



# Hadronic $B_{(s)}$ Decays

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# Outline:

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- **Measurements and Theoretical problems**
  - rare decays
  - direct CP asymmetry
  - polarizations
- **Possible Theoretical solution to problems and improvements**
- **Summary**



$B \rightarrow M_2 M_3$  **non-perturbative QCD**

**decay amplitude rely on factorization**

$$A = \langle f | H_{eff} | B \rangle = \frac{G_F}{\sqrt{2}} \sum_i V_{CKM}^i C_i(\mu) \langle M_2 M_3 | O_i(\mu) | B \rangle$$

- ✓ **Naïve factorization**
- ✓ **Generalized Factorization**
- ✓ **QCD factorization (QCDF)**
- ✓ **Soft-collinear effective theory (SCET)**
- ✓ **Perturbative QCD approach (PQCD)**



# Non-leptonic B decays after B factories

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- Experimental precision requires more **reliable and higher precision** theoretical calculations
- $\Phi(p_2) \times J(p_1, p_2) \times \phi(p_1)$
- $J(p_1, p_2) = 1 + \alpha_s + \alpha_s^2 + \dots$
- $\phi(p_1) = 1 + p_1/m_b + (p_1/m_b)^2 + \dots$



## QCDF and SCET has already gone to NLO or NNLO at $\alpha_s$ expansion

- Heavy-to-light currents at **NNLO** in SCET, G. Bell M. Beneke T. Huber X-Q Li NPB843 (2011) 143
- **NNLO** vertex corrections to non-leptonic B decays: Tree amplitudes, M. Beneke, T. Huber, XQ Li NPB832 (2010) 109 ...
- **However, the knowledge of power corrections is limited and numerically more important**



Perturbative QCD (PQCD) in B decays is going to next-to-leading order now

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- NLO correction to **pion form factor** in  $k_T$  factorization, H.-n. Li, Y.L. Shen, Y.M. Wang, H. Zou, PRD83 (2011) 054029
- NLO **time-like pion form factors** in  $k_T$  factorization, HC Hu, Hn Li, arXiv:1204.6708
- NLO corrections to  **$B \rightarrow \pi$  form factors** in  $k_T$  factorization, H.n. Li, Y.L. Shen, Y.M. Wang, PRD85 (2012) 074004



# Revisiting the $B \rightarrow \pi\rho, \pi\omega$ Decays in the perturbative QCD Approach Beyond the Leading Order, Zhou R, Gao XD, CDL, EPJC72 (2012) 1923 ( $10^{-6}$ )

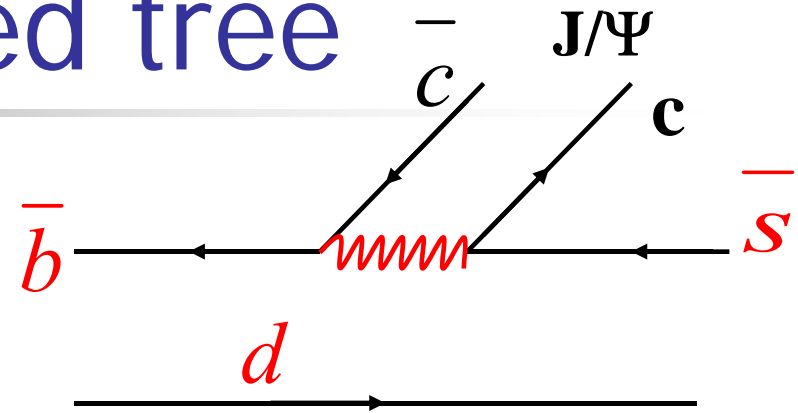
Mode	LO-PQCD	NLO-PQCD	QCDF-I [11]	QCDF-II [91]	SCET [92]	Data [55]
$\pi^\pm \rho^\mp$	41.3	$25.7^{+7.0+2.4+1.3+1.8}_{-5.5-1.9-2.0-1.6}$	$36.5^{+18.2+10.3+2.0+3.9}_{-14.7-8.6-3.5-2.9}$	$25.1^{+1.5+1.4}_{-2.2-1.8}$	$16.8^{+0.5+1.6}_{-0.5-1.5}$	$23 \pm 2.3$
$\pi^+ \rho^0$	9.0	$5.4^{+1.4+0.5+0.6+0.3}_{-1.1-0.3-0.5-0.0}$	$11.9^{+6.3+3.6+2.5+1.3}_{-5.0-3.1-1.2-1.1}$	$8.7^{+2.7+1.7}_{-1.3-1.4}$	$7.9^{+0.2+0.8}_{-0.1-0.8}$	$8.3 \pm 1.2$
$\rho^+ \pi^0$	14.1	$9.6^{+2.5+0.8+0.7+0.7}_{-2.1-0.7-1.3-0.6}$	$14.0^{+6.5+5.1+1.0+0.8}_{-5.5-4.3-0.6-0.7}$	$11.8^{+1.8+1.4}_{-1.1-1.4}$	$11.4^{+0.6+1.1}_{-0.6-0.9}$	$10.9 \pm 1.4$
$\rho^0 \pi^0$	0.15	$0.37^{+0.09+0.02+0.03+0.08}_{-0.08-0.01-0.05-0.02}$	$0.4^{+0.2+0.2+0.9+0.5}_{-0.2-0.1-0.3-0.3}$	$1.3^{+1.7+1.2}_{-0.6-0.6}$	$1.5^{+0.1+0.1}_{-0.1-0.1}$	$2.0 \pm 0.5$
$\pi^+ \omega$	8.4	$4.6^{+1.2+0.5+0.5+0.1}_{-0.9-0.4-0.4-0.1}$	$8.8^{+4.4+2.6+1.8+0.8}_{-3.5-2.2-0.9-0.9}$	$6.7^{+2.1+1.3}_{-1.0-1.1}$	$8.5^{+0.3+0.8}_{-0.3-0.8}$	$6.9 \pm 0.5$
$\pi^0 \omega$	0.22	$0.32^{+0.06+0.01+0.04+0.04}_{-0.05-0.02-0.07-0.04}$	$0.01^{+0.00+0.02+0.02+0.03}_{-0.00-0.00-0.00-0.00}$	$0.01^{+0.02+0.04}_{-0.00-0.02}$	$0.015^{+0.024+0.002}_{-0.000-0.002}$	$< 0.5$



# Problem 1:

## Color suppressed tree

- It is small, since the Wilson coefficients of Effective operators are small



- It may be enhanced by **soft contributions**, but hard to explain the difference between  $B^0 \rightarrow \pi^0 \pi^0$  and  $\rho^0 \rho^0$





# $B^0 \rightarrow \pi^0\pi^0, \pi^0\rho^0$ and $\rho^0\rho^0$ ( $10^{-6}$ )

	QCDF	PQCD	SCET	Exp.
$\pi^0\pi^0$	$0.3 \pm 0.5$ $1.1 \pm 1.1$	$0.29 \pm 0.50$	$0.84 \pm 0.46$	$1.62 \pm 0.31$
$\pi^0\rho^0$	$0.4 \pm 1.0$ $1.3 \pm 1.8$	$0.37 \pm 0.12$	$1.5 \pm 0.1$	$2.0 \pm 0.5$
$\rho^0\rho^0$	$0.9 \pm 1.6$	$0.92 \pm 1.10$	-	$0.73 \pm 0.28$




# Amplitude parametrization of $B \rightarrow K \pi$ decays

-  $A(B^+ \rightarrow K^0 \pi^+) = P'$ ,

$$A(B_d^0 \rightarrow K^+ \pi^-) = -P' \left( 1 + \frac{T'}{P'} e^{i\phi_3} \right),$$

$$\sqrt{2}A(B^+ \rightarrow K^+ \pi^0) = -P' \left[ 1 + \frac{P'_{ew}}{P'} + \left( \frac{T'}{P'} + \frac{C'}{P'} \right) e^{i\phi_3} \right],$$

$$\sqrt{2}A(B_d^0 \rightarrow K^0 \pi^0) = P' \left( 1 - \frac{P'_{ew}}{P'} - \frac{C'}{P'} e^{i\phi_3} \right),$$

$$\frac{T'}{P'} \sim \lambda, \quad \frac{P'_{ew}}{P'} \sim \lambda, \quad \frac{C'}{P'} \sim \lambda^2$$


$$(C_2/C_4)(V_{us}V_{ub}/V_{ts}V_{tb}) \gg (1/\lambda^2)(\lambda^5/\lambda^2) \gg \lambda$$



## $B \rightarrow K\pi$ puzzle

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- $K^+\pi^-$  and  $K^+\pi^0$  differ by sub-leading amplitudes  $P_{ew}$  and color suppressed tree (C). Their CP asymmetry are expected to be similar.

- Their data differ by  $5\sigma!$  A puzzle!

$$A_{CP}(K^+\pi^-) = (-9.8 \pm 1.3)\%$$

$$A_{CP}(K^+\pi^0) = (5.1 \pm 2.5)\%$$

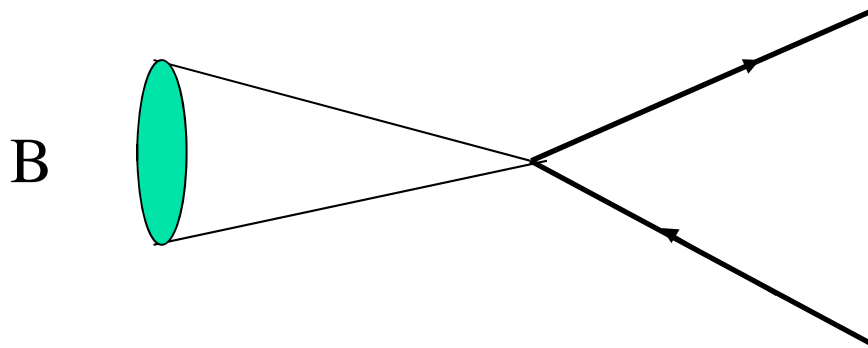
**Importance of power corrections**



The power suppressed terms,  
sometimes chiral enhanced

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**Questions No. 2: Annihilation type  
diagrams suppressed  $\sim 1/m_B$  10%**



- **Helicity suppressed: pseudo-scalar decays to two massless quarks**



# “Annihilation”

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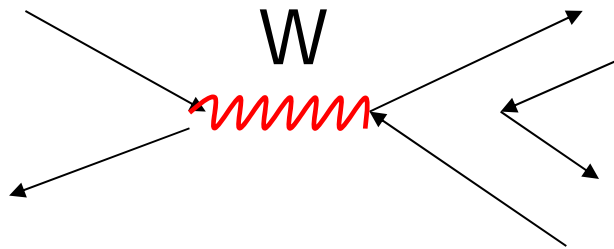
- W annihilation
- W exchange
- Time-like penguin
- Space-like penguin

Can not be **universal** for all decays,  
since not only one type

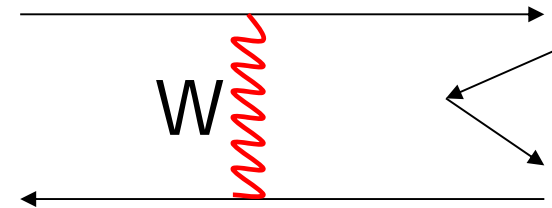
----sensitive to many parameters



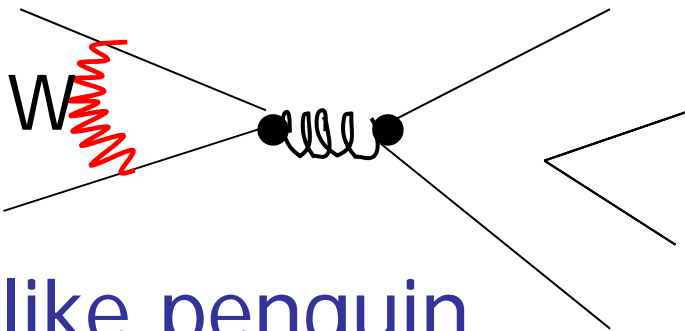
# Annihilation-Type diagrams



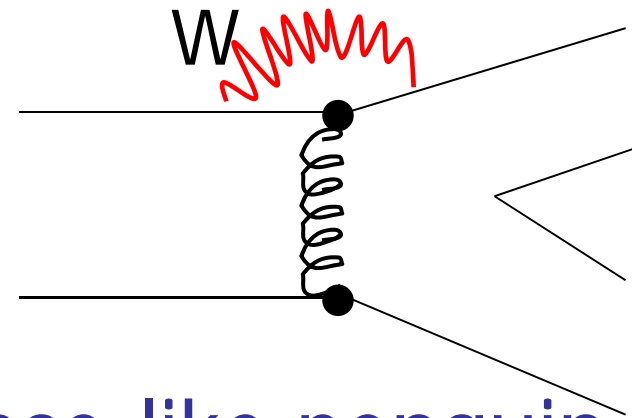
W annihilation(A)



W exchange(E)



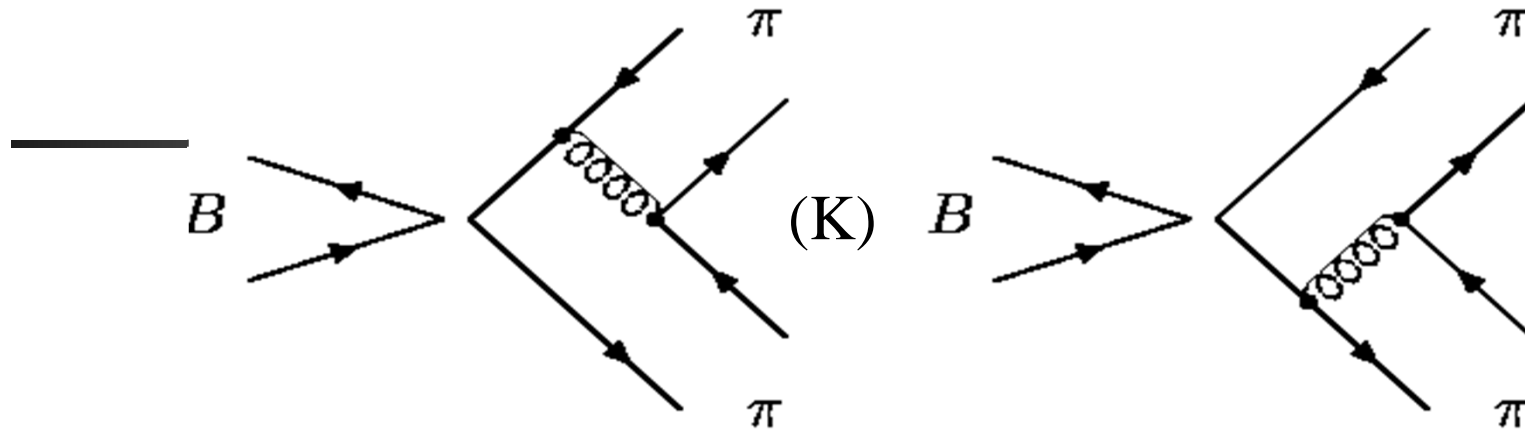
Time-like penguin



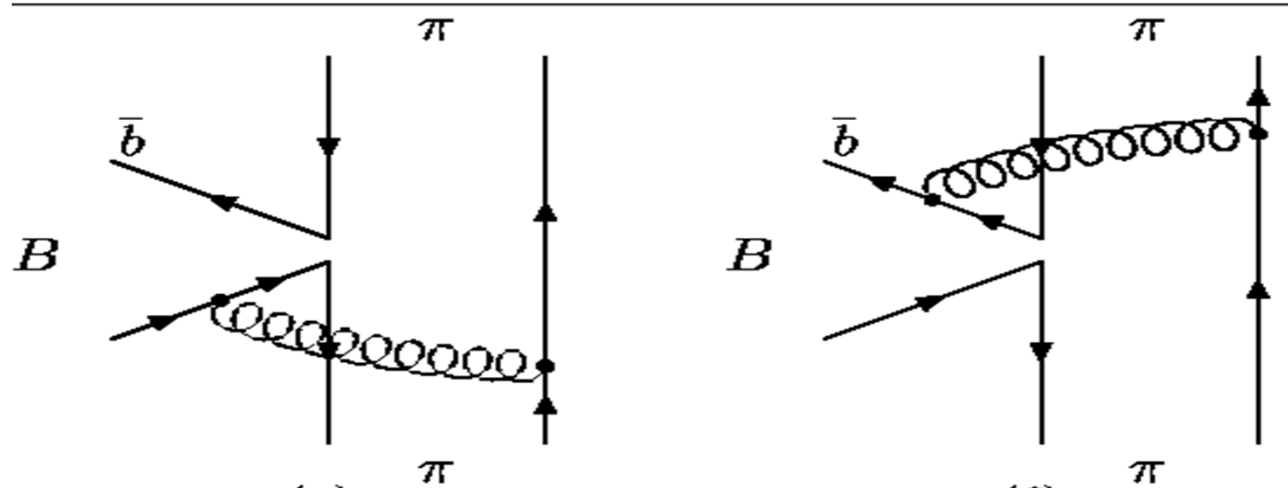
Space-like penguin



# PQCD Approach



Two diagrams cancel each other for tree operators, leaving only the other two





Gluon propagator  $\frac{i}{(k_1 - k_2)^2} = \frac{i}{-2xym_B^2}$

- In collinear factorization(QCDF and SCET), there is **endpoint singularity** in the annihilation type diagrams, not calculable, usually a **free parameter**
- However the transverse momentum at the endpoint is not negligible. In pQCD approach, we pick up back the **transverse momentum** of valence quarks to kill the divergence, Thus calculable

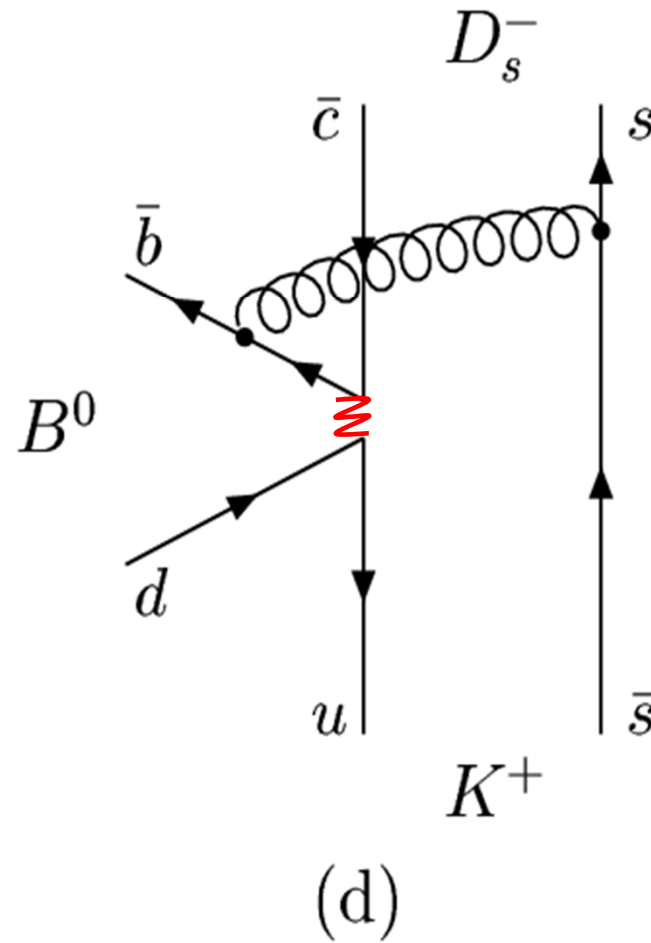
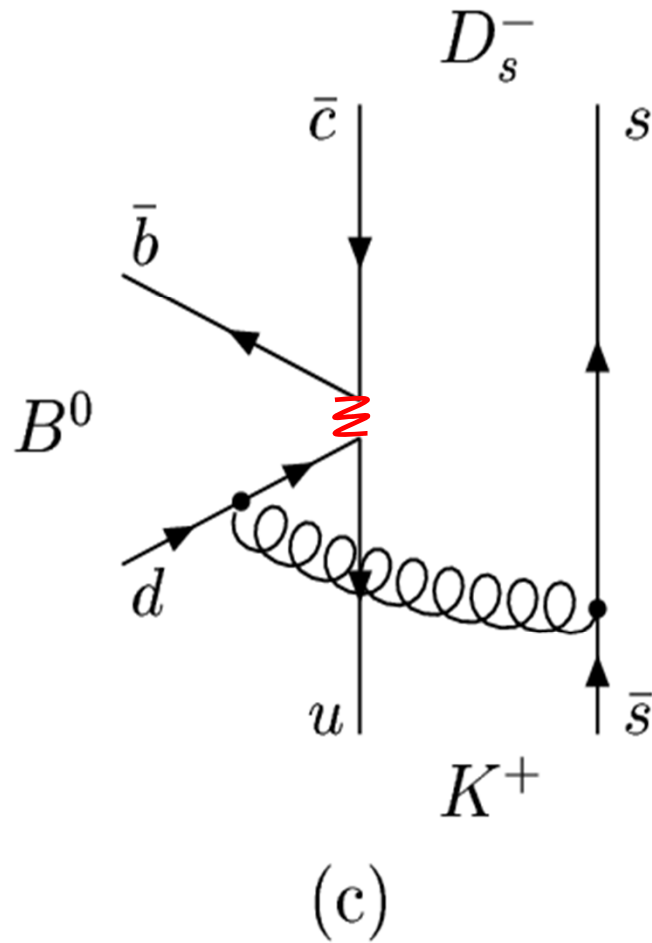
$$\frac{i}{(k_1 - k_2)^2} = \frac{i}{-2xym_B^2 - (k_1^T - k_2^T)^2}$$





# W Exchange Process

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# First annihilation type B decay

Results:

$$Br(B^0 \rightarrow D_S^- K^+) = (4.6_{-0.6}^{+0.8}) \times 10^{-5}$$

$$Br(B^0 \rightarrow D_S^{*-} K^+) = (2.7 \pm 0.6) \times 10^{-5}$$

Reported by Ukai in BCP4 (2001) before Exps:

Lü, Ukai, hep-ph/0210206, EPJC28 (2003) 305

$$Br(B^0 \rightarrow D_S^- K^+) = (4.6_{-0.6}^{+1.2} \pm 1.3) \times 10^{-5}, Belle$$

$$Br(B^0 \rightarrow D_S^- K^+) = (3.2 \pm 1.0 \pm 1.0) \times 10^{-5}, BaBar$$



# Penguin over tree

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- $B^0 \rightarrow K^+ \pi^-$  and  $B^0 \rightarrow \pi^+ \pi^-$  are dominated by **penguin (P)** and **tree (T)** operators, respectively
- In leading power,
- $|\mathbf{P/T}| \sim |f_K/f_\pi| \times |V_{ts}/V_{ub}| \times |\mathbf{a4/a1}|$   
 $= 158/132 \times 41.61/3.96 \times 0.045/1.05 = 0.5$

**Exp:**  $B(B^0 \rightarrow K^+ \pi^-)/B(B^0 \rightarrow \pi^+ \pi^-) = 18.2/4.6 = 4$



# Power Corrections

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- $(V-A)(V+A)$  **operator  $O_6$**  can be chirally enhanced when doing Fierz transformation in QCDF and pQCD.
- $a_6$  only slightly larger than  $a_4$ , QCDF needs very large chiral factor  $m_0 = m_K^2/m_s$ ,  $\Rightarrow$  small  $m_s$ .
- pQCD has additional chiral enhanced annihilation penguin contribution  $O_6$ , **does not need small  $m_s$**
- SCET/BPRS **without  $a_6$** , needs very large charming **penguin**



# Fierz transformation

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$$\langle P_1 P_2 | O_6 | B \rangle = -2 \sum_q (\langle P_1 | \bar{s} R q | 0 \rangle \langle P_2 | \bar{q} L b | B \rangle)$$

$$\langle K | \bar{s} \gamma^\mu \gamma^5 u | 0 \rangle = i f_K p^\mu$$

$$\langle P_1 P_2 | O_6 | B \rangle = R[P_1, P_2] \langle P_1 P_2 | O_4 | B \rangle$$

$$R[P_1, P_2] \equiv \frac{2M_{P_1}^2}{(m_q + m_s)(m_b - m_q)}$$



# The dominant contribution

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The biggest contribution for  $B \rightarrow \pi K$  in various approaches:

- Chiral enhanced penguin -- QCDF
- Chiral enhanced + annihilation penguin -- pQCD
- Charming penguin -- SCET

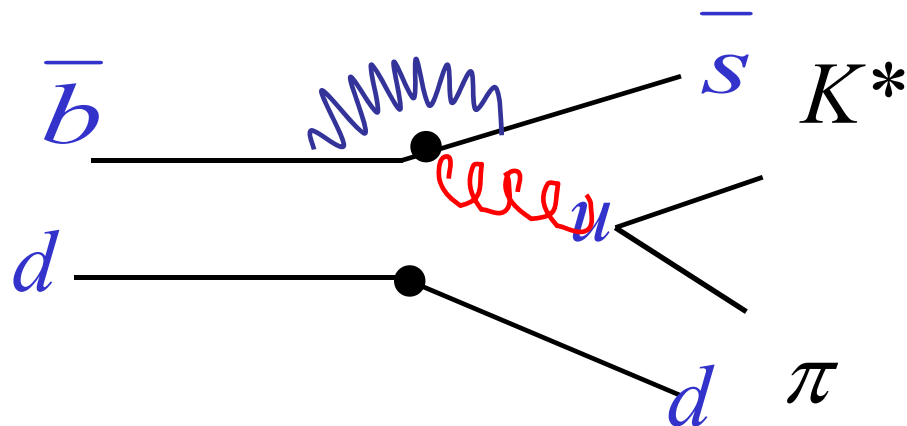


# B → VP decays

- Difficult for QCD to get large enough BRs
- No chiral enhanced penguin for  $B \rightarrow \pi K^*$

$$\langle K^* | \bar{s} \gamma^\mu u | 0 \rangle = m_{K^*} f_{K^*} \varepsilon^\mu$$

$$\langle K^* | \bar{s} (1 - \gamma^5) u | 0 \rangle = 0$$



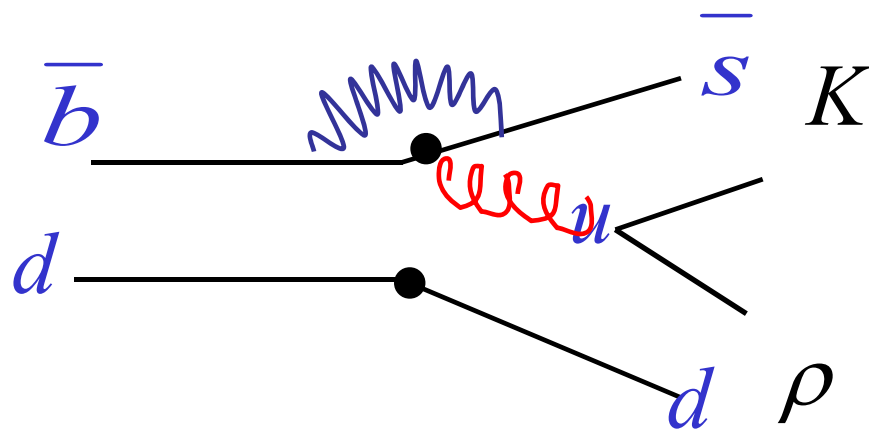


## Even worse for $B \rightarrow \rho K$

- Ordinary penguin canceled by chiral enhanced penguin (**minus sign**) for  $B \rightarrow \rho K$

$$\langle K\rho | o_6 | B \rangle = R \langle K\rho | o_4 | B \rangle$$

$$R = \frac{-2m_K^2}{(m_b + m_u)(m_s + m_u)}$$

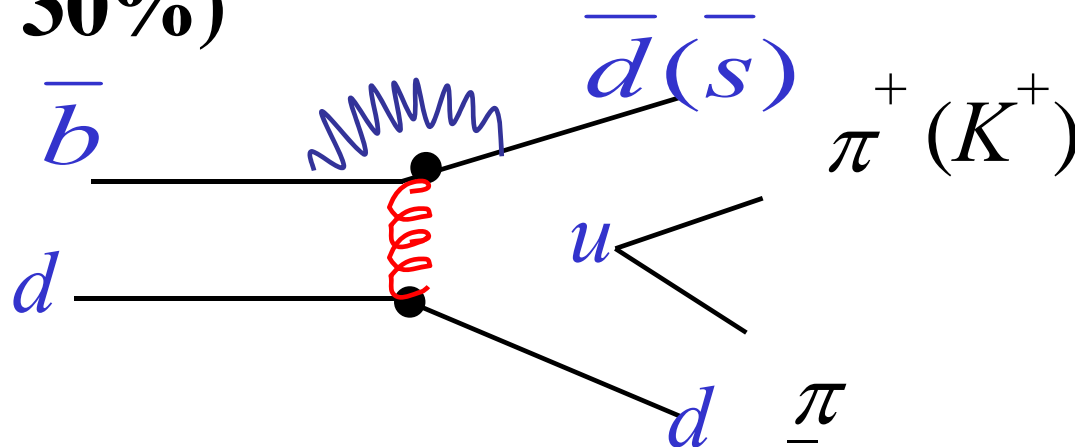






# No suppression for $O_6$

- Space-like penguin (annihilation)
- Become (s-p)(s+p) operator after Fiertz transformation **Chirally enhanced**
- No suppression, contribution “big” (20-30%)

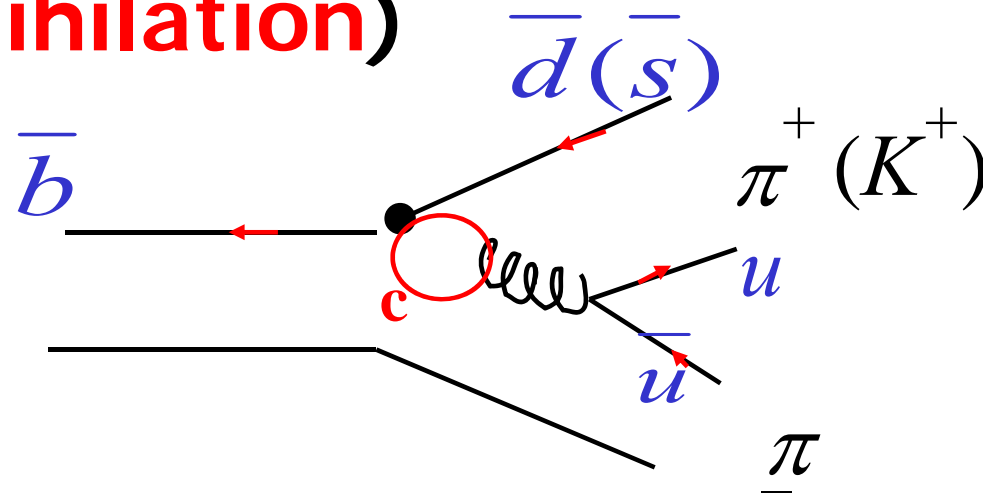


Calculable  
in pQCD  
approach



# Charming penguins in SCET

- Play the **similar role** as annihilation penguin, but not calculable
- Charming penguin appear always together with **space like penguin (annihilation)**





# CP Violation in $B \rightarrow \pi \pi (K)$ (*real prediction before exp.*)

CP(%)	FA	BBNS	PQCD (2001)	Exp (2004)
$\pi^+ K^-$	$+9 \pm 3$	$+5 \pm 9$	$-17 \pm 5$	$-11.5 \pm 1.8$
$\pi^+ K^0$	$1.7 \pm 0.1$	$1 \pm 1$	$-1.0 \pm 0.5$	$-2 \pm 4$
$\pi^0 K^+$	$+8 \pm 2$	$7 \pm 9$	$-13 \pm 4$	$+4 \pm 4$
$\pi^+ \pi^-$	$-5 \pm 3$	$-6 \pm 12$	$+30 \pm 10$	$+37 \pm 10$



# CP Violation in $B \rightarrow \pi \pi (K)$

Including large annihilation fixed from exp.

CP(%)	FA	Cheng HY	PQCD (2001)	Exp (2004)
$\pi^+ K^-$	$+9 \pm 3$	$-7.4 \pm 5.0$	$-17 \pm 5$	$-11.5 \pm 1.8$
$\pi^+ K^0$	$1.7 \pm 0.1$	$0.28 \pm 0.10$	$-1.0 \pm 0.5$	$-2 \pm 4$
$\pi^0 K^+$	$+8 \pm 2$	$4.9 \pm 5.9$	$-13 \pm 4$	$+4 \pm 4$
$\pi^+ \pi^-$	$-5 \pm 3$	$17 \pm 1.3$	$+30 \pm 10$	$+37 \pm 10$



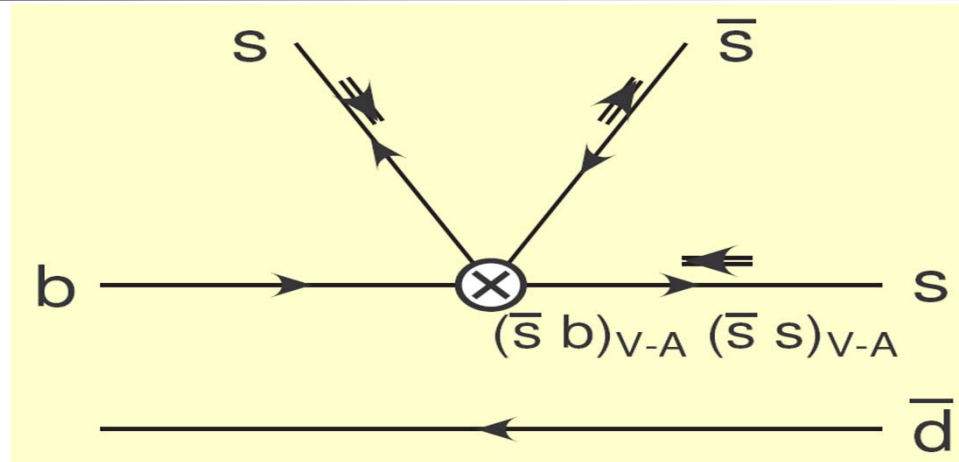
# Polarization of $B \rightarrow VV$ decays

Table 1 Longitudinal Polarization Fractions

Process	Belle	Babar	QCDF
$B^0 \rightarrow \phi K^{*0},$	$0.45 \pm 0.05 \pm 0.02$	$0.52 \pm 0.05 \pm 0.02$	0.91
$B^+ \rightarrow \phi K^{*+},$	$0.52 \pm 0.8 \pm 0.03$	$0.46 \pm 0.12 \pm 0.03$	0.91
$B^+ \rightarrow \rho^0 K^{*+},$		$0.78 \pm 0.12 \pm 0.03$	0.94
$B^+ \rightarrow \rho^+ K^{*0},$	$0.43 \pm 0.11^{+0.05}_{-0.07}$	$0.52 \pm 0.10 \pm 0.04$	0.95
$B^+ \rightarrow \rho^+ \rho^0,$	$0.95 \pm 0.11 \pm 0.02$	$0.97 \pm 0.04^{+0.03}_{-0.07}$	0.94
$B^+ \rightarrow \rho^+ \omega,$		$0.88 \pm 0.04^{+0.12}_{-0.15}$	
$B^0 \rightarrow \rho^+ \rho^-,$		$0.99 \pm 0.03^{+0.04}_{-0.03}$	0.95



# Helicity flip suppression of the transverse polarization amplitude



$$H_0 \gg H_- \gg H_+$$

**Naïve counting rule**

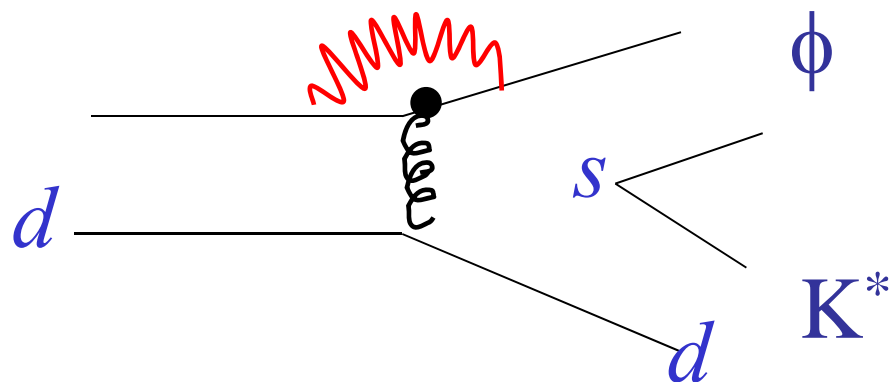
$$\mathbf{H}_\pm = \mathbf{M}_N \pm \mathbf{M}_T$$

$$R_L = \Gamma_L / \Gamma_{total} = \mathcal{O}(1), R_N \sim R_T = \mathcal{O}(m_V^2 / m_B^2)$$



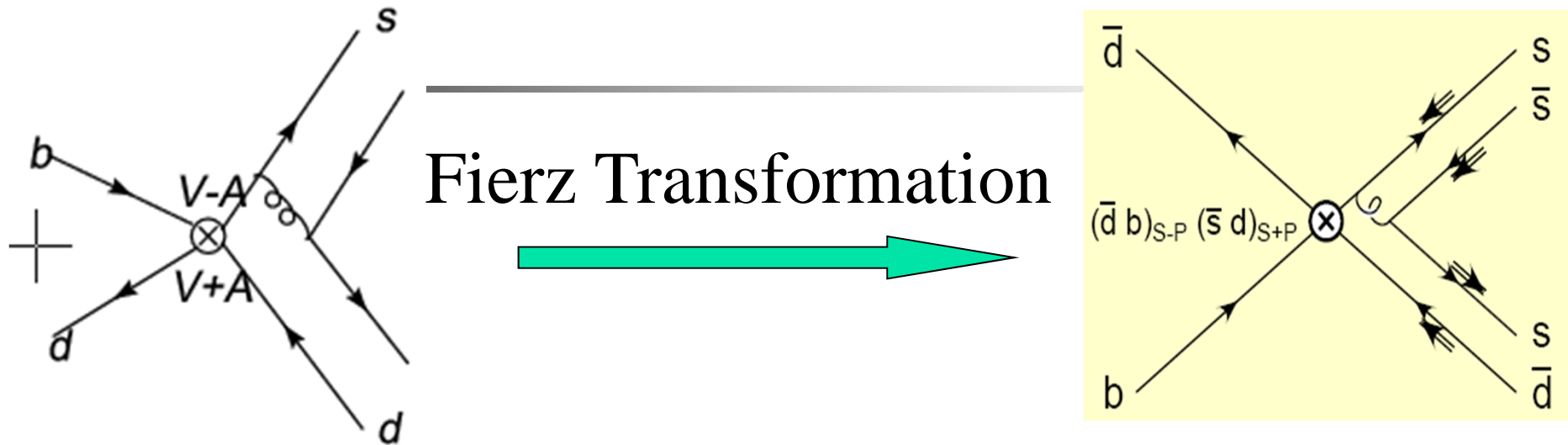
# Large transverse component in $B \rightarrow \phi K^*$ decays

- Annihilation can also enhance transverse contribution





# The annihilation diagram



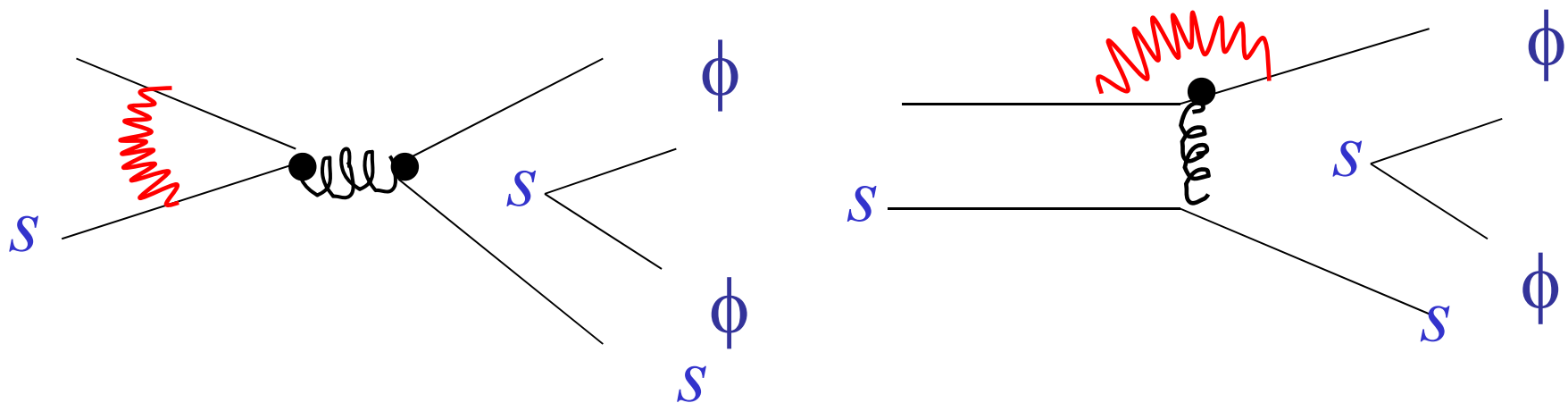
**The (S+P)(S-P) current can break the counting rule,  
The annihilation diagram contributes equally to the **three polarization amplitudes****





# Time-like and space-like penguin in $B_s \rightarrow \phi \phi$ decays

- Recently measured by LHCb arXiv:1204.2813
- $f_L = 0.365 \pm 0.022 \pm 0.012$

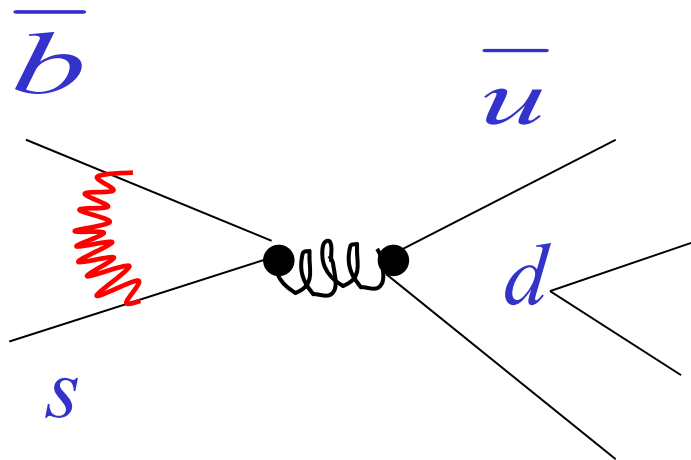




CDF Results

arXiv:1111.0485

First evidence:  $B_s^0 \rightarrow \pi^+ \pi^-$



**Pure  
annihilation**

$$\mathcal{B}(B_s^0 \rightarrow \pi^+ \pi^-) = (0.57 \pm 0.15 \text{ (stat.)} \pm 0.10 \text{ (syst.)}) \times 10^{-6}.$$



# The pQCD prediction given at 2007

$$\text{Br}(B_s^0 \rightarrow \pi^+ \pi^-) = 5.7 \times 10^{-7}$$

PHYSICAL REVIEW D **76**, 074018 (2007)

## Charmless nonleptonic $B_s$ decays to $PP$ , $PV$ , and $VV$ final states in the perturbative QCD approach

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*Institute of High Energy Physics, CAS, P.O. Box 918(4), 100049, People's Republic of China*

(Received 19 March 2007; published 17 October 2007)

We calculate the  $CP$ -averaged branching ratios and  $CP$ -violating asymmetries of a number of two-body charmless hadronic decays  $\bar{B}_s^0 \rightarrow PP, PV, VV$  in the perturbative QCD (pQCD) approach to leading order in  $\alpha_s$  (here  $P$  and  $V$  denote light pseudoscalar and vector mesons, respectively). The mixing-induced  $CP$  violation parameters are also calculated for these decays. We also predict the polarization fractions of  $B_s \rightarrow VV$  decays and find that the transverse polarizations are enhanced in some penguin-dominated decays such as  $\bar{B}_s^0 \rightarrow K^* \bar{K}^*, K^* \rho$ . Some of the predictions worked out here can already be confronted with the recently available data from the CDF Collaboration on the branching ratios for the decays  $\bar{B}_s^0 \rightarrow K^+ \pi^-, \bar{B}_s^0 \rightarrow K^+ K^-$  and the  $CP$  asymmetry in the decay  $\bar{B}_s^0 \rightarrow K^+ \pi^-$ , and are found to be in agreement within the current errors. A large number of predictions for the branching ratios,  $CP$  asymmetries, and vector-meson polarizations in  $\bar{B}_s^0$  decays, presented in this paper and compared with the already existing results in other theoretical frameworks, will be put to stringent experimental tests in forthcoming experiments at Fermilab, CERN LHC, and Super  $B$  factories.



## $B^0 \rightarrow K^+ K^-$ decay

**CDF:**  $\mathcal{B}(B^0 \rightarrow K^+ K^-) = (0.23 \pm 0.10 \text{ (stat.)} \pm 0.10 \text{ (syst.)}) \times 10^{-6}$

**LHCb:**  $\mathcal{BR}(B^0 \rightarrow K^+ K^-) = (0.14 \pm 0.06 \pm 0.07) \times 10^{-6}$

**Zhen-Jun Xiao et al.** recalculate this decay in pQCD recently, and find that

$$\text{Br}(B^0 \rightarrow K^+ K^-) = 1.5 \times 10^{-7}$$

**Well consistent with the data!**

◆ [hep-ph/1111.6264](https://arxiv.org/abs/hep-ph/1111.6264)



PHYSICAL REVIEW D 76, 074018 (2007)

# Charmless nonleptonic $B_s$ decays to $PP$ , $PV$ , and $VV$ final states in the perturbative QCD approach

Ahmed Ali<sup>1</sup> and Gustav Kramer<sup>2</sup>

**Direct CP violation in  $B_s^0 \rightarrow K \pi$**  *Hamburg, Germany*  
*22761 Hamburg, Germany*

Ying Li, Cai-Dian Lü, Yue-Long Shen, Wei Wang, and Yu-Ming Wang

➤ first evidence of direct CPV  
 in  $B_s$  decays

*4), 100049, People's Republic of China*  
*(dated 17 October 2007)*

Calculating asymmetries of a number of two-body

in  $\alpha_s$  (here  $P$  and  $V$  denote light pseudoscalar and  
 violation parameters are also calculated for these

$$A_{CP}(B_s^0) = (27 \pm 8 \pm 2)\%$$

Modes	Class	QCDF	SCET	This work	Experiment
$\bar{B}_s^0 \rightarrow K^+ \pi^-$	$T$	$-6.7^{+2.1+3.1+0.2+15.5}_{-2.2-2.9-0.4-15.2}$	$20 \pm 17 \pm 19 \pm 5$	$24.1^{+3.9+3.3+2.3}_{-3.6-3.0-1.2}$	$39 \pm 15 \pm 8$
$\bar{B}_s^0 \rightarrow K^0 \pi^0$	$C$	$41.6^{+16.6+14.3+7.8+40.9}_{-12.0-13.3-14.5-51.0}$	$76 \pm 26 \pm 27 \pm 17$	$59.4^{+1.8+7.4+2.2}_{-4.0-11.3-3.5}$	
$\bar{B}_s^0 \rightarrow K^+ K^-$	$P$	$4.0^{+1.0+2.0+0.5+10.4}_{-1.0-2.3-0.5-11.3}$	$-6 \pm 5 \pm 6 \pm 2$	$-23.3^{+0.9+4.9+0.8}_{-0.2-4.4-1.1}$	





# Pure annihilation type $B_c$ decays

Decay modes ( $\Delta S = 0$ )	BRs ( $10^{-8}$ )	Decay modes ( $\Delta S = 1$ )	BRs ( $10^{-8}$ )
$B_c \rightarrow \pi^+ \pi^0$	0	$B_c \rightarrow \pi^+ K^0$	$4.0^{+1.0}_{-0.6}(m_c)^{+2.3}_{-1.6}(a_i)^{+0.5}_{-0.3}(m_0)$
$B_c \rightarrow \pi^+ \eta$	$22.8^{+6.9}_{-4.6}(m_c)^{+7.2}_{-4.5}(a_i)^{+3.4}_{-4.2}(m_0)$	$B_c \rightarrow K^+ \eta$	$0.6^{+0.0}_{-0.0}(m_c)^{+0.6}_{-0.5}(a_i)^{+0.2}_{-0.1}(m_0)$
$B_c \rightarrow \pi^+ \eta'$	$15.3^{+4.6}_{-3.1}(m_c)^{+4.8}_{-3.0}(a_i)^{+2.2}_{-2.8}(m_0)$	$B_c \rightarrow K^+ \eta'$	$5.7^{+0.9}_{-0.9}(m_c)^{+1.0}_{-1.6}(a_i)^{+0.0}_{-0.3}(m_0)$
$B_c \rightarrow K^+ \bar{K}^0$	$24.0^{+2.4}_{-0.0}(m_c)^{+7.3}_{-6.0}(a_i)^{+6.8}_{-5.8}(m_0)$	$B_c \rightarrow K^+ \pi^0$	$2.0^{+0.5}_{-0.3}(m_c)^{+1.2}_{-0.8}(a_i)^{+0.3}_{-0.1}(m_0)$

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Decay modes ( $\Delta S = 0$ )	BRs ( $10^{-7}$ )	Decay modes ( $\Delta S = 1$ )	BRs ( $10^{-8}$ )
$B_c \rightarrow \pi^+ \rho^0$	$1.7^{+0.1}_{-0.0}(m_c)^{+0.1}_{-0.2}(a_i)^{+0.6}_{-0.3}(m_0)$	$B_c \rightarrow K^+ \rho^0$	$3.1^{+0.6}_{-0.8}(m_c)^{+1.2}_{-1.5}(a_i)^{+0.1}_{-0.2}(m_0)$
$B_c \rightarrow \bar{K}^0 K^{*+}$	$1.8^{+0.7}_{-0.1}(m_c)^{+4.1}_{-2.1}(a_i)^{+0.1}_{-0.0}(m_0)$	$B_c \rightarrow K^0 \rho^+$	$6.1^{+1.3}_{-1.5}(m_c)^{+2.5}_{-2.9}(a_i)^{+0.2}_{-0.3}(m_0)$
$B_c \rightarrow \pi^+ \omega$	$5.8^{+1.4}_{-2.2}(m_c)^{+1.1}_{-1.3}(a_i)^{+0.4}_{-1.2}(m_0)$	$B_c \rightarrow K^+ \omega$	$2.3^{+1.1}_{-0.3}(m_c)^{+1.8}_{-1.2}(a_i) \pm 0.1(m_0)$

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Decay modes ( $\Delta S = 0$ )	BRs ( $10^{-7}$ )	Decay modes ( $\Delta S = 1$ )	BRs ( $10^{-8}$ )
$B_c \rightarrow \rho^+ \pi^0$	$0.5^{+0.1}_{-0.1}(m_c)^{+0.3}_{-0.2}(a_i)^{+0.2}_{-0.3}(m_0)$	$B_c \rightarrow K^{*0} \pi^+$	$3.3^{+0.7}_{-0.2}(m_c)^{+0.4}_{-0.4}(a_i)^{+0.2}_{-0.1}(m_0)$
$B_c \rightarrow \rho^+ \eta$	$5.4^{+2.1}_{-1.2}(m_c)^{+0.9}_{-1.4}(a_i) \pm 0.0(m_0)$	$B_c \rightarrow K^{*+} \pi^0$	$1.6^{+0.4}_{-0.1}(m_c)^{+0.3}_{-0.1}(a_i)^{+0.1}_{-0.0}(m_0)$
$B_c \rightarrow \rho^+ \eta'$	$3.6^{+1.4}_{-0.8}(m_c)^{+0.6}_{-0.9}(a_i) \pm 0.0(m_0)$	$B_c \rightarrow K^{*+} \eta$	$0.9^{+0.1}_{-0.0}(m_c)^{+0.6}_{-0.2}(a_i) \pm 0.0(m_0)$
$B_c \rightarrow \bar{K}^{*0} K^+$	$10.0^{+0.5}_{-0.6}(m_c)^{+1.7}_{-3.3}(a_i)^{+0.0}_{-0.2}(m_0)$	$B_c \rightarrow K^{*+} \eta'$	$3.8 \pm 1.1(m_c)^{+1.0}_{-0.6}(a_i) \pm 0.0(m_0)$
		$B_c \rightarrow \phi K^+$	$5.6^{+1.1}_{-0.0}(m_c)^{+1.2}_{-0.9}(a_i)^{+0.3}_{-0.0}(m_0)$



# Pure annihilation type $B_c$ decays with longitudinal polarization fractions

TABLE IV. The pQCD predictions of branching ratios (BRs) and longitudinal polarization fractions (LPFs) for  $B_c \rightarrow VV$  modes.

Decay modes	BRs ( $10^{-7}$ )	LPFs (%)
$B_c \rightarrow \rho^+ \rho^0$	0	-
$B_c \rightarrow \rho^+ \omega$	$10.6^{+3.2}_{-0.2}(m_c)^{+2.1}_{-0.2}(a_i)$	$92.9^{+1.6}_{-0.1}(m_c)^{+1.2}_{-0.1}(a_i)$
$B_c \rightarrow \bar{K}^{*0} K^{*+}$	$10.0^{+0.6}_{-0.4}(m_c)^{+8.1}_{-4.8}(a_i)$	$92.0^{+0.5}_{-0.4}(m_c)^{+3.6}_{-7.1}(a_i)$
$B_c \rightarrow K^{*0} \rho^+$	$0.6^{+0.0}_{-0.0}(m_c)^{+0.2}_{-0.1}(a_i)$	$94.9^{+0.2}_{-0.2}(m_c)^{+2.0}_{-1.4}(a_i)$
$B_c \rightarrow K^{*+} \rho^0$	$0.3^{+0.0}_{-0.0}(m_c)^{+0.1}_{-0.1}(a_i)$	$94.9^{+0.2}_{-0.2}(m_c)^{+1.3}_{-1.4}(a_i)$
$B_c \rightarrow K^{*+} \omega$	$0.3^{+0.0}_{-0.0}(m_c)^{+0.0}_{-0.2}(a_i)$	$94.8^{+0.3}_{-0.2}(m_c)^{+1.1}_{-1.2}(a_i)$
$B_c \rightarrow \phi K^{*+}$	$0.5^{+0.0}_{-0.1}(m_c)^{+0.1}_{-0.3}(a_i)$	$86.4^{+0.0}_{-1.4}(m_c)^{+4.9}_{-9.0}(a_i)$



$B_c \rightarrow DD$  decays with contributions from both emission and annihilation diagrams calculated in Perturbative QCD approach (Zhou R, Zou ZT, CDL, arXiv: 1203.2303)

	channels	This work	Kiselev[4]	IKP[5]	IKS[7]	LC[8]	CF[10]
1	$B_c \rightarrow D^+ \bar{D}^0$	$32_{-6-1-4}^{+6+1+2}$	53	32	33	86	8.4
2	$B_c \rightarrow D^+ \bar{D}^{*0}$	$34_{-6-1-3}^{+7+2+3}$	75	83	38	75	7.5
3	$B_c \rightarrow D^{*+} \bar{D}^0$	$12_{-3-0-1}^{+3+1+0}$	49	17	9	30	84
4	$B_c \rightarrow D^{*+} \bar{D}^{*0}$	$34_{-8-1-0}^{+9+2+0}$	330	84	21	55	140
5	$B_c \rightarrow D_s^+ \bar{D}^0$	$2.3_{-0.4-0.1-0.2}^{+0.4+0.1+0.2}$	4.8	1.7	2.1	4.6	0.6
6	$B_c \rightarrow D_s^+ \bar{D}^{*0}$	$2.6_{-0.6-0.1-0.2}^{+0.4+0.1+0.1}$	7.1	4.3	2.4	3.9	0.53
7	$B_c \rightarrow D_s^{*+} \bar{D}^0$	$0.7_{-0.2-0.0-0.0}^{+0.1+0.0+0.0}$	4.5	0.95	0.65	1.8	5
8	$B_c \rightarrow D_s^{*+} \bar{D}^{*0}$	$2.8_{-0.6-0.1-0.0}^{+0.7+0.1+0.1}$	26	4.7	1.6	3.5	8.4





# Bc → DP decays calculated in Perturbative QCD approach (Zhou R, Zou ZT, CDL, arXiv: 1112.1257)

channels	Class	$\mathcal{BR}(10^{-7})$			$A_{CP}^{dir}(\%)$	
		This work	RCQM <sup>a</sup>	LFQM	This work	RCQM
$B_c \rightarrow D^0 \pi^+$	T	$26.7^{+3.1+6.0+0.8}_{-3.5-5.6-0.6}$	22.9	4.3	$-41.2^{+4.5+11.1+0.8}_{-4.6-7.8-1.2}$	6.5
$B_c \rightarrow D^+ \pi^0$	C,A	$0.82^{+0.24+0.55+0.06}_{-0.16-0.41-0.01}$	2.1	0.067	$2.3^{+6.3+1.4+15.0}_{-3.0-0.8-18.8}$	-1.9
$B_c \rightarrow D^0 K^+$	A,P	$47.8^{+17.2+2.2+5.4}_{-9.1-1.7-3.6}$	44.5	0.35	$-34.8^{+4.9+7.4+1.8}_{-2.6-3.7-1.3}$	-4.6
$B_c \rightarrow D^+ K^0$	A,P	$46.9^{+15.6+0.3+7.4}_{-12.3-0.3-4.6}$	49.3	–	$2.3^{+0.4+0.9+0.0}_{-0.2-0.5-0.0}$	-0.8
$B_c \rightarrow D^+ \eta$	C,A	$0.92^{+0.15+0.21+0.03}_{-0.15-0.25-0.00}$	–	0.087	$40.8^{+0.0+18.4+15.6}_{-2.9-14.0-13.5}$	–
$B_c \rightarrow D^+ \eta'$	C,A	$0.91^{+0.12+0.16+0.06}_{-0.10-0.20-0.03}$	–	0.048	$-14.0^{+0.6+4.6+15.9}_{-1.5-5.2-11.9}$	–
$B_c \rightarrow D_s^+ \pi^0$	C,P	$0.41^{+0.04+0.01+0.02}_{-0.04-0.02-0.02}$	–	0.0067	$46.7^{+1.4+6.3+2.5}_{-1.4-11.8-2.8}$	–
$B_c \rightarrow D_s^+ \bar{K}^0$	A,P	$2.1^{+0.9+0.3+0.3}_{-0.6-0.3-0.2}$	1.9	–	$54.3^{+6.9+5.3+0.0}_{-7.2-8.0-0.3}$	13.3
$B_c \rightarrow D_s^+ \eta$	A,P	$17.3^{+1.7+0.5+3.3}_{-1.8-0.6-1.2}$	–	0.009	$2.8^{+0.0+0.4+1.1}_{-0.1-0.7-1.2}$	–
$B_c \rightarrow D_s^+ \eta'$	A,P	$51.0^{+4.9+0.4+6.7}_{-5.4-0.3-3.5}$	–	0.0048	$1.1^{+0.1+0.2+0.7}_{-0.0-0.2-0.6}$	–



# Bc → DP decays calculated in Perturbative QCD approach (Zhou R, Zou ZT, CDL, arXiv: 1112.1257)

channels	Class	$\mathcal{BR}(10^{-7})$		$A_{CP}^{dir}(\%)$		$\mathcal{R}_T(\%)$
		This work	RCQM	This work	RCQM	This work
$B_c \rightarrow D^{*0} \rho^+$	T	$55.3^{+8.6+11.9+1.5}_{-5.0-11.1-1.4}$	59.7	$-24.1^{+3.0+4.2+0.4}_{-3.4-2.7-0.4}$	3.8	$16.4^{+2.5+2.0+0.3}_{-1.7-1.4-0.1}$
$B_c \rightarrow D^{*+} \rho^0$	C,A	$3.8^{+1.0+0.5+0.1}_{-0.8-0.6-0.1}$	13.0	$30.2^{+0.0+2.6+5.4}_{-1.5-5.8-7.6}$	-3.0	$54.3^{+1.8+4.0+0.5}_{-0.9-2.4-0.4}$
$B_c \rightarrow D^{*0} K^{*+}$	A,P	$161^{+59+5+11}_{-40-4-9}$	37.7	$-14.9^{+1.1+3.1+0.3}_{-0.8-1.7-0.1}$	-6.2	$52.6^{+1.5+2.3+1.3}_{-1.1-1.8-0.7}$
$B_c \rightarrow D^{*+} K^{*0}$	A,P	$172^{+57+1+11}_{-42-1-9}$	30.6	$0.4^{+0.0+0.0+0.0}_{-0.0-0.1-0.0}$	-0.8	$57.4^{+0.6+0.1+0.9}_{-0.7-0.1-0.4}$
$B_c \rightarrow D^{*+} \omega$	C,A	$2.4^{+0.4+0.9+0.2}_{-0.6-0.7-0.1}$	—	$-7.8^{+1.0+2.6+5.8}_{-0.0-3.4-5.0}$	—	$56.0^{+1.2+9.6+0.7}_{-0.5-6.5-0.7}$
$B_c \rightarrow D^{*+} \phi$	P	$0.004^{+0.001+0+0}_{-0-0-0.001}$	—	—	—	$11.4^{+22.3+0.0+5.3}_{-5.4-0.0-6.9}$
$B_c \rightarrow D_s^{*+} \rho^0$	C,P	$0.72^{+0.08+0.03+0.02}_{-0.08-0.03-0.03}$	—	$-29.3^{+1.3+7.6+1.4}_{-1.1-4.5-0.9}$	-3.0	$11.2^{+0.5+2.1+0.2}_{-0.3-1.4-0.1}$
$B_c \rightarrow D_s^{*+} \bar{K}^{*0}$	A,P	$4.3^{+1.3+0.4+0.3}_{-1.0-0.3-0.2}$	2.9	$6.2^{+0.1+1.3+0.0}_{-0.3-1.8-0.1}$	13.3	$68.8^{+2.1+3.9+0.8}_{-2.3-4.4-0.4}$
$B_c \rightarrow D_s^{*+} \omega$	C,P	$0.26^{+0.03+0.04+0.07}_{-0.01-0.05-0.04}$	—	$-21.3^{+5.3+6.8+7.9}_{-4.6-6.8-4.7}$	—	$49.5^{+8.8+2.1+4.4}_{-10.9-1.2-2.8}$
$B_c \rightarrow D_s^{*+} \phi$	A,P	$137.3^{+39.3+0.5+10.5}_{-27.8-0.5-7.5}$	38.8	$0.3^{+0.1+0.1+0.0}_{-0.1-0.1-0.0}$	-0.8	$67.5^{+2.1+0.1+1.4}_{-3.1-0.2-1.5}$



# $B \rightarrow PT$ decays

( $\Delta S=1$  modes)( $10^{-7}$ )

HY Cheng, KC Yang, PRD83,  
034001(2008) & ZT Zou, X Yu, CDL,  
**arXiv:1203.4120**

Decay Modes	class	This Work	ISGW2 [24]	QCDF [4]	Expt.
$B^+ \rightarrow K_2^{*0} \pi^+$	PA	$0.9^{+0.2}_{-0.2} {}^{+0.2}_{-0.2} {}^{+0.3}_{-0.2}$	...	$3.1^{+8.3}_{-3.1}$	$5.6^{+2.2}_{-1.4}$
$B^+ \rightarrow K_2^{*+} \pi^0$	PA	$0.4^{+0.1}_{-0.0} {}^{+0.1}_{-0.1} {}^{+0.1}_{-0.1}$	0.090	$2.2^{+4.7}_{-1.9}$	...
$B^+ \rightarrow a_2^0 K^+$	T,PA	$2.1^{+0.7}_{-0.6} {}^{+0.6}_{-0.5} {}^{+0.6}_{-0.5}$	0.31	$4.9^{+8.4}_{-4.2}$	$< 45$
$B^+ \rightarrow a_2^+ K^0$	PA	$3.1^{+0.9}_{-0.8} {}^{+0.9}_{-0.8} {}^{+1.1}_{-0.9}$	0.011	$8.4^{+16.1}_{-7.2}$	...
$B^+ \rightarrow f_2 K^+$	T,PA,P	$11.8^{+2.7}_{-2.4} {}^{+3.2}_{-2.8} {}^{+3.0}_{-2.7}$	0.34	$3.8^{+7.8}_{-3.0}$	$1.06^{+0.28}_{-0.29}$
$B^+ \rightarrow f' K^+$	P,PA	$3.8^{+0.4}_{-0.4} {}^{+0.9}_{-0.8} {}^{+1.0}_{-0.8}$	0.004	$4.0^{+7.4}_{-3.6}$	$< 7.7$
$B^+ \rightarrow K_2^{*+} \eta$	PA,P	$0.8^{+0.2}_{-0.2} {}^{+0.3}_{-0.2} {}^{+0.3}_{-0.3}$	0.031	$6.8^{+13.5}_{-8.7}$	$9.1 \pm 3.0$
$B^+ \rightarrow K_2^{*+} \eta'$	PA,P	$12.7^{+3.7}_{-3.2} {}^{+4.5}_{-3.5} {}^{+4.0}_{-3.5}$	1.41	$12.1^{+20.7}_{-12.1}$	$28.0^{+5.3}_{-5.0}$
$B^0 \rightarrow K_2^{*+} \pi^-$	PA	$1.0^{+0.2}_{-0.2} {}^{+0.2}_{-0.2} {}^{+0.3}_{-0.2}$	...	$3.3^{+8.5}_{-3.2}$	$< 6.3$
$B^0 \rightarrow K_2^{*0} \pi^0$	PA	$0.6^{+0.2}_{-0.1} {}^{+0.1}_{-0.1} {}^{+0.2}_{-0.1}$	0.084	$1.2^{+4.3}_{-1.3}$	$< 4.0$
$B^0 \rightarrow a_2^- K^+$	T,PA	$5.0^{+1.6}_{-1.4} {}^{+1.4}_{-1.1} {}^{+1.3}_{-1.0}$	0.58	$9.7^{+17.2}_{-8.1}$	...
$B^0 \rightarrow a_2^0 K^0$	PA	$2.0^{+0.5}_{-0.5} {}^{+0.4}_{-0.4} {}^{+0.6}_{-0.5}$	0.005	$4.2^{+8.3}_{-3.5}$	...
$B^0 \rightarrow f_2 K^0$	PA,P	$9.2^{+2.0}_{-1.8} {}^{+2.5}_{-2.1} {}^{+2.6}_{-2.2}$	0.005	$3.4^{+8.5}_{-3.1}$	$2.7^{+1.3}_{-1.2}$
$B^0 \rightarrow f'_2 K^0$	P,PA	$3.7^{+0.3}_{-0.4} {}^{+0.7}_{-0.8} {}^{+0.9}_{-0.9}$	0.00007	$3.8^{+7.3}_{-3.5}$	...
$B^0 \rightarrow K_2^{*0} \eta$	PA,P	$1.0^{+0.2}_{-0.2} {}^{+0.3}_{-0.2} {}^{+0.3}_{-0.3}$	0.029	$6.6^{+13.5}_{-8.7}$	$9.6 \pm 2.1$
$B^0 \rightarrow K_2^{*0} \eta'$	PA,P	$11.6^{+3.6}_{-2.9} {}^{+4.2}_{-3.1} {}^{+3.8}_{-3.1}$	1.30	$12.4^{+21.3}_{-12.4}$	$13.7^{+3.2}_{-3.1}$





$B \rightarrow PT$  ( $\Delta S=0$ )( $10^{-7}$ ) HY Cheng, KC Yang, PRD83,  
034001(2008) & ZT Zou, X Yu, CDL, [arXiv:1203.4120](https://arxiv.org/abs/1203.4120)

Decay Modes	class	This Work	ISGW2 [24]	QCDF [4]
$B^+ \rightarrow a_2^0 \pi^+$	T	29.1 $^{+12.8}_{-10.6}$ $^{+14.2}_{-10.4}$ $^{+3.1}_{-2.8}$	26.02	30 $^{+14}_{-12}$
$B^+ \rightarrow a_2^+ \pi^0$	T,C	0.3 $^{+0.0}_{-0.0}$ $^{+0.1}_{-0.1}$ $^{+0.0}_{-0.0}$	0.01	2.4 $^{+4.9}_{-3.1}$
$B^+ \rightarrow a_2^+ \eta$	C,PA,P	1.0 $^{+0.3}_{-0.3}$ $^{+0.4}_{-0.3}$ $^{+0.4}_{-0.3}$	2.94	1.1 $^{+2.8}_{-1.1}$
$B^+ \rightarrow a_2^+ \eta'$	C,PA,P	3.5 $^{+1.4}_{-1.0}$ $^{+1.6}_{-1.1}$ $^{+1.1}_{-0.8}$	13.1	1.1 $^{+4.7}_{-1.2}$
$B^+ \rightarrow f_2 \pi^+$	T	42.5 $^{+18.9}_{-15.4}$ $^{+18.9}_{-13.9}$ $^{+4.2}_{-3.9}$	28.74	27 $^{+14}_{-12}$
$B^+ \rightarrow f_2' \pi^+$	T	1.2 $^{+0.3}_{-0.2}$ $^{+0.4}_{-0.3}$ $^{+0.1}_{-0.1}$	0.37	0.09 $^{+0.24}_{-0.09}$
$B^+ \rightarrow K_2^{*+} \bar{K}^0$	PA,P	1.2 $^{+0.2}_{-0.2}$ $^{+0.2}_{-0.2}$ $^{+0.3}_{-0.3}$	$4.0 \times 10^{-4}$	4.4 $^{+7.4}_{-4.1}$
$B^+ \rightarrow \bar{K}_2^{*0} K^+$	PA	0.8 $^{+0.1}_{-0.1}$ $^{+0.2}_{-0.2}$ $^{+0.3}_{-0.2}$	...	1.2 $^{+5.2}_{-1.2}$
$B^0 \rightarrow a_2^- \pi^+$	T	98.9 $^{+35.1}_{-29.9}$ $^{+42.6}_{-32.0}$ $^{+5.8}_{-9.7}$	48.82	52 $^{+18}_{-18}$
$B^0 \rightarrow a_2^+ \pi^-$	T,PA	2.7 $^{+0.5}_{-0.3}$ $^{+0.8}_{-0.5}$ $^{+0.4}_{-0.3}$	...	2.1 $^{+4.3}_{-1.7}$
$B^0 \rightarrow a_2^0 \pi^0$	C	4.6 $^{+1.2}_{-1.0}$ $^{+1.6}_{-1.2}$ $^{+0.9}_{-0.7}$	0.003	2.4 $^{+4.2}_{-1.9}$
$B^0 \rightarrow a_2^0 \eta$	C,PA,P	0.6 $^{+0.1}_{-0.1}$ $^{+0.2}_{-0.1}$ $^{+0.1}_{-0.1}$	1.38	0.6 $^{+1.6}_{-0.5}$
$B^0 \rightarrow a_2^0 \eta'$	C,PA,P	1.8 $^{+0.6}_{-0.5}$ $^{+0.7}_{-0.6}$ $^{+0.4}_{-0.4}$	6.15	0.5 $^{+2.2}_{-0.4}$
$B^0 \rightarrow f_2 \pi^0$	C	2.8 $^{+0.7}_{-0.6}$ $^{+0.7}_{-0.6}$ $^{+0.6}_{-0.4}$	0.003	1.5 $^{+4.2}_{-1.4}$
$B^0 \rightarrow f_2' \pi^0$	P	0.2 $^{+0.0}_{-0.0}$ $^{+0.1}_{-0.1}$ $^{+0.0}_{-0.0}$	$4.0 \times 10^{-5}$	0.05 $^{+0.12}_{-0.05}$
$B^0 \rightarrow f_2 \eta$	C,P,PA	2.6 $^{+0.7}_{-0.5}$ $^{+0.8}_{-0.6}$ $^{+0.7}_{-0.6}$	1.52	1.7 $^{+2.3}_{-1.2}$
$B^0 \rightarrow f_2 \eta'$	C,PA,P	3.3 $^{+1.0}_{-0.8}$ $^{+1.1}_{-0.9}$ $^{+0.9}_{-0.9}$	6.8	1.3 $^{+2.2}_{-1.3}$
$B^0 \rightarrow f_2' \eta$	PA,P	0.08 $^{+0.03}_{-0.02}$ $^{+0.03}_{-0.03}$ $^{+0.01}_{-0.02}$	0.02	0.02 $^{+0.06}_{-0.03}$
$B^0 \rightarrow f_2' \eta'$	PA,P	0.09 $^{+0.00}_{-0.00}$ $^{+0.02}_{-0.02}$ $^{+0.02}_{-0.03}$	0.09	0.08 $^{+0.08}_{-0.05}$
$B^0 \rightarrow K_2^{*+} K^-$	PA	0.16 $^{+0.02}_{-0.03}$ $^{+0.03}_{-0.04}$ $^{+0.03}_{-0.03}$	...	0.3 $^{+0.7}_{-0.2}$
$B^0 \rightarrow K_2^{*-} K^+$	PA	0.9 $^{+0.1}_{-0.1}$ $^{+0.3}_{-0.1}$ $^{+0.2}_{-0.2}$	...	1.3 $^{+1.6}_{-1.0}$
$B^0 \rightarrow K_2^{*0} \bar{K}^0$	P,PA	1.5 $^{+0.3}_{-0.3}$ $^{+0.3}_{-0.3}$ $^{+0.5}_{-0.4}$	$3.0 \times 10^{-4}$	5.4 $^{+8.8}_{-4.9}$
$B^0 \rightarrow \bar{K}_2^{*0} K^0$	P,PA	0.8 $^{+0.1}_{-0.1}$ $^{+0.2}_{-0.1}$ $^{+0.3}_{-0.2}$	...	2.2 $^{+5.4}_{-2.2}$



# $B_{(s)} \rightarrow DT$ decays ( $10^{-7}$ ) (CKM suppressed by $V_{ub}$ )

Decay Modes	Class	This Work	SDV[14]	KLO[15]
$B^0 \rightarrow D^0 a_2^0$	C	$0.55^{+0.28}_{-0.20} +^{+0.15}_{-0.15} +^{+0.10}_{-0.08}$	0.34	...
$B^0 \rightarrow D^0 f_2$	C	$2.05^{+0.81}_{-0.71} +^{+0.34}_{-0.29} +^{+0.27}_{-0.24}$	0.36	...
$B^0 \rightarrow D^0 f_2'$	C	$0.038^{+0.015}_{-0.013} +^{+0.006}_{-0.006} +^{+0.005}_{-0.005}$	0.0071	...
$B^0 \rightarrow D^0 K_2^{*0}$	C	$41.8^{+17.4}_{-14.1} +^{+7.50}_{-7.04} +^{+5.75}_{-5.36}$	12	11
$B^0 \rightarrow D^+ a_2^-$	T	$15.2^{+7.82}_{-6.31} +^{+1.97}_{-2.62} +^{+1.96}_{-1.80}$	12	...
$B^0 \rightarrow D_s^+ a_2^-$	T	$521^{+249}_{-189} +^{+44}_{-60} +^{+72}_{-65}$	380	180
$B^0 \rightarrow D_s^+ K_2^{*-}$	E	$0.61^{+0.15}_{-0.14} +^{+0.12}_{-0.16} +^{+0.08}_{-0.07}$	...	...
$B^+ \rightarrow D^0 a_2^+$	C	$1.95^{+0.81}_{-0.70} +^{+0.41}_{-0.48} +^{+0.24}_{-0.24}$	0.73	...
$B^+ \rightarrow D^0 K_2^{*+}$	C	$37.3^{+14.3}_{-12.4} +^{+6.99}_{-8.32} +^{+5.10}_{-4.67}$	13	12
$B^+ \rightarrow D^+ a_2^0$	T	$9.40^{+4.59}_{-3.39} +^{+1.15}_{-1.62} +^{+1.20}_{-1.12}$	6.5	...
$B^+ \rightarrow D^+ f_2$	T	$12.9^{+6.31}_{-5.31} +^{+0.90}_{-1.42} +^{+1.60}_{-1.50}$	6.9	...
$B^+ \rightarrow D^+ f_2'$	T	$0.24^{+0.12}_{-0.09} +^{+0.02}_{-0.02} +^{+0.03}_{-0.03}$	1.4	...
$B^+ \rightarrow D^+ K_2^{*0}$	A	$5.27^{+1.78}_{-1.65} +^{+0.69}_{-0.66} +^{+0.72}_{-0.66}$	...	...
$B^+ \rightarrow D_s^+ a_2^0$	T	$280^{+134}_{-110} +^{+23}_{-33} +^{+38}_{-35}$	200	94
$B^+ \rightarrow D_s^+ f_2$	T	$299^{+149}_{-122} +^{+26}_{-33} +^{+41}_{-37}$	220	100
$B^+ \rightarrow D_s^+ f_2'$	T,A	$4.12^{+1.69}_{-1.98} +^{+1.62}_{-0.78} +^{+0.57}_{-0.51}$	4.3	1.2
$B^+ \rightarrow D_s^+ \bar{K}_2^{*0}$	A	$0.34^{+0.12}_{-0.10} +^{+0.06}_{-0.06} +^{+0.04}_{-0.04}$	...	...
$B_s \rightarrow D^0 a_2^0$	E	$3.87^{+1.35}_{-1.19} +^{+0.69}_{-0.95} +^{+0.53}_{-0.48}$	...	...
$B_s \rightarrow D^0 f_2$	E	$6.26^{+2.29}_{-1.99} +^{+1.00}_{-1.18} +^{+0.53}_{-0.48}$	0.15	...
$B_s \rightarrow D^0 f_2'$	C	$25.5^{+12.5}_{-11.4} +^{+4.00}_{-3.35} +^{+3.5}_{-3.2}$	10	...
$B_s \rightarrow D^0 \bar{K}_2^{*0}$	C	$1.42^{+0.69}_{-0.55} +^{+0.24}_{-0.19} +^{+0.18}_{-0.17}$	0.46	...
$B_s \rightarrow D^+ a_2^-$	E	$8.06^{+3.03}_{-2.68} +^{+1.43}_{-1.99} +^{+1.11}_{-1.00}$	...	...
$B_s \rightarrow D^+ K_2^{*-}$	T	$11.2^{+5.61}_{-4.51} +^{+0.60}_{-0.71} +^{+1.46}_{-1.33}$	8.3	...
$B_s \rightarrow D_s^+ K_2^{*-}$	T	$206^{+115}_{-95} +^{+16}_{-26} +^{+28}_{-25}$	260	...

ZT Zou,  
X Yu,  
CDL,  
arXiv:12  
05.2971  
**Poster  
session**



# $B_{(s)} \rightarrow D^* T$ decays ( $10^{-7}$ ) (CKM suppressed by $V_{ub}$ )

Decay Modes	Class	Branching Ratio			$R_T$
		This Work	SDV[14]	KLO[15]	
$B^0 \rightarrow D^{*0} a_2^0$	C	$1.34^{+0.64}_{-0.53} +^{0.25}_{-0.19} +^{0.17}_{-0.15}$	0.50	...	$47^{+3.7}_{-4.5} +^{1.4}_{-1.6}$
$B^0 \rightarrow D^{*0} f_2$	C	$2.70^{+1.22}_{-1.02} +^{0.43}_{-0.30} +^{0.36}_{-0.33}$	0.53	...	$26^{+3.7}_{-4.0} +^{1.3}_{-1.1}$
$B^0 \rightarrow D^{*0} f_2'$	C	$0.052^{+0.023}_{-0.02} +^{0.008}_{-0.005} +^{0.007}_{-0.006}$	0.01	...	$26^{+3.7}_{-4.0} +^{1.3}_{-1.1}$
$B^0 \rightarrow D^{*0} K_2^{*0}$	C	$60.5^{+25.3}_{-21.3} +^{10.6}_{-9.15} +^{8.30}_{-7.56}$	19	18	$22^{+2.7}_{-3.2} +^{1.0}_{-0.5}$
$B^0 \rightarrow D^{*+} a_2^-$	T	$21.6^{+10.6}_{-8.60} +^{2.48}_{-3.10} +^{2.90}_{-2.50}$	18	...	$28^{+1.6}_{-1.5} +^{1.7}_{-1.6}$
$B^0 \rightarrow D_s^{*+} a_2^-$	T	$688^{+321}_{-267} +^{55}_{-79} +^{94}_{-86}$	367	291	$26^{+1.2}_{-0.9} +^{0.4}_{-0.7}$
$B^0 \rightarrow D_s^{*+} K_2^{*-}$	E	$0.57^{+0.13}_{-0.13} +^{0.12}_{-0.11} +^{0.07}_{-0.07}$	...	...	$12^{+1.6}_{-1.6} +^{3.2}_{-2.5}$
$B^+ \rightarrow D^{*0} a_2^+$	C	$4.46^{+2.01}_{-1.65} +^{0.73}_{-0.61} +^{0.58}_{-0.53}$	1.1	...	$41^{+3.8}_{-3.6} +^{2.3}_{-1.6}$
$B^+ \rightarrow D^{*0} K_2^{*+}$	C	$72.1^{+28.3}_{-23.7} +^{11.8}_{-9.38} +^{10.}_{-9.00}$	21	19	$35^{+4.0}_{-3.6} +^{0.9}_{-1.0}$
$B^+ \rightarrow D^{*+} a_2^0$	T	$14.0^{+6.51}_{-5.45} +^{1.03}_{-1.34} +^{1.80}_{-1.65}$	9.6	...	$25^{+1.5}_{-1.0} +^{0.3}_{-0.1}$
$B^+ \rightarrow D^{*+} f_2$	T	$15.1^{+8.83}_{-6.27} +^{1.42}_{-2.15} +^{2.01}_{-1.90}$	10	...	$25^{+1.3}_{-1.0} +^{1.8}_{-1.8}$
$B^+ \rightarrow D^{*+} f_2'$	T	$0.29^{+0.15}_{-0.11} +^{0.03}_{-0.04} +^{0.02}_{-0.02}$	0.21	...	$25^{+1.3}_{-1.0} +^{1.8}_{-1.8}$
$B^+ \rightarrow D^{*+} K_2^{*0}$	A	$18.2^{+4.77}_{-5.15} +^{0.21}_{-2.15} +^{2.00}_{-2.70}$	...	...	$82^{+2.1}_{-2.9} +^{3.8}_{-2.7}$
$B^+ \rightarrow D_s^{*+} a_2^0$	T	$330^{+155}_{-127} +^{27}_{-37} +^{45}_{-42}$	196	155	$26^{+1.2}_{-0.8} +^{0.4}_{-0.7}$
$B^+ \rightarrow D_s^{*+} f_2$	T	$385^{+203}_{-156} +^{31}_{-44} +^{52}_{-48}$	207	167	$25^{+1.1}_{-0.8} +^{0.7}_{-0.8}$
$B^+ \rightarrow D_s^{*+} f_2'$	A	$21.6^{+6.77}_{-6.03} +^{1.00}_{-2.32} +^{3.00}_{-2.70}$	4.0	2.0	$83^{+5.2}_{-5.3} +^{1.9}_{-1.9}$
$B^+ \rightarrow D_s^{*+} \bar{K}_2^{*0}$	A	$1.25^{+0.36}_{-0.34} +^{0.06}_{-0.16} +^{0.16}_{-0.15}$	...	...	$81^{+1.6}_{-1.8} +^{3.7}_{-3.3}$
$B_s \rightarrow D^{*0} a_2^0$	E	$2.68^{+0.91}_{-0.81} +^{0.70}_{-0.63} +^{0.37}_{-0.33}$	...	...	$21^{+2.6}_{-3.0} +^{4.8}_{-3.9}$
$B_s \rightarrow D^{*0} f_2$	E	$5.06^{+1.93}_{-1.65} +^{0.84}_{-0.98} +^{0.71}_{-0.62}$	0.24	...	$14^{+2.0}_{-2.1} +^{2.0}_{-1.6}$
$B_s \rightarrow D^{*0} f_2'$	C	$36.2^{+17.3}_{-14.4} +^{5.62}_{-5.51} +^{4.90}_{-4.60}$	16	...	$17^{+2.9}_{-3.1} +^{1.6}_{-1.1}$
$B_s \rightarrow D^{*0} \bar{K}_2^{*0}$	C	$2.06^{+1.03}_{-0.83} +^{0.33}_{-0.31} +^{0.25}_{-0.24}$	0.7	...	$21^{+3.0}_{-3.6} +^{0.8}_{-0.6}$
$B_s \rightarrow D^{*+} a_2^-$	E	$5.36^{+1.82}_{-1.59} +^{1.41}_{-1.27} +^{0.74}_{-0.66}$	...	...	$21^{+2.6}_{-3.0} +^{4.8}_{-3.9}$
$B_s \rightarrow D^{*+} K_2^{*-}$	T	$14.8^{+7.42}_{-5.93} +^{0.90}_{-0.85} +^{1.94}_{-1.77}$	12	...	$26^{+1.2}_{-1.0} +^{0.1}_{-0.2}$
$B_s \rightarrow D_s^{*+} K_2^{*-}$	T	$332^{+172}_{-138} +^{20}_{-31} +^{46}_{-41}$	261	...	$34^{+1.9}_{-1.6} +^{1.5}_{-1.0}$





# $B_{(s)} \rightarrow \underline{D}T$ decays ( $10^{-5}$ ) (CKM enhanced by $V_{cb}$ )

Decay Modes	Class	This Work	SDV[14]	KLO[15]
$B^0 \rightarrow \bar{D}^0 a_2^0$	C	$12.3^{+3.21}_{-2.99} \quad +3.08 \quad +0.67$	8.2	4.8
$B^0 \rightarrow \bar{D}^0 f_2$	C	$9.46^{+2.52}_{-2.29} \quad +3.64 \quad +0.51$	8.8	5.3
$B^0 \rightarrow \bar{D}^0 f_2'$	C	$0.18^{+0.04}_{-0.05} \quad +0.06 \quad -0.09$	0.17	0.062
$B^0 \rightarrow \bar{D}^0 K_2^{*0}$	C	$1.45^{+0.41}_{-0.38} \quad +0.29 \quad -0.09$	0.81	0.68
$B^0 \rightarrow D^- a_2^+$	T	$39.8^{+15.3}_{-12.6} \quad +12.5 \quad +2.15$	...	...
$B^0 \rightarrow D^- K_2^{*+}$	T	$1.16^{+0.50}_{-0.40} \quad +0.52 \quad +0.06$	...	...
$B^0 \rightarrow D_s^- K_2^{*+}$	E	$6.06^{+1.73}_{-1.65} \quad +0.43 \quad +0.32$	...	...
$B^+ \rightarrow \bar{D}^0 a_2^+$	T,C	$41.5^{+16.5}_{-12.6} \quad +13.0 \quad +2.24$	18	10
$B^+ \rightarrow \bar{D}^0 K_2^{*+}$	T,C	$3.33^{+1.33}_{-1.02} \quad +0.87 \quad +0.20$	0.87	0.73
$B_s \rightarrow \bar{D}^0 a_2^0$	E	$0.11^{+0.04}_{-0.04} \quad +0.01 \quad +0.01$	...	...
$B_s \rightarrow \bar{D}^0 f_2$	E	$0.14^{+0.04}_{-0.05} \quad +0.01 \quad +0.01$	0.0099	...
$B_s \rightarrow \bar{D}^0 f_2'$	C	$1.36^{+0.53}_{-0.43} \quad +0.22 \quad +0.08$	0.67	...
$B_s \rightarrow \bar{D}^0 \bar{K}_2^{*0}$	C	$20.3^{+7.70}_{-6.41} \quad +3.98 \quad +1.00$	11	...
$B_s \rightarrow D^- a_2^+$	E	$0.23^{+0.08}_{-0.08} \quad +0.02 \quad +0.01$	...	...
$B_s \rightarrow D_s^- a_2^+$	T	$11.3^{+5.53}_{-4.43} \quad +6.11 \quad +0.61$	...	...
$B_s \rightarrow D_s^- K_2^{*+}$	T,E	$1.97^{+0.81}_{-0.69} \quad +0.72 \quad +0.12$	...	...



# $B_{(s)} \rightarrow \underline{D}^* T$ decays ( $10^{-5}$ ) (CKM enhanced by $V_{cb}$ )

Decay Modes	Class	Branching Ratio			$R_T$
		This Work	SDV[14]	KLO[15]	
$B^0 \rightarrow \bar{D}^{*0} a_2^0$	C	$39.3^{+13.6+2.05+2.15}_{-11.1-0.50-1.34}$	12	7.8	$73^{+4.6+9.0}_{-4.3-8.1}$
$B^0 \rightarrow \bar{D}^{*0} f_2$	C	$38.2^{+13.9+1.97+2.10}_{-11.6-1.22-1.30}$	13	8.4	$70^{+5.9+9.4}_{-5.6-6.3}$
$B^0 \rightarrow \bar{D}^{*0} f_2'$	C	$0.72^{+0.26+0.02+0.04}_{-0.22-0.03-0.03}$	0.26	0.11	$70^{+5.9+9.4}_{-5.6-6.3}$
$B^0 \rightarrow \bar{D}^{*0} K_2^{*0}$	C	$5.32^{+1.69+0.79+0.32}_{-1.42-0.66-0.22}$	1.3	1.1	$71^{+1.8+8.6}_{-1.6-8.8}$
$B^0 \rightarrow D^{*-} a_2^+$	T	$29.6^{+11.5+9.58+1.62}_{-9.86-10.1-1.01}$	...	...	$3^{+0.2+0.4}_{-0.2-0.4}$
$B^0 \rightarrow D^{*-} K_2^{*+}$	T	$1.15^{+0.49+0.46+0.07}_{-0.37-0.43-0.04}$	...	...	$7^{+0.4+0.3}_{-0.2-0.3}$
$B^0 \rightarrow D_s^{*-} K_2^{*+}$	E	$4.55^{+1.32+0.48+0.25}_{-1.14-0.51-0.16}$	...	...	$22^{+3.0+7.6}_{-3.4-6.7}$
$B^+ \rightarrow \bar{D}^{*0} a_2^+$	T,C	$80.6^{+29.0+3.67+4.41}_{-24.1-3.12-2.75}$	26	17	$58^{+3.7+13.8}_{-3.2-10.2}$
$B^+ \rightarrow \bar{D}^{*0} K_2^{*+}$	T,C	$6.81^{+2.36+0.34+0.42}_{-1.93-0.30-0.27}$	1.4	1.2	$57^{+1.4+14.1}_{-1.3-11.6}$
$B_s \rightarrow \bar{D}^{*0} a_2^0$	E	$0.09^{+0.03+0.01+0.01}_{-0.03-0.01-0.01}$	...	...	$26^{+4.0+6.5}_{-4.4-6.8}$
$B_s \rightarrow \bar{D}^{*0} f_2$	E	$0.21^{+0.08+0.01+0.01}_{-0.07-0.02-0.01}$	0.016	...	$11^{+0.6+2.2}_{-0.6-1.0}$
$B_s \rightarrow \bar{D}^{*0} f_2'$	C	$5.02^{+2.06+0.39+0.30}_{-1.70-0.50-0.20}$	1.1	...	$71^{+1.1+7.5}_{-1.0-7.2}$
$B_s \rightarrow \bar{D}^{*0} \bar{K}^{*0}$	C	$70.1^{+28.8+4.10+3.83}_{-23.9-5.82-2.40}$	17	...	$68^{+1.7+8.5}_{-1.3-7.9}$
$B_s \rightarrow D^{*-} a_2^+$	E	$0.18^{+0.06+0.01+0.01}_{-0.06-0.03-0.01}$	...	...	$26^{+4.0+6.5}_{-4.4-6.8}$
$B_s \rightarrow D_s^{*-} a_2^+$	T	$11.5^{+5.59+5.79+0.63}_{-4.44-4.70-0.40}$	...	...	$6^{+0.1+0.2}_{-0.1-0.3}$
$B_s \rightarrow D_s^{*-} K_2^{*+}$	T,E	$1.73^{+0.75+0.65+0.11}_{-0.60-0.58-0.06}$	...	...	$11^{+0.9+2.3}_{-0.9-2.4}$





# Summary (1)

- 
- The theoretical calculation for hadronic B decays is a tough work
  - The theoretical calculation has been reached the **NLO or NNLO** order at  $\alpha_s$  expansion
  - More and more **power correction** have been taken into account
  - **A lot of work** is still missing to be done



## Summary (2)

- **Color favored tree (T)** – We do know what we expected to know
- **color suppressed Tree (C)** – We do not exactly know what we expected to know
- **Annihilations (A,E,PA,SP)** – We do know now what we did not expect to know
- **Penguins (P, P<sub>EW</sub>)** – We hope we really understand so many kinds of



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谢谢!

*Thanks!*



# Nambu-Goldstone boson

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(H.-n. Li, S. Mishima, e-Print: arXiv:0901.1272)

- Pion as a qq bound state and as a **massless Nambu-Goldstone boson**?
- **Massless boson** => huge spacetime=> large separation between qq => high mass under confinement => contradiction!
- Reconciliation: **leading qq state like that in rho, higher Fockstate gives soft cloud**