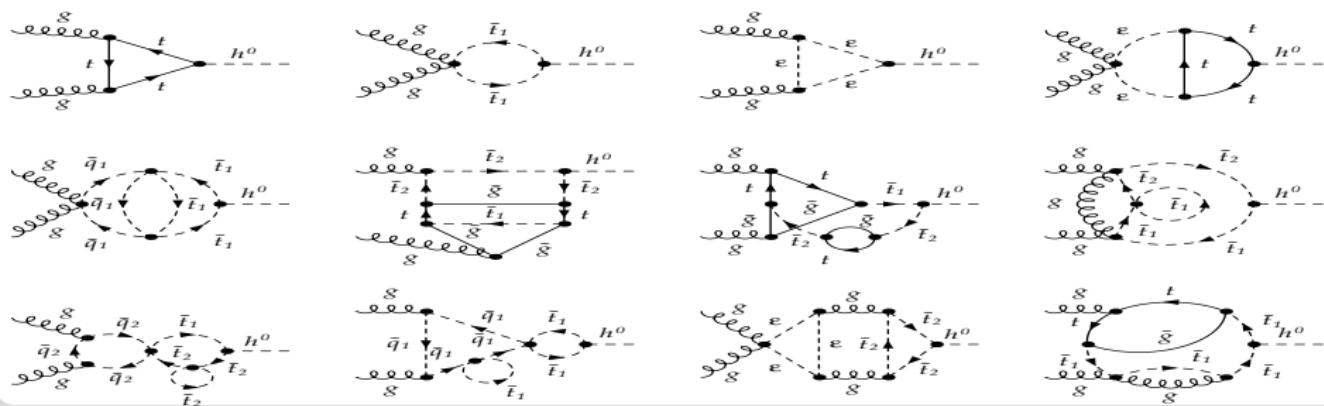


Low-energy theorem and Higgs boson production in the MSSM

Mid-Term meeting LHCPhenoNet, Ravello, September 2012

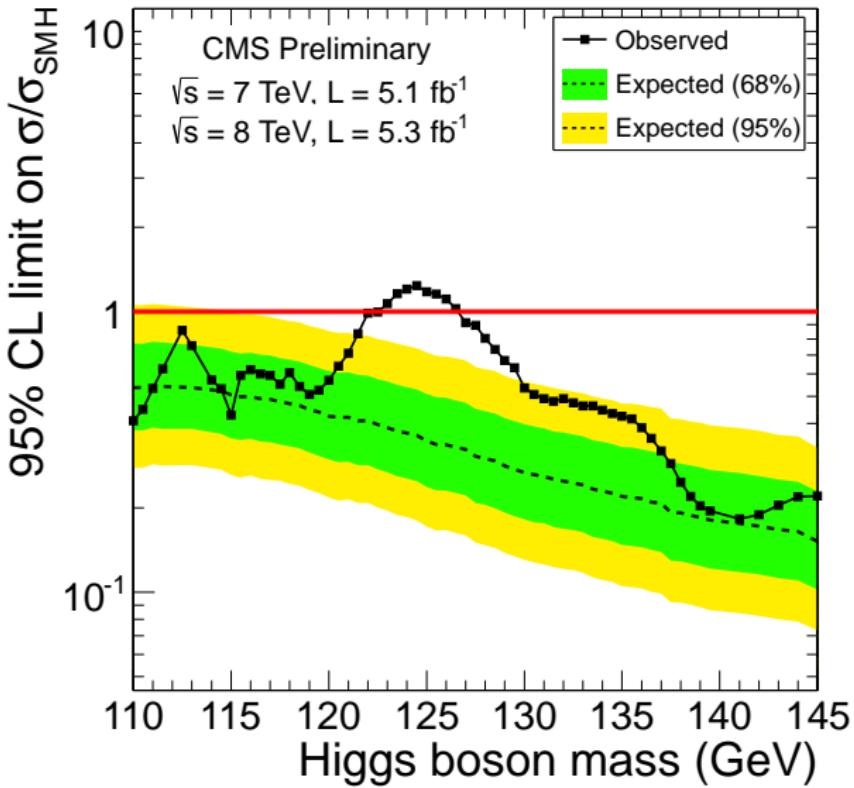
Matthias Steinhauser — TTP Karlsruhe | in collaboration with Alexander Kurz, Alexey Pak, Nikolai Zerf



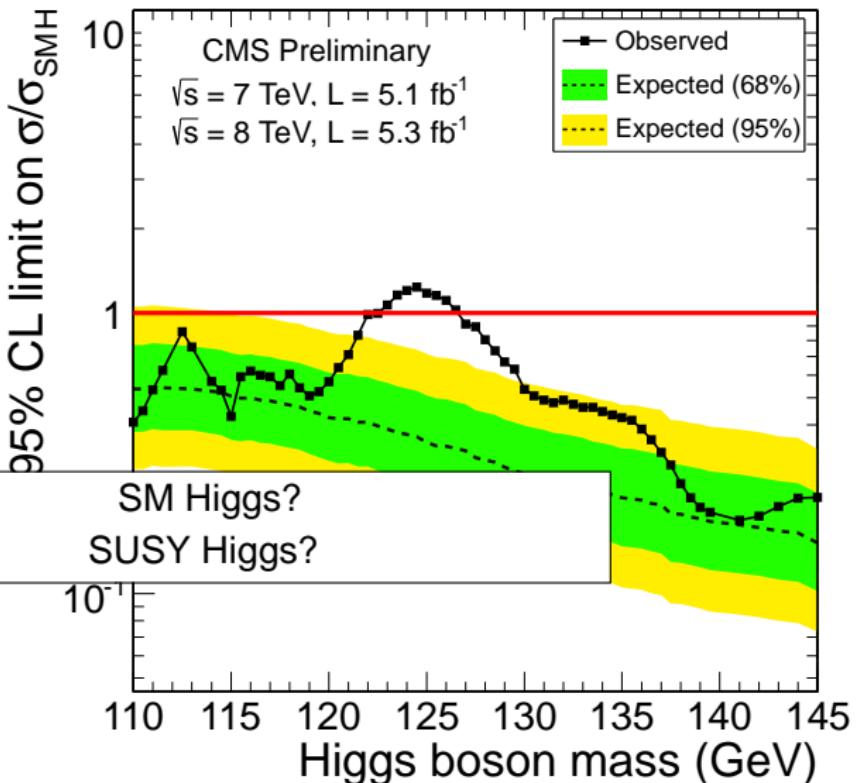
Outline

- Introduction
- $gg \rightarrow H$ in the SM
- Excursion: decoupling for α_s
- LET
- $gg \rightarrow H$ in the MSSM
- Conclusions

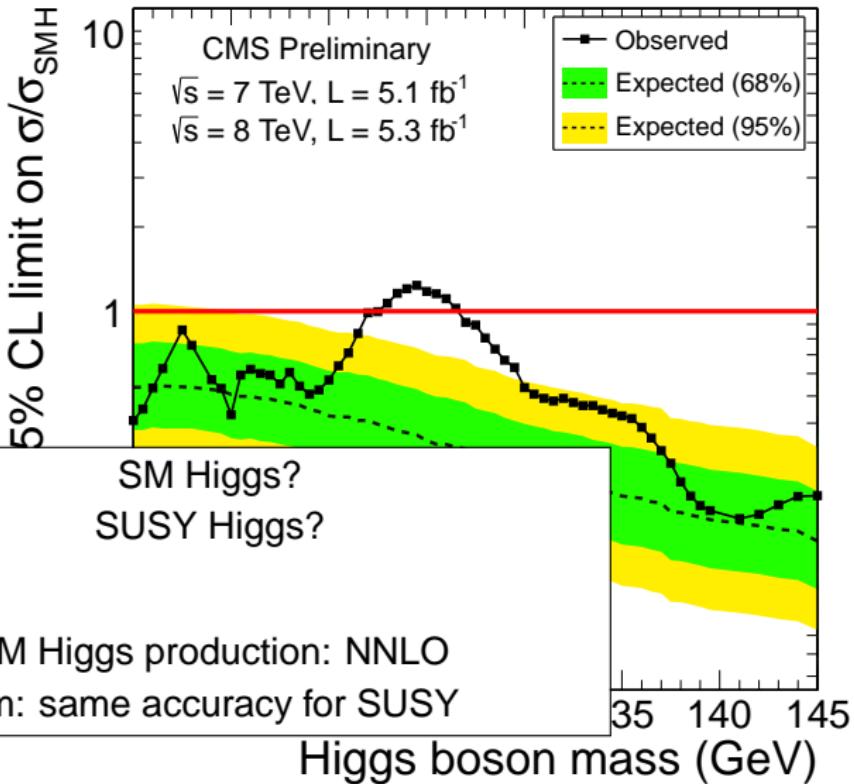
LHC discovery



LHC discovery



LHC discovery



Gluon fusion

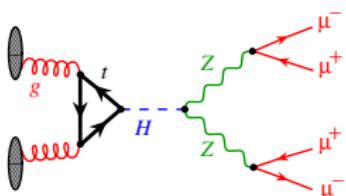
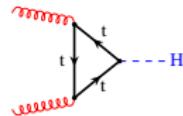
- largest cross section

- $gg \rightarrow H \rightarrow \gamma\gamma$

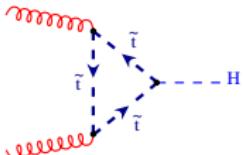
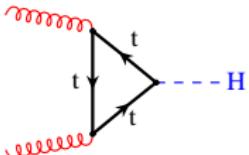
- $gg \rightarrow H \rightarrow ZZ \rightarrow 4\mu$

- $gg \rightarrow H \rightarrow WW \rightarrow l\nu l\nu$

...

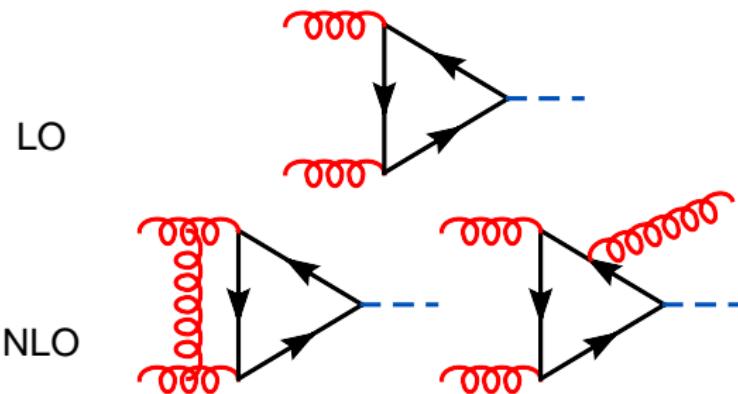


- sensitive to heavy particles, supersymmetry



⇒ later more

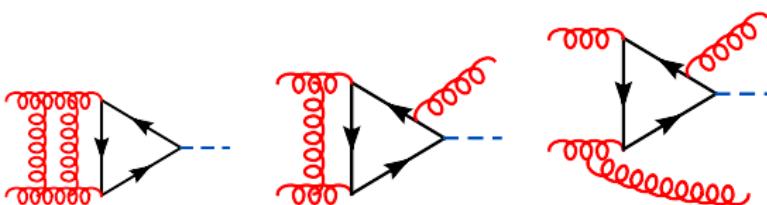
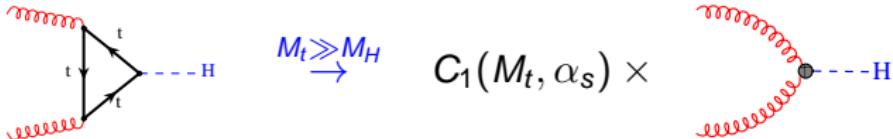
II. SM: $gg \rightarrow H$ at LO and NLO



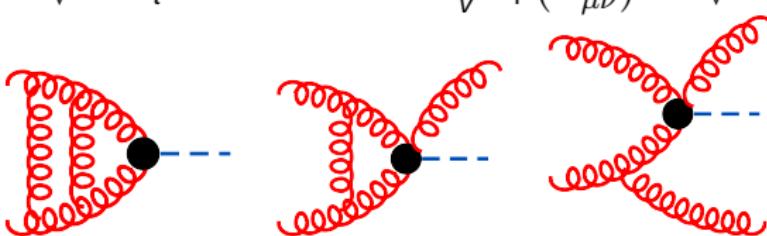
complete dependence on \hat{s}, M_t, M_H

[Dawson'91; Spira,Djouadi,Graudenz,Zerwas'91'95]

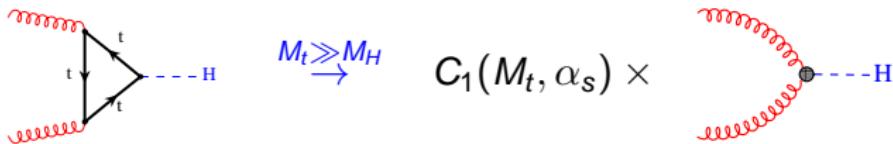
Effective theory



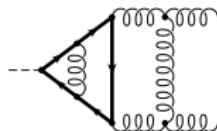
$$\Downarrow \quad M_t \rightarrow \infty: \quad \mathcal{L}^{\text{eff}} = \frac{H}{v} C_1 (G_{\mu\nu}^a)^2 \quad \Downarrow$$



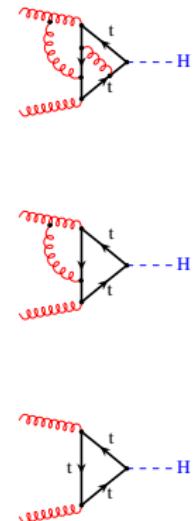
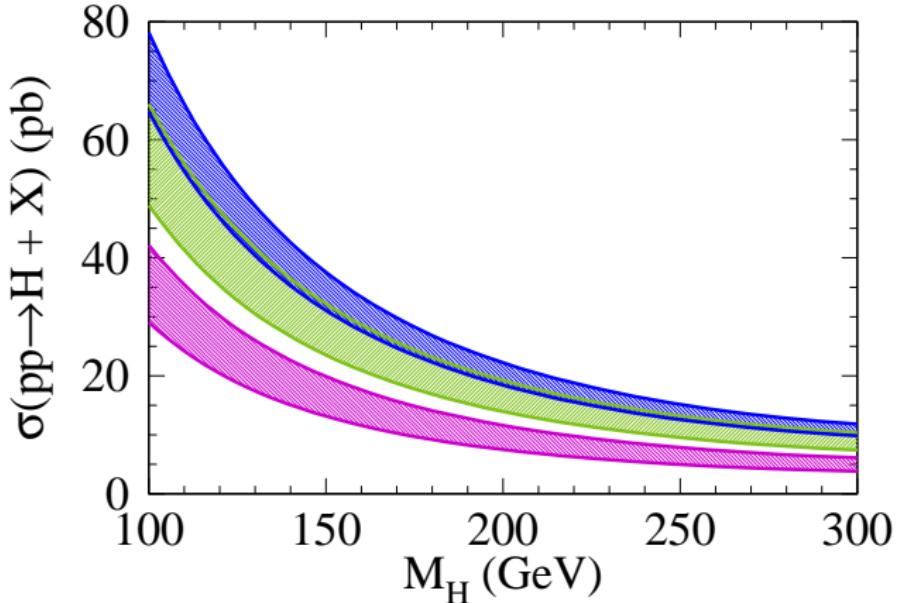
Effective theory



- 1. step: $C(M_t, \alpha_s)$ to NNLO
- 2. step: real and virtual corrections within effective theory



Gluon fusion to NNLO



NNLO $M_t \rightarrow \infty$

NNLO finite M_t effects: $\lesssim 1\%$ [Harlander,Ozeren'09; Harlander,Mantler,Marzani,Ozeren'10;

[Harlander,Kilgore'02; Anastasiou,Melnikov'02; Ravindran,Smith,v.Neerven'03]

Pak,Rogal,Steinhauser'09'11]

Beyond fixed order:

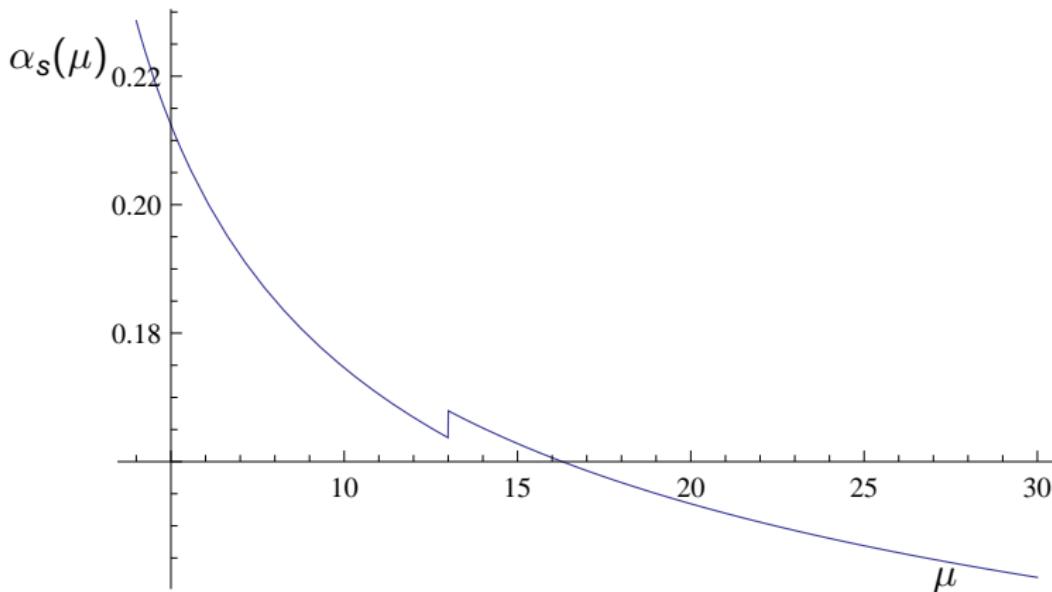
- soft-gluon resummation to NNLL
[\rightarrow is “simulated” for $\mu = M_H/2$ in NNLO]
- soft-gluon resummation to N^3LL
[Moch,Vogt'05; Ravindran'05'06; Idilbi,Ji,Yuan'06]
- “ π^2 -Resummation”
[Ahrens,Becher,Neubert,Yang'08]

all based on approximation for $M_t \rightarrow \infty$

III. Decoupling of heavy masses from running of α_s

$\alpha_s(\mu)$ for $4 \text{ GeV} \leq \mu \leq 30 \text{ GeV}$; 4-loop analysis

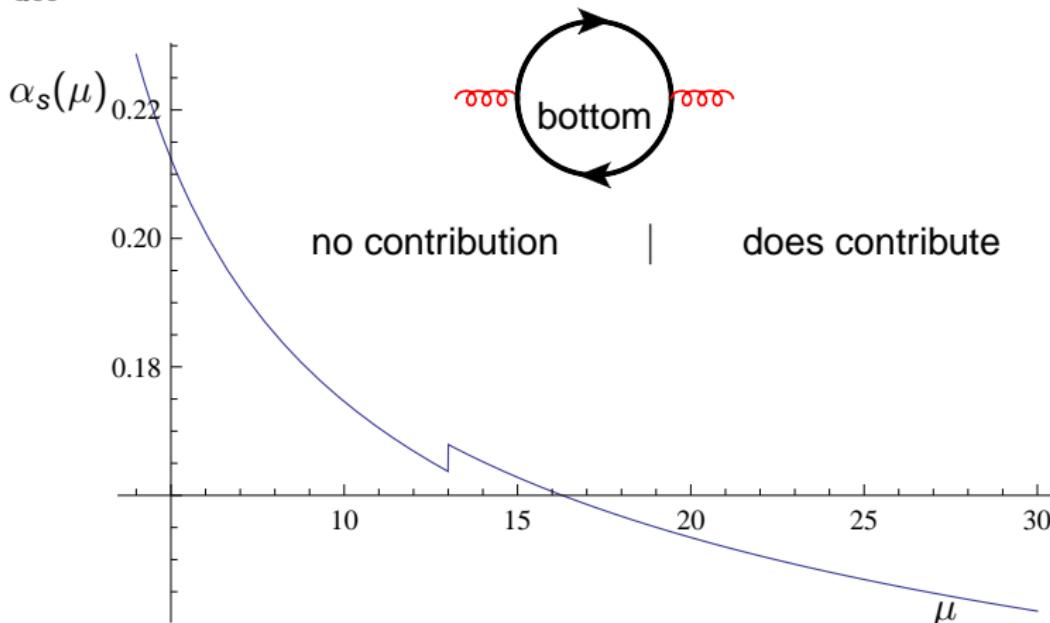
$\mu_{\text{dec}} = 13 \text{ GeV}$



III. Decoupling of heavy masses from running of α_s

$\alpha_s(\mu)$ for $4 \text{ GeV} \leq \mu \leq 30 \text{ GeV}$; 4-loop analysis

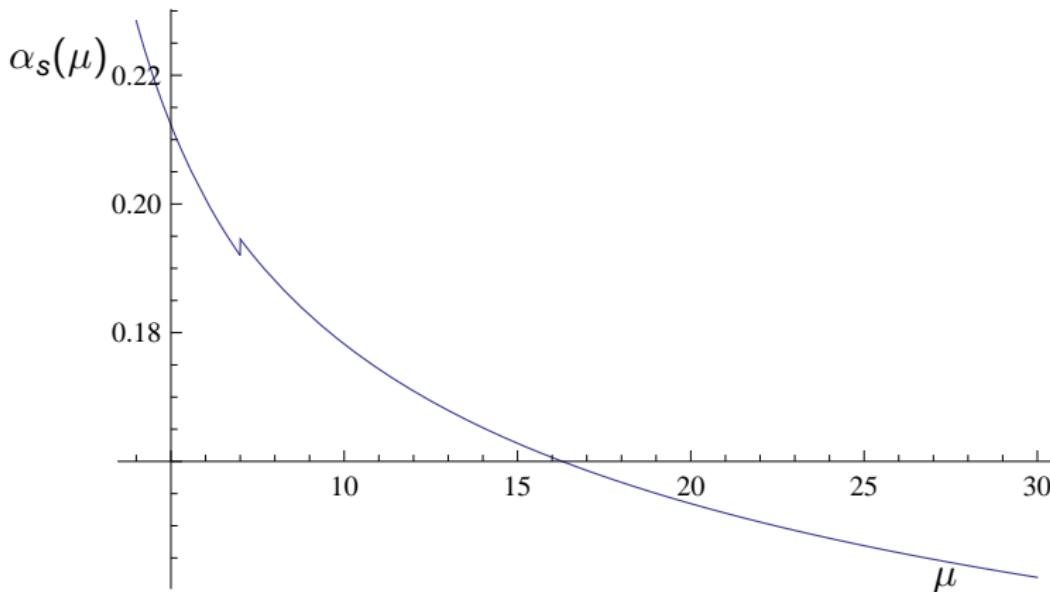
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III. Decoupling of heavy masses from running of α_s

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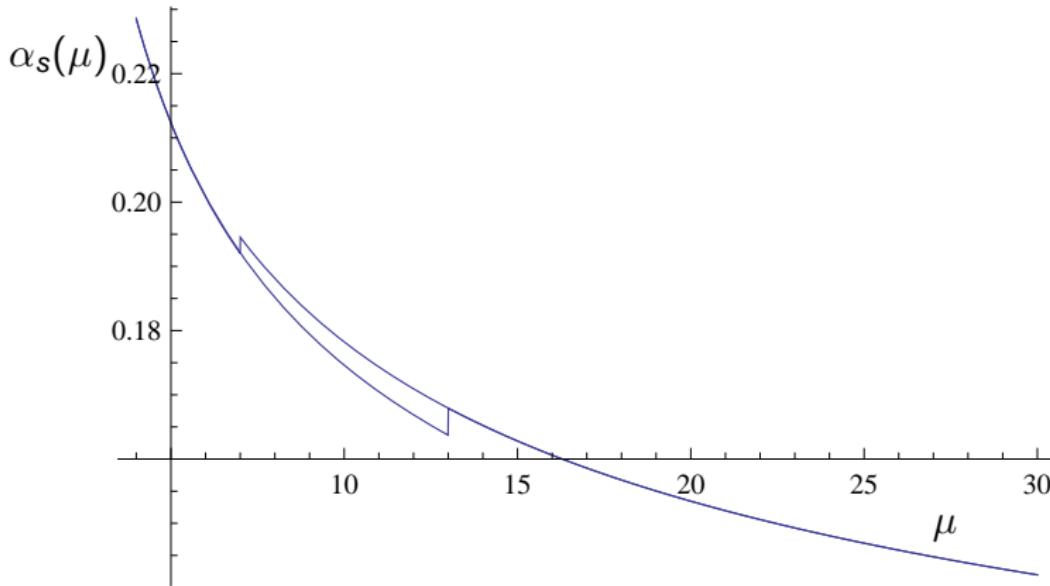
$\mu_{\text{dec}} = 7 \text{ GeV}$



III. Decoupling of heavy masses from running of α_s

$\alpha_s(\mu)$ for $4 \text{ GeV} \leq \mu \leq 30 \text{ GeV}$; 4-loop analysis

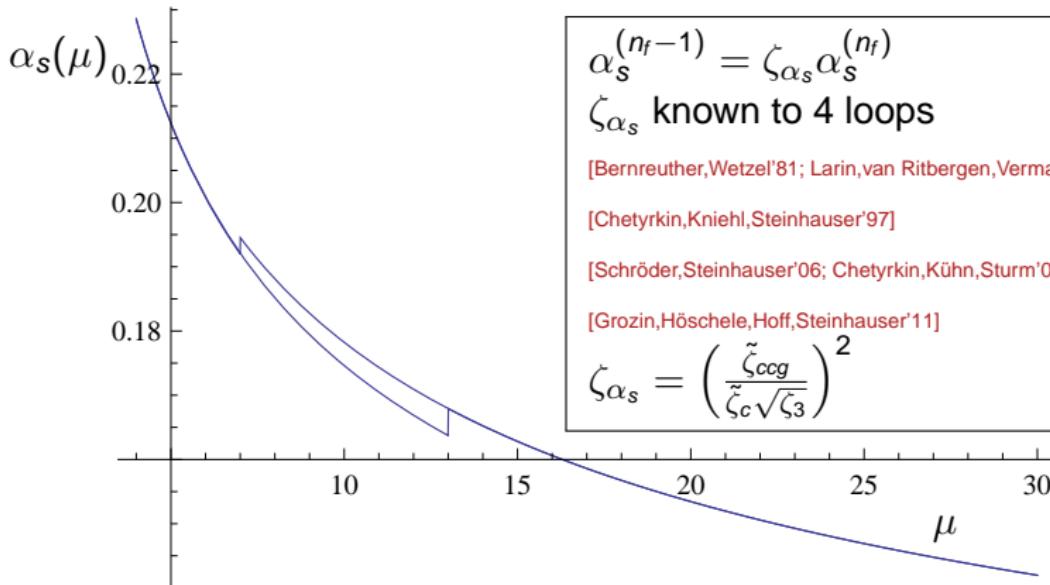
$\mu_{\text{dec}} = 7 \text{ GeV}$ and $\mu_{\text{dec}} = 13 \text{ GeV}$



III. Decoupling of heavy masses from running of α_s

$\alpha_s(\mu)$ for $4 \text{ GeV} \leq \mu \leq 30 \text{ GeV}$; 4-loop analysis

$\mu_{\text{dec}} = 7 \text{ GeV}$ and $\mu_{\text{dec}} = 13 \text{ GeV}$



Decoupling of SUSY particles

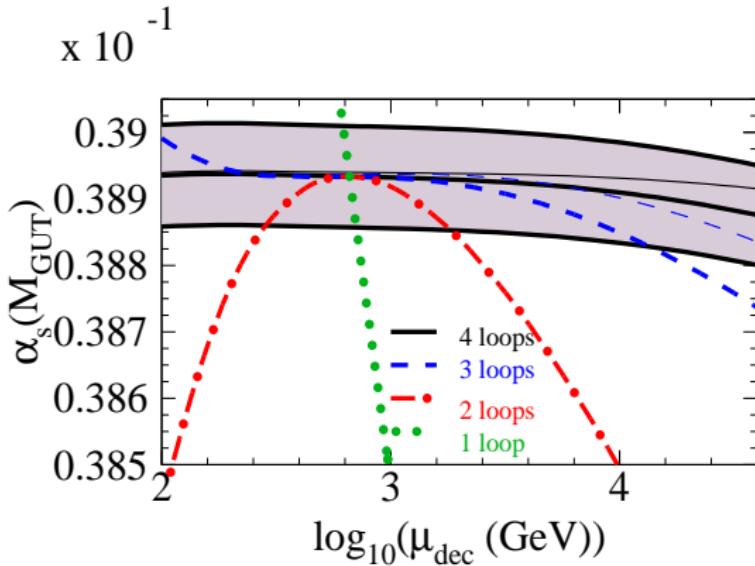
$$\alpha_s^{(5)}(\mu_{\text{dec}}) = \zeta_{\alpha_s}(\mu_{\text{dec}}) \alpha_s^{\text{(SQCD)}}(\mu_{\text{dec}})$$

$$\alpha_s^{(5),\overline{\text{MS}}}(M_Z) \xrightarrow{\text{run.}} \alpha_s^{(5),\overline{\text{MS}}}(\mu_{\text{dec}}) \xrightarrow{\text{dec.}} \alpha_s^{\text{(SQCD)}}(\mu_{\text{dec}}) \xrightarrow{\text{run.}} \alpha_s^{\text{(SQCD)}}(M_{\text{GUT}})$$

Decoupling of SUSY particles

$$\alpha_s^{(5)}(\mu_{\text{dec}}) = \zeta_{\alpha_s}(\mu_{\text{dec}}) \alpha_s^{(\text{SQCD})}(\mu_{\text{dec}})$$

$$\alpha_s^{(5),\overline{\text{MS}}} (M_Z) \xrightarrow{\text{run.}} \alpha_s^{(5),\overline{\text{MS}}} (\mu_{\text{dec}}) \xrightarrow{\text{dec.}} \alpha_s^{(\text{SQCD})} (\mu_{\text{dec}}) \xrightarrow{\text{run.}} \alpha_s^{(\text{SQCD})} (M_{\text{GUT}})$$



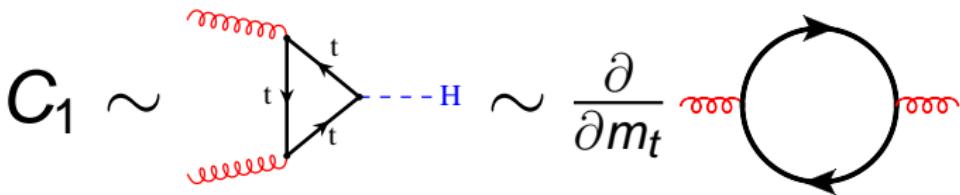
n -loop running
and
 $(n - 1)$ -loop
decoupling

[Harlander,Mihaila,Steinhauser'05; Bauer,Mihaila,Salomon'08; Kurz,Zerf,Steinhauser'12]

IV. Low-energy theorem in the SM

$$\mathcal{L}^{\text{eff}} = \frac{H}{v} C_1 (G_{\mu\nu}^a)^2$$

- naive:

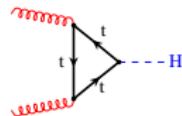


- works at 1 and 2 loops

[Ellis, Gaillard, Nanopoulos'75; Shifman et al.'79; Kniehl, Spira'95; ...]

LET: all-order result

$$\mathcal{L}^{\text{eff}} = \frac{H}{v} C_1 (G_{\mu\nu}^a)^2$$

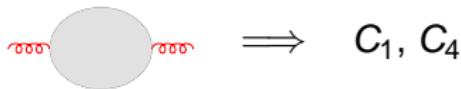


→ consider all dim.-4 operators (containing 2 external gluons) [Spiridonov'84]

$$\mathcal{O}_1 = (G_{\mu\nu}^a)^2$$

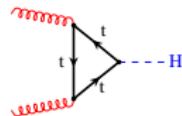
$$\mathcal{O}_4 = G_\nu^a \left(\nabla_\mu^{ab} G^{b\mu\nu} + g_s^0 \sum_{i=1}^{n_l} \bar{\psi}_{q_i}^0 \frac{\lambda^a}{2} \gamma^\nu \psi_{q_i}^0 \right) - \partial_\mu \bar{c}^a \partial^\mu c^a$$

$$\mathcal{O}_5 = (\nabla_\mu^{ab} \partial^\mu \bar{c}^b) c^a$$



LET: all-order result

$$\mathcal{L}^{\text{eff}} = \frac{H}{v} C_1 (G_{\mu\nu}^a)^2$$



→ consider all dim.-4 operators (containing 2 external gluons) [Spiridonov'84]

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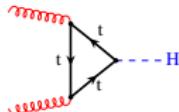
$$\mathcal{O}_4 = G_\nu^a \left(\nabla_\mu^{ab} G^{b\mu\nu} + g_s^0 \sum_{i=1}^{n_l} \bar{\psi}_{q_i}^0 \frac{\lambda^a}{2} \gamma^\nu \psi_{q_i}^0 \right) - \partial_\mu \bar{c}^a \partial^\mu c^a$$

$$\mathcal{O}_5 = (\nabla_\mu^{ab} \partial^\mu \bar{c}^b) c^a$$

$$\zeta_3 \iff \text{---} \circlearrowleft \text{---} \implies C_1, C_4$$

LET: all-order result

$$\mathcal{L}^{\text{eff}} = \frac{H}{v} C_1 (G_{\mu\nu}^a)^2$$



⇒ consider all dim.-4 operators (containing 2 external gluons) [Spiridonov'84]

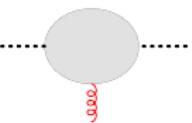
$$\mathcal{O}_1 = (G_{\mu\nu}^a)^2$$

$$\mathcal{O}_4 = G_\nu^a \left(\nabla_\mu^{ab} G^{b\mu\nu} + g_s^0 \sum_{i=1}^{n_l} \bar{\psi}_{q_i}^0 \frac{\lambda^a}{2} \gamma^\nu \psi_{q_i}^0 \right) - \partial_\mu \bar{c}^a \partial^\mu c^a$$

$$\mathcal{O}_5 = (\nabla_\mu^{ab} \partial^\mu \bar{c}^b) c^a$$

$$\zeta_3 \iff \text{---} \circlearrowleft \text{---} \Rightarrow C_1, C_4$$


$$\tilde{\zeta}_c \iff \text{---} \circlearrowleft \text{---} \Rightarrow C_4, C_5$$


$$\tilde{\zeta}_{ccg} \iff \text{---} \circlearrowleft \text{---} \Rightarrow C_5$$


LET: all-order result

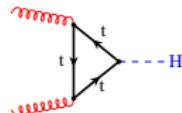
$$\mathcal{L}^{\text{eff}} = \frac{H}{v} C_1 (G_{\mu\nu}^a)^2$$

⇒ consider all dim.-4 operators (containing 2 external gluons) [Spiridonov'84]

$$\mathcal{O}_1 = (G_{\mu\nu}^a)^2$$

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$$\mathcal{O}_5 = (\nabla_\mu^{ab} \partial^\mu \bar{c}^b) c^a$$



$$\zeta_3 \iff \text{---} \circlearrowleft \text{---} \Rightarrow C_1, C_4$$

$$\tilde{\zeta}_c \iff \text{.....} \circlearrowleft \text{.....} \Rightarrow C_4, C_5$$

$$\tilde{\zeta}_{ccg} \iff \text{.....} \circlearrowleft \text{.....} \Rightarrow C_5$$

⇒ solve for C_1 :

$$C_1 = -m_t \frac{\partial}{\partial m_t} \ln \zeta_{\alpha_s}$$

[Chetyrkin,Kniehl,Steinhauser'97]

C_1 from LET (SM)

ζ_{α_s} to 3 loops $\Rightarrow C_1$ to 3 loops (NNLO) [Chetyrkin,Kniehl,Steinhauser'97]

$C_1 = -m_t \frac{\partial}{\partial m_t} \ln \zeta_{\alpha_s}$ $\Rightarrow C_1$ to 4 loops (N^3LO)

ζ_{α_s} to 4 loops $\Rightarrow C_1$ to 4 loops (N^3LO)

[Schröder,Steinhauser'06; Chetyrkin,Kühn,Sturm'06]

$C_1 = -m_t \frac{\partial}{\partial m_t} \ln \zeta_{\alpha_s}$ $\Rightarrow C_1$ to 5 loops (N^4LO)

(up to n_f -dependent coefficient in 5-loop β function)

$\zeta_{\alpha_s}(m_1, m_2)$ $\Rightarrow C_1$ for “many heavy quarks” (NNLO)

[Grozin,Höschele,Hoff,Steinhauser'11]

(3 loops)

(4^{th} generation; see also [Anastasiou,Boughezal,Furlan'10])

LET and the MSSM

$$C_1^0 = D_h^0 \ln \zeta_{\alpha_s}^0$$

[Kurz,Steinhauser,Zerf'12]

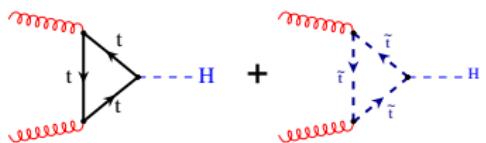
$$D_h^0 = D_t^0 + D_{\tilde{q}}^0 + V_t^0 \frac{\partial}{\partial m_t^0} + (\Lambda_\varepsilon^0)^2 \frac{\partial}{\partial (m_\varepsilon^0)^2}$$

$$D_t = V_{11}^{\tilde{t}} \frac{\partial}{\partial m_{t_1}^2} + V_{22}^{\tilde{t}} \frac{\partial}{\partial m_{t_2}^2} + \frac{V_{12}^{\tilde{t}} + V_{21}^{\tilde{t}}}{2(m_{t_1}^2 - m_{t_2}^2)} \frac{\partial}{\partial \theta_t}$$

...

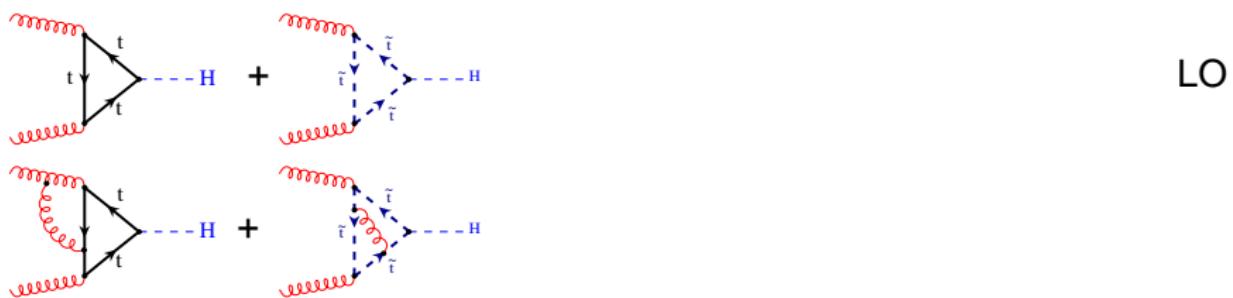
- NLO (2 loops): [Degrassi,Slavich'08; Mihaila,Reisser'10]
- “0”: bare quantities
- check of 3-loop ggh vertex calculation (\Rightarrow later more)

V. Gluon fusion in SUSY

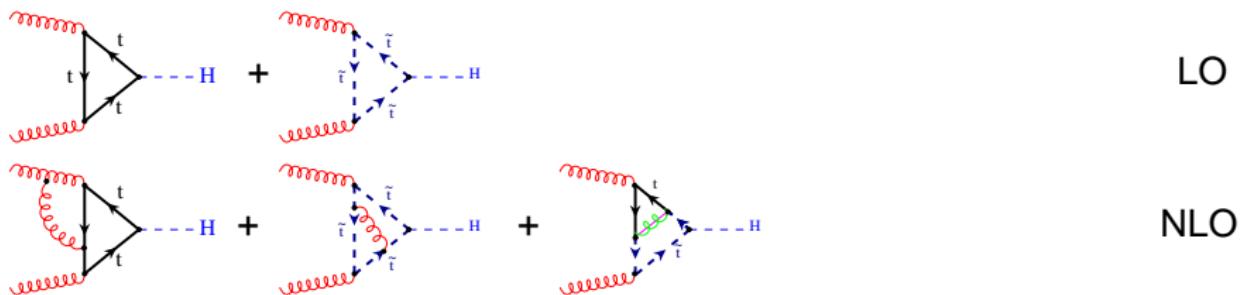


LO

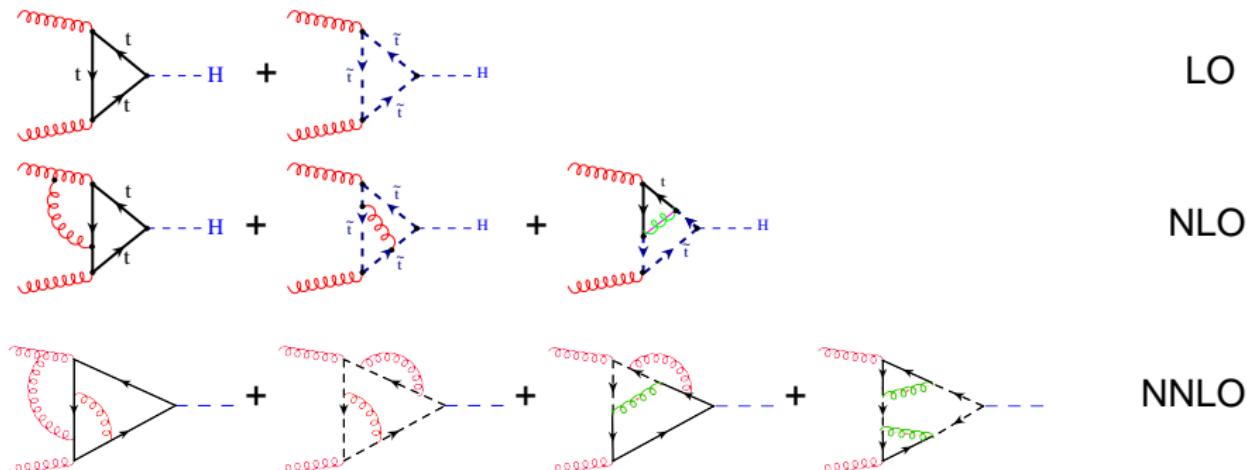
V. Gluon fusion in SUSY



V. Gluon fusion in SUSY

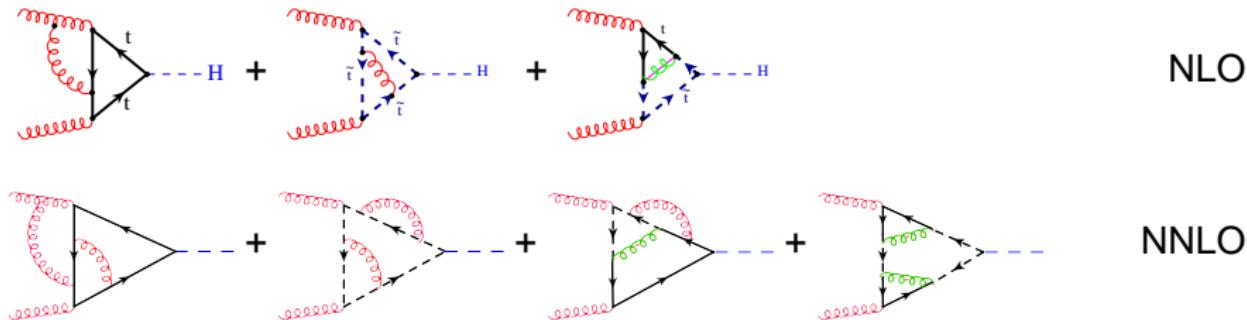


V. Gluon fusion in SUSY



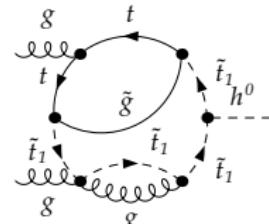
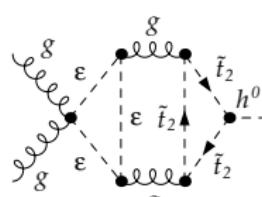
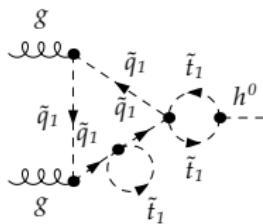
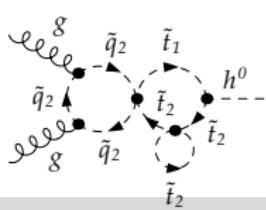
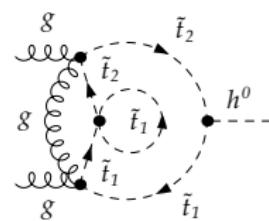
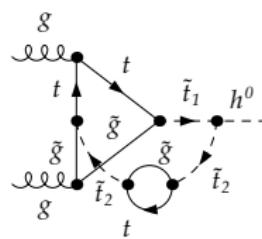
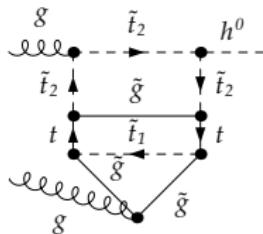
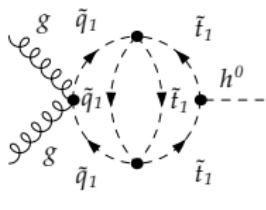
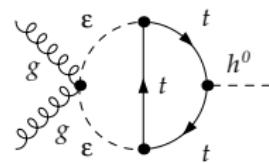
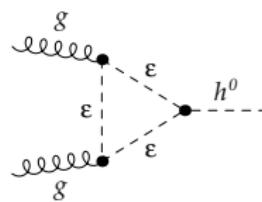
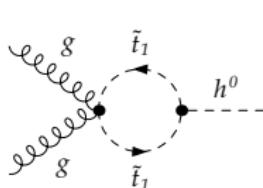
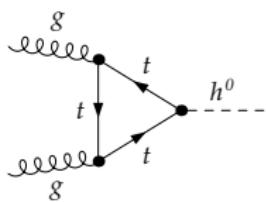
+ 57936 more Feynman diagrams

V. Gluon fusion in SUSY

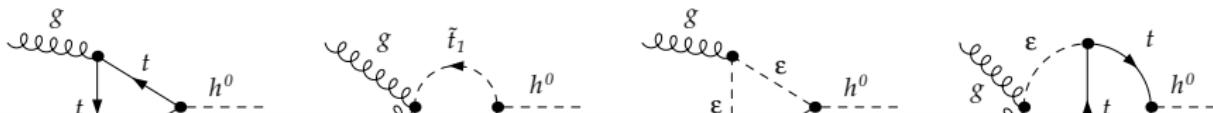


- NLO SUSY-QCD, $M_H \ll M_{\text{SUSY}}$, (t, \tilde{t}) sector [Harlander, Steinhauser'03'04; Degrassi, Slavich'08]
- NLO squark loops [Aglietti, Bonciani, Degrassi, Vivini'07; Mühlleitner, Spira'08]
- NLO SUSY-QCD, (b, \tilde{b}) sector [Anastasiou et al.'07; Degrassi, Slavich'11; Harlander, Hofmann, Mantler'11]
- NLO SUSY-QCD “full theory” [Anastasiou, Bucherer, Daleo'07; Mühlleitner, Rzehak, Spira]
- NNLO SUSY-QCD, $M_H \ll M_{\text{SUSY}}$, (t, \tilde{t}) sector [Pak, Steinhauser, Zerf'10'12]

$\sigma(gg \rightarrow h + X)$ in the MSSM to NNLO



$\sigma(gg \rightarrow h + X)$ in the MSSM to NNLO



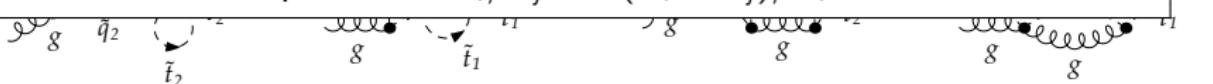
- mass scales: $m_t, m_{\tilde{t}1}, m_{\tilde{t}2}, m_{\tilde{q}}, m_{\tilde{g}}$
- exact calculation not possible
- consider hierarchies

$$(h1) \quad m_{\tilde{q}} \approx m_{\tilde{t}1} \approx m_{\tilde{t}2} \approx m_{\tilde{g}} \gg m_t$$

$$(h2) \quad m_{\tilde{q}} \approx m_{\tilde{t}2} \approx m_{\tilde{g}} \gg m_{\tilde{t}1} \gg m_t$$

$$(h3) \quad m_{\tilde{q}} \approx m_{\tilde{t}1} \approx m_{\tilde{g}} \gg m_{\tilde{t}2} \approx m_t$$

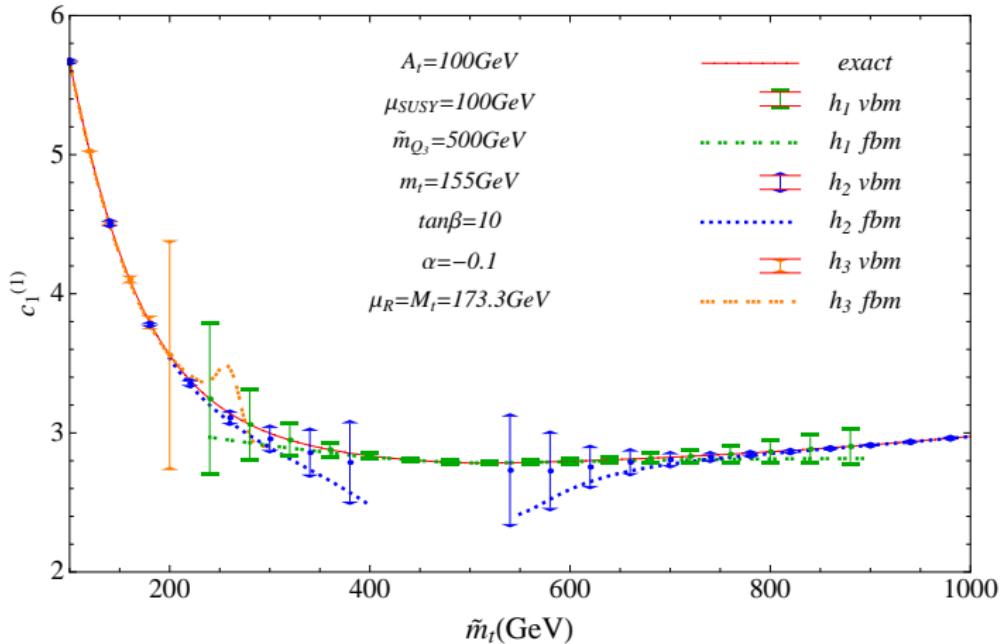
- requirements:
 - good approximation at 2 loops
 - h.o. expansion in m_i/m_j and $(m_i - m_j)/m_i$ "small"



Some comments ...

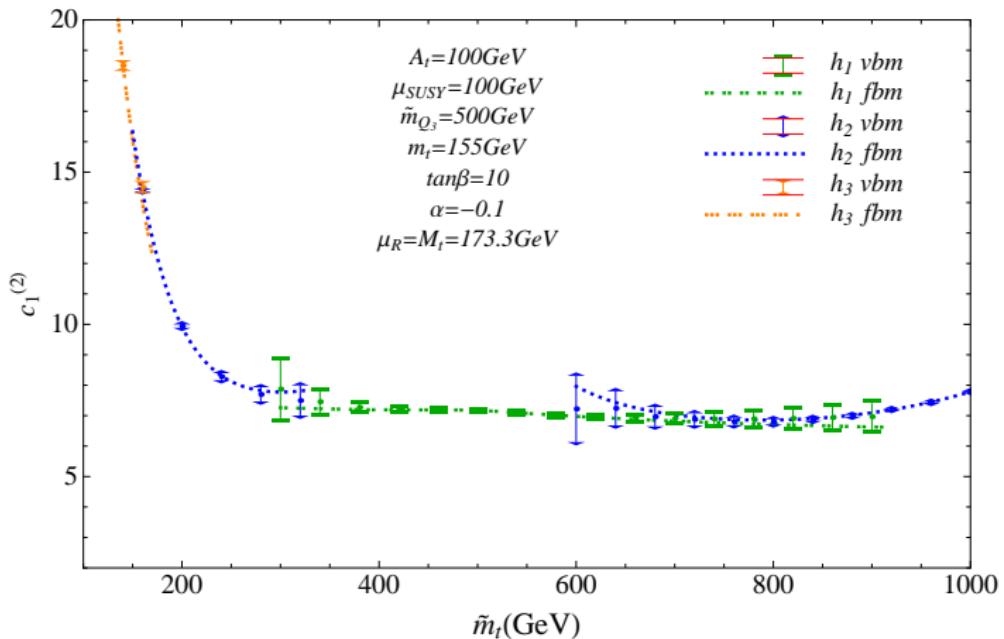
- $\overline{\text{DR}}$ scheme
- Dimensional Reduction (DRED), ε scalar
- $\mathcal{L}_\varepsilon = -\frac{1}{2} (M_\varepsilon^0)^2 \varepsilon_\sigma^{0,a} \varepsilon_\sigma^{0,a} - \frac{\phi^0}{v^0} (\Lambda_\varepsilon^0)^2 \varepsilon_\sigma^{0,a} \varepsilon_\sigma^{0,a}$
- renormalization of M_ε and Λ_ε
- integrate out M_ε together with m_t and SUSY particles
 - ⇒ $M_\varepsilon \neq 0$ but small
 - ⇒ match: MSSM ($\overline{\text{DR}}$) → SM ($\overline{\text{MS}}$)

C_1 at 2 loops



[Pak,Zerf,Steinhauser'12]

C_1 at 3 loops

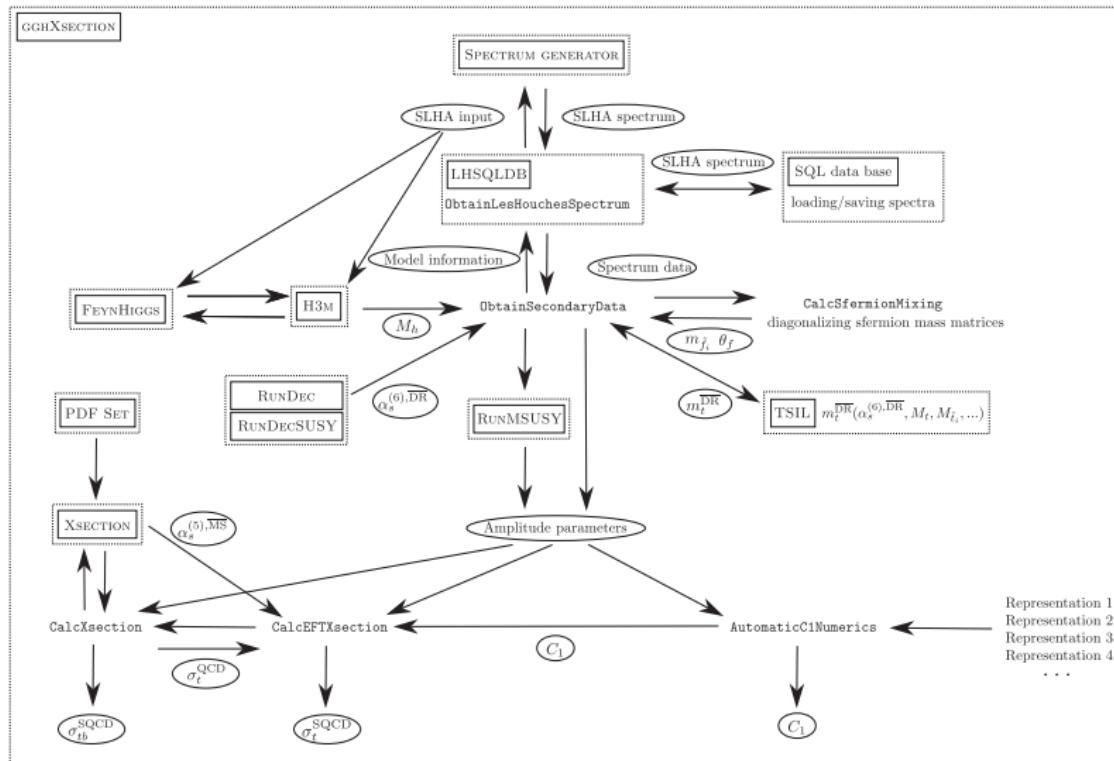


[Pak,Zerf,Steinhauser'12]

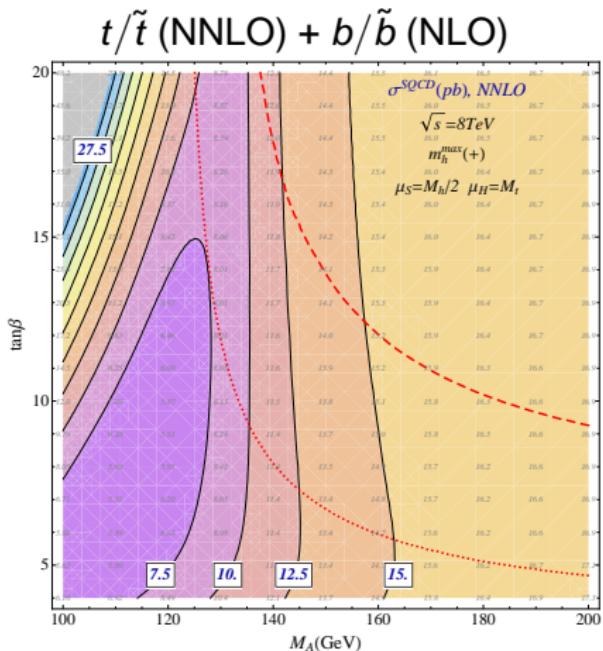
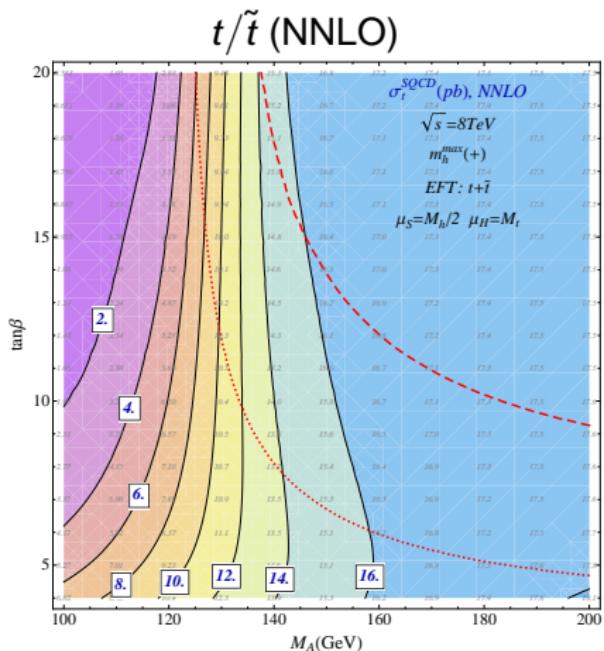
$$\begin{aligned}\sigma(pp \rightarrow h + X) = & (1 + \delta^{\text{EW}}) \times \\ & \left[\sigma_{tb}(\mu_s) \Big|_{\text{NLO}} - \sigma_t(\mu_s) \Big|_{\text{NLO}} + \sigma_t(\mu_s, \mu_h) \Big|_{\text{NNLO}} \right]\end{aligned}$$

- δ^{EW} : NLO electroweak corrections: [Actis, Passarino, Sturm, Uccirati'08]
- σ_{tb} : NLO top and bottom contribution [Degrassi, Slavich'11; Harlander, Hofmann, Mantler'11]
- NNLO: top/stop/sbottom ($m_b = 0$)
- separate hard (μ_h) and soft scale (μ_s):
 $\mathcal{O}_1 \rightarrow \tilde{\mathcal{O}}_1 = \frac{\beta^{(5)}}{\alpha_s^{(5)}} \mathcal{O}_1$ RG invariant $\Leftrightarrow \tilde{C}_1 = \frac{\alpha_s^{(5)}}{\beta^{(5)}} C_1$ RG invariant
 $\Leftrightarrow \tilde{\mathcal{O}}_1 \leftrightarrow \mu_s, \tilde{C}_1 \leftrightarrow \mu_h$
 \Leftrightarrow resum $\ln(\mu^2/M_h^2)$ terms

Numerical setup

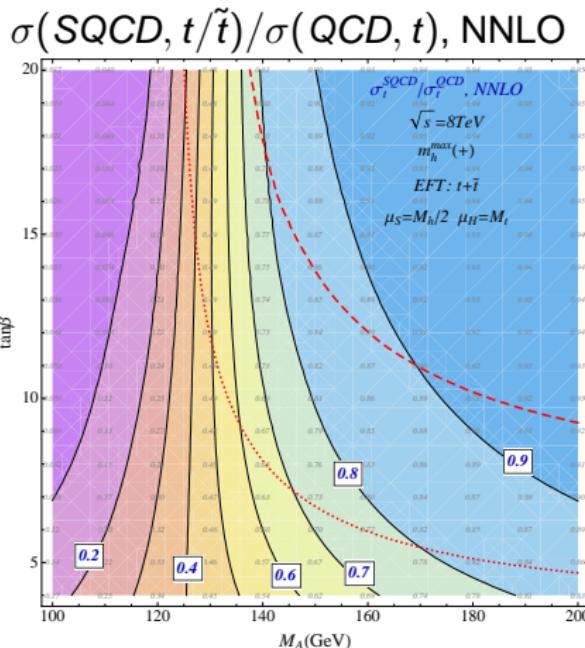
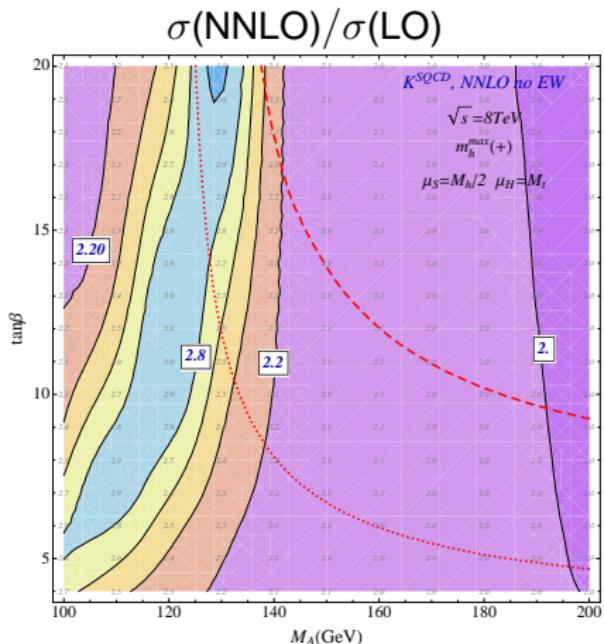


m_h^{\max} : $\sigma(pp \rightarrow h + X)$ in pb



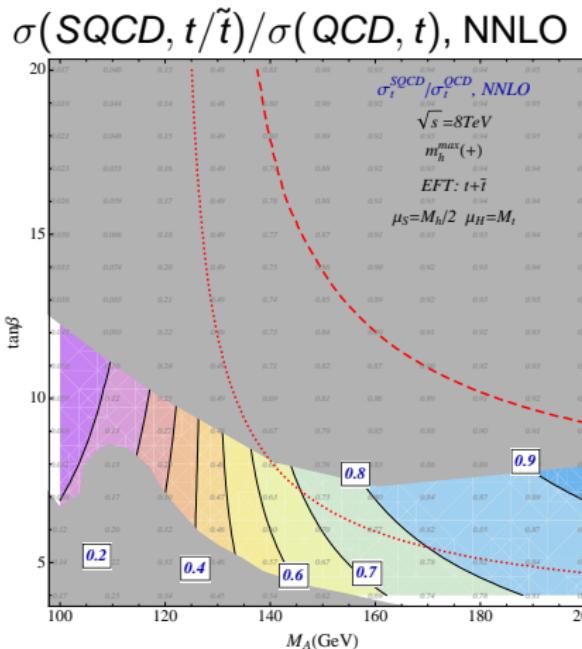
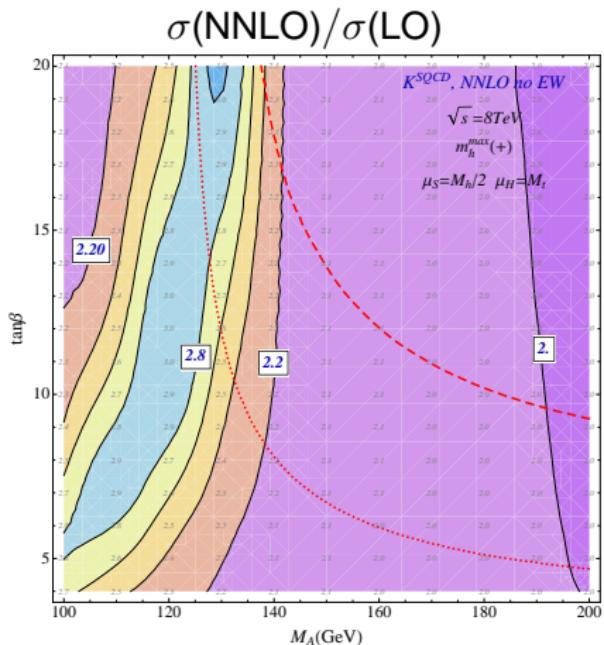
[Pak,Zerf,Steinhauser'12]

m_h^{\max} : $\sigma(pp \rightarrow h + X)$ in pb



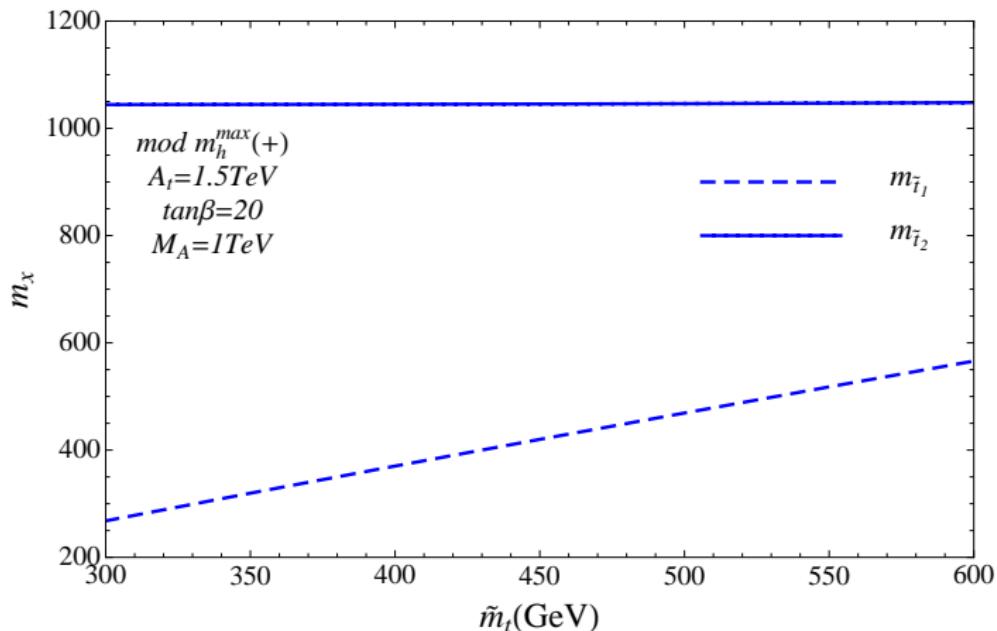
[Pak,Zerf,Steinhauser'12]

m_h^{\max} : $\sigma(pp \rightarrow h + X)$ in pb



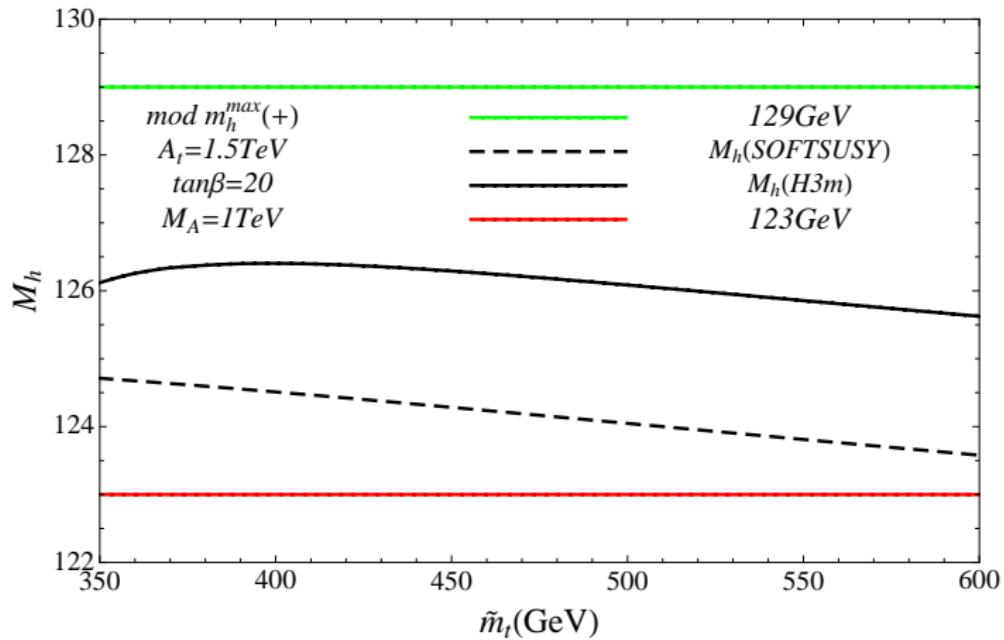
[Pak,Zerf,Steinhauser'12]

modified m_h^{\max}



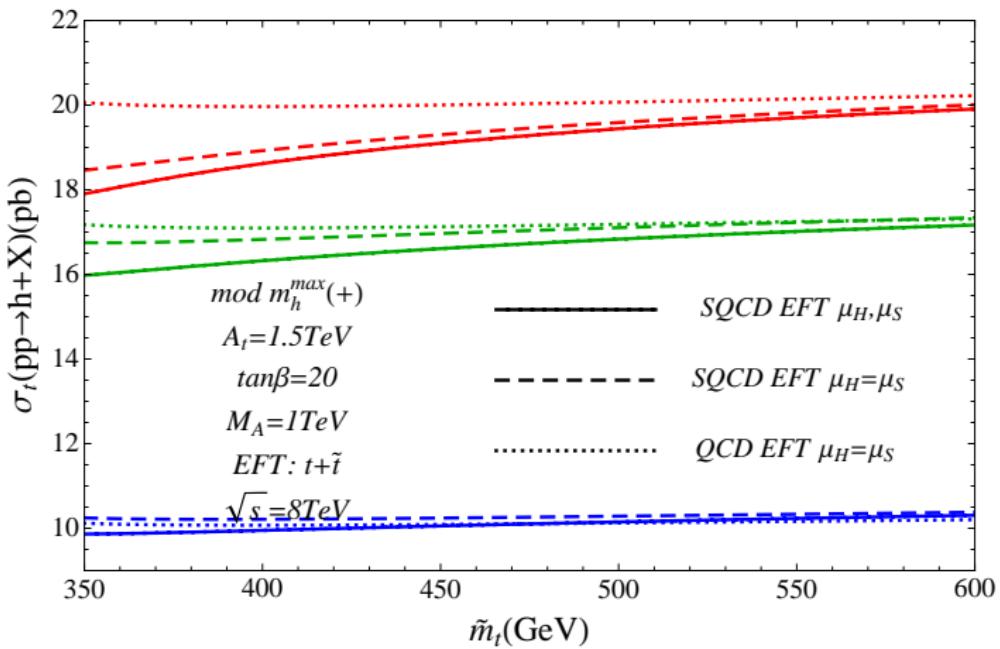
[Pak,Zerf,Steinhauser'12]

modified m_h^{\max}



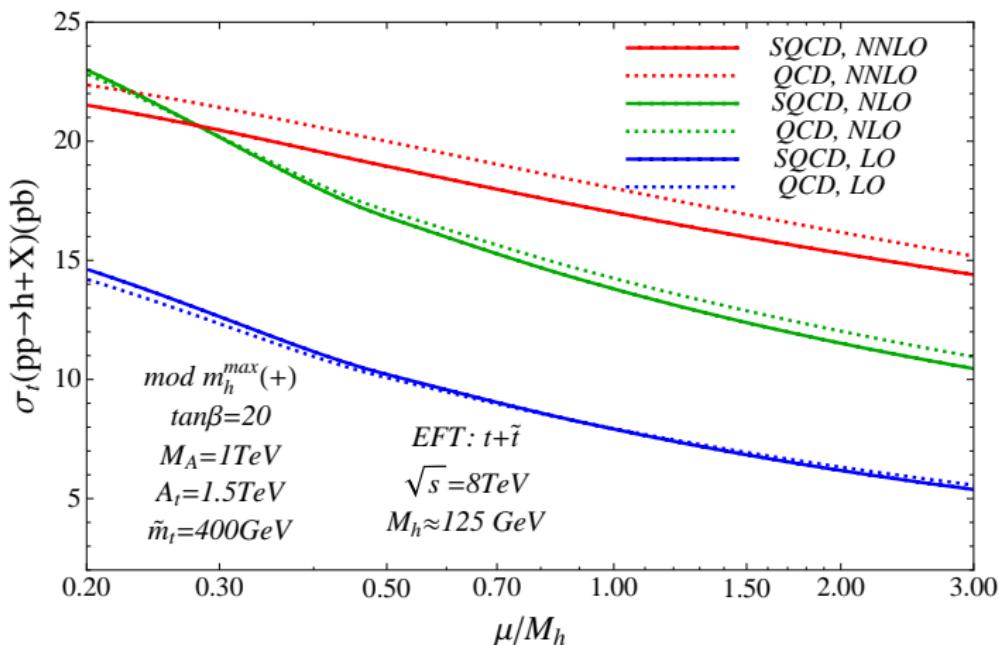
[Pak,Zerf,Steinhauser'12]

modified m_h^{\max}



[Pak,Zerf,Steinhauser'12]

modified m_h^{\max}



[Pak,Zerf,Steinhauser'12]

Conclusions

- 3-loop corrections to C_1 in the MSSM
- all-order LET ($C_1 \leftrightarrow \zeta_{\alpha_s}$)
- renormalization of evanescent operators
- NNLO corrections to $gg \rightarrow h$ in the MSSM
- flexible numerical setup
- calculation easily extendable (NMSSM, ...)