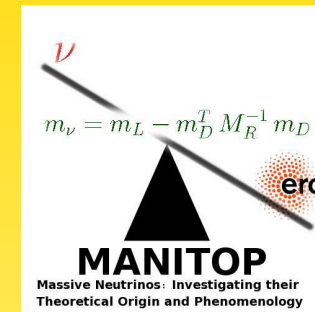


Status of the PMNS Matrix



WERNER RODEJOHANN
(MPIK, HEIDELBERG)

24/05/12



Flavor Physics & CP Violation

FPCCP 2012 May 21-25, 2012, University of Science and Technology of China, Hefei

The aim of the conference is to discuss experimental and theoretical developments related to the physics of heavy flavors and CP violation. Among the topics included will be CP violation, rare decays, flavor beyond the standard model and at the energy frontier, spectroscopy, CDM elements, neutrino physics, and the potential for studies of heavy-flavor decays to help unravel any new physics seen directly at LHC.

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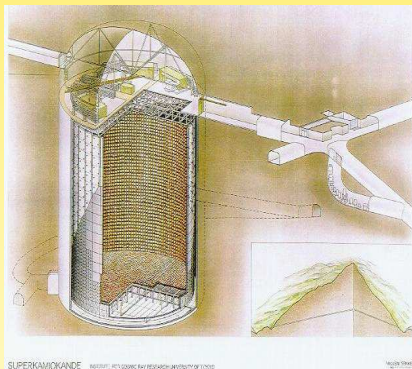
Status of the PMNS Matrix

- What are the parameters of U ?
 - θ_{12}
 - θ_{23}
 - θ_{13}
- Impact on θ_{13} on Phenomenology and Models

$$U = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13} \end{pmatrix} =$$

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{atmospheric and LBL accelerator}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{SBL reactor}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{solar and LBL reactor}}$$

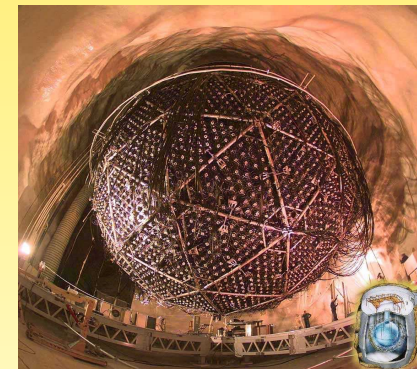
atmospheric and
LBL accelerator



SBL reactor



solar and
LBL reactor



$$U = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13} \end{pmatrix} =$$

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{atmospheric and LBL accelerator}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{SBL reactor}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{solar and LBL reactor}}$$

atmospheric and
LBL accelerator

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \\ 0 & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

$$(\sin^2 \theta_{23} = \frac{1}{2})$$

$$\Delta m_{\text{A}}^2$$

SBL reactor

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$(\sin^2 \theta_{13} = 0)$$

$$\Delta m_{\text{A}}^2$$

solar and

LBL reactor

$$\begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$(\sin^2 \theta_{12} = \frac{1}{3})$$

$$\Delta m_{\odot}^2$$

$$U = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13} \end{pmatrix} =$$

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{atmospheric and LBL accelerator}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{SBL reactor}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{solar and LBL reactor}}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \\ 0 & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

$$(\sin^2 \theta_{23} = \frac{1}{2})$$

$$\Delta m_{\text{A}}^2$$

$$\begin{pmatrix} 1 & 0 & \epsilon e^{-i\delta} \\ 0 & 1 & 0 \\ -\epsilon e^{i\delta} & 0 & 1 \end{pmatrix}$$

$$(\sin^2 \theta_{13} = \epsilon^2)$$

$$\Delta m_{\text{A}}^2$$

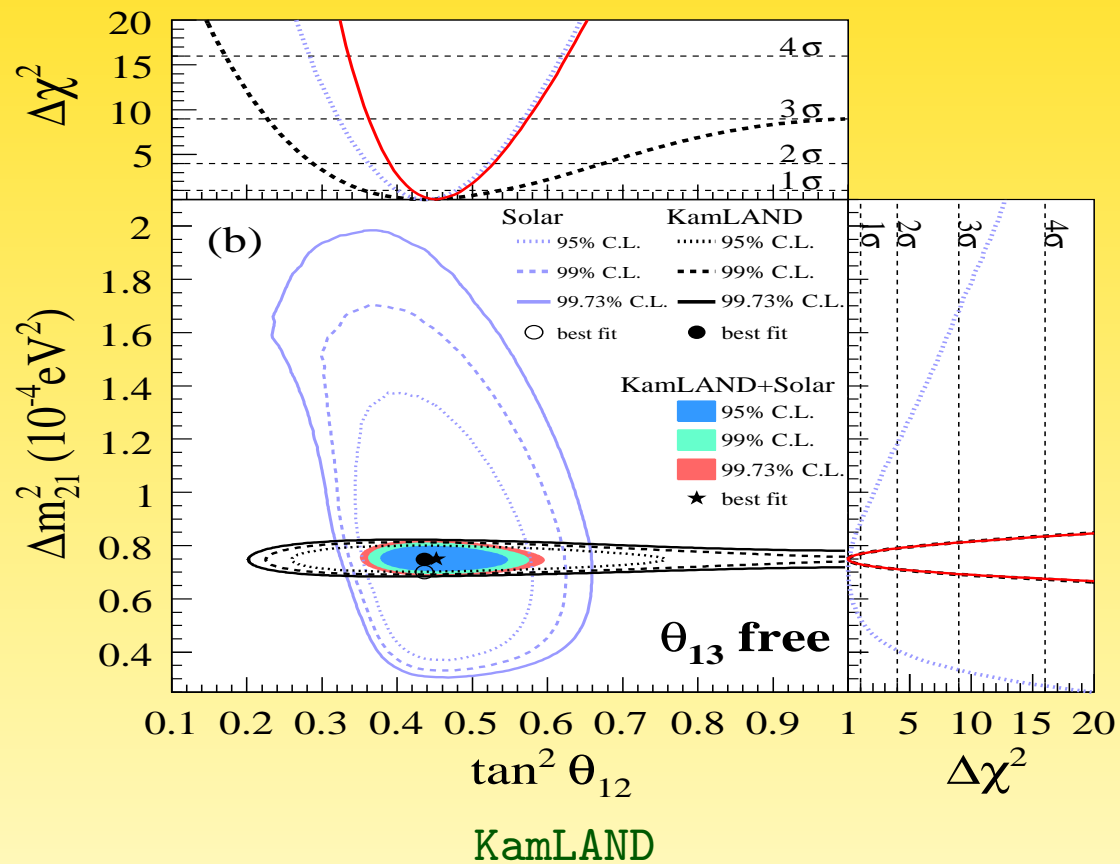
$$\begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$(\sin^2 \theta_{12} = \frac{1}{3})$$

$$\Delta m_{\odot}^2$$

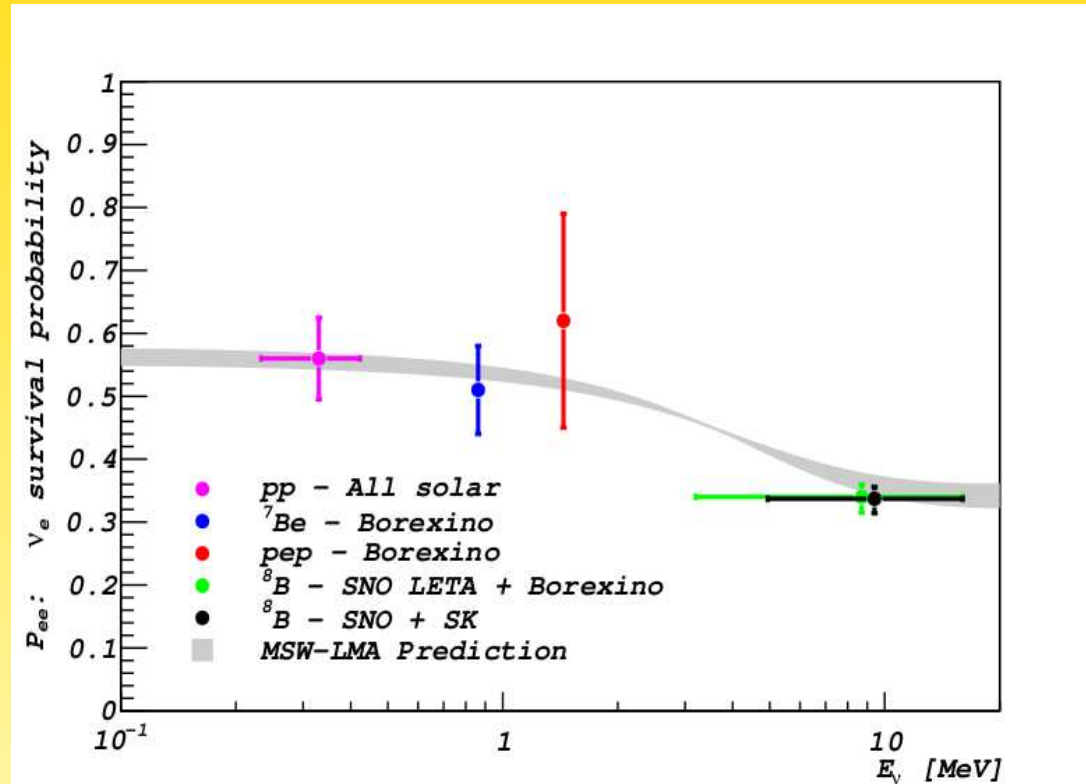
θ_{12} : status

P_{ee} tested in solar and long baseline reactor (KamLAND) experiments



θ_{12} : status

no sign of new physics so far \leftrightarrow consistent with LMA



transition from $1 - \frac{1}{2} \sin^2 2\theta_{12}$ to $\sin^2 \theta_{12}$

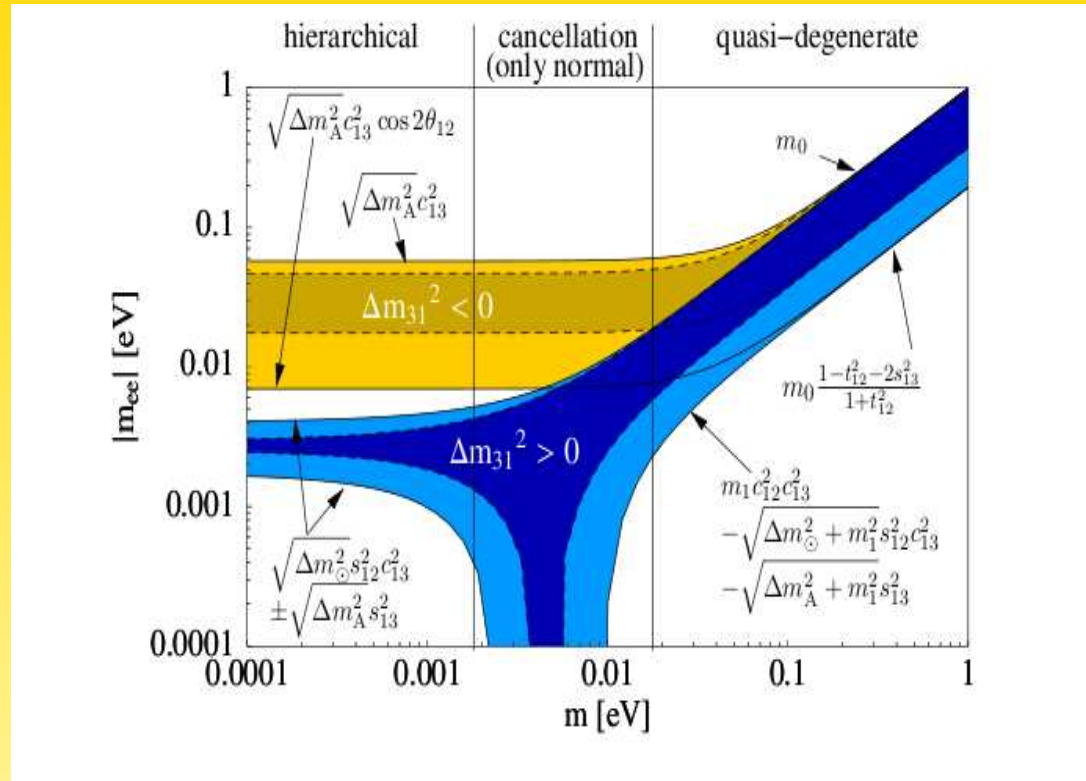
Borexino

θ_{12} : status

$\sin^2 \theta_{12} = 0.320^{+0.015}_{-0.017}$ (0.05)	Tortola, Valle, Vanegas
$\sin^2 \theta_{12} = 0.312^{+0.019}_{-0.018}$ (0.063)	Fogli, Lisi <i>et al.</i>
$\sin^2 \theta_{12} = 0.321^{+0.016}_{-0.016}$ (0.062)	Gonzalez-Garcia <i>et al.</i>

- remarkably stable
- 3σ -precision about 15 %
- tri-bimaximal value: $\sin^2 \theta_{12} = \frac{1}{3}$ marginally within 1σ
- hexagonal value: $\sin^2 \theta_{12} = \frac{1}{4}$ a bit outside 3σ
- golden ratio values: $\cot \theta_{12} = \varphi$ and $\cos \theta_{12} = \varphi/2$ marginally within 3σ
- QLC value: $\sin^2(\pi/4 - \theta_C)$ marginally within 3σ
- no improvement in sight

θ_{12} : a phenomenological aspect



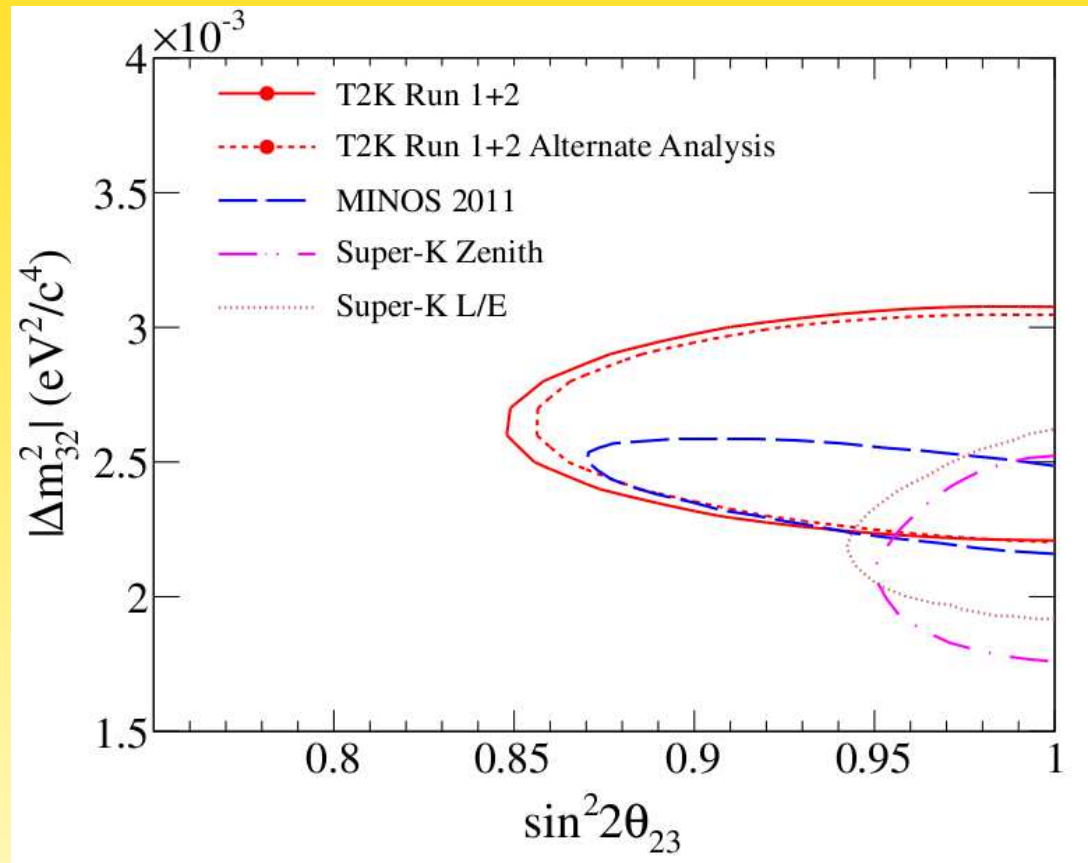
Ruling out the inverted hierarchy: go below

$$|m_{ee}|_{\min}^{\text{IH}} = (1 - |U_{e3}|^2) \sqrt{|\Delta m_A^2|} (1 - 2 \sin^2 \theta_{12})$$

θ_{12} -uncertainty introduces factor 3 uncertainty in lifetime...

θ_{23} : status

$P_{\mu\mu}$ or $P_{\mu\tau}$ tested in atmospheric and long baseline accelerator (K2K, T2K, MINOS) experiments



T2K

θ_{23} : status

Signs of new physics:

- different Δm^2 for neutrinos and anti-neutrinos. . .
- faster than light OPERA neutrinos. . .

. . .all went away. . .

θ_{23} : status

$$\sin^2 \theta_{23} = 0.49^{+0.08}_{-0.05} \quad (0.15) \quad (0.10)$$

Tortola, Valle, Vanegas

$$\sin^2 \theta_{23} = 0.466^{+0.073}_{-0.058} \quad (0.178) \quad (0.135)$$

Fogli, Lisi *et al.*

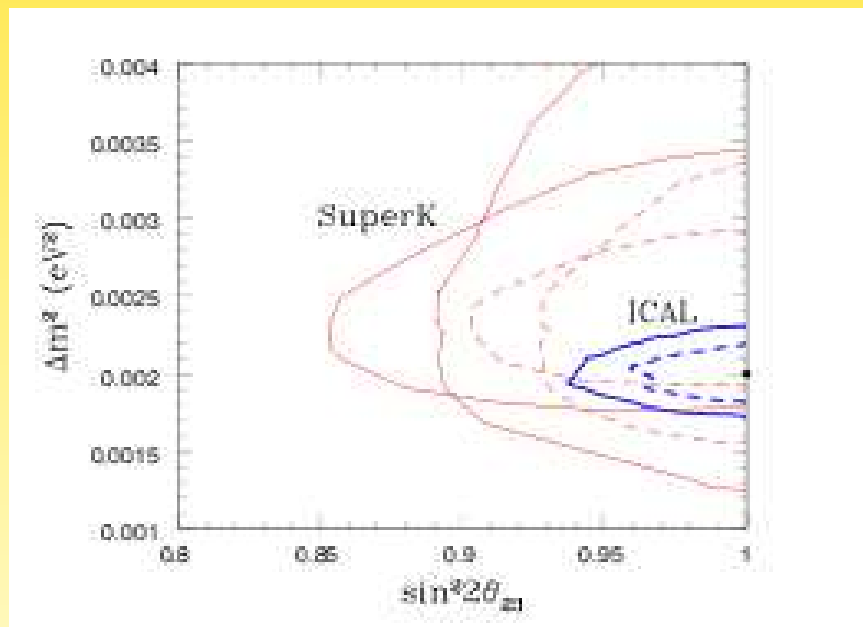
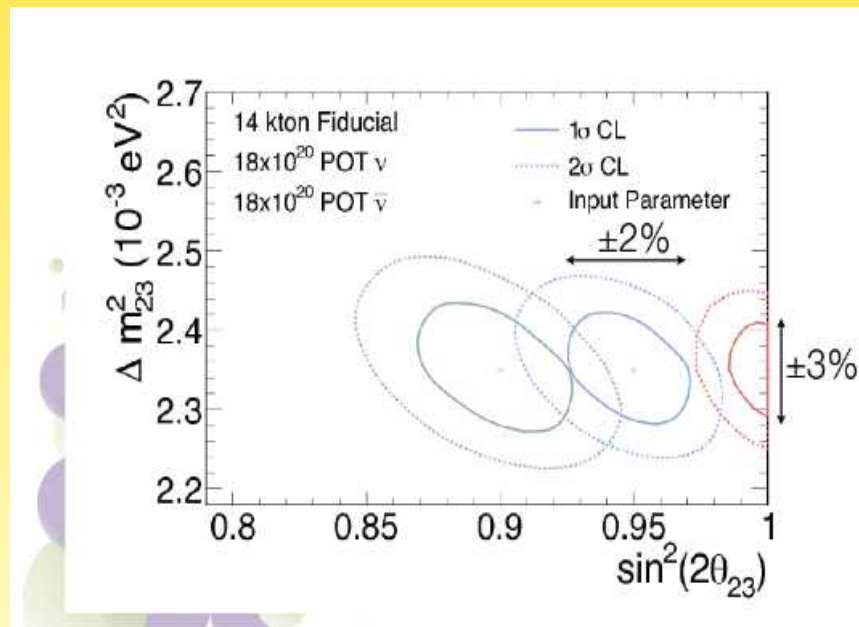
$$\sin^2 \theta_{23} = 0.462^{+0.08}_{-0.05} \quad (0.18) \quad (0.13)$$

Gonzalez-Garcia *et al.*

- somewhat stable
- 3σ -precision about 30 %
- at 1σ : some sensitivity to mass hierarchy (fragile)
- at 1σ : some sensitivity to octant (fragile)
- maximal mixing: $\sin^2 \theta_{23} = \frac{1}{2}$ within 1σ
- improvement in sight

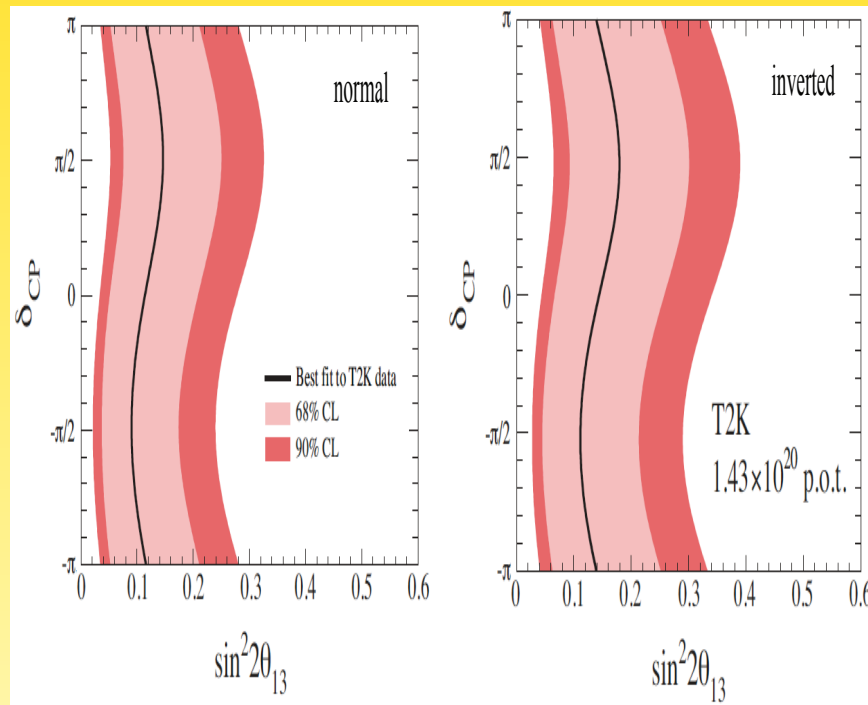
θ_{23} : perspectives

- 2-flavor analysis in vacuum goes with $\sin^2 2\theta_{23}$, hence octant and hierarchy determination needs necessarily 3-flavor and long-baseline analysis
- June, Neutrino2012 in Kyoto: updates of MINOS, T2K, IceCube
- future projects: **No ν A**, LBNE, LBNO, T2HK, **INO**,...

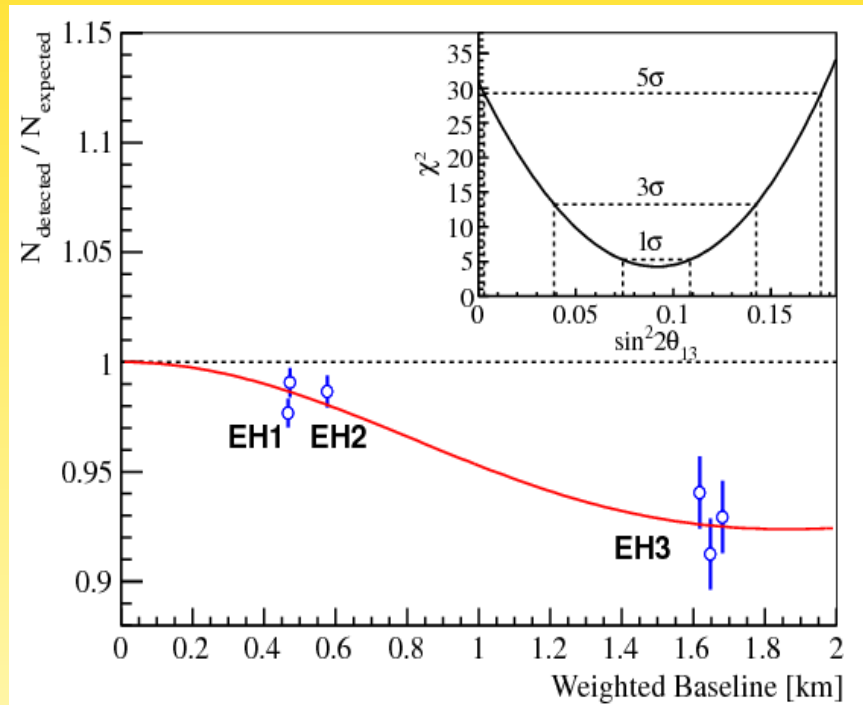


θ_{13} : status

P_{ee} or $P_{\mu e}$ tested in long baseline accelerator (T2K, MINOS) and reactor (Double Chooz, Reno, Daya Bay) experiments



T2K



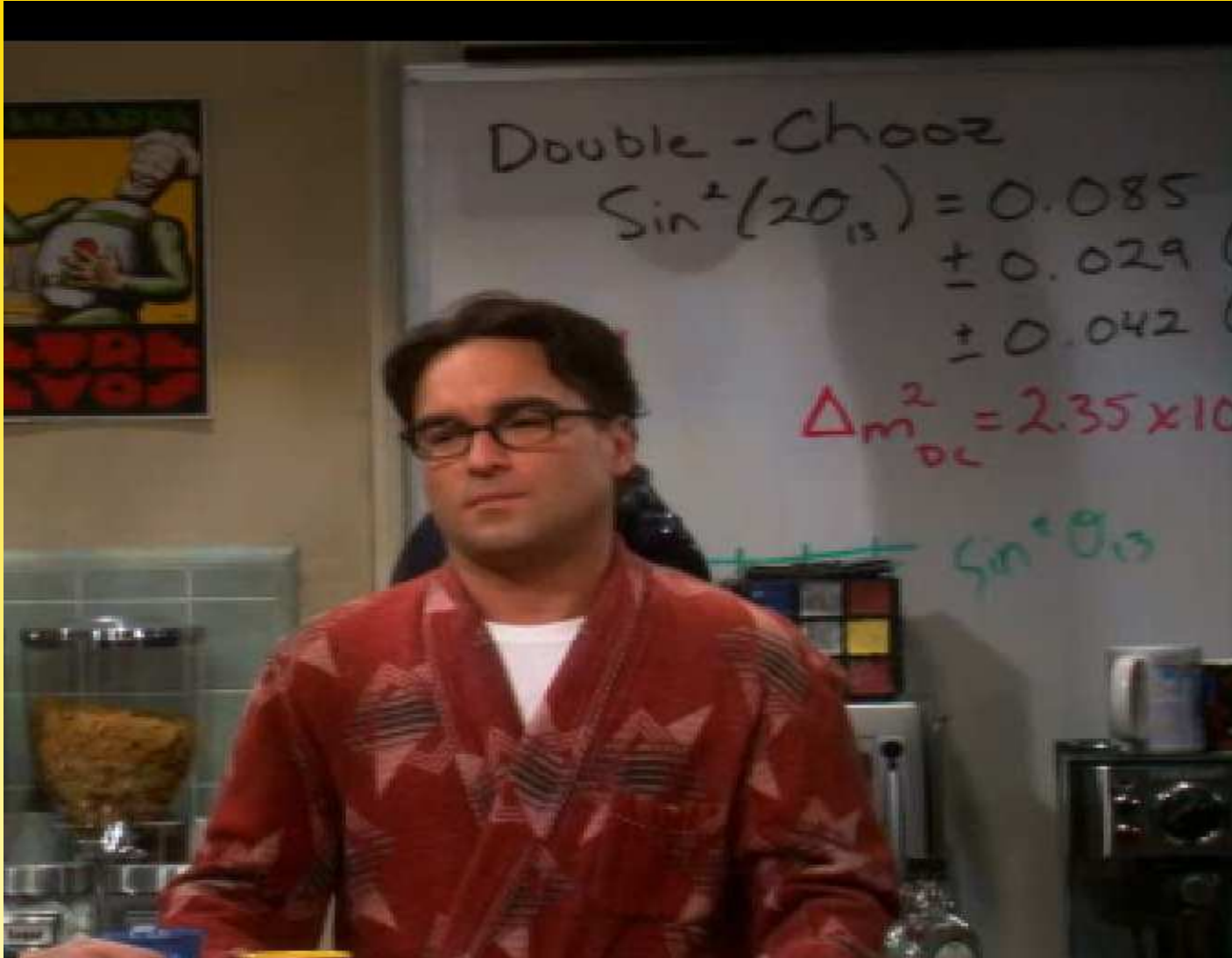
Daya Bay

θ_{13} : status

Double Chooz: $\sin^2 2\theta_{13} = 0.086 \pm 0.051 \neq 0$ at 1.9σ

Daya Bay: $\sin^2 2\theta_{13} = 0.092 \pm 0.017 \neq 0$ at 5.2σ

RENO: $\sin^2 2\theta_{13} = 0.113 \pm 0.023 \neq 0$ at 4.9σ



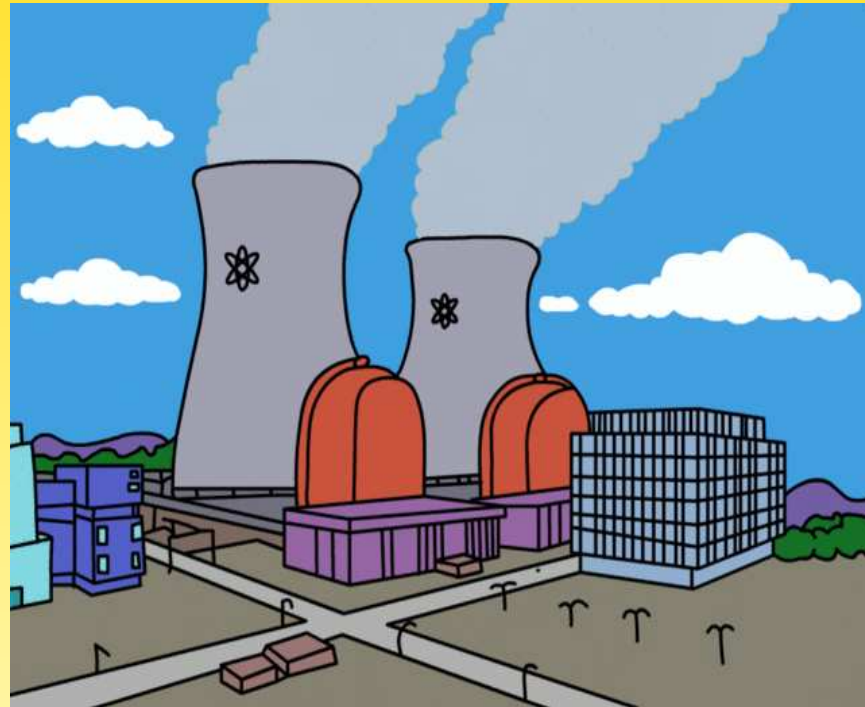
at least, Double Chooz are the only ones who made it to Big Bang Theory...

Year of the dragon



should rather be...

...Year of the reactor!



θ_{13} : status

Double Chooz: $\sin^2 2\theta_{13} = 0.086 \pm 0.051 \neq 0$ at 1.9σ

Daya Bay: $\sin^2 2\theta_{13} = 0.092 \pm 0.017 \neq 0$ at 5.2σ

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θ_{13} : status

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RENO: $\sin^2 2\theta_{13} = 0.113 \pm 0.023 \neq 0$ at 4.9σ

statistics experts, please close your eyes:

θ_{13} : status

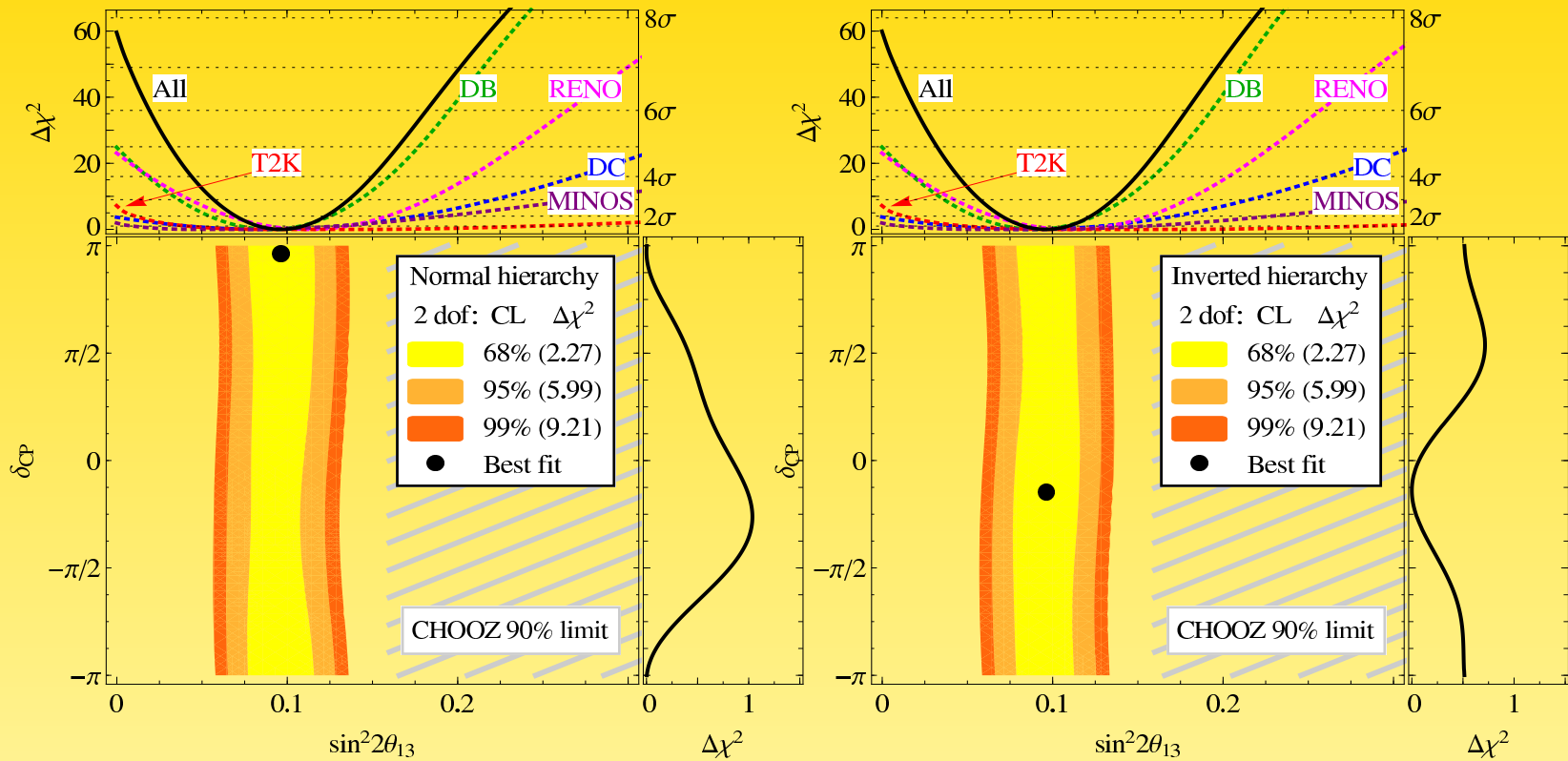
Double Chooz: $\sin^2 2\theta_{13} = 0.086 \pm 0.051 \neq 0$ at 1.9σ

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RENO: $\sin^2 2\theta_{13} = 0.113 \pm 0.023 \neq 0$ at 4.9σ

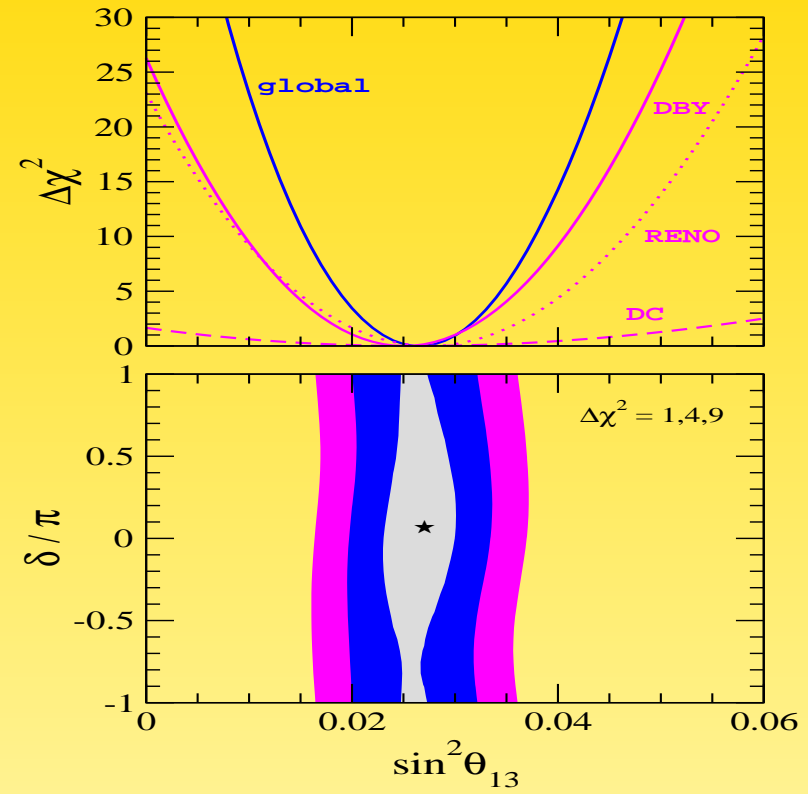
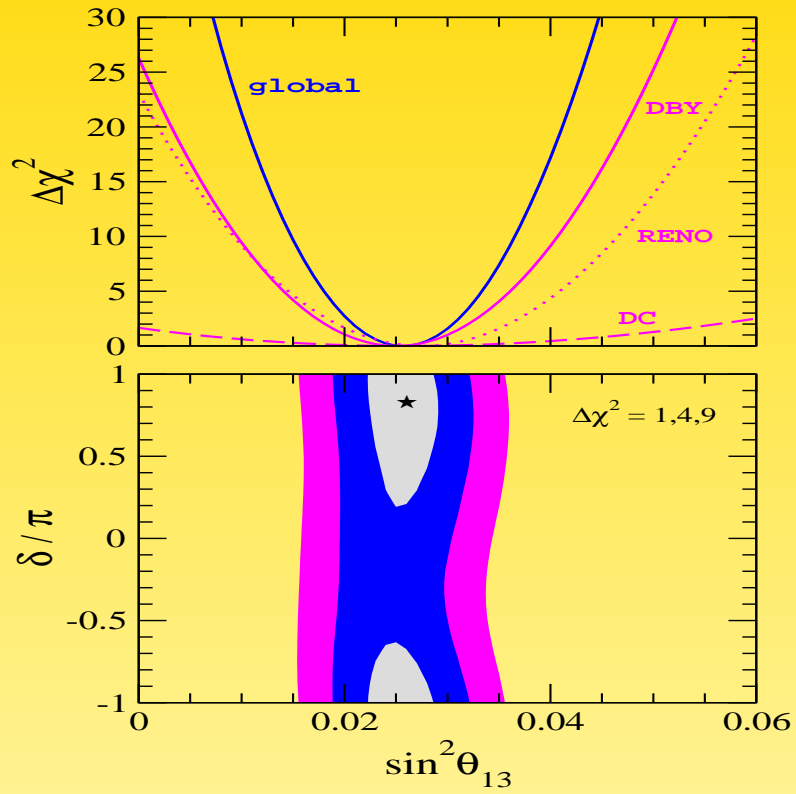
statistics experts, please close your eyes:

$$\sqrt{1.9^2 + 5.2^2 + 4.9^2} \simeq 7.4$$



non-zero θ_{13} ruled out at 7.7σ

Machado, Minakata, Nunokawa, Zukanovich Funchal



non-zero θ_{13} ruled out at 8σ

Tortola, Valle, Vanegas

θ_{13} is large!

with SBL data:

$$\begin{aligned}\sin^2 \theta_{13} &= 0.022^{+0.0033}_{-0.0030} \\ \sin^2 2\theta_{13} &= 0.086 \pm 0.012 \\ \theta_{13} &= (8.5^{+0.62}_{-0.61})^\circ\end{aligned}$$

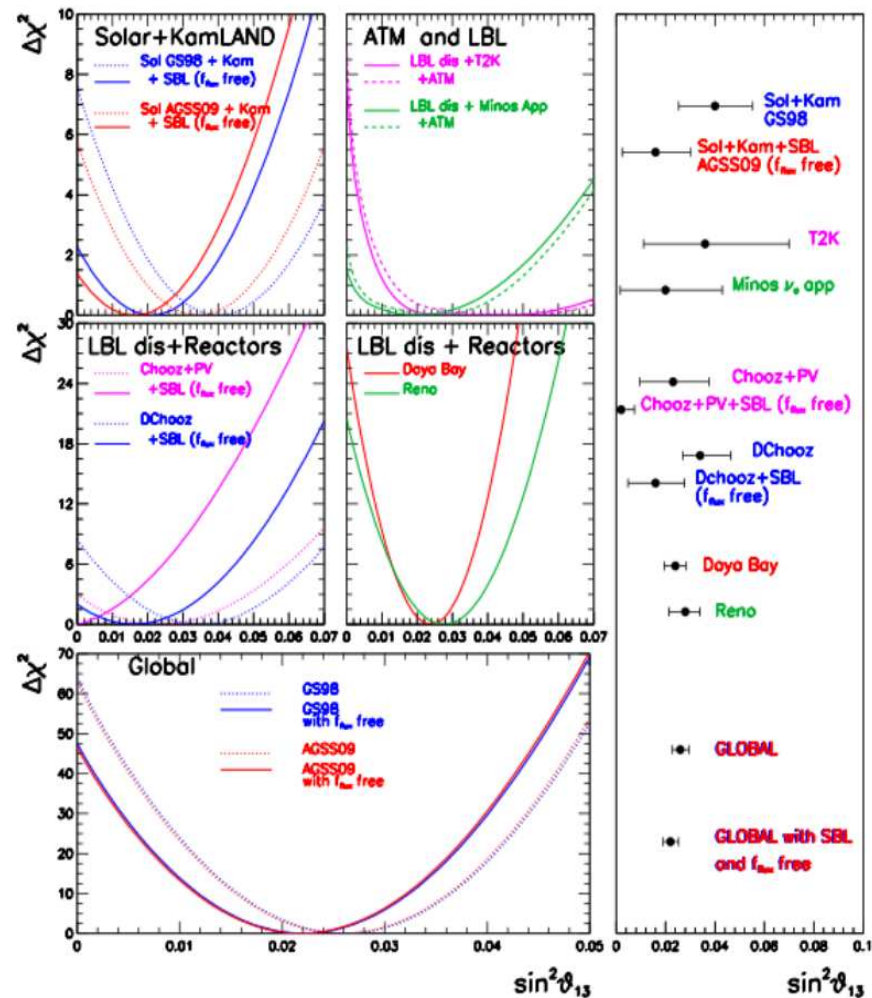
6.9 σ significance

without SBL data using
2011 flux pred.:

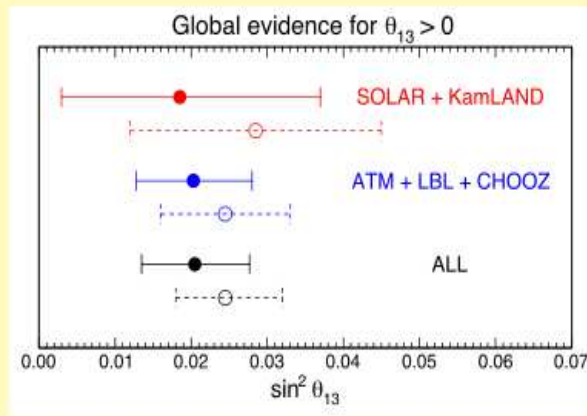
$$\begin{aligned}\sin^2 \theta_{13} &= 0.026^{+0.0034}_{-0.0032} \\ \sin^2 2\theta_{13} &= 0.101^{+0.013}_{-0.012} \\ \theta_{13} &= (9.3 \pm 0.59)^\circ\end{aligned}$$

8.0 σ significance

Gonzalez-Garcia, Maltoni,
Salvado, TS, in prep.



Comparing 2011 evidence for $\sin^2\theta_{13} > 0$...



$$\sin^2\theta_{13} = 0.021 \pm 0.007 \quad (\text{old reactor fluxes})$$

$$\sin^2\theta_{13} = 0.025 \pm 0.007 \quad (\text{new reactor fluxes})$$

... with new (2012) SBL reactor data:

Double Chooz (far detector): $\sin^2\theta_{13} = 0.022 \pm 0.013$

Daya Bay (near + far detectors): $\sin^2\theta_{13} = 0.024 \pm 0.004$

RENO (near + far detectors): $\sin^2\theta_{13} = 0.029 \pm 0.006$

we find a **spectacular** agreement!


θ_{13} : status

$$\sin^2 2\theta_{13} = 0.096 \pm 0.040 \quad \text{Minakata *et al.*}$$

$$\sin^2 2\theta_{13} = 0.094 \pm 0.046 \quad \text{Fogli, Lisi *et al.*}$$

$$\sin^2 2\theta_{13} = 0.086 \pm 0.036 \quad \text{Gonzalez-Garcia *et al.*}$$

$$\sin^2 2\theta_{13} = 0.103 \pm 0.060 \quad \text{Tortola *et al.*}$$

- 3σ -precision about 40 %
- mean is $\theta_{13} = 8.8^\circ$ 
- astonishingly close to $\sin \theta_C / \sqrt{2}$!
- astonishingly close to $\sqrt{m_s / m_b}$?
- $\sin^2 \theta_{13} = 0$ ruled out by $\gtrsim 7\sigma$
- precision dominated by reactors

Remarks

- open questions:
 - θ_{23} : non-maximal? octant?
 - mass ordering: normal? inverted?
 - CP phase(s): δ ? Majorana phases?
 - neutrino mass scale?
 - neutrino nature?
- hierarchy seems possible within 1 – 2 decades
- full CP coverage not without new facilities (not necessarily easier with large θ_{13})
- next generation experiments presumably not able to do it individually
- far future: β -beams? neutrino factory?

θ_{13} : phenomenological aspects

minimal flavor violation in the lepton sector

(Cirigliano, Grinstein, Isidori, Wise)

$$\text{BR}(\mu \rightarrow e\gamma) \propto |(m_\nu m_\nu^\dagger)_{e\mu}|^2$$

this quantity can be zero if and only if CP conservation and

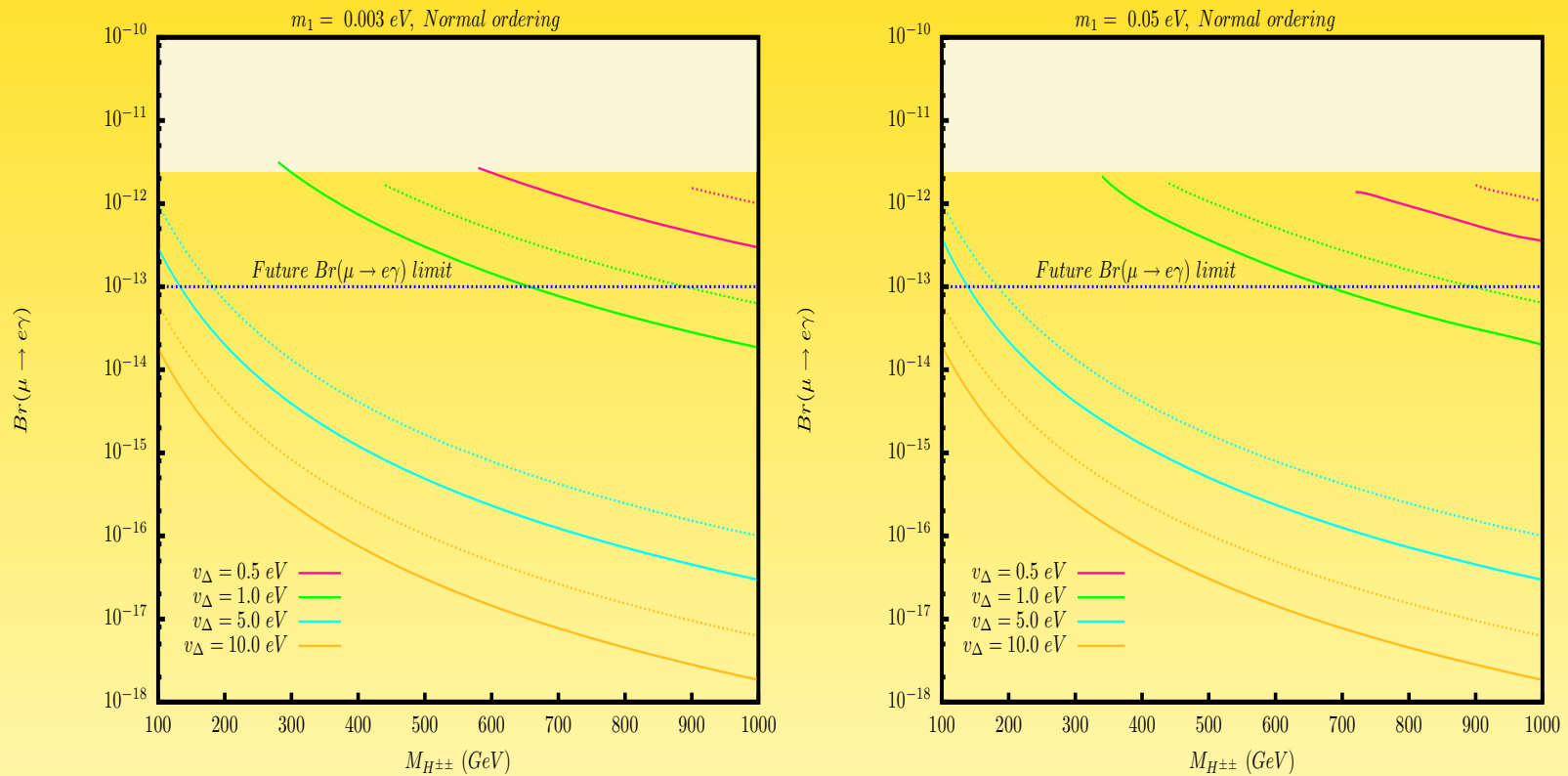
$$|U_{e3}| = \frac{1}{2} \frac{R \sin 2\theta_{12} \cot \theta_{23}}{1 \mp R \sin^2 \theta_{12}} \simeq 0.014$$

With large θ_{13} , the decay is guaranteed!

Chakraborty, Ghosh, W.R.

Lower limit on $\mu \rightarrow e\gamma$

explicitly realized in type II seesaw with Higgs triplet



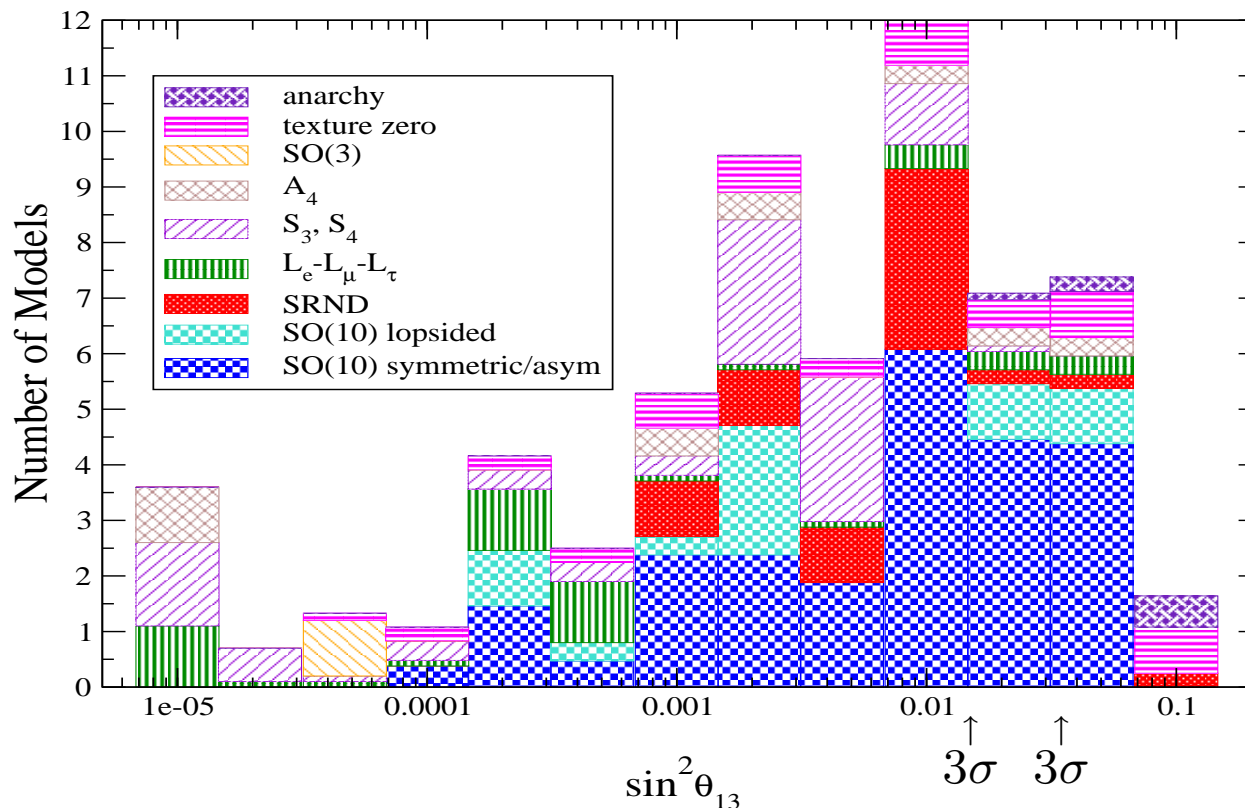
Chakraborty, Ghosh, W.R.

Model impact...

Xing, 1106.3244; Qui, Ma, 1106.3284; He, Zee, 1106.4359; Zheng, Ma, 1106.4040; Zhou, 1106.4808; Araki, 1106.5211; Haba, Takahashi, 1106.5926; Morisi, Patel, Peinado, 1107.0696, Chao, Zheng, 1107.0738; Zhang, Zhou, 1107.1097; Chu, Dhen, Hambye, 1107.1589; Toorop, Feruglio, Hagedorn, 1107.3486; Antusch, Maurer, 1107.3728; Rodejohann, Zhang, Zhou, 1107.3970; Ahn, Cheng, Oh, 1107.4549; Marzocca, Petcov, Romanino, Spinrath, 1108.0614; Ge, Dicus, Repko, 1108.0964; Riazuddin, 1108.1469; Ludl, Morisi, Peinado, 1109.3393; Verma, 1109.4228; Meloni, 1110.5210; Kitabayashi, Yasue, 1110.5162; He, Majee, 1111.2293; Rashed, 1111.3072; Buchmuller, Domcke, Schmitz, 1111.3872; King, Luhn, 1112.1959; Eby, Frampton, 1112.2675; Heeck, Rodejohann, 1112.3628; Gupta, Joshipura, Patel, 1112.6113; Damanik, arXiv:1201.2747; Ding, 1201.3279; Ishimori, Kobayashi, 1201.3429; Dev, Gautam, Singh, 1201.3755; Ahn, Okada, 1201.4436; Rodejohann, Tanimoto, Watanabe, 1201.4936; Dev, Dutta, Mohapatra, Severson, 1202.4012; BenTov, Zee, 1202.4234; Zhang, Ma, 1202.4258; Dorame, Morisi, Peinado, Valle, Rojas, 1203.0155; Dev, Kumar, Verma, Gupta, Gautam, 1203.1403; Cooper, King, Luhn, 1203.1324; Zhang, Zheng, Ma, 1203.1563; Siyeon, 1203.1593; Xing, 1203.1672; Wu, 1203.2382; Branco, Felipe, Joaquim, Serodio, 1203.2646; He, Xu, 1203.2908; Zhang, Ma, 1203.2906; Meloni, 1203.3126; Ahn, Kang, 1203.4185; Fritzscht, 1203.4460; Varzielas, Ross, 1203.6636; de Gouvea, Murayama, 1204.1249; Fukugita, Shimizu, Tanimoto, Yanagida, 1204.2389; Ishimori, Khalil, Ma, 1204.2705; Meloni, Blankenburg, 1204.2706; Minkowski, 1204.4376; Kitabayashi, Yasue, 1204.4523; Zhang, Ma, 1204.6604; King, 1205.0506; Zhou, 1205.0761; Ma, 1205.0766; Antusch, Gross, Maurer, Sluka, 1205.1051; Adhikary, Chakraborty, Ghosal, 1205.1355; Harigaya, Ibe, Yanagida, 1205.2198; Hagedorn, King, Luhn, 1205.3114...

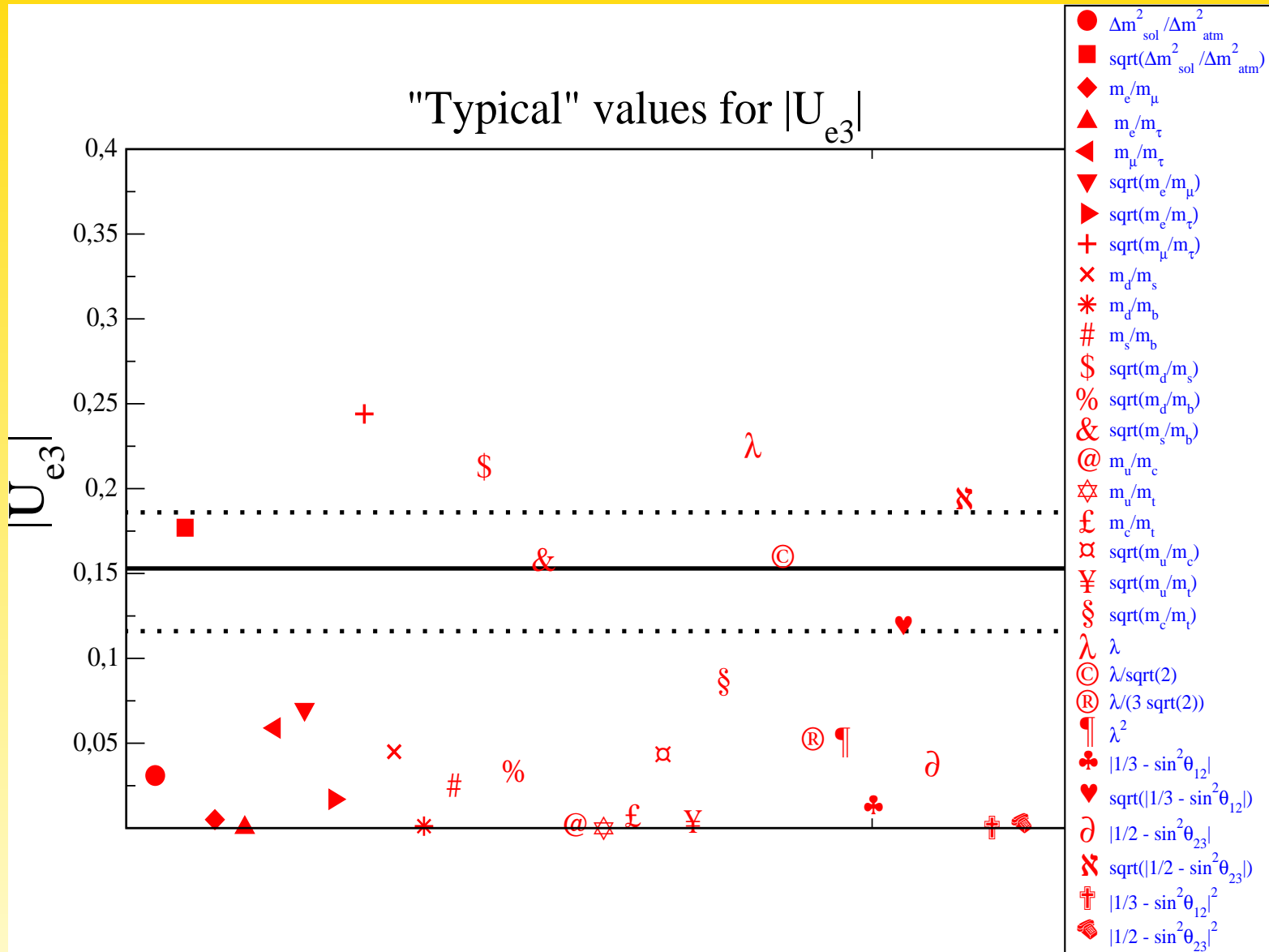
What's that good for?

Predictions of All 63 Models



Albright, Chen

θ_{13} : status



Impact on flavor symmetry models

Almost all models, $\mathcal{O}(500)$, were designed to generate tri-bimaximal mixing:

$$U_{\text{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

Harrison, Perkins, Scott (2002)

corresponding to

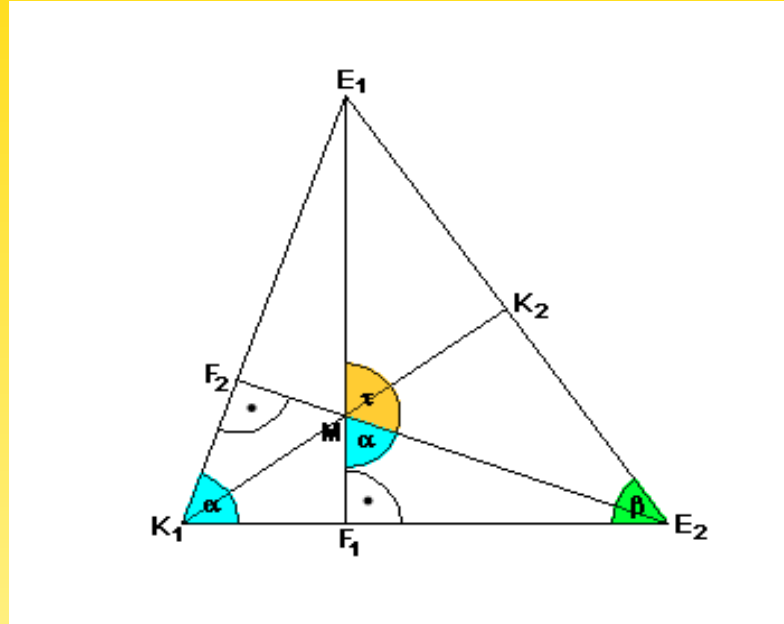
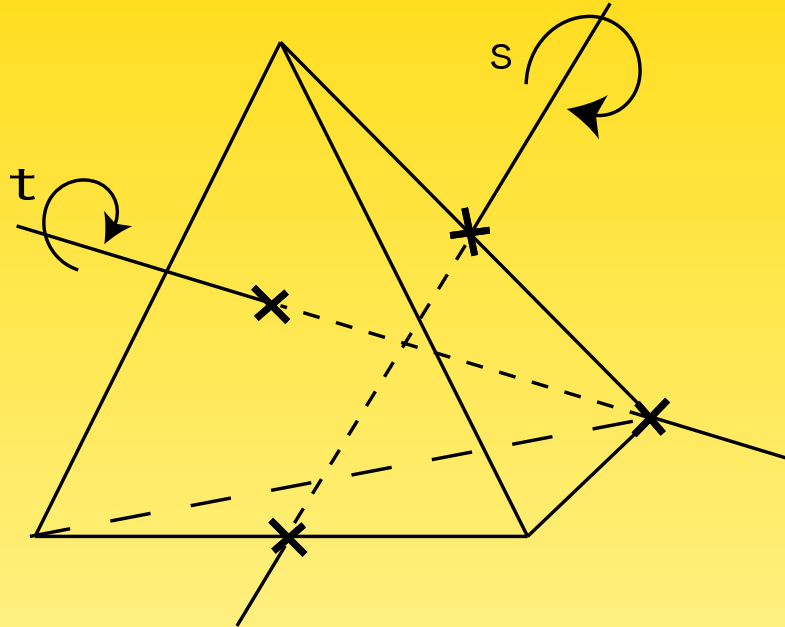
$$\sin^2 \theta_{12} = \frac{1}{3} \quad \text{a bit outside } 1\sigma$$

$$\sin^2 \theta_{23} = \frac{1}{2} \quad \text{well within } 1\sigma$$

$$\sin^2 \theta_{13} = 0 \quad \text{wrong by } \gtrsim 7\sigma$$

those models typically base on A_4

A_4 is isomorphic to rotation group of tetrahedron:



- smallest group with 3-dim irrep.
- has 3 one-dimensional irreps. $1, 1', 1''$
- angle between two faces: $\alpha = 2\theta_{\text{TBM}}$, where $\sin^2 \theta_{\text{TBM}} = \frac{1}{3}$

The Zoo (of A_4 models)

Type	L_i	ℓ_i^c	ν_i^c	Δ	References
A1				-	[1-14] [15] [#]
A2	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	-	$\underline{1}, \underline{1}', \underline{1}'', \underline{3}$	[16-18]
A3				$\underline{1}, \underline{3}$	[19]
B1	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{3}$	-	[4, 20-27] [#] [28-30]* [31-45]
B2				$\underline{1}, \underline{3}$	[46] [#]
C1				-	[2, 47, 48]
C2	$\underline{3}$	$\underline{3}$	-	$\underline{1}$	[49, 50] [51] [#]
C3				$\underline{1}, \underline{3}$	[52]
C4				$\underline{1}, \underline{1}', \underline{1}'', \underline{3}$	[53]
D1				-	[54, 55] [#] [56, 57]* [58]
D2	$\underline{3}$	$\underline{3}$	$\underline{3}$	$\underline{1}$	[59] [60]*
D3				$\underline{1}'$	[61]*
D4				$\underline{1}', \underline{3}$	[62]*
E	$\underline{3}$	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	-	[63, 64]
F	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{3}$	$\underline{3}$	$\underline{1}$ or $\underline{1}'$	[65]
G	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{1}, \underline{1}', \underline{1}''$	-	[66]
H	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	-	-	[67]
I	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{1}, \underline{1}, \underline{1}$	-	[68]*
J	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{3}$	-	[12, 39, 69, 70]
K	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{1}, \underline{1}$	$\underline{1}$	[71]*
L	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{1}$	-	[72]*
M	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{1}, \underline{1}'', \underline{1}'$	$\underline{3}, \underline{1}$	-	[73, 74]
N	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{1}, \underline{1}'', \underline{1}'$	$\underline{3}, \underline{1}', \underline{1}''$	-	[75]

Barry, W.R., updated regularly on

http://www.mpi-hd.mpg.de/personalhomes/jamesb/Table_A4.pdf

In a model one has to decide which fermions transform as what:

Field	l	e^c	μ^c	τ^c	$h_{u,d}$	φ	φ'	ξ	φ_0	φ'_0	ξ_0	θ
A_4	3	1	$1''$	$1'$	1	3	3	1	3	3	1	1
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	1	ω	ω	1
$U(1)_{\text{FN}}$	0	4	2	0	0	0	0	0	0	0	0	-1
$U(1)_R$	1	1	1	1	0	0	0	0	2	2	2	0

Altarelli, Feruglio

Due to VEV alignment, A_4 is broken to

- Z_2 in m_ν from $\varphi' = (v', v', v')$
- Z_3 in m_ℓ from $\varphi = (v, 0, 0)$
- accidental μ - τ symmetry \Rightarrow two Z_2 fix m_ν completely

Impact on flavor symmetry models

now θ_{13} is non-zero and large!

- corrections to TBM are naturally occurring in flavor symmetry models
- but: give similar corrections to ALL ANGLES...
- possibilities:
 - tune them to have $\delta\theta_{13} \gg \delta\theta_{12}$
 - start with non-zero θ_{13}

Alternative I: flavor symmetries and non-zero U_{e3}

$G_f = \Delta(96)$, generated by $S^2 = (ST)^3 = T^8 = (ST^{-1}ST)^3 = \mathbb{1}$ with

$$S = \frac{1}{2} \begin{pmatrix} 0 & \sqrt{2} & \sqrt{2} \\ \cdot & -1 & 1 \\ \cdot & \cdot & -1 \end{pmatrix} \quad \text{and} \quad T = \begin{pmatrix} e^{6i\pi/4} & 0 & 0 \\ \cdot & e^{7i\pi/4} & 0 \\ \cdot & \cdot & e^{3i\pi/4} \end{pmatrix}$$

assumption (1): charged leptons invariant under $G_e = Z_3$; neutrinos under

$$G_\nu = Z_2 \times Z_2$$

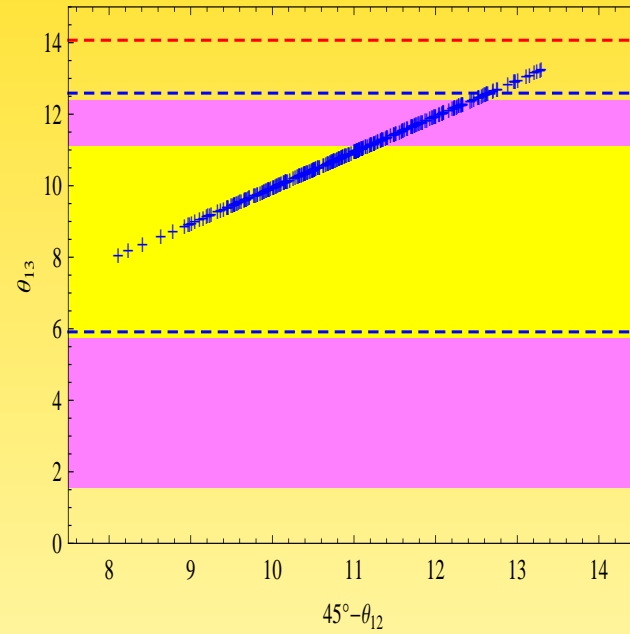
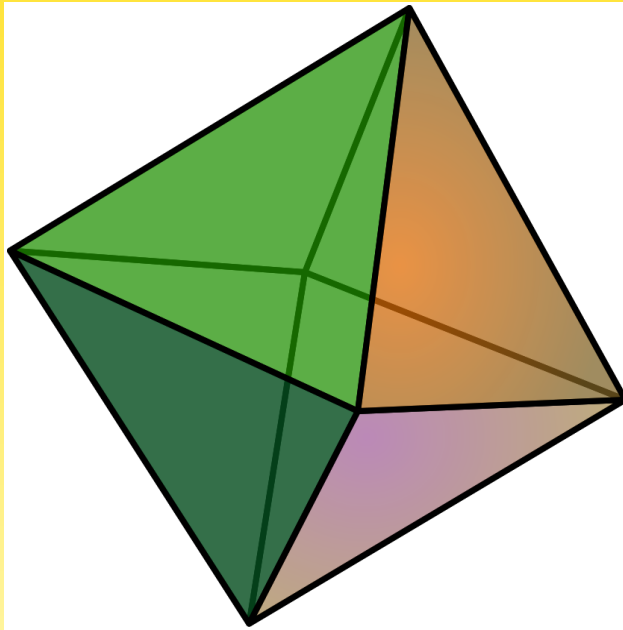
assumption (2): $G_e = ST$ and $G_\nu = \{S, ST^4ST^4\}$

$$|U| = \sqrt{\frac{1}{3}} \begin{pmatrix} \frac{1}{2}(\sqrt{3} + 1) & 1 & \frac{1}{2}(\sqrt{3} - 1) \\ \frac{1}{2}(\sqrt{3} - 1) & 1 & \frac{1}{2}(\sqrt{3} + 1) \\ 1 & 1 & 1 \end{pmatrix}$$

Toorop, Feruglio, Hagedorn

Alternative I: flavor symmetries and non-zero U_{e3}

octahedral group O_h , isomorphic to $S_4 \times Z_2$



He, Xu

Alternative II: $|U_{e3}| = \theta_C/\sqrt{2}$ from GUTs

$$U_\nu = \begin{pmatrix} * & * & 0 \\ * & * & \sqrt{\frac{1}{2}} \\ * & * & \sqrt{\frac{1}{2}} \end{pmatrix} \quad \text{and} \quad U_\ell = \begin{pmatrix} 1 & \lambda & 0 \\ -\lambda & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow |U_{e3}| = \frac{\lambda}{\sqrt{2}}$$

natural framework: $m_{\text{up}} \simeq \text{diag}$ and relate down quarks to charged leptons

$$Y_d = \begin{pmatrix} d & b \\ a & c \end{pmatrix} \Rightarrow \left\{ \begin{array}{l} m_\ell = \begin{pmatrix} c_d d & c_b b \\ c_a a & c_c c \end{pmatrix} \\ m_\ell = \begin{pmatrix} c_d d & c_a a \\ c_b b & c_c c \end{pmatrix} \end{array} \right. \begin{array}{l} \text{Pati-Salam} \\ SU(5) \end{array} \Rightarrow |U_{e3}| = \left| \frac{c_c}{c_b} \right| \frac{\theta_C}{\sqrt{2}}$$

Clebsch-Gordan factors depending on GUT breaking

Antusch et al.

Alternative III: Gatto-Sartori-Tonin for leptons

$$\sqrt{\frac{\Delta m_{\odot}^2}{\Delta m_{\text{A}}^2}} = 0.160 \dots 0.190 \quad \leftrightarrow \quad |U_{e3}| = 0.122 \dots 0.190$$

can be arranged in flavor symmetry models, e.g. S_3

	$(\overline{L}_1, \overline{L}_2)$	\overline{L}_3	(ν_{R_1}, ν_{R_2})	e_R	μ_R	τ_R	(ϕ_1, ϕ_2)	χ
S_3	2^*	1_S	2	1_S	1_S	1_S	2	1_S
Z_3	ω	ω	ω	ω	1	ω^2	ω	ω

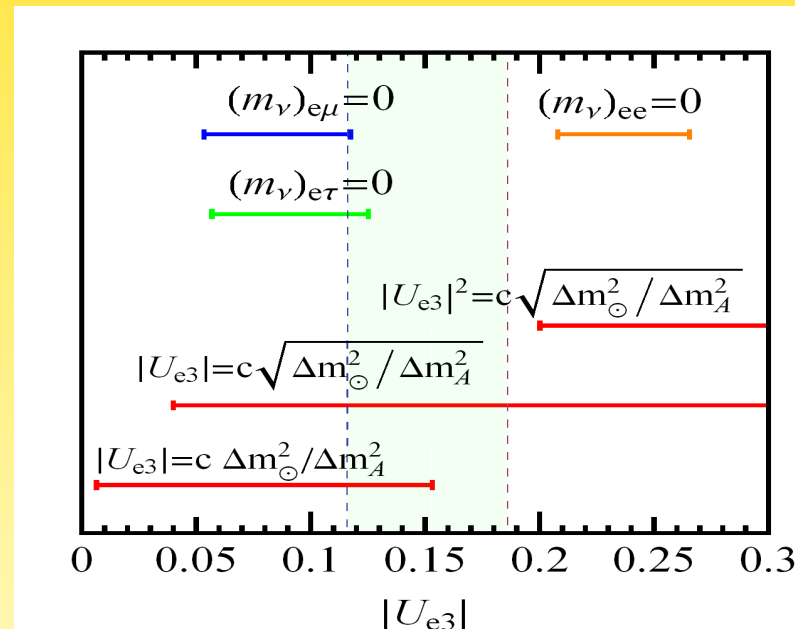
	$(\overline{L}_1, \overline{L}_2)$	\overline{L}_3	(ν_{R_1}, ν_{R_2})	(e_R, μ_R)	τ_R	η_1^-	η_2^+	η_3^+	η_4^-	(ξ_1^+, ξ_2^+)
D_4	2	1_1	2	2	1_1	1_1	1_2	1_3	1_4	2
Z_2	$+$	$+$	$+$	$-$	$+$	$-$	$+$	$+$	$-$	$+$

W.R., Tanimoto, Watanabe

in 3-flavor framework: relations receive order one factors

Example, $m_1 = 0$ and $(m_\nu)_{e\tau} = 0$:

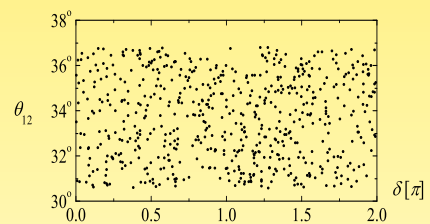
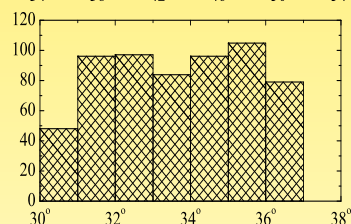
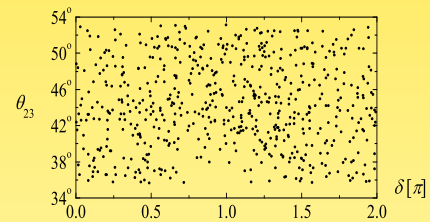
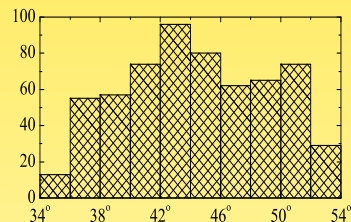
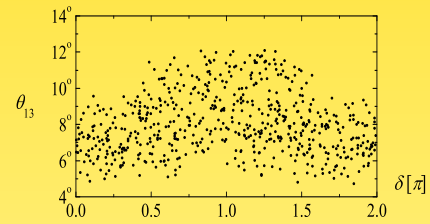
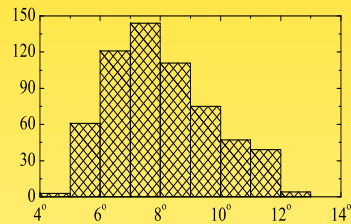
$$|U_{e3}| \simeq \frac{1}{2} \sqrt{\frac{\Delta m_\odot^2}{\Delta m_A^2}} \sin 2\theta_{12} \tan \theta_{23} = 0.084_{-0.027}^{+0.041}$$



W.R., Tanimoto, Watanabe

Alternative IV: texture zeros

$$m_\nu = \begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix} \Rightarrow |U_{e3}|^2 \simeq \frac{1}{4} \frac{\sin^2 2\theta_{12}}{\tan^2 \theta_{23} \cos 2\theta_{12}} \frac{\Delta m_{\odot}^2}{\Delta m_A^2}$$



Fritzsch, Xing, Zhou

Summary

$$|U| = \begin{pmatrix} 0.779 \dots 0.848 & 0.510 \dots 0.604 & 0.122 \dots 0.190 \\ 0.183 \dots 0.568 & 0.385 \dots 0.728 & 0.613 \dots 0.794 \\ 0.200 \dots 0.576 & 0.408 \dots 0.742 & 0.589 \dots 0.775 \end{pmatrix}$$

$$|V| = \begin{pmatrix} 0.97428 \pm 0.00015 & 0.2253 \pm 0.0007 & 0.00347^{+0.00016}_{-0.00012} \\ 0.2252 \pm 0.0007 & 0.97345^{+0.00015}_{-0.00016} & 0.0410^{+0.0011}_{-0.0007} \\ 0.00862^{+0.00026}_{-0.00020} & 0.0403^{+0.0011}_{-0.0007} & 0.999152^{+0.000030}_{-0.000045} \end{pmatrix}$$

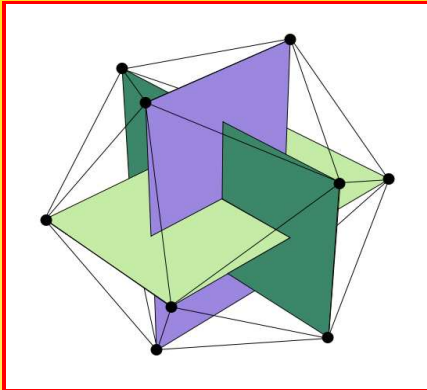
- precision
- unitarity
- CP
- test all elements directly

Summary

- $\theta_{13} \neq 0$ at $\gtrsim 7\sigma$
- factor $\lesssim 2$ below Chooz limit
- makes it possible to test:
 - mass ordering
 - CP violation
 - impact on planning of next generation experiments
 - impact on model building
- no sign new physics (except for sterile neutrinos)

Alternatives to TBM: focus on θ_{12}

- Golden Ratio 1: $\cot \theta_{12} = \varphi = (1 + \sqrt{5})/2 \Rightarrow \sin^2 \theta_{12} \simeq 0.276$ $\leftrightarrow A_5$



Cartesian coordinates of 12 icosahedron vertices:

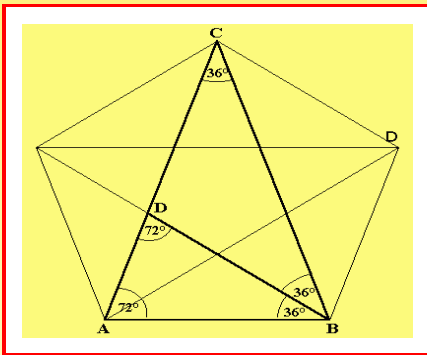
$$(0, \pm 1, \pm \varphi)$$

$$(\pm 1, \pm \varphi, 0)$$

$$(\pm \varphi, 0, \pm 1)$$

(Datta, Ling, Ramond; Kajiyama, Raidal, Strumia; Everett, Stuart)

- Golden Ratio 2: $\cos \theta_{12} = \varphi/2 \Rightarrow \sin^2 \theta_{12} = \sin^2 \pi/5 \simeq 0.345$ $\leftrightarrow D_{10}$



$$\overline{AD} = \varphi \overline{AB}$$

(W.R., Adulpravitchai, Blum, W.R.)

Degeneracies

Expand 3 flavor oscillation probabilities in terms of $R = \Delta m_{\odot}^2 / \Delta m_{\text{A}}^2$ and $|U_{e3}|$:

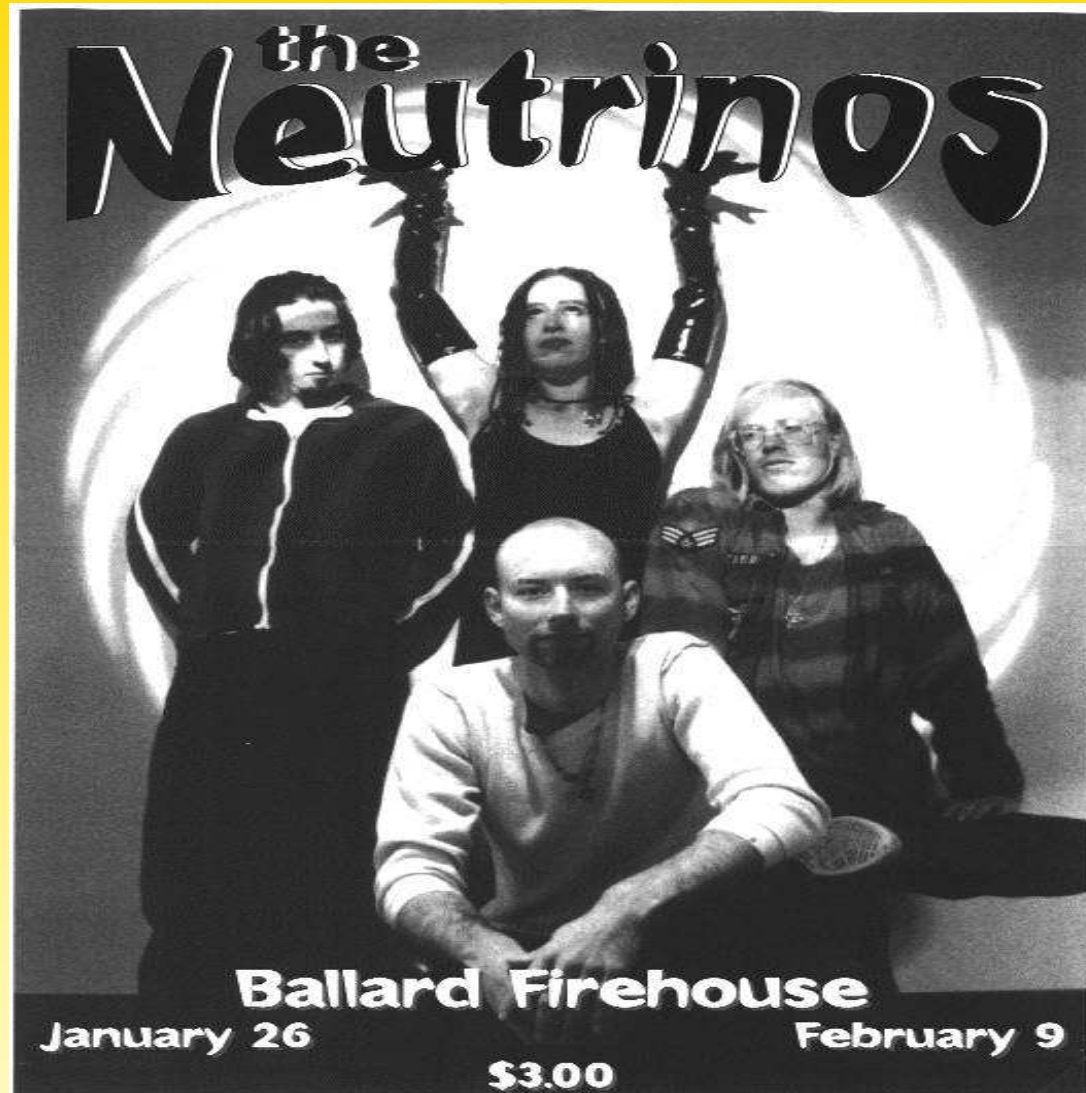
$$\begin{aligned}
 P(\nu_e \rightarrow \nu_\mu) \simeq & \sin^2 2\theta_{13} \sin^2 \theta_{23} \frac{\sin^2 (1-\hat{A})\Delta}{(1-\hat{A})^2} + R^2 \sin^2 2\theta_{12} \cos^2 \theta_{23} \frac{\sin^2 \hat{A}\Delta}{\hat{A}^2} \\
 & + \sin \delta \sin 2\theta_{13} R \sin 2\theta_{12} \cos \theta_{13} \sin 2\theta_{23} \sin \Delta \frac{\sin \hat{A}\Delta \sin (1-\hat{A})\Delta}{\hat{A}(1-\hat{A})} \\
 & + \cos \delta \sin 2\theta_{13} R \sin 2\theta_{12} \cos \theta_{13} \sin 2\theta_{23} \cos \Delta \frac{\sin \hat{A}\Delta \sin (1-\hat{A})\Delta}{\hat{A}(1-\hat{A})}
 \end{aligned}$$

with $\hat{A} = 2\sqrt{2} G_F n_e E / \Delta m_{\text{A}}^2$ and $\Delta = \frac{\Delta m_{\text{A}}^2}{4E} L$

- $\theta_{23} \leftrightarrow \pi/2 - \theta_{23}$ degeneracy
- θ_{13} - δ degeneracy
- δ - $\text{sgn}(\Delta m_{\text{A}}^2)$ degeneracy

Solutions: more channels, different L/E , high precision,...

Is the PMNS matrix 4×4 ?



Which one is sterile?

Motivation

- particle physics
 - LSND/MiniBooNE
 - Gallium experiments
 - reactor anomaly
- cosmology
 - CMB
 - BBN
- astrophysics
 - r -process nucleosynthesis in supernovae

Light Sterile Neutrinos: A White Paper

K. N. Abazajian¹, M. A. Acero,² S. K. Agarwalla,³ A. A. Aguilar-Arevalo,² C. H. Albright,^{4,5} S. Antusch,⁶ C. A. Argüelles,⁷ A. B. Balantekin,⁸ G. Barenboim^{a,3}, V. Barger,⁸ P. Bernardini,⁹ F. Bezrukov,¹⁰ O. E. Bjaelde,¹¹ S. A. Bogacz,¹² N. S. Bowden,¹³ A. Boyarsky,¹⁴ A. Bravar,¹⁵ D. Bravo Berguño,¹⁶ S. J. Brice,⁵ A. D. Bross,⁵ B. Caccianiga,¹⁷ F. Cavanna,^{18,19} E. J. Chun,²⁰ B. T. Cleveland,²¹ A. P. Collin,²² P. Coloma,¹⁶ J. M. Conrad,²³ M. Cribier,²² A. S. Cucoanes,²⁴ J. C. D'Olivo,² S. Das,²⁵ A. de Gouvêa,²⁶ A. V. Derbin,²⁷ R. Dharmapalan,²⁸ J. S. Diaz,²⁹ X. J. Ding,¹⁶ Z. Djurcic,³⁰ A. Donini,^{31,3} D. Duchesneau,³² H. Ejiri,³³ S. R. Elliott,³⁴ D. J. Ernst,³⁵ A. Esmaili,³⁶ J. J. Evans,^{37,38} E. Fernandez-Martinez,³⁹ E. Figueroa-Feliciano,²³ B. T. Fleming^{a,18}, J. A. Formaggio^{a,23}, D. Franco,⁴⁰ J. Gaffiot,²² R. Gandhi,⁴¹ Y. Gao,⁴² G. T. Garvey,³⁴ V. N. Gavrin,⁴³ P. Ghoshal,⁴¹ D. Gibin,⁴⁴ C. Giunti,⁴⁵ S. N. Gninenko,⁴³ V. V. Gorbachev,⁴³ D. S. Gorbunov,⁴³ R. Guenette,¹⁸ A. Guglielmi,⁴⁴ F. Halzen,^{46,8} J. Hamann,¹¹ S. Hannestad,¹¹ W. Haxton,^{47,48} K. M. Heeger,⁸ R. Henning,^{49,50} P. Hernandez,³ P. Huber^{b,16}, W. Huelsnitz,^{34,51} A. Ianni,⁵² T. V. Ibragimova,⁴³ Y. Karadzhov,¹⁵ G. Karagiorgi,⁵³ G. Keefer,¹³ Y. D. Kim,⁵⁴ J. Kopp^{a,5}, V. N. Korneukhov,⁵⁵ A. Kusenko,^{56,57} P. Kyberd,⁵⁸ P. Langacker,⁵⁹ Th. Lasserre^{a,22,40}, M. Laveder,⁶⁰ A. Letourneau,²² D. Lhuillier,²² Y. F. Li,⁶¹ M. Lindner,⁶² J. M. Link^{b,16}, B. L. Littlejohn,⁸ P. Lombardi,¹⁷ K. Long,⁶³ J. Lopez-Pavon,⁶⁴ W. C. Louis^{a,34}, L. Ludhova,¹⁷ J. D. Lykken,⁵ P. A. N. Machado,^{65,66} M. Maltoni,³¹ W. A. Mann,⁶⁷ D. Marfatia,⁶⁸ C. Mariani,^{53,16} V. A. Matveev,^{43,69} N. E. Mavromatos,^{70,39} A. Melchiorri,⁷¹ D. Meloni,⁷² O. Mena,³ G. Mention,²² A. Merle,⁷³ E. Meroni,¹⁷ M. Mezzetto,⁴⁴ G. B. Mills,³⁴ D. Minic,¹⁶ L. Miramonti,¹⁷ D. Mohapatra,¹⁶ R. N. Mohapatra,⁵¹ C. Montanari,⁷⁴ Y. Mori,⁷⁵ Th. A. Mueller,⁷⁶ H. P. Mumm,⁷⁷ V. Muratova,²⁷ A. E. Nelson,⁷⁸ J. S. Nico,⁷⁷ E. Noah,¹⁵ J. Nowak,⁷⁹ O. Yu. Smirnov,⁶⁹ M. Obolensky,⁴⁰ S. Pakvasa,⁸⁰ O. Palamara,^{18,52} M. Pallavicini,⁸¹ S. Pascoli,⁸² L. Patrizii,⁸³ Z. Pavlovic,³⁴ O. L. G. Peres,³⁶ H. Pessard,³² F. Pietropaolo,⁴⁴ M. L. Pitt,¹⁶ M. Popovic,⁵ J. Pradler,⁸⁴ G. Ranucci,¹⁷ H. Ray,⁸⁵ S. Razzaque,⁸⁶ B. Rebel,⁵ R. G. H. Robertson,^{87,78} W. Rodejohann^{a,62}, S. D. Rountree,¹⁶ C. Rubbia,^{39,52} O. Ruchayskiy,³⁹ P. R. Sala,¹⁷ K. Scholberg,⁸⁸ T. Schwetz^{a,62}, M. H. Shaevitz,⁵³ M. Shaposhnikov,⁸⁹ R. Shrock,⁹⁰ S. Simone,⁹¹ M. Skorokhvatov,⁹² M. Sorel,³ A. Sousa,⁹³ D. N. Spergel,⁹⁴ J. Spitz,²³ L. Stanco,⁴⁴ I. Stancu,²⁸ A. Suzuki,⁹⁵ T. Takeuchi,¹⁶ I. Tamborra,⁹⁶ J. Tang,^{97,98} G. Testera,⁸¹ X. C. Tian,⁹⁹ A. Tonazzo,⁴⁰ C. D. Tunnell,¹⁰⁰ R. G. Van de Water,³⁴ L. Verde,¹⁰¹ E. P. Veretenkin,⁴³ C. Vignoli,⁵² M. Vivier,²² R. B. Vogelaar,¹⁶ M. O. Wascko,⁶³ J. F. Wilkerson,^{49,102} W. Winter,⁹⁷ Y. Y. Y. Wong^{a,25}, T. T. Yanagida,⁵⁷ O. Yasuda,¹⁰³ M. Yeh,¹⁰⁴ F. Yermia,²⁴ Z. W. Yokley,¹⁶ G. P. Zeller,⁵ L. Zhan,⁶¹ and H. Zhang⁶²

¹University of California, Irvine

²Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México

³Instituto de Física Corpuscular, CSIC and Universidad de Valencia

⁴Northern Illinois University

⁵Fermi National Accelerator Laboratory

⁶University of Basel

^aSection editor

^bEditor and corresponding author (pahuber@vt.edu and jmlink@vt.edu)

Remarks

- majority of experiments does not require sterile neutrinos
- oscillation experiments: $\Delta m^2 \simeq 1 \text{ eV}^2$ vs. cosmology: $m_s \lesssim 1 \text{ eV}$
- appearance-disappearance tension
- all anomalies explained by the same thing?
- are they all real?

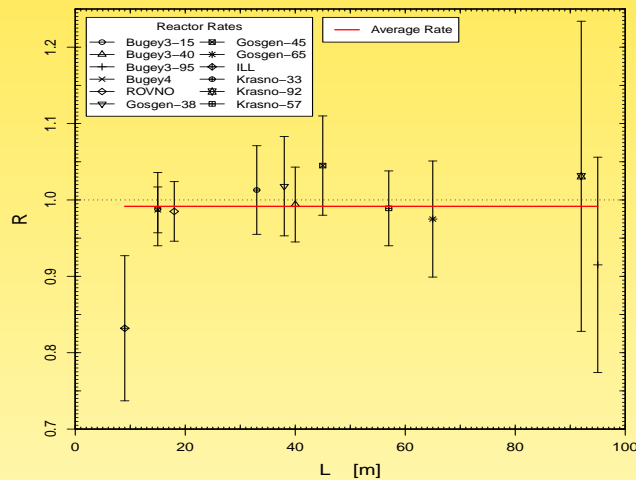
	$\Delta m_{41}^2 [\text{eV}^2]$	$ U_{e4} $	$ U_{\mu 4} $	$\Delta m_{51}^2 [\text{eV}^2]$	$ U_{e5} $	$ U_{\mu 5} $
3+2/2+3	0.47	0.128	0.165	0.87	0.138	0.148
1+3+1	0.47	0.129	0.154	0.87	0.142	0.163

or $\Delta m_{41}^2 = 1.78 \text{ eV}^2$ and $|U_{e4}|^2 = 0.151$

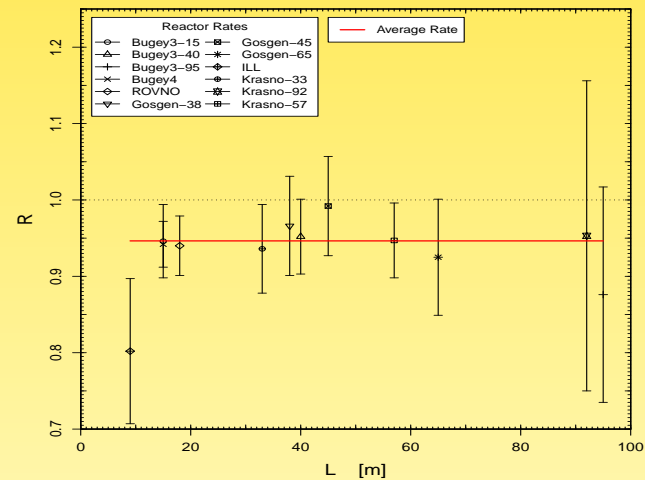
Kopp, Maltoni, Schwetz

Reactor anomaly

- fission yield per isotope
- β decay branching ratios (allowed, forbidden)
- β shape (corrections: QED, weak magnetism, Coulomb)
- extraction from electron spectra



$$R_{\text{old}} = 0.992 \pm 0.024$$



$$R_{\text{new}} = 0.946 \pm 0.024$$

θ_{13} : phenomenological aspects

possibility to distinguish normal vs. inverted with supernovas

Fluxes arriving at the Earth

$$F_{\nu_e} = \rho F_{\nu_e}^0 + (1 - \rho) F_{\nu_x}^0, \quad F_{\bar{\nu}_e} = \bar{\rho} F_{\bar{\nu}_e}^0 + (1 - \bar{\rho}) F_{\nu_x}^0$$

ρ at low, intermediate, high energies

		Phase A ($L_{\nu_e} \gtrsim L_{\nu_x}$)			Phase C ($L_{\nu_e} \gtrsim L_{\nu_x}$)		
NH	$\sin^2 \theta_{13} \gtrsim 10^{-3}$	0	0	0	0	0	s^2
	$\sin^2 \theta_{13} \lesssim 10^{-5}$	s^2	s^2	s^2	s^2	s^2	0
IH	$\sin^2 \theta_{13} \gtrsim 10^{-3}$	s^2	0	0	s^2	0	$c^2 (s^2)$
	$\sin^2 \theta_{13} \lesssim 10^{-5}$	s^2	0	0	s^2	0	$c^2 (s^2)$

$\bar{\rho}$ at low, intermediate, high energies

		Phase A ($L_{\nu_e} \gtrsim L_{\nu_x}$)			Phase C ($L_{\nu_e} \gtrsim L_{\nu_x}$)		
NH	$\sin^2 \theta_{13} \gtrsim 10^{-3}$	c^2	c^2	c^2	c^2	c^2	0
	$\sin^2 \theta_{13} \lesssim 10^{-5}$	c^2	c^2	c^2	c^2	c^2	0
IH	$\sin^2 \theta_{13} \gtrsim 10^{-3}$	0	c^2	c^2	0	$c^2 [0]$	$s^2 (0)$
	$\sin^2 \theta_{13} \lesssim 10^{-5}$	c^2	0	0	c^2	0 [c^2]	$s^2 (c^2)$

$$s^2 \equiv \sin^2 \theta_{12}, \quad c^2 \equiv \cos^2 \theta_{12}$$

(), [] : non-adiabatic swaps



Dighe