Introduction to relativistic kinematics and the concept of mass

Mass is one of the most fundamental concepts in physics.

When a new particle is discovered (e.g. the Higgs boson), the first question a physicist will ask is, **'What is its mass?'**

Classical physics ($v \ll c$)

$T = mv^2/2 \longrightarrow m = 2T/v^2$

$p = mv \longrightarrow m = p/v$

$T = p^2/2m \longrightarrow m = p^2/2T$

Knowing any 2 of *T*, *p* and *v*, one can calculate *m*.

Same is true in relativity but we need the generalised formulae.

Before that: a brief discussion of '$E = mc^2$'

Einstein's equation: '$E = mc^2$'

$E_0 = m\,c^2$ $E = m\,c^2$ $E_0 = m_0\,c^2$ $E = m_0\,c^2$

where c = velocity of light in vacuo

E = total energy of free body

E_0 = rest energy of free body

m_0 = rest mass

m = mass

Q1: Which equation most rationally follows from special relativity and expresses one of its main consequences and predictions?

Q2: Which of these equations was first written by Einstein and was considered by him a consequence of special relativity?

The correct answer to both questions is: $E_0 = mc^2$

(Poll carried out by Lev Okun among professional physicists in 1980s showed that the majority preferred 2 or 3 as the answer to both questions.)

‘This choice is caused by the confusing terminology widely used in the popular science literature and in many textbooks. According to this terminology a body at rest has a *proper mass* or *rest mass* m_0, whereas a body moving with speed v has a *relativistic mass* or *mass* m, given by

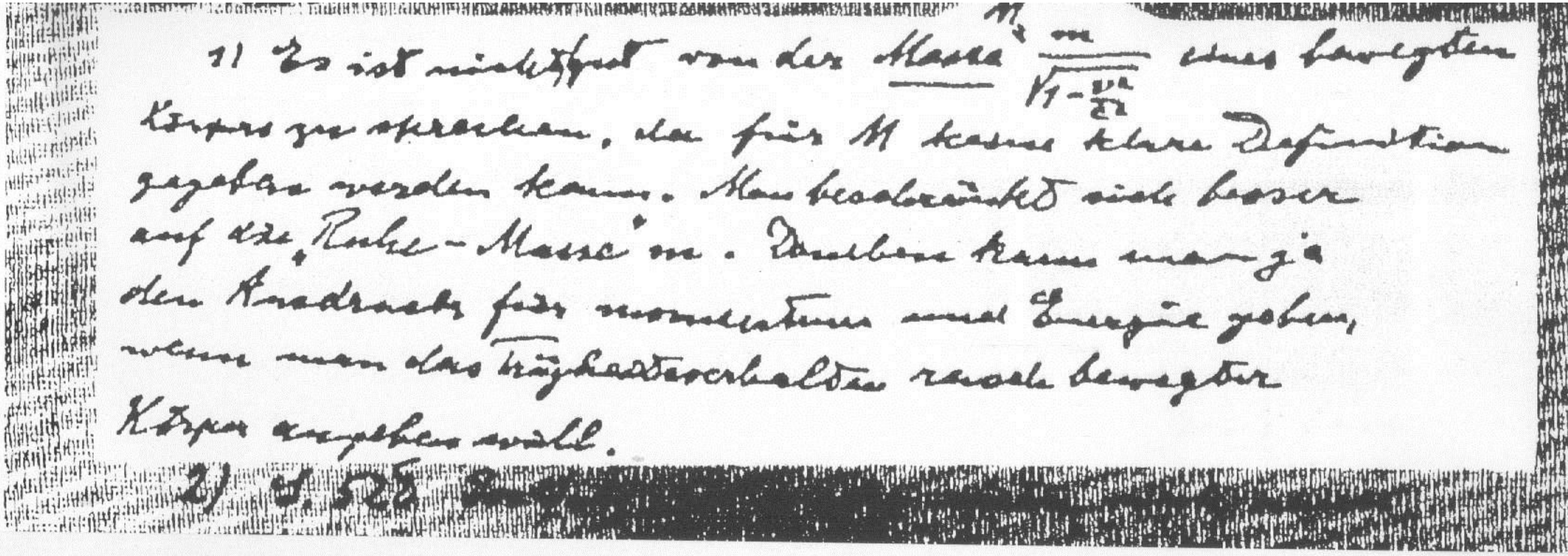
$$m = \frac{E}{c^2} = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

‘ … this terminology had some historical justification at the start of our century, but it has no justification today.

‘Today, particle physicists only use the term *mass*. According to this rational terminology the terms *rest mass* and *relativistic mass* are redundant and misleading.

There is only one mass in physics, *m,* which does not depend on the reference frame.

‘As soon as you reject the *relativistic mass* there is no need to call the other mass *rest mass* and to mark it with a subscript 0.’



1) Es ist nicht gut, von der Masse $M = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}}$ eines bewegten Körpers zu sprechen, da für M keine klare Definition gegeben werden kann. Man beschränkt sich besser auf die „Ruhe-Masse" m. Daneben kann man ja den Ausdruck für momentum und Energie geben, wenn man das Trägheitsverhalten rasch bewegter Körper angeben will.

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Letter from Albert Einstein to Lincoln Barnett, 19 June 1948

It is not good to introduce the concept of mass $m = \frac{E}{c^2} = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ of a moving body for which no clear definition can be given.

It is better to introduce no other mass concept than the *rest mass m*. Instead of introducing *m*, it is better to mention the expression for the momentum and energy of a body in motion.

The two fundamental equations of relativistic kinematics

(Relativistic generalisations of $E = p^2/2m$ and $p = mv$.)

Conservation of energy and momentum are close to the heart of physics. Discuss how they are related to 2 deep symmetries of nature.

All this is looked after in special relativity if we define energy and momentum as follows:

$E^2 = p^2c^2 + m^2c^4$ and $\boldsymbol{p} = \boldsymbol{v} \dfrac{E}{c^2}$

where E = total energy
$\boldsymbol{p}$ = momentum
$\boldsymbol{v}$ = velocity
m = ordinary mass as in Newtonian mechanics

Next: hope to persuade you to accept these equations.

* $E_0 = mc^2$

Consider $E^2 - p^2c^2 = m^2c^4$. For the situation when the particle is at rest (v = o), the energy E is the rest energy E_0 and p = 0.

$$\text{So, } E_0 = mc^2$$

* Show that, for $v \ll c$, $E = mc^2 + p^2/2m = mc^2 + mv^2/2$

Firstly, when $v \ll c$, $p \approx v \dfrac{E_0}{c^2} = v\, m$. Also,

$$E = (p^2c^2 + m^2c^4)^{1/2} = mc^2 (1 + p^2c^2/m^2c^4)^{1/2} = mc^2 (1 + p^2c^2/2m^2c^4 + \ldots)$$

For v<<c, $p^2c^2 \ll m^2c^4$.

So, $E = mc^2 + p^2/2m = E_0 + mv^2/2$

Rest energy (E_0); Newtonian kinetic energy ($mv^2/2$)

The relativistic equations for *p* and *E* reduce to the Newtonian ones for *v<<c*; so the *m* in them is the Newtonian mass.

	E (in MeV)	p_x (in MeV/c)	p_y (in MeV/c)	p_z (in MeV/c)	m(in MeV/c^2)
γ_1	82	5.569610402	81.6979929509	-4.2915484119	
γ_2	177	-3.3303482436	79.459701904	158.1267357734	
	259	2.2392621584	161.157694855	153.8351873221	132.056496

π^0 !

Now move to a frame moving with a speed of 0.5c in the +ve y-direction:

$$p'_y = \gamma\left(p_y - \frac{v}{c^2}E\right) \quad \text{and} \quad E' = \gamma\left(E - v\,p_\mu\right)$$

(LORENTZ TRANSFORMATION)

Putting in the numbers yields:

γ_1	47.517085	5.569610402	46.993992	-4.2915484119	
γ_2	158.50591	-3.3303482436	-10.438838	158.1267357734	
	206.02376	2.2392622	36.555154	153.83518	132.05769

$\rightarrow \pi^0$

So: the "effective mass"

$$M(\gamma_1\gamma_2) = \sqrt{\frac{\left(E(\gamma_1)+E(\gamma_2)\right)^2 - \left(p(\gamma_1)+p(\gamma_2)\right)^2 c^2}{c^4}}$$

is unchanged ("INVARIANT") when one goes into a different Lorentz frame of reference.