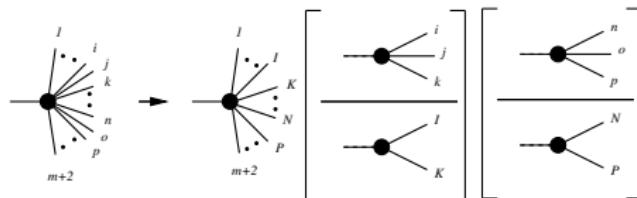
- Three possible colour ordering of double unresolved particles



(a) S^b colour connect $X_4^0 \otimes A_m^0$



(b) S^c colour almost connect $X_3^0 \otimes X_3^0 \otimes A_m^0$



(c) S^d colour not connect $X_3^0 \otimes X_3^0 \otimes A_m^0$

- Antenna function form physical matrix elements

$$X_4^0(i, j, k, l) = S_{ijkl, IL} \frac{|\mathcal{M}_{ijkl}^0|^2}{|\mathcal{M}_{IL}^0|^2} \quad X_3^0(i, j, k) = S_{ijk, IL} \frac{|\mathcal{M}_{ijkL}^0|^2}{|\mathcal{M}_{IL}^0|^2}$$

Matrix Element Splitting to Antenna Functions

- Antenna function at one-loop

$$A_{m+1}^1 \rightarrow X_3^0 \otimes A_m^1 + X_3^1 \otimes A_m^0$$

Z. Bern, L.D. Dixon, D. Dunbar, D. Kosower; S. Catani, M. Grazzini; P. Uwer
V. Del Duca, W Kilgore, C.R. Schmidt

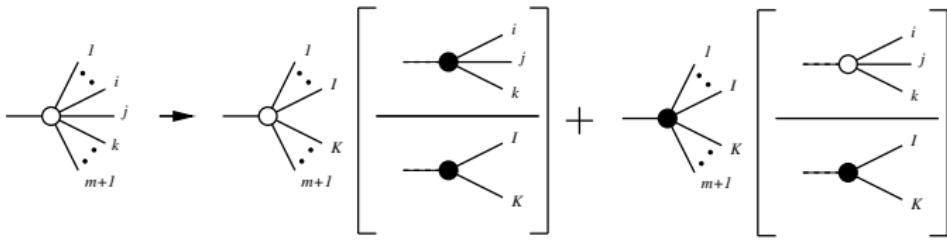


Figure: VS terms for one-loop single-unresolved limits

$$X_{ijk}^1 = S_{ijk,IK} \frac{|\mathcal{M}_{ijk}^1|^2}{|\mathcal{M}_{IK}^0|^2} - X_{ijk}^0 \frac{|\mathcal{M}_{IK}^1|^2}{|\mathcal{M}_{IK}^0|^2}$$

- Obtained from the colour-ordered renormalised one-loop three-parton matrix elements (at scale of s_{ijk})

Matrix Element Splitting to Antenna Functions

- Antenna function form physical matrix elements

$A, \tilde{A}, B, C \sim \gamma^* \rightarrow q\bar{q} + \text{partons}$ (hard quark – antiquark pair)

$D, E, \tilde{E} \sim \tilde{\chi} \rightarrow \tilde{g} + \text{partons}$ (hard quark – gluon pair)

$F, G, \tilde{G}, H \sim H \rightarrow \text{partons}$ (hard gluon – gluon pair)

A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, 05

- Complete set of Antenna tool box

phase config. \otimes *colour order* \otimes *parton types*

$$[FF, IF, II] \otimes [X_3^0, X_4^0, X_3^1] \otimes [A \sim H]$$

- All antenna functions are analytically integrable

- Final-Final X_3^0, X_4^0 and X_3^1 A. Gehrmann-De Ridder, T. Gehrmann, N. Glover
- Initial-Final X_3^0, X_4^0 and X_3^1 A. Daleo, T. Gehrmann, A. Gehrmann-De Ridder, G. Luisoni, D. Maitre
- Initial-Initial X_3^0, X_4^0 and X_3^1 R. Boughezal, A. Daleo, A. Gehrmann-De Ridder, T. Gehrmann, D. Maitre, P.F. Monni, M. Ritzmann

Antenna Subtraction Terms for $d\sigma_{NNLO}^S$

- Colour ordering of $gg \rightarrow gggH$ matrix element

$$\begin{aligned} \sum_{\text{colour \& spin}} |\mathcal{M}|^2 &= \sum_{P(i,j,k,l)} A_{5H}^0(1,i,j,k,l,H) \\ &= 2 \underbrace{\sum_{P(i,j,k)} A_{5H}^0(\textcolor{red}{1}, \textcolor{red}{2}, i, j, k, H)}_{\mathbb{X}_{5H}^0(\textcolor{red}{1}, \textcolor{red}{2}, 3, 4, 5, H)} + 2 \underbrace{\sum_{P(i,j,k)} A_{5H}^0(\textcolor{red}{1}, i, \textcolor{red}{2}, j, k, H)}_{\mathbb{Y}_{5H}^0(\textcolor{red}{1}, 3, \textcolor{red}{2}, 4, 5, H)} \end{aligned}$$

- Topology structure (colour leading)

$$\mathbb{X} \sim \dots F I I F \dots$$

$$\mathbb{Y} \sim \dots F I F I F \dots$$

$$\mathbb{Z} \sim \dots F I F F I F \dots$$

...

N. Glover, J. Pires, 10

$$d\sigma_{NNLO}^{RR} = \mathcal{N}_{NNLO}^{RR} d\Phi_4(p_3, \dots, p_H; p_1, p_2) \frac{2}{3!}$$

$$[\mathbb{X}_{5H}^0(\textcolor{red}{1}, \textcolor{red}{2}, 3, 4, 5, H) + \mathbb{Y}_{5H}^0(\textcolor{red}{1}, 3, \textcolor{red}{2}, 4, 5, H)]$$

Antenna Subtraction Terms for $d\sigma_{NNLO}^{RR,S}$

- Double unresolved subtraction structure

$$d\sigma_{NNLO}^S = d\sigma_{NNLO}^{S,a} + d\sigma_{NNLO}^{S,b} + d\sigma_{NNLO}^{S,c} + d\sigma_{NNLO}^{S,d}$$

- $d\sigma_{NNLO}^{S,a}$ one unresolved parton with Jet function missing additional parton

$$d\sigma_{NNLO}^{S,a} \sim X_3^0 \otimes A_{m+1}^0 \otimes \mathcal{J}_{m-2}^{(m-1)}$$

- $d\sigma_{NNLO}^{S,d}$ two colour-unconnected unresolved partons

$$d\sigma_{NNLO}^{S,d} \sim X_3^0 \otimes X_3^0 \otimes A_m^0 \otimes \mathcal{J}_{m-2}^{(m-2)}$$

- $d\sigma_{NNLO}^{S,b}$ genuine colour-connected double unresolved partons

$$d\sigma_{NNLO}^{S,b} \sim X_4^0 \otimes A_m^0 \otimes \mathcal{J}_{m-2}^{(m-2)} - \sum X_3^0 \otimes X_3^0 \otimes A_m^0 \otimes \mathcal{J}_{m-2}^{(m-2)}$$

- $d\sigma_{NNLO}^{S,c}$ two almost colour-unconnected unresolved partons

- Contains $X_3^0 \otimes X_3^0 \otimes A_m^0 \otimes \mathcal{J}_{m-2}^{(m-2)}$ structure and large angle soft subtraction terms
- Predictable (related to $d\sigma_{NNLO}^{S,b}$) → automated construction

Antenna Subtraction Terms for $d\sigma_{NNLO}^{RR,S}$

- Colour connection of X_4^0 antenna

$$A_4^0(1_q, 3_g, 4_g, 2_{\bar{q}}) \sim (\widetilde{134}, \widetilde{243}), (\widetilde{143}, \widetilde{234})$$

$$D_4^0(1_q, 3_g, 4_g, 5_g) \sim (\widetilde{134}, \widetilde{543}), (\widetilde{154}, \widetilde{345}), (\widetilde{135}, \widetilde{453}), (\widetilde{153}, \widetilde{435})$$

$$\begin{aligned} F_4^0(3_g, 4_g, 5_g, 6_g) \sim & (\widetilde{345}, \widetilde{654}), (\widetilde{365}, \widetilde{456}), (\widetilde{436}, \widetilde{563}), (\widetilde{543}, \widetilde{634}) \\ & (\widetilde{346}, \widetilde{564}), (\widetilde{364}, \widetilde{546}), (\widetilde{435}, \widetilde{653}), (\widetilde{453}, \widetilde{635}) \end{aligned}$$

- Symmetry properties

$$A_4^0(1_q, 3_g, 4_g, 2_{\bar{q}}) = A_4^0(2_q, 4_g, 3_g, 1_{\bar{q}})$$

$$D_4^0(1_q, 3_g, 4_g, 5_g) = D_4^0(1_q, 5_g, 4_g, 3_g)$$

$$F_4^0(3_g, 4_g, 5_g, 6_g) = F_4^0(6_g, 5_g, 4_g, 3_g) = F_4^0(4_g, 5_g, 6_g, 3_g) = \dots$$

$$\tilde{A}_4^0(1_q, 3_g, 4_g, 2_{\bar{q}}) = \tilde{A}_4^0(1_q, 4_g, 3_g, 2_{\bar{q}}) = \tilde{A}_4^0(2_q, 4_g, 3_g, 1_{\bar{q}})$$

- Spurious poles form non-complete sub-group

$$D_4^0(1_q, 3_g, 4_g, 5_g) \xrightarrow{1||3||5} \tilde{P}_{135 \rightarrow Q}(w, x, y)$$

Antenna Subtraction Terms for $d\sigma_{NNLO}^{RR,S}$

- Over subtraction form IF $X_4^0(1, i, j, k)$

- $q\bar{q} \rightarrow g \cdots g(H) \sim \mathbb{X}$

$$\mathbb{X} \sim \cdots g \underbrace{g \quad g \quad g}_{D_4^0(\textcolor{red}{q}, g_n, g_m, \textcolor{red}{g}_l)} q \chi \quad \underbrace{\bar{q} \chi \quad g \quad g \quad g}_{D_4^0(\bar{q}, g_i, g_j, \textcolor{red}{g}_k)} g \cdots$$

$$\mathbb{X} \sim \cdots D_4^0(q, g_n, g_m, g_l) + D_4^0(\bar{q}, g_i, g_j, g_k) - \tilde{A}_4^0(\textcolor{red}{q}, \textcolor{red}{g}_l, \bar{q}, \textcolor{red}{g}_k) \cdots$$

- $gg \rightarrow g \cdots g(H) \sim \mathbb{X} + \mathbb{Y} + \cdots$

$$\mathbb{X} \sim \cdots F_4^0(g_1, g_i, g_j, g_k) + F_4^0(g_2, g_n, g_m, g_l) - F_4^0(\textcolor{red}{g}_1, \textcolor{red}{g}_l, \textcolor{red}{g}_2, \textcolor{red}{g}_k) \cdots$$

$$\mathbb{Y} \sim \cdots g \underbrace{g \quad g}_{F_4^0(\textcolor{red}{g}_2, g_n, g_m, \textcolor{red}{g}_l)} \underbrace{g \quad g \chi}_{F_4^0(g_1, g_o, g_2, g_i) + F_4^0(g_1, g_o, g_2, g_n)} \underbrace{g \quad g \chi}_{F_4^0(g_1, g_i, g_j, \textcolor{red}{g}_k)} \underbrace{g \quad g}_{F_4^0(g_1, g_i, g_j, \textcolor{red}{g}_k)} g \cdots$$

$$\mathbb{Y} \sim \cdots F_4^0(g_1, g_o, g_2, g_i) + F_4^0(g_1, g_o, g_2, g_n)$$

$$+ F_4^0(g_2, g_n, g_m, g_l) + F_4^0(g_2, g_n, g_m, g_l) - F_4^0(\textcolor{red}{g}_1, \textcolor{red}{g}_l, \textcolor{red}{g}_2, \textcolor{red}{g}_k) \cdots$$

Application to $gg \rightarrow gggH$ Process

- \mathbb{X} topology

$$d\sigma_{NNLO}^{S^b, \mathbb{X}} = \mathcal{N}_{NNLO}^{RR} d\Phi_4(p_3, \dots, p_H; p_1, p_2) \frac{2}{3!} \sum_{(i,j,k)} A_{5H}^0(1, 2, i, j, k, H) \mathcal{J}_2^{(4)}(p_3, \dots, p_H)$$
$$d\sigma_{NNLO}^{S^{b_1}} + d\sigma_{NNLO}^{S^{b_2}} = \mathcal{N}_{NNLO}^{RR} d\Phi_4(p_3, \dots, p_H; p_1, p_2) \frac{2}{3!} \sum_{P_c(i,j,k)} \left[\begin{array}{l} +F_4^0(2, i, j, k) A_{3H}^0(1, \bar{2}, \widetilde{(ijk)}, H) \mathcal{J}_2^{(2)}(p_{(i,j,k)}, p_H) \\ +F_4^0(1, k, j, i) A_{3H}^0(\bar{1}, 2, \widetilde{(ijk)}, H) \mathcal{J}_2^{(2)}(p_{(i,j,k)}, p_H) \\ -F_4^0(1, i, 2, k) A_{3H}^0(\bar{1}, \bar{2}, \widetilde{j}, H) \mathcal{J}_2^{(2)}(p_{\bar{j}}, p_H) \end{array} \right]$$

Application to $gg \rightarrow gggH$ Process

- \mathbb{X} topology

$$d\sigma_{NNLO}^{S^b, \mathbb{X}} = \mathcal{N}_{NNLO}^{RR} d\Phi_4(p_3, \dots, p_H; p_1, p_2) \frac{2}{3!} \sum_{(i,j,k)} A_{5H}^0(1, 2, i, j, k, H) \mathcal{J}_2^{(4)}(p_3, \dots, p_H)$$

$$d\sigma_{NNLO}^{S^{b_1}} + d\sigma_{NNLO}^{S^{b_2}} = \mathcal{N}_{NNLO}^{RR} d\Phi_4(p_3, \dots, p_H; p_1, p_2) \frac{2}{3!} \sum_{P_c(i,j,k)} \left[\begin{array}{l} +F_4^0(2, i, j, k) A_{3H}^0(1, \bar{2}, \widetilde{(ijk)}, H) \mathcal{J}_2^{(2)}(p_{(i,j,k)}, p_H) \\ +F_4^0(1, k, j, i) A_{3H}^0(\bar{1}, 2, \widetilde{(ijk)}, H) \mathcal{J}_2^{(2)}(p_{(i,j,k)}, p_H) \\ -F_4^0(1, i, 2, k) A_{3H}^0(\bar{1}, \bar{2}, \widetilde{j}, H) \mathcal{J}_2^{(2)}(p_{\bar{j}}, p_H) \\ \\ - (f_3^0(2, i, j) F_3^0(\bar{2}, \widetilde{(ij)}, k) + f_3^0(i, j, k) F_3^0(2, \widetilde{(ij)}, \widetilde{(jk)}) + f_3^0(j, k, 2) F_3^0(\bar{2}, i, \widetilde{(jk)})) \\ A_{3H}^0(1, \bar{2}, \widetilde{(i,j,k)}, H) \mathcal{J}_2^{(2)}(p_{(i,j,k)}, p_H) \\ \\ - (f_3^0(1, k, j) F_3^0(\bar{1}, \widetilde{(kj)}, i) + f_3^0(k, j, i) F_3^0(1, \widetilde{(kj)}, \widetilde{(ji)}) + f_3^0(j, i, 1) F_3^0(\bar{1}, k, \widetilde{(ij)})) \\ A_{3H}^0(\bar{1}, 2, \widetilde{(i,j,k)}, H) \mathcal{J}_2^{(2)}(p_{(i,j,k)}, p_H) \\ \\ + (F_3^0(1, i, 2) F_3^0(\bar{1}, \bar{2}, k) + F_3^0(1, k, 2) F_3^0(\bar{1}, \bar{2}, i)) A_{3H}^0(\bar{1}, \bar{2}, \widetilde{\bar{j}}, H) \mathcal{J}_2^{(2)}(p_{\widetilde{\bar{j}}}, p_H) \end{array} \right]$$

Application to $gg \rightarrow gggH$ Process

- \mathbb{X} topology

$$d\sigma_{NNLO}^{S^b, \mathbb{X}} = \mathcal{N}_{NNLO}^{RR} d\Phi_4(p_3, \dots, p_H; p_1, p_2) \frac{2}{3!} \sum_{(i,j,k)} A_{5H}^0(1, 2, i, j, k, H) \mathcal{J}_2^{(4)}(p_3, \dots, p_H)$$

$$d\sigma_{NNLO}^{S^{b_1}} + d\sigma_{NNLO}^{S^{b_2}} = \mathcal{N}_{NNLO}^{RR} d\Phi_4(p_3, \dots, p_H; p_1, p_2) \frac{2}{3!} \sum_{P_c(i,j,k)} \left[\begin{array}{l} + F_4^0(2, i, j, k) A_{3H}^0(1, \bar{2}, \widetilde{(ijk)}, H) \mathcal{J}_2^{(2)}(p_{(i,j,k)}, p_H) \\ + F_4^0(1, k, j, i) A_{3H}^0(\bar{1}, 2, \widetilde{(ijk)}, H) \mathcal{J}_2^{(2)}(p_{(i,j,k)}, p_H) \\ - F_4^0(1, i, 2, k) A_{3H}^0(\bar{1}, \bar{2}, \widetilde{j}, H) \mathcal{J}_2^{(2)}(p_{\bar{j}}, p_H) \\ \\ - (\textcolor{red}{f}_3^0(2, i, j) F_3^0(\bar{2}, \widetilde{(ij)}, k) + f_3^0(i, j, k) F_3^0(2, \widetilde{(ij)}, \widetilde{(jk)}) + \textcolor{red}{f}_3^0(j, k, 2) F_3^0(\bar{2}, i, \widetilde{(jk)})) \\ A_{3H}^0(1, \bar{2}, \widetilde{(i, j, k)}, H) \mathcal{J}_2^{(2)}(p_{(i,j,k)}, p_H) \\ \\ - (\textcolor{red}{f}_3^0(1, k, j) F_3^0(\bar{1}, \widetilde{(kj)}, i) + f_3^0(k, j, i) F_3^0(1, \widetilde{(kj)}, \widetilde{(ji)}) + \textcolor{red}{f}_3^0(j, i, 1) F_3^0(\bar{1}, k, \widetilde{(ij)})) \\ A_{3H}^0(\bar{1}, 2, \widetilde{(i, j, k)}, H) \mathcal{J}_2^{(2)}(p_{(i,j,k)}, p_H) \\ \\ + (\textcolor{red}{F}_3^0(1, i, 2) F_3^0(\bar{1}, \bar{2}, k) + F_3^0(1, k, 2) F_3^0(\bar{1}, \bar{2}, i)) A_{3H}^0(\bar{1}, \bar{2}, \widetilde{\bar{j}}, H) \mathcal{J}_2^{(2)}(p_{\bar{j}}, p_H) \end{array} \right]$$

Application to $gg \rightarrow gggH$ Process

- \mathbb{Y} topology

$$d\sigma_{NNLO}^{S^b, \mathbb{Y}} = \mathcal{N}_{NNLO}^{RR} d\Phi_4(p_3, \dots, p_H; p_1, p_2) \frac{2}{3!} \sum_{(i,j,k)} A_{5H}^0(1, i, 2, j, k, H) \mathcal{J}_2^{(4)}(p_3, \dots, p_H)$$

$$d\sigma_{NNLO}^{S^{b_1}} + d\sigma_{NNLO}^{S^{b_2}} = \mathcal{N}_{NNLO}^{RR} d\Phi_4(p_3, \dots, p_H; p_1, p_2) \frac{2}{3!} \sum_{P_c(i,j,k)} \left[\begin{array}{l} + F_4^0(1, i, 2, j) A_{3H}^0(\bar{1}, \bar{2}, \tilde{k}, H) \mathcal{J}_2^{(2)}(p_{(\tilde{k})}, p_H) \\ + F_4^0(1, i, 2, k) A_{3H}^0(\bar{1}, \bar{2}, \tilde{j}, H) \mathcal{J}_2^{(2)}(p_{(\tilde{j})}, p_H) \\ + F_4^0(1, k, j, 2) A_{3H}^0(\bar{1}, \tilde{i}, \bar{2}, H) \mathcal{J}_2^{(2)}(p_{\tilde{i}}, p_H) \\ + F_4^0(1, j, k, 2) A_{3H}^0(\bar{1}, \tilde{i}, \bar{2}, H) \mathcal{J}_2^{(2)}(p_{\tilde{i}}, p_H) \\ \\ \left\{ - (F_3^0(1, i, 2) F_3^0(\bar{1}, \bar{2}, \tilde{j}) + F_3^0(1, j, 2) F_3^0(\bar{1}, \tilde{i}, \bar{2})) A_{3H}^0(\bar{\bar{1}}, \bar{\bar{2}}, \tilde{k}, H) \mathcal{J}_2^{(2)}(p_{(\tilde{k})}, p_H) \right. \\ \left. - (F_3^0(1, i, 2) F_3^0(\bar{1}, \bar{2}, \tilde{k}) + F_3^0(1, k, 2) F_3^0(\bar{1}, \tilde{i}, \bar{2})) A_{3H}^0(\bar{\bar{1}}, \bar{\bar{2}}, \tilde{j}, H) \mathcal{J}_2^{(2)}(p_{(\tilde{j})}, p_H) \right. \\ \left. - (f_3^0(1, k, j) F_3^0(\bar{1}, \widetilde{(kj)}, 2) + f_3^0(2, j, k) F_3^0(1, \widetilde{(kj)}, \bar{2})) A_{3H}^0(\bar{1}, \tilde{i}, \bar{2}, H) \mathcal{J}_2^{(2)}(p_{(\tilde{i})}, p_H) \right. \\ \left. - (f_3^0(1, j, k) F_3^0(\bar{1}, \widetilde{(jk)}, 2) + f_3^0(2, k, j) F_3^0(1, \widetilde{(jk)}, \bar{2})) A_{3H}^0(\bar{1}, \tilde{i}, \bar{2}, H) \mathcal{J}_2^{(2)}(p_{(\tilde{i})}, p_H) \right] \end{array} \right]$$

Application to $q\bar{q} \rightarrow gggH$ Process

- Only \mathbb{X} topology

$$d\sigma_{NNLO}^{S^b, \mathbb{X}} = \mathcal{N}_{NNLO}^{RR} d\Phi_4(p_3, \dots, p_H; p_1, p_2) \frac{1}{3!} \sum_{(i,j,k)} A_{5H}^0(1_q, i, j, k, 2_{\bar{q}}, H) \mathcal{J}_2^{(4)}(p_3, \dots, p_H)$$

$$d\sigma_{NNLO}^{S^{b_1}} + d\sigma_{NNLO}^{S^{b_2}} = \mathcal{N}_{NNLO}^{RR} d\Phi_4(p_3, \dots, p_H; p_1, p_2) \frac{1}{3!} \sum_{P_c(i,j,k)} \left[\begin{array}{l} + D_4^0(1_q, i, j, k) B_{3H}^0(\bar{1}_q, \widetilde{(ijk)}, 2_{\bar{q}}, H) \mathcal{J}_2^{(2)}(p_{(i,j,k)}, p_H) \\ + D_4^0(2_{\bar{q}}, k, j, i) B_{3H}^0(1_q, \widetilde{(ijk)}, \bar{2}_{\bar{q}}, H) \mathcal{J}_2^{(2)}(p_{(i,j,k)}, p_H) \\ - \tilde{A}_4^0(1_q, k, i, 2_{\bar{q}}) B_{3H}^0(\bar{1}_q, \widetilde{j}, \bar{2}_{\bar{q}}, H) \mathcal{J}_2^{(2)}(p_{\bar{j}}, p_H) \\ - (d_3^0(1_q, i, j) D_3^0(\bar{1}, \widetilde{(ij)}, k) + f_3^0(i, j, k) D_3^0(1_q, \widetilde{(ij)}, \widetilde{(jk)}) + d_3^0(1_q, k, j) D_3^0(\bar{1}_q, i, \widetilde{(jk)})) \\ B_{3H}^0(\bar{1}_q, \widetilde{(i, j, k)}, 2_{\bar{q}}, H) \mathcal{J}_2^{(2)}(p_{(i,j,k)}, p_H) \\ - (d_3^0(2_{\bar{q}}, k, j) D_3^0(\bar{2}_{\bar{q}}, \widetilde{(kj)}, i) + f_3^0(k, j, i) D_3^0(2_{\bar{q}}, \widetilde{(kj)}, \widetilde{(ji)}) + d_3^0(2_{\bar{q}}, i, j) D_3^0(\bar{2}_{\bar{q}}, k, \widetilde{(ij)})) \\ B_{3H}^0(1, \widetilde{(i, j, k)}, \bar{2}_{\bar{q}}, H) \mathcal{J}_2^{(2)}(p_{(i,j,k)}, p_H) \\ + (A_3^0(1, i, 2) A_3^0(\bar{1}, \widetilde{k}, \bar{2}) + A_3^0(1, k, 2) A_3^0(\bar{1}, \widetilde{i}, \bar{2})) B_{3H}^0(\bar{\bar{1}}, \widetilde{\bar{j}}, \bar{\bar{2}}, H) \mathcal{J}_2^{(2)}(p_{\widetilde{\bar{j}}}, p_H) \end{array} \right]$$

Future Work

- Short term
 - Numerical testing of subtraction terms
 - Tackle subleading colour terms systematically
 - Include remaining channels:
 - $q\bar{q} \rightarrow Q\bar{Q}gH$ process
 - $q\bar{q} \rightarrow q\bar{q}gH$ process
 - Automated double unresolved counterterms
- Long term
 - Monte Carlo integration of counterterms
 - Real-Virtual subtraction terms + integration
 - Combine with Double-Virtual contribution to final program