Restriction on the energy and luminosity of e+e- storage rings due to beamstrahlung

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Introduction

- Everybody knows that at storage rings energy losses due to synchrotron radiation in bending magnets $\Delta E \propto E^4 / R$ per turn, therefore for energies higher than LEP-2 one should use linear colliders: ILC or CLIC on the energy $2E_0$=200-500 GeV (up to 1000 and 3000 GeV).

- After LEP-2 there was a proposal of e+e- ring with 2E=90-400 GeV in VLHC tunnel (Sen, Norem, 2002).

- Most recently A.Blondel and F.Zimmermann (arXiv:1112.2518) have proposed to construct a storage ring collider in the LHC ring on the energy 2E=240 GeV for study of the Higgs boson.

- Inspired by the above suggestion K.Oide has proposed (13 Feb. 2012) SuperTRISTAN on the energy 2E=240-500 GeV (with $2\pi R$=40-60 km) with crab-waist collisions.

- If all correct, such ring collider would be be easier, cheaper than LC and can provide a higher luminosity. That means the end of ILC!

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Beamstrahlung

However, comparing rings and linear colliders K.Oide very correctly noted in his transperences "Beamstrahlung-free, Ring better than Linear? Needs detailed calculation."

I learned about Oide's report at KEK from BINP colleagues and immediately checked this issue, first by my simulation code for LC-PLC, then analytically. It became clear that beamstruhlung is very important for considered collider parameters. My conclusions I reported at BINP at the end of February and then published arXiv:1203.6563 (March 29).

Two weeks later I found a 34 years old conference paper (working group report)
J.E.Augustin, N.Dikansky Y.Derbenev, J.Rees, B.Richter, A.Skrinsky, M.Tigner and H.Wiedemann, Limitations on Performance of e+ e- Storage rings and Linear Colliding Beam Systems at High Energy, 1st Workshop on Possibilities and Limitations of Accelerators and Detectors 15-21 Oct 1978. eConf C781015, (1978) 009.
This never cited paper is devoted exactly to consideration of beamstrahlung at high energy storage rings.

We realized early in the study that a phenomenon which has been negligible in electron-positron systems designed to date would become important at these higher energies - synchrotron radiation from a particle being deflected by the collective electromagnetic field of the opposing bunch - and we dubbed this phenomenon "beam-strahlung." During the rest of the week we investigated the scaling laws for these two colliding-beam systems taking beam-strahlung into consideration.

    In the following sections, we shall first review briefly the formulae relevant to beam-strahlung; then we shall apply them to storage rings to obtain scaling laws; and finally we shall derive some scaling laws for the system of colliding linac beams. Before embarking on that program, however, we wish to put the reader on notice that our intent throughout will be to estimate, not to calculate in detail. Our results are therefore approximate and should be taken in that spirit.

In fact, Augustin et al. have estimated, an additional beam energy spread due to beamstrahlung.

Being not aware about above report I also calculated the beam energy spread, but found that even more important is the emission of single photons in the tail of the beamstrahlung spectra. Namely this process determines the beam lifetime (when the increase of the energy spread is still acceptable).

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(Initial) parameters of rings under study (Blondel-Zimmermann, Oide)

TABLE I. Parameters of LEP and several recently proposed storage-ring colliders [6, 7]. "STR" refers to "SuperTRISTAN" [7]. Use of the crab-waist collision scheme [11, 12] is denoted by "cr-w". The luminosities and the numbers of bunches for all projects are normalized to the total synchrotron-radiation power of 100 MW. Beamstrahlung-related quantities derived in this paper are listed below the double horizontal line.

	LEP	LEP3	DLEP	STR1	STR2	STR3 cr-w	STR4 cr-w	STR5 cr-w	STR6 cr-w
$2E_0$, GeV	209	240	240	240	240	240	400	400	500
Circumference, km	27	27	53	40	60	40	40	60	80
Beam current, mA	4	7.2	14.4	14.5	23	14.7	1.5	2.7	1.55
Bunches/beam	4	3	60	20	49	15	1	1.4	2.2
N, 10^{11}	5.8	13.5	2.6	6	6	8.3	12.5	25.	11.7
σ_z, mm	16	3	1.5	3	3	1.9	1.3	1.4	1.9
ε_x, nm	48	20	5	23.3	24.6	3	2	3.2	3.4
ε_y, nm	0.25	0.15	0.05	0.09	0.09	0.011	0.011	0.017	0.013
β_x, mm	1500	150	200	80	80	26	20	30	34
β_y, mm	50	1.2	2	2.5	2.5	0.25	0.2	0.32	0.26
σ_x, μm	270	54	32	43	44	8.8	6.3	9.8	10.7
σ_y, μm	3.5	0.42	0.32	0.47	0.47	0.05	0.047	0.074	0.06
SR power, MW	22	100	100	100	100	100	100	100	100
Energy loss/turn, GeV	3.4	7	3.47	3.42	2.15	3.42	33.9	18.5	32.45
$\mathcal{L}$, 10^{34} cm^{-2}s^{-1}	0.013	1.3	1.6	1.7	2.7	17.6	4	7	2.2
$E_{c,max}/E_0$, 10^{-3}	0.09	6.3	4.2	3.5	3.4	38	194	232	91
n_γ/electron	0.09	1.1	0.37	0.61	0.6	4.2	8.7	11.3	4.8
lifetime(SR@IP), s (Eq. 4)	$\sim\infty$	0.02	0.3	0.2	0.4	0.005	0.001	0.0005	0.005
$\mathcal{L}_{corr}$, 10^{34} cm^{-2}s^{-1}	0.013	0.2	0.4	0.5	0.8	0.46	0.02	0.03	0.024

The critical energy of beamstrahlung is too high. If the electron loses more than about 1% of its energy (1.2% at LEP2), it will leave the beam. The beam lifetimes at these parameters are very short. One can decrease N and simultaneously increase the number of bunches, then L will be much lower.

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Beam lifetime due to beamstrahlung

Electron loses the beam after emission of beamstrahlung photon with an energy greater than the threshould energy $E_{th}=\eta E_0$, where $\eta \sim \mathcal{O}(0.01)$. These photons have energies larger than the critical energy

$$E_c = \hbar\omega_c = \hbar\frac{3\gamma^3 c}{2\rho};$$

The spectrum per unit length at $u = E_\gamma / E_c \gg 1$

$$\frac{dn}{dx} = \sqrt{\frac{3\pi}{2}}\frac{\alpha\gamma}{2\pi\rho}\frac{e^{-u}}{\sqrt{u}}du,$$

The number of photons on collision length l with $E_\gamma > \eta E_0$

$$n_\gamma(E_\gamma \geq \eta E_0) \approx \frac{\alpha^2\eta l}{\sqrt{6\pi}r_e\gamma u^{3/2}}e^{-u}; \quad u = \frac{\eta E_0}{E_c},$$

$l \approx \sigma_z/2$ for head-on and $l \approx \beta_y/2$ for crab-waist collisions

The beam lifetime depends exponentially on the critical energy (prop. to the beam field). And inversely, the critical energy (acceptable beam parameters, luminosity) depend logarithmically on the beam lifetime, therefore we can make useful approximations in calculation of the lifetime.

Let us assume that the electron crosses the region with strongest field with 10% probability. The average number of collisions n_{col} before an electron leaves the beam is found from $0.1 n_{col} n_\gamma = 1$, that gives

$$n_{col} \approx 10 \frac{\sqrt{6\pi} r_e \gamma u^{3/2}}{\alpha^2 \eta l} e^u; \quad \tau = n_{col} \frac{2\pi R}{c}.$$

Assuming (in front of the exponent) E_0=150 GeV, l=1 mm, η=0.01, 2πR=50 km and the beam lifetime 30 min we get

$$u = \eta E_0 / E_c \approx 8.5; \quad E_c \approx 0.12 \eta E_0 \sim 0.1 \eta E_0. \qquad *$$

Note, the accuracy of this expression is quite good for any ring collider, because it depends logarithmically on approximate values we used.

The maximum effective field for Gaussian beams $B \approx 2eN/\sigma_x \sigma_z$, then the (max) critical energy

$$\frac{E_c}{E_0} = \frac{3\gamma r_e^2 N}{\alpha \sigma_x \sigma_z}.$$

Eq. * imposes the following restriction on the beam parameters:

$$\boxed{\frac{N}{\sigma_x \sigma_z} < 0.1\eta \frac{\alpha}{3\gamma r_e^2}} \qquad (**)$$

This formula is the basis for the following discussions.

In collision of Gaussian beams the average number of beamstrahlung photons $\langle n_\gamma \rangle \approx 2.16 N\alpha r_e / \sigma_x$, their average energy $\langle E_\gamma \rangle \approx 0.31\langle E_c \rangle$ and $\langle E_c \rangle \approx 0.42 E_{c,\max}$, that gives $\langle E_\gamma \rangle \approx 0.13 E_{c,\max}$

Hence the beam lifetime is determined by photons with energy 8.5/0.13 = 65 times greater than $\langle E_\gamma \rangle$.
  With account of (**)

$$\langle n_\gamma \rangle = \frac{0.07\eta\alpha^2 \sigma_z}{r_e \gamma} \approx \frac{0.067(\sigma_z/\mathrm{mm})}{(E_0/100\,\mathrm{GeV})}\left(\frac{\eta}{0.01}\right)$$

$$\langle E_\gamma \rangle \approx 0.13 \times 0.1\eta E_0 \approx 1.3 \times 10^{-4} E_0 \left(\frac{\eta}{0.01}\right)$$

The beam energy spread

The energy spread due to beamstrahlung

$$\frac{\sigma_E^2}{E_0^2} = \frac{\tau_{\text{s}}}{4E_0^2}\dot{n}_\gamma\langle E_\gamma^2\rangle$$

where the damping time (due to radiation in bending magnets)

$$\tau_{\text{s}} \approx T_{\text{rev}}E_0/\Delta E_{\text{rev}}, \ \dot{n}_\gamma = \langle n_\gamma\rangle/T_{\text{rev}}, \qquad \langle E_\gamma^2\rangle \approx 4.3\langle E_\gamma\rangle^2$$

which gives with account of (**)

$$\frac{\sigma_E^2}{E_0^2} \approx \frac{\langle n_\gamma\rangle\langle E_\gamma\rangle^2}{E_0\Delta E_{\text{rev}}} \approx \frac{1.15\cdot10^{-9}(\sigma_z/\text{mm})}{(E_0/100\,\text{GeV})(\Delta E_{\text{rev}}/E_0)}\left(\frac{\eta}{0.01}\right)^3$$

For E_0=120 GeV, σ_z=5 mm, $\Delta E_{\text{rev}}/E_0$=0.05

$$\left(\frac{\sigma_E}{E}\right)_{\text{BS}} \approx 3\cdot10^{-4}\left(\frac{\eta}{0.01}\right)^{3/2}$$

The energy spread due to radiation in bending magnets

$$\left(\frac{\sigma_E^2}{E_0^2}\right)_{\text{SR}} = \frac{55\sqrt{3}}{128\pi\alpha J_{\text{s}}}\frac{mc^2}{E_0}\frac{\Delta E_{\text{rev}}}{E_0} = \frac{0.016}{J_{\text{s}}E_0(\text{GeV})}\frac{\Delta E_{\text{rev}}}{E_0}$$

For E_0=120 GeV, $\Delta E_{rev}/E_0$=0.05, J_s=1.5

$$\left(\frac{\sigma_E}{E_0}\right)_{SR} \approx 2\cdot 10^{-3}$$

For the given example BS and SR energy spreads become equal at η~0.035. In general, $\sigma_{BS}/\sigma_{SR} \propto E_0/\Delta E_{\text{rev}}$ and in large rings or at low energies the energy spread due to beamstrahlung could be dominant.

The lifetime due to the beam energy spread

In order to have acceptable lifetime due to the energy spread one needs $\eta > 6(\sigma_E)_{BS.}$

If the lifetime due to single beamstrahlung is kept large (30 min) (condition **), then the BS energy spread contributes to the lifetime only if (follows from previous formulas)

$$\eta > \frac{2.5}{\sigma_z(\text{mm})} \left(\frac{\Delta E_{\text{rev}}}{10\,\text{GeV}} \right)$$

For typical cases this value is very large ($\eta > 0.3$) compared with a reasonable $\eta \sim 0.01\text{-}0.03$. Therefore the beam lifetime due to beamstrahlungis is always determined by emission of single photons.

Head-on and "crab-waist" collision schemes

Below we consider two collision schemes: head-on and crab-waist.
In the crab-waist scheme the beams collide at an angle $\theta >> \sigma_x / \sigma_z$
This scheme allows a higher luminosity, if it is determined by the tune shift.
For head-on collisions the tune shift ($\xi_y \leq 0.1 - 0.15$) and the luminosity

$$\xi_y = \frac{N r_e \beta_y}{2\pi\gamma\sigma_x\sigma_y} \approx \frac{N r_e \sigma_z}{2\pi\gamma\sigma_x\sigma_y} \text{ for } \beta_y \approx \sigma_z \qquad \mathcal{L} \approx \frac{N^2 f}{4\pi\sigma_x\sigma_y} \approx \frac{N f \gamma \xi_y}{2 r_e \sigma_z}$$

For the crab-waist scheme

$$\xi_y = \frac{N r_e \beta_y^2}{\pi\gamma\sigma_x\sigma_y\sigma_z} \text{ for } \beta_y \approx \sigma_x/\theta \qquad \mathcal{L} \approx \frac{N^2 f}{2\pi\sigma_y\sigma_z\theta} \approx \frac{N^2 \beta_y f}{2\pi\sigma_x\sigma_y\sigma_z} \approx \frac{N f \gamma \xi_y}{2 r_e \beta_y}$$

In the crab-waist scheme one can make $\beta_y << \sigma_z$, therefore the luminosity is higher. Nf is determined by SR power. The only free parameters in L are σ_z (for head-on) and β_y (crab-waist), they are constrained by beamstrahlung condition

$$\frac{N}{\sigma_x\sigma_z} < 0.1\eta\frac{\alpha}{3\gamma r_e^2}$$

Comparing above equations one can find the minimum beam energy when beamstrahlung becomes important.

For head-on collisions

$$\gamma_{\min} = \left(\frac{0.1\eta\alpha\sigma_z^2}{6\pi r_e \xi_y \sigma_y} \right)^{1/2} \propto \frac{\sigma_z^{3/4}}{\xi_y^{1/2} \varepsilon_y^{1/4}}$$

For "crab-waist" collisions

$$\gamma_{\min} = \left(\frac{0.1\eta\alpha\beta_y^2}{3\pi r_e \xi_y \sigma_y} \right)^{1/2} \propto \frac{2^{1/2}\beta_y^{3/4}}{\xi_y^{1/2} \varepsilon_y^{1/4}}$$

In the crab-waist scheme the beamstrahlung become important at much low energies because $\beta_y <<< \sigma_z$. For typical values of parameters in Table 1 $E_{min} \sim 70$ GeV for head-on collisions and $E_{min} \sim 20$ GeV for "crab-waist".

For considered colliders with $2E_0 > 240$ GeV beamstrahlung is important in both schemes.

Luminosities with account of beamstrahlung

For head-on collisions

$$\mathcal{L} \approx \frac{(Nf)N}{4\pi\sigma_x\sigma_y}, \; \xi_y \approx \frac{Nr_e\sigma_z}{2\pi\gamma\sigma_x\sigma_y}, \; \frac{N}{\sigma_x\sigma_z} \equiv k \approx 0.1\eta\frac{\alpha}{3\gamma r_e^2} \qquad \sigma_y \approx \sqrt{\varepsilon_y\sigma_z}$$

This can be rewritten as

$$\mathcal{L} \approx \frac{(Nf)k\sigma_z}{4\pi\sigma_y}, \qquad \xi_y \approx \frac{kr_e\sigma_z^2}{2\pi\gamma\sigma_y}, \qquad \sigma_y \approx \sqrt{\varepsilon_y\sigma_z}$$

One can see that in beamstrahlung dominated regime the luminosity is proportional to the bunch length and its maximum value is determined by the tune shift. Together these equations give

$$\mathcal{L} \approx \frac{Nf}{4\pi}\left(\frac{0.1\eta\alpha}{3}\right)^{2/3}\left(\frac{2\pi\xi_y}{\gamma r_e^5\varepsilon_y}\right)^{1/3}$$

$$\sigma_{z,\text{opt}} = \varepsilon_y^{1/3}\left(\frac{6\pi\gamma^2 r_e\xi_y}{0.1\eta\alpha}\right)^{2/3}$$

Luminosities with account of beamstrahlung

Similarly for the crab-waist collisions

$$\mathcal{L} \approx \frac{(Nf)N\beta_y}{2\pi\sigma_x\sigma_y\sigma_z}, \quad \xi_y \approx \frac{Nr_e\beta_y^2}{\pi\gamma\sigma_x\sigma_y\sigma_z}, \quad \frac{N}{\sigma_x\sigma_z} \equiv k \approx 0.1\eta\frac{\alpha}{3\gamma r_e^2} \qquad \sigma_y \approx \sqrt{\varepsilon_y\beta_y}$$

Substituting, we obtain

$$\mathcal{L} \approx \frac{(Nf)k\beta_y}{2\pi\sigma_y}, \quad \frac{kr_e\beta_y^2}{\pi\gamma\sigma_y} \approx \xi_y, \quad \sigma_y \approx \sqrt{\varepsilon_y\beta_y}$$

These relations are similar to those for head on collisions if to replace σ_z by β_y and k by 2k. The corresponding solutions are

$$\mathcal{L} \approx \frac{Nf}{4\pi}\left(\frac{0.2\eta\alpha}{3}\right)^{2/3}\left(\frac{2\pi\xi_y}{\gamma r_e^5\varepsilon_y}\right)^{1/3}$$

$$\beta_{y,\mathrm{opt}} = \varepsilon_y^{1/3}\left(\frac{3\pi\gamma^2 r_e\xi_y}{0.1\eta\alpha}\right)^{2/3}$$

In the beamstrahlung dominated regime the luminosities in crab-waist and head-on collisions are practically the same (difference $2^{2/3}$) !

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As soon as the crab-waist gives no profit at high energies, further we will consider only the head-on scheme.

The optimum bunch length in practical unites

$$\frac{\sigma_{z,\text{opt}}}{\text{mm}} \approx \frac{2\xi_y^{2/3}}{\eta^{2/3}}\left(\frac{\varepsilon_y}{\text{nm}}\right)^{1/3}\left(\frac{E_0}{100\,\text{GeV}}\right)^{4/3}; \qquad \text{typically several mm to cm}$$

The maximum luminosity with account of beamstraнlung

$$\mathcal{L} \approx h\frac{N^2 f}{4\pi\sigma_x\sigma_y} = h\frac{Nf}{4\pi}\left(\frac{0.1\eta\alpha}{3}\right)^{2/3}\left(\frac{2\pi\xi_y}{\gamma r_e^5\varepsilon_y}\right)^{1/3}$$

where h is the hourglass loss factor, $f=n_b c/2\pi R$. SR power in rings

$$P = 2\delta E\frac{cNn_b}{2\pi R} = \frac{4e^2\gamma^4 cNn_b}{3RR_b}.$$

Finally, the luminosity

$$\mathcal{L} \approx h\frac{(0.1\eta\alpha)^{2/3}PR}{32\pi^2\gamma^{13/3}r_e^3}\left(\frac{R_b}{R}\right)\left(\frac{6\pi\xi_y r_e}{\varepsilon_y}\right)^{1/3}$$

In practical units

$$\frac{\mathcal{L}}{10^{34}\,\text{cm}^{-2}\text{s}^{-1}} \approx \frac{100h\eta^{2/3}\xi_y^{1/3}}{(E_0/100\,\text{GeV})^{13/3}(\varepsilon_y/\text{nm})^{\frac{1}{3}}}$$

$$\times \left(\frac{P}{100\,\text{MW}}\right)\left(\frac{2\pi R}{100\,\text{km}}\right)\frac{R_b}{R}$$

The beamstrahlung suppresses the luminosity by a factor $(E_{min}/E_0)^{4/3}$
for the energies above E_{min}, which is about 70 GeV for head-on
and 20 GeV for crab-waist schemes.

Beamstrahlung and the tune-shift determine σ_z and the combination N/σ_x.
Assuming P=100 MW, h=0.8, ξ_y =0.15, η=0.01 and other parameters from
Table 1 we obtain optimized parameters for projects from Table 1:

TABLE II. Realistically achievable luminosities and other beam parameters for the projects listed in Table I at synchrotron-radiation power $P = 100$ MW. Only the parameters that differ from those in Table I are shown.

	LEP	LEP3	DLEP	STR1	STR2	STR3 cr-w	STR4 cr-w	STR5 cr-w	STR6 cr-w
$2E_0$, GeV	209	240	240	240	240	240	400	400	500
Circumference, km	27	27	53	40	60	40	40	60	80
Bunches/beam	~ 2	~ 7	70	24	53	240	36	45	31
N, 10^{11}	33	5.9	2.35	3.9	4.	0.4	0.34	0.6	0.65
σ_z, mm	8.1	8.1	5.7	6.9	6.9	3.4	6.7	7.8	9.6
σ_y, μm	1.4	1.1	0.53	0.78	0.78	0.19	0.27	0.36	0.35
$\mathcal{L}$, 10^{34} cm^{-2}s^{-1}	0.47	0.31	0.89	0.55	0.83	1.1	0.12	0.16	0.087

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Conclusions

❖ Luminosities at high energy storage rings are suppressed due to beamstrahlung (influences via the beam lifetime).

❖ Attainable luminosities at "the Higgs energy" $2E_0 = 240$ GeV at e+e- storage rings and linear colliders are comparable.

❖ However, for $2E_0 = 400\text{-}500$ GeV (needed for the measurement of the Higgs self-coupling) the storage ring luminosities would be a factor of 15-25 smaller than desired.