# Heavy Quarkonium Production and NRQCD

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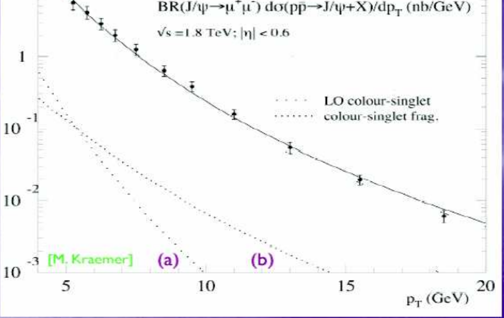
Flavor Physics and CP Violation 2012
May 21, 2012. Hefei, CHINA

# Introduction

- Perturbative and non-perturbative QCD, hadronization, factorization
- Color-singlet and Color-octet mechanism was proposed based on NRQCD since b and c-quark is heavy.
- Clear signal to detect $J/\psi$.
- heavy quarkonium production is a good place to testify these theoretical framework.
- $J/\psi$ photoproduction at HERA
- $J/\psi$ production at the B factories
- $J/\psi$ production and polarization at the Tevatron
- $J/\psi$ production at the LHC
- LO theoretical predication were given before more than 15 years
- NLO theoretical predications were given within last 5 years.
- It seems that the QCD NLO calculations can adequately describe the experimental data.
- But there are still many difficulties.




PRL **99,** 132001 (2007) PHYSICAL REVIEW LETTERS week ending 28 SEPTEMBER 2007

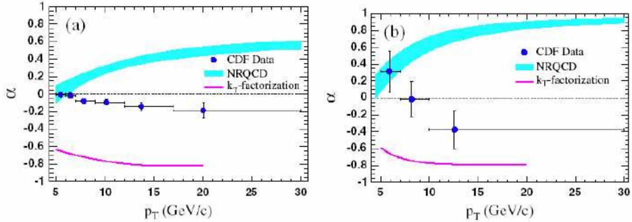



FIG. 4 (color online). Prompt polarizations as functions of $p_T$: (a) $J/\psi$ and (b) $\psi(2S)$. The band (line) is the prediction from NRQCD [4] (the $k_T$-factorization model [9]).

# $e^+e^- \to J/\psi + \eta_c$

## Experimantal Data

BELLE: $\sigma[J/\psi + \eta_c] \times B^{\eta_c}[\geq 2] = (25.6 \pm 2.8 \pm 3.4)\ \mathrm{fb}$
*BARAR*: $\sigma[J/\psi + \eta_c] \times B^{\eta_c}[\geq 2] = (17.6 \pm 2.8^{+1.5}_{-2.1})\ \mathrm{fb}$
[Abe et al.(2002), Pakhlov(2004), Aubert et al.(2005)]

## LO NRQCD Predictions

$2.3 \sim 5.5\ \mathrm{fb}$
[Braaten and Lee(2003), Liu et al.(2003), Hagiwara et al.(2003)]

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## LO NRQCD Predictions

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## NLO QCD corrections

$K \equiv \sigma^{NLO}/\sigma^{LO} \sim 2$

First given in PRL96, (2006) Y. J. Zhang, Y. J. Gao and K. T. Chao

Confirmed by the analytic result in PRD77, (2008), B. Gong and J. X. Wang

## Relativistic corrections

$K \sim 2$

PRD67, (2007) E. Braaten and J. Lee

AIP Conf. Proc. (2007), G.T. Bodwin, D. Kang, T. Kim, J. Lee and C. Yu

PRD75, (2007), Z. G. He, Y. Fan and K. T. Chao

PRD77,(2008),G.T. Bodwin, J. Lee and C. Yu

# $e^+e^- \longrightarrow J/\psi + J/\psi$

## Problem

LO NRQCD prediction indicates that the cross section of this process is large than that of $J/\psi + \eta_c$ production by a factor of 1.8, but no evidence for this process was found at the B factories.
PRL90, (2003) G. T. Bodwin, E. Braaten and J. Lee
PRD70, (2004), K. Abe, et al

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PRD70, (2004), K. Abe, et al

## NLO QCD corrections

- Greatly decreased, with a K factor ranging from $-0.31 \sim 0.25$ depending on the renormalization scale.
- Might explain the situation.

PRL100, (2008) B. Gong and J. X. Wang

## LO NRQCD Predictions:

| | |
|---|---|
| $e^+e^- \to J/\psi + c\bar{c}$ | $0.07 \sim 0.20$pb |
| $e^+e^- \to J/\psi + gg$ | $0.15 \sim 0.3$pb |
| $e^+e^- \to J/\psi^{(8)}(^3P_J, {}^1S_0) + g$ | $0.3 \sim 0.8$pb |

PRL76,(1996), E. Braaten and Y. C. Chen, PLB577,(2003), K.Y. Liu, Z.G. He and K.T. chao, ....

## Experimental Data:

| | |
|---|---|
| *BARAR* | $\sigma[e^+e^- \to J/\psi + X] = (2.54 \pm 0.21 \pm 0.21)$ pb |
| CLEO | $\sigma[e^+e^- \to J/\psi + X] = (1.9 \pm 0.20)$ pb |
| BELLE | $\sigma[e^+e^- \to J/\psi + X] = (1.45 \pm 0.10 \pm 0.13)$ pb |
| | $\sigma[e^+e^- \to J/\psi + c\bar{c} + X] = (0.87^{+0.21}_{-0.19} \pm 0.17)$ pb |

[Aubert et al.(2001), Aubert et al.(2005), Briere et al.(2004), Abe et al.(2002a), Abe et al.(2002)]

## New BELLE Data

$$\sigma[e^+e^- \to J/\psi + X] = (1.17 \pm 0.02 \pm 0.07) \text{ pb}$$

$$\sigma[e^+e^- \to J/\psi + c\bar{c}] = (0.74 \pm 0.08^{+0.09}_{-0.08}) \text{ pb}$$

$$\sigma[e^+e^- \to J/\psi + X_{\mathrm{non}-c\bar{c}}] = (0.43 \pm 0.09 \pm 0.09) \text{ pb}$$

[Pakhlov et al.(2009)]

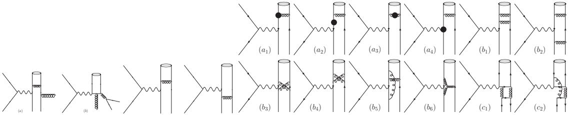
$$\sigma^{(1)} = \sigma^{(0)} \left\{ 1 + \frac{\alpha_s(\mu)}{\pi} \left[ a(\hat{s}) + \beta_0 \ln\left(\frac{\mu}{2m_c}\right) \right] \right\}$$

| $m_c$(GeV) | $\alpha_s(\mu)$ | $\sigma^{(0)}$(pb) | $a(\hat{s})$ | $\sigma^{(1)}$(pb) | $\sigma^{(1)}/\sigma^{(0)}$ |
|---|---|---|---|---|---|
| 1.4 | 0.267 | 0.341 | 2.35 | 0.409 | 1.20 |
| 1.5 | 0.259 | 0.308 | 2.57 | 0.373 | 1.21 |
| 1.6 | 0.252 | 0.279 | 2.89 | 0.344 | 1.23 |

Consistent results from two group:
PRL102, (2009) Y. Q. Ma, Y. J. Zhang and K. T. Chao
PRL102, (2009) B. Gong and J. X. Wang

Relativistic Correction enchance results about a factor 1.3 from two group:
PRD81, (2010) Z. G. He, Y. Fan and K. T. Chao
PRD82, (2010). Y. Jia

# $e^+e^- \to J/\psi + c\bar{c}$



$$\sigma^{(1)} = \sigma^{(0)} \left\{ 1 + \frac{\alpha_s(\mu)}{\pi} \left[ a(\hat{s}) + \beta_0 \ln\left( \frac{\mu}{2m_c} \right) \right] \right\}$$

| $m_c$(GeV) | $\alpha_s(\mu)$ | $\sigma^{(0)}$(pb) | $a(\hat{s})$ | $\sigma^{(1)}$(pb) | $\sigma^{(1)}/\sigma^{(0)}$ |
|---|---|---|---|---|---|
| 1.4 | 0.267 | 0.224 | 8.19 | 0.380 | 1.70 |
| 1.5 | 0.259 | 0.171 | 8.94 | 0.298 | 1.74 |
| 1.6 | 0.252 | 0.129 | 9.74 | 0.230 | 1.78 |

Cross sections with different charm quark mass $m_c$ with the renormalization scale $\mu = 2m_c$ and $\sqrt{s} = 10.6\ \mathrm{GeV}$. The former result given by PRL98, (2007) Y. J. Zhang and K. T. Chao
confirmed by PRD80, (2009) B. Gong and J. X. Wang

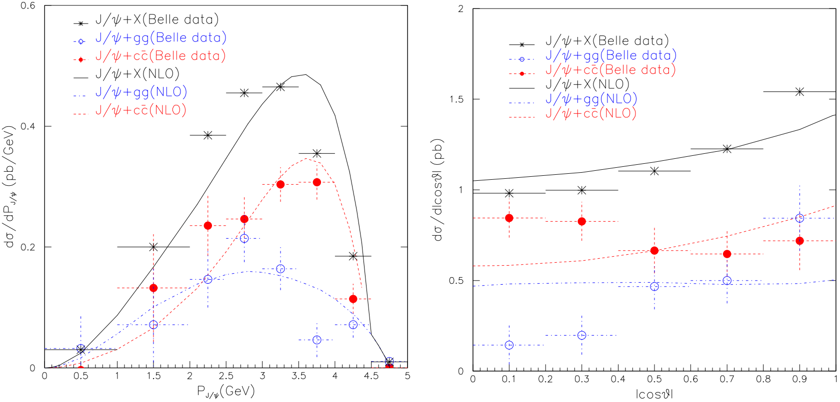
# More about the scale and comparision with data

Use Brodsky, Lepage and Mackenzie (BLM) scale setting [Brodsky et al.(1983)]

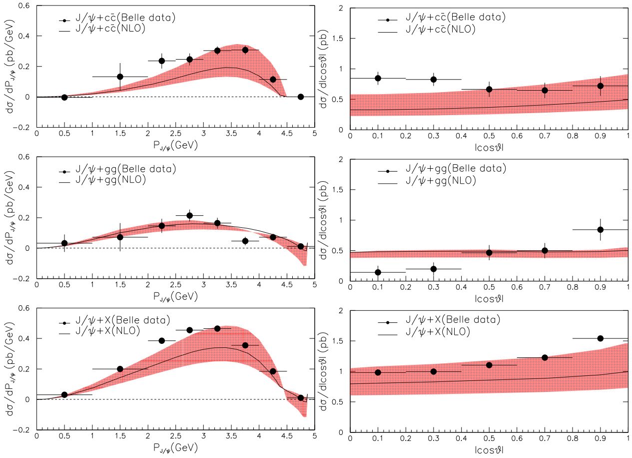
$$\sigma^{(1)} = \sigma^{(0)}(\mu^*)[1 + \frac{\alpha_s(\mu^*)}{\pi} b(\hat{s})].$$

| $m_c$(GeV) | $\alpha_s(\mu^*)$ | $\sigma^{(0)}$(pb) | $b(\hat{s})$ | $\sigma^{(1)}$(pb) | $\sigma^{(1)}/\sigma^{(0)}$ | $\mu^*$(GeV) |
|---|---|---|---|---|---|---|
| 1.4 | 0.348 | 0.381 | 3.77 | 0.540 | 1.42 | 1.65 |
| 1.5 | 0.339 | 0.293 | 4.31 | 0.429 | 1.47 | 1.72 |
| 1.6 | 0.332 | 0.222 | 4.90 | 0.337 | 1.52 | 1.79 |

Cross sections with different charm quark mass $m_c$. The renormalization scale $\mu = \mu^* \sim m_c$.

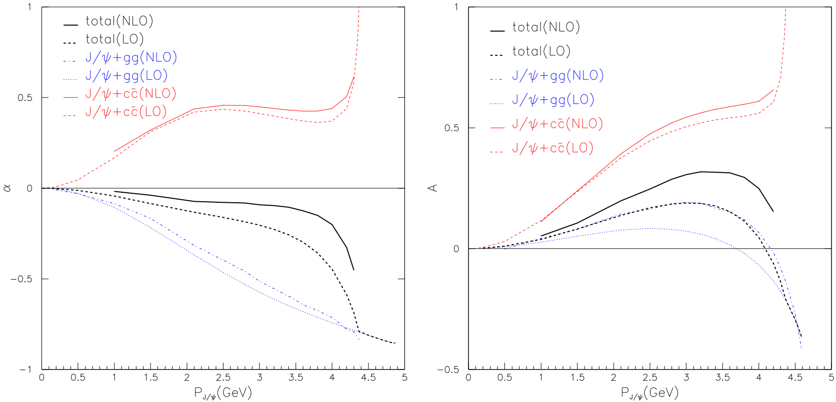



Momentum distribution of inclusive $J/\psi$ production with $\mu = \mu^*$ and $m_c = 1.4$ GeV is taken for the $J/\psi cc$ channel. The contribution from the feed-down of $\psi'$ has been added to all curves by multiplying a factor of 1.29.




Momentum and angular distributions of inclusive $J/\psi$ production.

The contribution from the feed-down of $\psi'$ has been added to all curves by multiplying a factor of 1.29.




Polarization parameter $\alpha$ and angular distribution parameter $A$ of $J/\psi$ as functions of $p$.

# Constraint for color-octect matrix element of $c\bar{c}({}^1S_0^8, 3P_J^8)$

$$\sigma[e^+e^- \to J/\psi + X_{\mathrm{non}-c\bar{c}}] \quad = \quad (0.43 \pm 0.09 \pm 0.09)\ \mathrm{pb}$$

$$\sigma[e^+e^- \to J/\psi + X_{\mathrm{non}-c\bar{c}}]^{color-singleTh} \quad > \quad (0.43)\ \mathrm{pb}$$

$$\sigma[e^+e^- \to J/\psi + X_{\mathrm{non}-c\bar{c}}]^{color-octetTh} \quad > \quad (0.6)\ \mathrm{pb}$$

From the contribution of $e^+e^- \to J/\psi({}^1S_0^8, 3P_J^8) + g$ at NLO

PRD81, (2010) Y. J. Zhang, Y. Q. Ma, K. Wang and K. T. Chao

# Introduction

In last five years, there were a few very important progresses in the next-to-leading Order (NLO) QCD correction calculation:

- QCD Correction to color-singlet $J/\psi$ production
- QCD Correction to color-singlet $J/\psi$ polarization
- QCD Correction to color-octet $J/\psi({}^1S_0^8,\ {}^3S_1^8)$ production and polariozation
- QCD Correction to color-octet $J/\psi({}^1S_0^8,\ {}^3S_1^8, {}^3P_J^8)$ production
- QCD Correction to color-octet $J/\psi({}^1S_0^8,\ {}^3S_1^8, {}^3P_J^8)$ polarization
- QCD Correction to $\chi_{cJ}({}^3S_1^8, {}^3P_J^1)$ production

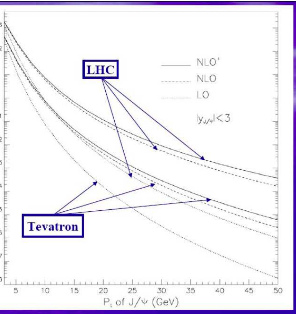
Untill now, there are:

$p_t$ distributioin of $J/\psi$ yield for prompt $J/\psi$ hadroproduction at QCD NLO
$p_t$ distributioin of $J/\psi$ polarization for direct $J/\psi$ hadroproduction at QCD NLO

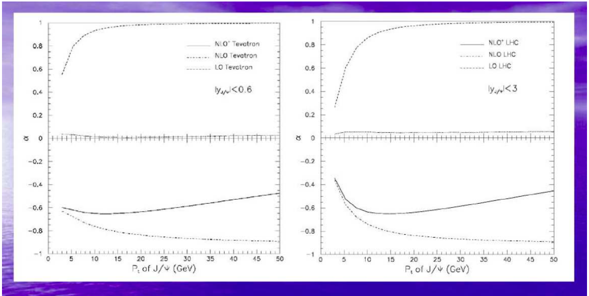
feeddown of $\chi_{cJ}$ about $20-30\%$ to prompt $J/\psi$ production and very improtant.
We need:
$p_t$ distributioin of $J/\psi$ polarization for prompt $J/\psi$ hadroproduction at QCD NLO

It is well known that the uncertainties in $p_t$ distribution of charmonium hadroproduction yield from charm-quak mass $m_c$, NRQCD scale $\mu_\Lambda$, renormalization scale $\mu_r$ and factorization scale $\mu_f$ are large at small $p_t$ range. And recent work on relativistic correction to $J/\psi$ hadroproduction shows that the correction is negtive and large at small $p_t$ range ($p_t < 10 GeV.$). For large $p_t$ range, the large logarithm term $ln(p_t/m_c)$ will appear even with the default choice of all the scales $\mu_r = \mu_f = p_t, \mu_\Lambda = m_c$ and it may ruin the result without proper treatment of this term. In the other side, it very clearly show in the previous fitting and also this fitting that the experimental data for $p_t < 7$ and $p_t > 15$ can not be represeted very welll simultaneously.

# QCD Correction to color-singlet $J/\psi$ production




Transverse momentum distribution of J/ $\psi$ production
NLO$^{+}$ : contribution from $J/\psi + c\bar{c}$ is included




Transverse momentum distribution of J/ $\psi$ polarization parameter $\alpha$

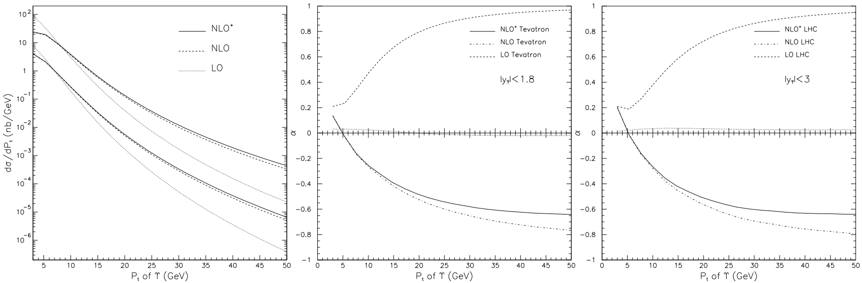
**$J/\psi$ polarization status drastically changes from transverse polarization dominant at LO into longitudinal polarization dominant at NLO**

$P_t$ distribution of $J/\psi$ production at QCD NLO was calculated in PRL98,252002 (2007), J. Campbell, F. Maltoni F. Tramontano

Some technique problems must be solved to calculate $J/\psi$ polarization

$P_t$ distribution of $J/\psi$ polarization at QCD NLO was calculated in PRL100,232001 (2008), B. Gong and J. X. Wang

# QCD Correction to color-singlet Υ production




Υ polarization drastically changes from transverse polarization dominant at LO into longitudinal polarization dominant at NLO

$P_t$ distribution of Υ polarization at QCD NLO was calculated with detail in PRD78 074011 (2008), B. Gong and J. X. Wang

Partly NNLO calculation for Υ production calculated by PRL101, 152001(2008), P. Artoisenet, John M. Campbell, J.P. Lansberg, F. Maltoni, F. Tramontano

## NLO QCD corrections to J/ $\psi$ production via S-wave color octet states

3 tree processes at LO — At NLO

$$g(p_1) + g(p_2) \to J/\psi\left[{}^1S_0^{(8)}, {}^3S_1^{(8)}\right](p_3) + g(p_4), \quad (267, 413)$$

$$g(p_1) + q(p_2) \to J/\psi\left[{}^1S_0^{(8)}, {}^3S_1^{(8)}\right](p_3) + q(p_4), \quad (49, 111)$$

$$q(p_1) + \bar{q}(p_2) \to J/\psi\left[{}^1S_0^{(8)}, {}^3S_1^{(8)}\right](p_3) + g(p_4). \quad (49, 111)$$

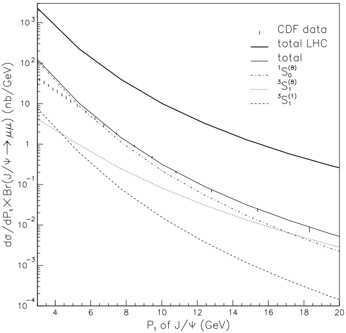
Real Correction (8 processes at NLO)

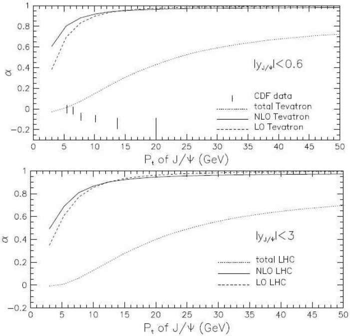
$$gg \to J/\psi\left[{}^1S_0^{(8)}, {}^3S_1^{(8)}\right]gg, \quad gg \to J/\psi\left[{}^1S_0^{(8)}, {}^3S_1^{(8)}\right]q\bar{q},$$

$$gq \to J/\psi\left[{}^1S_0^{(8)}, {}^3S_1^{(8)}\right]gq, \quad q\bar{q} \to J/\psi\left[{}^1S_0^{(8)}, {}^3S_1^{(8)}\right]gg,$$

$$q\bar{q} \to J/\psi\left[{}^1S_0^{(8)}, {}^3S_1^{(8)}\right]q\bar{q}, \quad q\bar{q} \to J/\psi\left[{}^1S_0^{(8)}, {}^3S_1^{(8)}\right]q'\bar{q}',$$

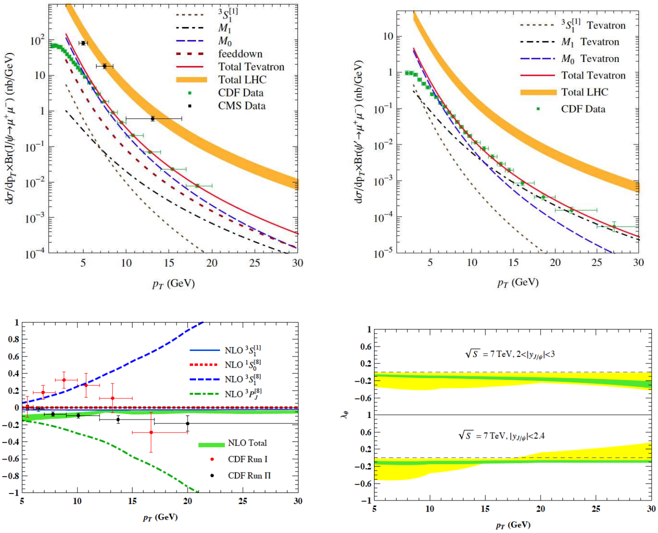
$$qq \to J/\psi\left[{}^1S_0^{(8)}, {}^3S_1^{(8)}\right]qq, \quad qq' \to J/\psi\left[{}^1S_0^{(8)}, {}^3S_1^{(8)}\right]qq',$$



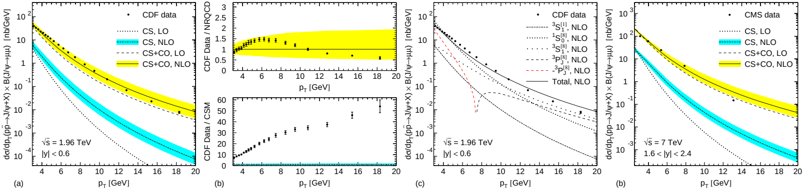

To fit the Tevatron $P_t$ distribution give more $\langle \mathcal{O}_8^{\psi}(^1S_0)\rangle = 0.075\ \mathrm{GeV}^3$ and less $\langle \mathcal{O}_8^{\psi}(^3S_1)\rangle = 0.0021\ \mathrm{GeV}^3$ than they are at LO fitting The experimental data with $p_t < 6$ GeV have to abandon
PLB673:197,2009, Erratum-ibid.693:612,2010 , B. Gong X. Q. Li and J. X. Wang

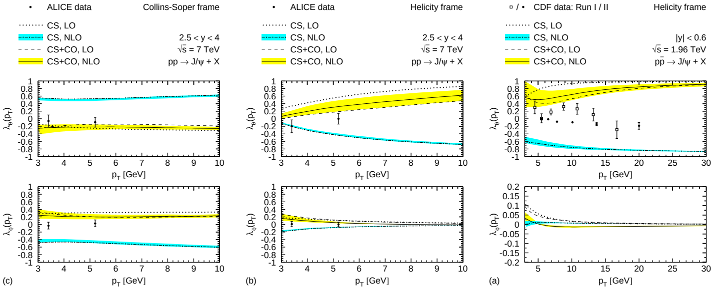
Correction to color-octet $J/\psi(^1S_0^8,\ ^3S_1^8, ^3P_J^8)$ production was done recently and gave almost the same prediction for $p_t$ distribution as before without calculation of polarization, by
PRL 106, 042002,2011, Yan-Qing Ma, Kai Wang, Kuang-Ta Chao
PRL 106, 022003,2011, Mathias Butenschoen, Bernd A. Kniehl




PRL 106, 042002,2011, Yan-Qing Ma, Kai Wang, Kuang-Ta Chao
PRL 108, ??????,2012 Kuang-Ta Chao, Yan-Qing Ma, Hua-Sheng Shao, Kai Wang, Yu-Jie Zhang

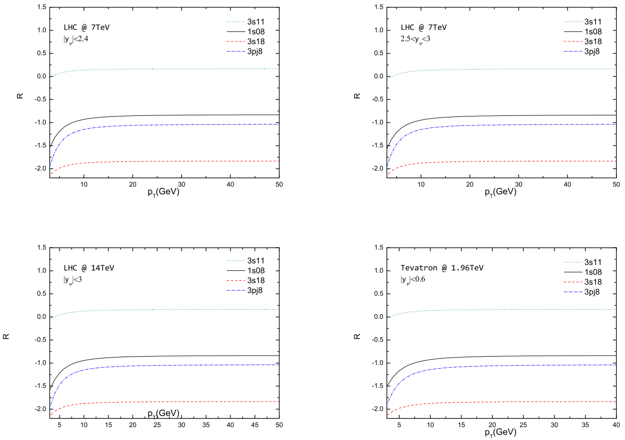
# QCD Correction to color-octet $J/\psi({}^1S_0^8, {}^3S_1^8, {}^3P_J^8)$ production and polarization



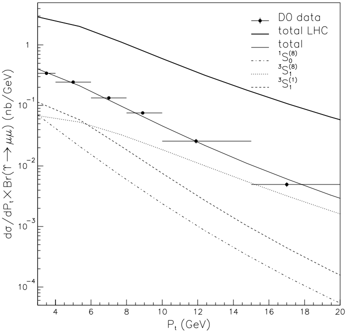



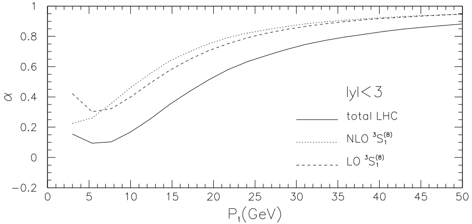

PRL 106, 022003,2011, Mathias Butenschoen, Bernd A. Kniehl
PRL 108, 172002,2012, Mathias Butenschoen, Bernd A. Kniehl

# Relativistic Correction to color-octet $J/\psi({}^1S_0^8,\ {}^3S_1^8, {}^3P_J^8)$ production



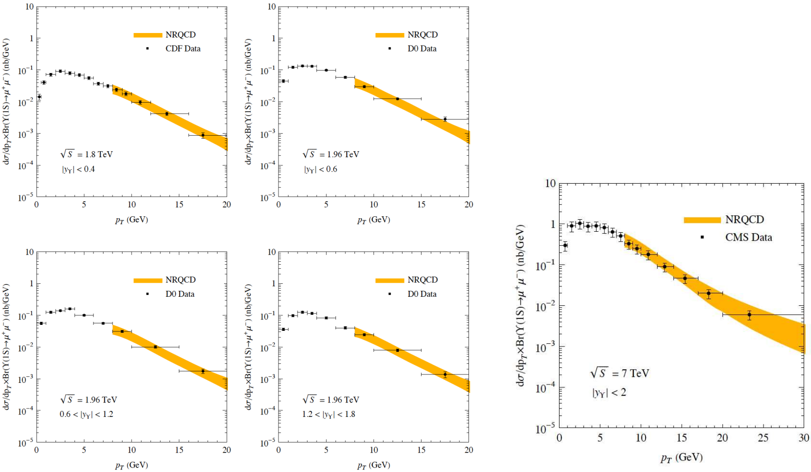

ArXiv:1203.0207, gng-Zhi Xu, Yi-Jie Li, Kui-Yong Liu, and Yu-Jie Zhang



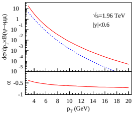

PRD 83:114021,2011, B. Gong, J. X. Wang and H. F. Zhang

# QCD Correction to color-octet $\Upsilon({}^1S_0^8, {}^3S_1^8, {}^3P_J^8)$ production



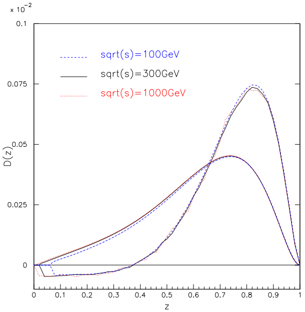

ArXiv:1201.6012 Kai Wang, Yan-Qing Ma, Kuang-Ta Chao

The main point is to extended the fragmentation factorization
from: one-parton fragment into hadron
to: two-parton fragment into hadron
There will be more fragmentation function needed in this scheme.




ArXiv 1109.1520, Zhong-Bo Kang, Jian-Wei Qiu and George Sterman

# Fragmentation function of $c \to J/\psi$ at QCD NLO




ArXiv:1102.0118, B. Gong, R. Li and J. X. Wang

# Summary

- For B-factories: NRQCD at NLO of $\alpha_s$ and $v$ can well described $J/\psi$ production data. strong constraint to **the values of color-octect matrix element of** $c\bar{c}({}^1S_0^8, 3P_J^8)$ **to almost zero**. The dominant part $c\bar{c}({}^3S_1^8)$ for hadronproduction is still there.
- The polarization problem for $J/psi$ hadroproduction is still there even when the QCD NLO fitting and prediction are archived.
- The prediction on the polarization of prompt $J/\psi$ hadroproduction at QCD NLO is strongly needed.
- The more precision experimental measurements at LHC are needed to clarify the situation.
- More theoretical Progresses are needed on relativistic coorection, .... to solve the polarization pzzle.

Thank you!

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